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### THE LINK BETWEEN ASYMMETRIC AND SYMMETRIC OPTIMAL PORTFOLIOS IN FADS MODELS

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**Abstract.** We study a financial market where asymmetric information, mispricing and jumps exist, and link the random optimal portfolios of informed and uninformed investors to the deterministic optimal portfolio of the symmetric market, where no mispricing exists. In particular, we show that under quadratic approximation, the expectation of the random optimal portfolio in the asymmetric market is equal to the optimal deterministic portfolio in the symmetric market. We also compute variance bounds of the optimal portfolios for investors having logarithmic preferences, and prove that the variance of optimal portfolios are bounded above by a simple function of the mean–reversion speed, level of mispricing, and the variance of the continuous component of the return process of the asset.

**Keywords:** Asymmetric information; Lévy processes; Logarithmic utility; Mispricing; Random optimal portfolios; Variance bounds.

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# 1. Introduction

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Investors in financial markets do not all share the same information set, and indeed, some are more informed than others. This naturally leads to asymmetric information which affects trading strategy and asset prices. Understanding the link between asset mispricing and asymmetric information is a topic of ongoing interest in financial economics. Many studies have been conducted on asymmetric information and its effects on asset price, including Biais *et al.* [1], Mendel and Shleifer [11], Serrano-Padial [13], and Vayanos and Wang [16]. Mispricing models for assets under asymmetric information are studied in Brunnermeier [2], Buckley *et al.* [4, 6], Buckley and Long [5], Guasoni [9], Shiller [14], Summers [15] and Wang [17].

The classic Black-Scholes option pricing model assumes that returns follow geometric Brownian motion. However, the return processes differ from this benchmark in at least three important ways. First, asset prices jump, leading to non-normal return innovations. Second, return volatilities vary stochastically over time. Third, returns and their volatilities are correlated, often negatively for equities. Carr and Wu [7] show that time-changed Lévy processes can simultaneously address these three issues. Lévy processes have gained more acceptance in the modelling of asset returns as the processes capture jumps of finite and infinite activity and variation, and as such, are good candidates for modelling asset returns.

Buckley *et al.* [6] incorporate Lévy jumps in asset pricing by developing a mispricing model of an asset under asymmetric information in a non-Guassian market driven by a jump-diffusion. The jumps are driven by a pure jump Lévy process, while the mispricing is modelled by a continuous Ornstein–Uhlenbeck process. Utility is logarithmic, as in the purely continuous geometric Brownian motion case of Guasoni [9]. The authors obtain random optimal portfolios and maximum expected logarithmic utilities for both the informed and uninformed investors, including asymptotic excess utilities and show that the optimal portfolio of each investor may contain excess asset holdings over its Merton optimal. They obtain approximation results which suggest that jumps reduce the excess asymptotic utility of the informed investor relative to the uninformed investor, and hence jump risk could be helpful for market efficiency as an indirect reducer of information asymmetry. Their study also suggests that investors should pay more attention to the overall variance of the asset pricing process when jumps exist in mispricing models. Moreover, if there is too little or too much mispricing, then the informed investor has

no utility advantage in the long run. Buckley and Long [5] present a discontinuous mispricing model under asymmetric information where both the asset/stock and mispricing jump, while Buckley [3] reports on the long-run excess optimal power utility of informed investors.

In this paper, we link the random optimal portfolios of the investors in the asymmetric market to the symmetric market, and show that the random optimal portfolio is equal to the deterministic optimal portfolio plus noise under quadratic approximation. This leads to purely deterministic optimal portfolios which result from Lévy jump-diffusion markets having strictly deterministic market coefficients. We also report the mean and variance of these random optimal portfolios in asymmetric markets for both investors, and provide bounds on the volatilities of each portfolio.

The rest of the paper is organized as follows: A literature review is presented in the next section. The model is briefly reviewed in Section 3. We present asymmetric optimal portfolios for both investors, and link these random portfolios to the symmetric deterministic optimal portfolio in Section 4. We also report expectation and variance bounds, along with their surfaces, for each asymmetric optimal portfolio. Section 5 concludes the paper.

