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FIXED POINT THEOREMS FOR CONTRACTIONS IN COMPLETE $E-$FUZZY METRIC SPACES

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Abstract. Many problems in mathematics are to find fixed point of the mapping $T : X \to X$, that is $T(x) = x$ and to study the existence and uniqueness of these points. These results are applied in differential and integral equations. Because of this, many authors proved existence and uniqueness of fixed point of mappings under some contractions in different spaces. The aim of this paper is to study some fixed point theorems for contractions in complete E-fuzzy metric spaces by using ψ function and ϕ function. The uniqueness and continuity of this fixed point is studied. Also an example is given to illustrate the main theorem and presented some applications in integral equations of obtained results. The theorems in this study generalized some well-known comparable results and introduced a novel approach to show the existence and uniqueness of solution of integral equations.

Keywords: fuzzy metric space; G-metric space; E-fuzzy metric space; complete E-fuzzy metric space; ψ map; ϕ map

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1. INTRODUCTION

The concept of fuzzy sets was introduced by L A Zadeh [1] in 1965. With the concept of

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fuzzy sets almost all existing branches of mathematics were fuzzified. Fuzzy metric was introduced by Kramosil and Michalek [2]. George and Veeramani [3] modified this concept and several results were studied. Many authors proved several fixed point theorems for contraction in fuzzy metric spaces.

The notion of 2-metric spaces was introduced by Gahler in 1963 [4] inspired by the map that maps vertices of a triangle to its area. 2-metric spaces are not topologically equivalent to metric spaces and there is no relationship between the results obtained in these two spaces. Dhage [5] introduced *D*- metric space in 1992 and presented an example, D(x,y,z) is the perimeter of a triangle formed with vertices x,y,z in R^2. Mustafa and Sims [6] showed that most of topological properties of *D*-metric spaces were not correct and this led them to generalize the metric space by defining *G*-metric from $X \times X \times X$ to $[0,\infty)$ in 2005 [7]. W Shatanawi [8] studied some fixed point theorems using ϕ contraction in Complete *G*-metric spaces. Many fixed point theorems established in this *G*-metric spaces and these results are used to find the existence of solutions of integral equations [9, 10, 11, 13]. Following this *G*-metric space Sun and Yang [14] introduced Generalized fuzzy metric space called *Q*-fuzzy metric space and established some fixed point theorems in generalized *Q*-fuzzy metric space. In 2017 Sukanya K P and Magie Jose [15] modified this concept and defined generalized *E*- fuzzy metric spaces and obtained a Hausdorff topology. Some fixed point theorems were obtained in this *E*-fuzzy metric spaces [16].

The purpose of this paper is to prove some fixed point theorems using contractions in generalized *E*-fuzzy metric spaces. Some fixed point theorems are established using the contraction defined by a continuous, non decreasing map $\psi : (0,1] \to (0,1]$ with $\psi(t) > t$, $\psi(1) = 1$ and $\lim_{n\to\infty} \psi^n(t) = 1$ for all $t \in (0,1)$ and some other fixed point theorems are proved using contraction related to the function $\phi : (0,\infty) \to (0,\infty)$ such that $\phi(t) > t$ and $\lim_{n\to\infty} \phi^n(t) = 0$ for all $t > 0$. Also provide an example to illustrate the result and give some applications in integral equations of main result. These results generalize and improve some existing results.

2. PRELIMINARIES

Definition 2.1. [12] A continuous $t-$norm is a function * :[0,1]$\times$ [0,1]$\to$[0,1] which satisfies the following properties:

(1) $a*b=b*a$ (commutative) and $(a*b)*c=a*(b*c)$ (associative),

(2) * is continuous,

(3) for all a ∈ [0,1], $a*1=a$,

(4) for all a, b,c,d ∈ [0,1] and a ≤ c and b ≤ d, a*b ≤ c*d.

Definition 2.2. [15] A 3-tuple $(X,E,*)$ is called a E- fuzzy metric space if X is an arbitrary set,* is a continuous t-norm and E is a fuzzy set on $X^3\times(0,\infty)$ satisfying the following conditions for each $x,y,z,a\in X$ and $t,s>0$,

(1) $E(x,y,z,t)>0$ and $E(x,x,y,t)\geq E(x,y,z,t)$ for all $x,y,z\in X$ with $z\neq y$,

(2) $E(x,y,z,t)=1$, for all $t>0$ if and only if $x=y=z$,

(3) $E(x,y,z,t)=E(p(x,y,z),t)$ (symmetry), where p is a permutation function,

(4) $E(x,a,z,t)*E(a,y,z,s)\leq E(x,y,z,t+s)$,

(5) $E(x,y,z,.):(0,\infty)\to[0,1]$ is continuous,

Example 2.1. [16] Let $X=R$ and G is a $G-$metric on X. The $t-$ norm is $a*b=ab$ for all $a,b\in[0,1]$. For each $t>0$ and $E(x,y,z,t)=[exp(\frac{G(x,y,z)}{t})]^{-1}$. Then $(X,E,*)$ is a E-fuzzy metric space.

Lemma 2.1. [15] If $(X,E,*)$ be a E-fuzzy metric space, then $E(x,y,z,t)$ is non decreasing with respect to t, $\forall x,y,z\in X$.

Definition 2.3. [16] $(X,E,*)$ be a E-fuzzy metric space. An open ball center x_0 and radius r is given by $B_E(x_0,r,t)=\{x\in X;E(x_0,x,x,t)>1-r\}$.

Definition 2.4. [16] $(X,E,*)$ be a E-fuzzy metric space. A sequence (x_n) in X converges to a point $x\in X$ if $E(x_n,x,x,t)\to 1$ as $n\to\infty$.

Proposition 2.1. [16] If (x_n) is a sequence in $(X,E,*)$ converges to $x\in X$, then

(1) $E(x_n,x_n,x,t)\to 1$ as $n\to\infty$

(2) $E(x_n,x_m,x,t)\to 1$ as $n\to\infty$

(3) $E(x_n,x_m,x,t)>1-\varepsilon$ for $\varepsilon>0$ and $m,n\geq n_0$.

Definition 2.5. [16] A sequence (x_n) in $(X,E,*)$ is said to be a Cauchy sequence if for each $0<\varepsilon<1$ and $t>0$ there exists $n_0\in N$ such that $E(x_m,x_n,x_l,t)>1-\varepsilon$ for each $l,m,n\geq n_0$.

Proposition 2.2. [17] A sequence (x_n) in $(X,E,*)$ is said to be a Cauchy sequence if for each $0<\varepsilon<1$ and $t>0$ there exists $n_0 \in N$ such that $E(x_m,x_m,x_n,t)>1-\varepsilon$ for each $m,n \geq n_0$.

Definition 2.6. [16] A E-fuzzy metric space in which every Cauchy sequence is convergent is said to be a complete E-fuzzy metric space.

Definition 2.7. [16] $(X_1,E_1,*)$ and $(X_2,E_2,*)$ be two generalized E-fuzzy metric spaces. A function $f:X_1 \to X_2$ is said to be continuous at a point $a \in X_1$ if $\forall \varepsilon>0$ there exists $\delta>0$ such that $E_2(f(x),f(a),f(a),s>1-\varepsilon$ whenever $E_1(x,a,a,t)>1-\delta$, $t,s>0$.

Proposition 2.3. [17] Let $(X_1,E_1,*)$ and $(X_2,E_2,*)$ be E-fuzzy metric spaces. A function $f:X_1 \to X_2$ is continuous at a point $x \in X_1$ if and only if every convergent sequence (x_n) converges to x in X_1, $(f(x_n))$ converges to $f(x)$ in X_2.

3. Fixed Point Theorems for Contractive Mapping Satisfying ψ - Maps

In this section some fixed point theorems in complete E-fuzzy metric space are proved using the ψ map, which is defined below.

Let $\psi:(0,1] \to (0,1]$ be a continuous, non decreasing mapping with following properties:

(1) $\psi(t)>t$ for all $t \in (0,1)$

(2) $\psi(1)=1$

(3) $\lim_{n\to\infty} \psi^n(t)=1$ for all $t \in (0,1)$

Theorem 3.1. Let $(X,E,*)$ be a complete E-fuzzy metric space and the map $T:X \to X$ satisfies

$$E(T(x),T(y),T(z),t) \geq \psi E(x,y,z,t), \forall x,y,z \in X \text{ and } t>0.$$

Then T has a unique fixed point and T is continuous at that point.