## 2. Related literature

Discrete-time mispricing (fads) models for stocks under asymmetric information were first introduced by Shiller [14] and Summers [15] as plausible alternatives to the efficient market or constant expected returns assumption (see, Fama [8]). Brunnermeier [2] presents an extensive review of asset pricing under asymmetric information mainly in the discrete setting. He shows how information affects trading activity, and that expected return depends on the information set or filtration of the investor. These models show that past prices still carry valuable information, which can be exploited using technical (chart) analysis that uses part or all of past prices to predict future prices.

Wang [17] presents the first continuous-time asset pricing model under asymmetric information, and obtained optimal portfolios for both the informed and uninformed investors. In his paper, investors have different information concerning the future growth rate of dividends, which satisfies a mean-reverting Ornstein-Uhlenbeck process. Informed investors know the future dividend growth rate, while the uninformed investors do not. All investors observe current dividend payments and stock prices. The growth rate of dividends determines the rate of appreciation of stock prices, and stock price changes provide signals about the future growth of dividends. Uninformed investors rationally extract information about the economy from prices, as well as dividends.

Guasoni [9] extends Summers [15] models to the purely continuous random setting. He studies a continuous-time version of these models both from the point of view of informed investor, who observe both fundamental and market values, and from that of uninformed investor, who only observe market prices. He specifies the asset price in the larger filtration of the informed investor, and then derive its decomposition in the smaller filtration of the uninformed investor using the Hitsuda [10] representation of Gaussian processes. Uninformed investors have a non-Markovian dynamics, which justifies the use of technical analysis in optimal trading strategies. For both types of investors, he solves the problem of maximization of expected logarithmic utility from terminal wealth, and obtains an explicit formula for the additional logarithmic utility of informed agents.

Buckley *et al.* [4] generalize the theory of mispricing models of stocks under asymmetric information, to investors having preferences from the power utility class (Constant Relative Risk Averse) instead of logarithmic preferences as in Guasoni [9], which is a limiting case of the relative risk aversion being one. They allow the stock price dynamic to move continuously as geometric Brownian motion, while the mispricing process remains as a continuous mean-reverting Ornstein–Uhlenbeck process. They present log-linear representations of maximum expected utilities of the investors, and report the excess utility of the informed investor. They obtain analogous but more general results which includes those of Guasoni [9] as a special case of the relative risk aversion approaching one. We direct the reader to Buckley and Long [5], and Buckley *et al.* [6] and the references therein, for a complete review of the literature.

### 3. The model

We begin by briefly reviewing the model presented in Buckley *et al.* [6]. Consider an economy with two types of investors: informed investors, who observe both fundamental and market values, and uninformed investors, who only observe market values. Investors have information banks that are modelled by two different filtrations—the less informed investor has a filtration  $\mathcal{H}^0$ , corresponding to the natural evolution of the market, while the better informed investor has the larger filtration  $\mathcal{H}^1$ , which contains the information of the uninformed investor. Although each investor observes the same asset price *S*, its dynamic depends on the filtration of the observer. For both investors, the asset has log returns dynamic:

$$d(\log S_t) = (\mu_t - \frac{1}{2}\sigma_t^2)dt + \sigma_t dY_t + dX_t, t \in [0, T],$$
(1)  

$$Y_t = pW_t + qU_t, p^2 + q^2 = 1, p \ge 0, q \ge 0,$$
(2)  

$$dU_t = -\lambda U_t dt + dB_t, \lambda > 0, U_0 = 0,$$
(2)  

$$X_t = \int_0^t \int_{\mathbb{R}} xN(ds, dx),$$

where *W* and *B* are independent standard Brownian motions independent of *X*, while  $U = (U_t)$  is a mean-reverting Ornstein–Uhlenbeck process with rate  $\lambda$ , and *X* is a pure jump Lévy process having a  $\sigma$ -finite Lévy measure *v* on  $\mathscr{B}(\mathbb{R} - \{0\})$ , with triple  $(\zeta, 0, v)$ , where  $\zeta = \int_{|x|<1} xv(dx)$ . *N* is a Poisson random measure on  $\mathbb{R}_+ \times \mathscr{B}(\mathbb{R} - \{0\})$  that is linked to the asset. It counts the jumps of *X* in the time interval (0, t). The proportion of mispricing in the asset is given by  $q^2$ .