Proof. Choose $x_0 \in X$, consider a sequence (x_n) in X such that $x_n=T(x_{n-1}), n \in \mathcal{N}$. First we prove (x_n) is a Cauchy sequence. For $m,n \in \mathcal{N}$ with $m>n$,

$$\begin{aligned} E(x_n,x_m,x_m,t) &= E(T(x_{n-1}),T(x_{m-1}),T(x_{m-1}),t) \\ &\geq \psi E(x_{n-1},x_{m-1},x_{m-1},t) \\ &\geq \psi E(T(x_{n-2}),T(x_{m-2}),T(x_{m-2}),t) \end{aligned}$$

$$\begin{aligned} &\geq \psi^2 E(x_{n-2}, x_{m-2}, x_{m-2}, t) \\ &\vdots \\ &\geq \psi^n E(x_0, x_{m-n}, x_{m-n}, t) \\ &\to 1 \text{ as } n \to \infty, \text{ since } \psi^n(t) \to 1 \text{ as } n \to \infty. \end{aligned}$$

Hence (x_n) is a Cauchy sequence in $(X, E, *)$. Since X is a complete $E-$fuzzy metric space, (x_n) converges to some $x' \in X$. To prove $T(x') = x'$. We first prove $T(x_n) \to T(x')$. For

$$\begin{aligned} E(T(x'), T(x_n), T(x_n), t) &\geq \psi E(x', x_n, x_n, t) \\ &> E(x', x_n, x_n, t), \text{ since } \psi(t) > t \\ &\to 1 \text{ as } n \to \infty \end{aligned}$$

Hence $T(x_n) \to T(x')$.

$$\begin{aligned} \text{Now } E(x_n, T(x_n), T(x_n), t) &= E(T(x_{n-1}), T(x_n), T(x_n), t) \\ &\geq \psi E(x_{n-1}, x_n, x_n, t) \\ &\geq \psi E(T(x_{n-2}), T(x_{n-1}), T(x_{n-1}), t) \\ &\geq \psi^2 E(x_{n-2}, x_{n-1}, x_{n-1}, t) \\ &\vdots \\ &\geq \psi^n E(x_0, x_1, x_1, t). \end{aligned}$$

Letting $n \to \infty$ and since E is continuous on its variable, we get $E(x', T(x'), T(x'), t) \to 1$. Hence x' is a fixed point of T. To prove uniqueness, suppose x_1 is another fixed point of T with $x' \neq x_1$.

$$\begin{aligned} E(x', x_1, x_1, t) &= E(T(x'), T(x_1), T(x_1), t) \\ &\geq \psi E(x', x_1, x_1, t) \\ &> E(x', x_1, x_1, t). \end{aligned}$$

Which is a contradiction. Thus T has a unique fixed point x'. Also we have proved $(x_n) \to x' \implies T(x_n) \to T(x')$. Therefore T is continuous at x'. □

Corollary 3.1. Let $(X,E,*)$ be a complete E-fuzzy metric space and the map $T:X\to X$ satisfies

$$E(T^m(x),T^m(y),T^m(z),t)\geq \psi E(x,y,z,t) \text{ for } m\in\mathscr{N},\forall x,y,z\in X \text{ and } t>0.$$

Then T has a unique fixed point.

Proof. Using above theorem we can conclude that T^m has unique fixed point x'. Since $T(x')=T(T^m(x'))=T^{m+1}(x')=T^m(T(x'))$. Thus $T(x')$ is also a fixed point of T^m. By uniqueness of fixed point, we get $T(x')=x'$. □

Theorem 3.2. Let $(X,E,*)$ be a complete $E-$ fuzzy metric space and suppose that the map $T:X\to X$ satisfies

$$E(T(x),T(y),T(z),t)\geq \psi \min\{E(x,y,z,t),E(x,T(x),T(x),t),E(y,T(y),T(y),t),$$
$$E(T(x),y,z,t)\},\forall x,y,z\in X \text{ and } t>0.$$

Then T has a unique fixed point and T is continuous at that point.