The return process of the asset has three components; a continuous component  $\mu_t^* = \mu_t - \frac{1}{2}\sigma_t^2$ , a diffusive component  $\sigma_t dY_t$  which is random, and a discontinuous component  $X_t$ , which is also random. The continuous component of the asset's return  $\mu_t^*$  and the volatility  $\sigma_t$ , are assumed to be deterministic functions with  $\sigma = \lim_{t\to\infty} \sigma_t = \sigma_\infty > 0$ . Generally,  $\sigma_t$  will be taken to be a constant. The process  $Y = (Y_t)_{t\geq 0}$ , the continuous random component of excess return, and the mean–reverting O–U process  $U = (U_t)_{t\geq 0}$  representing the mispricing, are defined exactly as in Guasoni [9], where U satisfies the Langevin stochastic differential equation (2). This admits a unique solution  $U_t = U_0 e^{-\lambda t} + \int_0^t e^{-\lambda(t-s)} dB_s = \int_0^t e^{-\lambda(t-s)} dB_s$  with  $\mathbf{E}U_t = 0$  and  $\mathbf{E}U_t^2 = \mathbf{Var}(U_t) = (1 - e^{-2\lambda t})/2\lambda$ . Applying Itô's formula to (1) yields percentage return:

$$\frac{dS_t}{S_{t-}} = \mu_t dt + \sigma_t dY_t + \int_{\mathbb{R}} (e^x - 1)N(dt, dx), t \in [0, T].$$

Now let  $i \in \{0, 1\}$ . Buckley *et al.* [6] show that under asymmetric information, the percentage returns dynamics for the *i*-th investor depends on the filtration  $\mathcal{H}^i$  and can be written as

$$\frac{dS_t}{S_{t-}} = \mu_t^i dt + \sigma_t dB_t^i + \int_{\mathbb{R}} (e^x - 1)N(dt, dx), \tag{3}$$

where  $\mu_t^i = \mu_t + v_t^i \sigma_t$ ,  $v_t^0 = -\lambda \int_0^t e^{-\lambda(t-u)} (1+\gamma(u)) dB_u^0$ ,  $v_t^1 = -\lambda q U_t = -\lambda q \int_0^t e^{-\lambda(t-u)} dB_u$ ,  $1 + \gamma(u) = (1-p^2)/(1+p \tanh(p\lambda u))$  and *B* and  $B^i$  are  $\mathcal{H}^i$ -adapted standard Brownian motions. Furthermore, Buckley *et al.* [6] show that to maximize the expected logarithmic utility from terminal wealth, an investor must hold an optimal portfolio given by the following:

**Lemma 1.** [6] Let  $\pi \in [0, 1]$  and defined  $G(\pi) = \int_{\mathbb{R}} \log(1 + \pi(e^x - 1))v(dx)$ , where v(.) is the Lévy measure of the jump process.

(1) If 
$$\int_{\mathbb{R}} (e^{\pm x} - 1)^2 v(dx) < \infty$$
, then  $G''(\pi) < 0$ . That is, G is strictly concave on [0, 1].

(2) Let  $i \in \{0,1\}$ . For the *i*-th investor, there is an unique optimal portfolio  $\pi^{*,i} \in [0,1]$  for the asset with dynamics (1), given by  $\pi_t^{*,i} = \frac{\theta_t^i}{\sigma_t} + \frac{G'(\pi_t^{*,i})}{\sigma_t^2}$ , where  $\theta_t^i = \frac{\mu_t^i - r_t}{\sigma_t}$  is the Sharpe ratio (market price of risk) of the asset for the *i*-th investor,  $r_t$  is the risk-free interest rate and  $\pi^{*,i} \in [0,1] \subset \mathscr{A}_i(x)$ , the admissible set.

### 4. Linkage of asymmetric and symmetric optimal portfolios

Consider the situation of a symmetric financial market with no mispricing, where both investors have equal information and observe the fundamental value of the asset. In this case, q = 0, and the asset price *S* has percentage return dynamic:

$$\frac{dS_t}{S_{t-}} = \mu_t dt + \sigma_t dW_t + \int_{\mathbb{R}} (e^x - 1)N(dt, dx).$$
(4)

Lemma 1 gives the optimal portfolio,  $\pi_t^{*,i}$ , for each investor as  $\pi_t^{*,i} = [\mu_t^i - r_t + G'(\pi_t^{*,i})]/\sigma_t^2$ . Thus, for the symmetric market (4), the deterministic optimal portfolio that maximizes expected utility of terminal wealth is  $\pi_t^* = [\mu_t - r_t + G'(\pi_t^*)]/\sigma_t^2$ , which must be solved numerically.

We now link  $\pi^{*,i}$ , the random optimal portfolio in the asymmetric market with dynamic given by (3), to  $\pi_t^*$ , the deterministic optimal portfolio in the symmetric market, with corresponding dynamic (4). **Theorem 1.** Let  $i \in \{0,1\}$  and T > 0 be the investment horizon. Assume that G is restricted to [0,1] and  $\int_{\mathbb{R}} (e^{\pm x} - 1)^k v(dx) < \infty$  for some integer  $k \ge 2$ . (1) There exists a process  $\eta^i$  between  $\pi^*$  and  $\pi^{*,i}$ , such that for all  $t \in [0,T]$ ,  $\pi_t^{*,i} = \pi_t^* + \frac{v_t^i \sigma_t}{\sigma_t^2 + |G''(\eta_t^i)|}$ . That is,  $\pi_t^{*,i} = \pi_t^* + \varepsilon_t^i$ , where  $\varepsilon$  represents noise. (2) Under quadratic approximation of G at  $\pi_t^*, \pi_t^{*,i} \approx \pi_t^* + \frac{v_t^i \sigma_t}{\sigma_t^2 - G''(\pi_t^*)}$  and  $\mathbf{E}[\pi_t^{*,i}] \approx \pi_t^*$ .

**Proof.** (1) If  $\int_{\mathbb{R}} (e^{\pm x} - 1)^2 v(dx) < \infty$ , then  $G''(\pi)$  exists for all  $\pi \in [0, 1]$ , and by Lemma 1, optimal portfolios exist for both symmetric and asymmetric markets, given respectively by

$$\pi_t^* = \frac{\mu_t - r_t + G'(\pi_t^*)}{\sigma_t^2} = \frac{\mu_t - r_t}{\sigma_t^2} + \frac{G'(\pi_t^*)}{\sigma_t^2},$$

and

$$\pi_t^{*,i} = \frac{\mu_t^i - r_t + G'(\pi_t^{*,i})}{\sigma_t^2} = \frac{\upsilon_t^i}{\sigma_t} + \frac{\mu_t - r_t}{\sigma_t^2} + \frac{G'(\pi_t^{*,i})}{\sigma_t^2}$$

Let the portfolios  $\beta^{*,i}$ , be defined by:  $\beta_t^{*,i} := \pi_t^* + \frac{v_t^i}{\sigma_t} = \frac{\mu_t - r_t}{\sigma_t^2} + \frac{G'(\pi_t^*)}{\sigma_t^2} + \frac{v_t^i}{\sigma_t}$ . Since *G'* is a differentiable function, then by the Mean Value theorem, there exists a process  $\eta^i$  between  $\pi^*$  and  $\pi^{*,i}$  such that for all  $t \in [0, T]$ ,

$$\pi_t^{*,i} - \beta_t^{*,i} = \frac{G'(\pi_t^{*,i}) - G'(\pi_t^{*})}{\sigma_t^2} = \frac{(\pi_t^{*,i} - \pi_t^{*})G''(\eta_t^{i})}{\sigma_t^2}.$$

Since  $G''(\pi) < 0$  for all  $\pi$ , then  $\pi_t^{*,i} - (\pi_t^* + \frac{\upsilon_t^i}{\sigma_t}) = -\frac{(\pi_t^{*,i} - \pi_t^*)|G''(\eta_t^i)|}{\sigma_t^2}$ . Rearranging, we get

$$\pi_t^{*,i}\left(\sigma_t^2 + |G''(\eta_t^i)|\right) = v_t^i \sigma_t + \pi_t^*\left(\sigma_t^2 + |G''(\eta_t^i)|\right)$$

whence,  $\pi_t^{*,i} = \pi_t^* + \frac{\upsilon_t^i \sigma_t}{\sigma_t^2 + |G''(\eta_t^i)|}.$ 

(2)  $\mathbf{E}(v_t^i) = 0$ , and under quadratic approximation of *G* at  $\pi_t^*$ ,  $G''(\eta_t^i) \approx G''(\pi_t^*)$ . Therefore by Part (1),

$$\mathbf{E}[\pi_t^{*,i}] = \pi_t^* + \mathbf{E}\left(\frac{\upsilon_t^i \,\sigma_t}{\sigma_t^2 + |G''(\eta_t^i)|}\right) \approx \pi_t^* + \sigma_t \frac{\mathbf{E}(\upsilon_t^i)}{\sigma_t^2 - G''(\pi_t^*)} = \pi_t^*. \quad \Box$$

Observe from Theorem 1 that the random portfolio is the deterministic symmetric optimal portfolio plus noise which is driven by the diffusive volatility of the asset return and the mispricing processes. That is, the investors hold a random excess amount of assets that depends

on the level of mispricing, volatility, mean-reversion speed, and symmetric optimal portfolio. Moreover, under quadratic approximation, the expected value of the random asymmetric optimal portfolio is the symmetric optimal portfolio for each investor. Thus, the deterministic symmetric optimal portfolio is the benchmark for the random optimal asymmetric portfolios.

Another advantage of our main result in Theorem 1 is that it also allows us to easily compute upper bounds on the expected squared deviations of the asymmetric and symmetric optimal portfolios for each investor.

**Proposition 1.** Let  $i \in \{0, 1\}$ ,  $p \in [0, 1]$  and T > 0 be the investment horizon. Assume that *G* is restricted to [0, 1] and  $\int_{\mathbb{R}} (e^{\pm x} - 1)^k v(dx) < \infty$  for some integer  $k \ge 2$ . Then, for each  $t \in [0, T]$ ,

(1)  $\mathbf{E}[\pi_t^{*,i} - \pi_t^*]^2 \leq \frac{\lambda}{2\sigma_t^2}(1+p)(1+(-1)^i p)(1-e^{-2\lambda t})$ , where  $\lambda$  is the mean-reversion rate of the mispricing process.

(2)  $\operatorname{Var}[\pi_t^{*,i}] \leq L_i$ , where  $L_i = \frac{\lambda}{2\sigma_t^2}(1+p)(1+(-1)^i p)$  are the variance bounds.

**Proof.** (1) By Theorem 1,  $\pi_t^{*,i} = \pi_t^* + \frac{v_t^i \sigma_t}{\sigma_t^2 + |G''(\eta_t^i)|}$ . The expected squared deviation of  $\pi_t^{*,i}$  from the symmetric optimal portfolio is  $\mathbf{E}[\pi_t^{*,i} - \pi_t^*]^2 = \mathbf{E}\left[\frac{v_t^i \sigma_t}{\sigma_t^2 + |G''(\eta_t^i)|}\right]^2 \leq \frac{\mathbf{E}[v_t^i]^2}{\sigma_t^2}$ . When i = 1, it is easy to show that

$$\mathbf{E}[v_t^1]^2 = \frac{\lambda}{2}(1-p^2)(1-e^{-2\lambda t}) = \frac{\lambda}{2}(1+p)(1-p)(1-e^{-2\lambda t}).$$

When i = 0,  $v_t^0 = -\lambda \int_0^t e^{-\lambda(t-u)} (1 + \gamma(u)) dB_u^0$ , and by Itô isometry,

$$\mathbf{E}[v_t^0]^2 = \lambda^2 \int_0^t e^{-2\lambda(t-u)} (1+\gamma(u))^2 du.$$
 (5)

But  $1 + \gamma(u) = \frac{1-p^2}{1+p\tanh(p\lambda u)}$ , where  $\tanh(u) \in (-1,1)$ , for each  $p \in [0,1]$ . Therefore, we have  $0 \le 1 + \gamma(u) \le \frac{1-p^2}{1-p} = 1 + p$ , whence  $(1 + \gamma(u))^2 \le (1+p)^2$ , and by (5)

$$\begin{split} \mathbf{E}[v_t^0]^2 &\leq \lambda^2 \int_0^t e^{-2\lambda(t-u)} (1+p)^2 du = \lambda^2 (1+p)^2 e^{-2\lambda t} \int_0^t e^{2\lambda u} du \\ &= \lambda^2 (1+p)^2 e^{-2\lambda t} \frac{(e^{2\lambda t}-1)}{2\lambda} = \frac{\lambda}{2} (1+p)^2 (1-e^{-2\lambda t}) \\ &= \frac{\lambda}{2} (1+p) (1+(-1)^0 p) (1-e^{-2\lambda t}). \end{split}$$

Therefore, for  $i \in \{0, 1\}$ ,

$$\mathbf{E}[v_t^i]^2 \le \frac{\lambda}{2}(1+p)(1+(-1)^i p)(1-e^{-2\lambda t}),$$

and

$$\mathbf{E}[\pi_t^{*,i} - \pi_t^*]^2 \le \frac{\lambda}{2\sigma_t^2}(1+p)(1+(-1)^i p)(1-e^{-2\lambda t}).$$

(2) The variance of the asymmetric optimal portfolio is

$$\begin{aligned} \mathbf{Var}[\pi_t^{*,i}] &\leq \mathbf{E}[\pi_t^{*,i} - \pi_t^*]^2 \\ &\leq \frac{\lambda}{2\sigma_t^2} (1+p)(1+(-1)^i p)(1-e^{-2\lambda t}) \\ &\leq \frac{\lambda}{2\sigma_t^2} (1+p)(1+(-1)^i p) = L_i \ . \end{aligned}$$

We first observe that the variance bounds  $L_0 = \lambda (1+p)^2/(2\sigma_t^2)$  and  $L_1 = \lambda q^2/(2\sigma_t^2)$  satisfy  $L_1 \leq L_0$  for all values of the parameters. This is expected as the informed investor is exposed to the less uncertainty. We also note that when there is no mispricing (i.e.  $q^2 = 0$  and hence  $L_1 = 0$ ), there is no variability in the optimal portfolio of the informed investor. In this case, the investor holds the Merton [12] optimal portfolio. The variance bound for the uninformed investor will be at its maximum in this event. Similarly, if there is 100% mispricing (i.e.  $q^2 = 1$ ), the variance bound for the optimal portfolio of the uninformed investor is at its minimum value of  $L_0 = \lambda/(2\sigma_t^2)$  which coincides with the maximum variance bound  $L_1 = \lambda/(2\sigma_t^2)$  of the optimal portfolio of the informed investor.

Another interesting observation of our study is that optimal portfolios for both investors have little variability as the mean-reversion speed  $\lambda \rightarrow 0$ . Thus at slow speeds, both investors hold the expected value of their random portfolios which is its Merton optimal portfolio. In practice, investors assume that  $\sigma_t$ , the continuous component of the asset's volatility, is constant. However, take note that the variances of the optimal portfolios of both investors are extremely sensitive to the variance of the asset, being inversely related thereto. Plots of variance bounds surfaces  $L_0(\lambda, \sigma, q^2)$  and  $L_1(\lambda, \sigma, q^2)$  for the uninformed and informed investor, respectively, are presented below in Figures (1)-(2), where  $q^2$  is the proportion of mispricing in the market.

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FIGURE 1. Variance Bounds Surfaces  $L_0(\lambda, \sigma, q^2)$  and  $L_1(\lambda, \sigma, q^2)$  when Asset Volatility  $\sigma = 24\%$ .



FIGURE 2. Variance Bounds Surfaces  $L_0(\lambda, \sigma, q^2)$  and  $L_1(\lambda, \sigma, q^2)$  when Asset Volatility  $\sigma = 50\%$ .

### 5. Conclusion

When asymmetric information, mispricing/fads and jumps exist in the price of a financial asset, optimal portfolios of both informed and uninformed investors are random. However, in a symmetric market, the portfolios of both investors are identical and equal to the deterministic Merton optimal portfolio. We show that under quadratic approximation, these random portfolios are noisy representations of the deterministic optimal portfolio of the symmetric market. Moreover, their expectations are the deterministic Merton optimal portfolio. We also show that the variance of each portfolio is bounded above by a simple function of the mean reversion speed, level of fads/mispricing, and variance of the continuous component of the return process of the asset. There is an asymmetric relation between the variance bounds and return volatility. However, the variance bounds correlate positively with mean reversion speed and the level of mispricing/fads.

### **Conflict of Interests**

The authors declare that there is no conflict of interests.

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