Proof. Choose $x_0\in X$, consider a sequence (x_n) in X such that $x_n=T(x_{n-1}),n\in\mathscr{N}$. To prove (x_n) is a Cauchy sequence in X, for $m,n\in\mathscr{N}$ with $m>n$,

$$\begin{aligned}E(x_n,x_m,x_m,t) &= E(T(x_{n-1}),T(x_{m-1}),T(x_{m-1}),t)\\ &\geq \psi\min\{E(x_{n-1},x_{m-1},x_{m-1},t),E(x_{n-1},T(x_{n-1}),T(x_{n-1}),t),\\ &\quad E(x_{m-1},T(x_{m-1}),T(x_{m-1}),t),E(T(x_{n-1}),x_{m-1},x_{m-1},t)\}\end{aligned}$$

Case 1:

$$\min\{E(x_{n-1},x_{m-1},x_{m-1},t),E(x_{n-1},T(x_{n-1}),T(x_{n-1}),t),E(x_{m-1},T(x_{m-1}),T(x_{m-1}),t),$$
$$E(T(x_{n-1}),x_{m-1},x_{m-1},t)\}=E(x_{n-1},x_{m-1},x_{m-1},t)$$

$$\begin{aligned}\text{Then } E(x_n,x_m,x_m,t) &\geq \psi E(x_{n-1},x_{m-1},x_{m-1},t)\\ &\geq \psi^2E(x_{n-2},x_{m-2},x_{m-2},t)\\ &\vdots\\ &\geq \psi^nE(x_0,x_{m-n},x_{m-n},t)\to 1 \text{ as } n\to\infty.\end{aligned}$$

Case 2:

$$\min\{E(x_{n-1},x_{m-1},x_{m-1},t),E(x_{n-1},T(x_{n-1}),T(x_{n-1}),t),E(x_{m-1},T(x_{m-1}),T(x_{m-1}),t),$$
$$E(T(x_{n-1}),x_{m-1},x_{m-1},t)\} = E(x_{n-1},T(x_{n-1}),T(x_{n-1}),t)$$

$$\begin{aligned}\text{Then } E(x_n,x_m,x_m,t) &\geq \psi E(x_{n-1},T(x_{n-1}),T(x_{n-1}),t)\\ &\geq \psi E(x_{n-1},x_n,x_n,t)\\ &\geq \psi^2 E(x_{n-2},x_{n-1},x_{n-1},t)\\ &\vdots\\ &\geq \psi^n E(x_0,x_1,x_1,t) \to 1 \text{ as } n\to\infty\end{aligned}$$

Case 3:

$$\min\{E(x_{n-1},x_{m-1},x_{m-1},t),E(x_{n-1},T(x_{n-1}),T(x_{n-1}),t),E(x_{m-1},T(x_{m-1}),T(x_{m-1}),t),$$
$$E(T(x_{n-1}),x_{m-1},x_{m-1},t)\} = E(x_{m-1},T(x_{m-1}),T(x_{m-1}),t)$$

$$\begin{aligned}\text{Then } E(x_n,x_m,x_m,t) &\geq \psi E(x_{m-1},T(x_{m-1}),T(x_{m-1}),t)\\ &\geq \psi E(x_{m-1},x_m,x_m,t)\\ &\geq \psi E(x_m,x_{m-1},x_m,t)\\ &\geq \psi^2 E(x_{m-1},x_{m-2},x_{m-1},t)\\ &\geq \psi^3 E(x_{m-2},x_{m-3},x_{m-2},t)\\ &\vdots\\ &\geq \psi^n E(x_{m-(n-1)},x_{m-n},x_{m-(n-1)},t)\\ &\to 1 \text{ as } n\to\infty\end{aligned}$$

Case 4:

$$\min\{E(x_{n-1},x_{m-1},x_{m-1},t),E(x_{n-1},T(x_{n-1}),T(x_{n-1}),t),E(x_{m-1},T(x_{m-1}),T(x_{m-1}),t),$$
$$E(T(x_{n-1}),x_{m-1},x_{m-1},t)\} = E(T(x_{n-1}),x_{m-1},x_{m-1},t)$$

$$\begin{aligned}\text{Then } E(x_n,x_m,x_m,t) &\geq \psi E(T(x_{n-1}),x_{m-1},x_{m-1},t)\\ &\geq \psi E(x_n,x_{m-1},x_{m-1},t)\\ &\geq \psi^2 E(T(x_{n-1}),x_{m-2},x_{m-2},t)\end{aligned}$$

$$\begin{aligned} &\geq \psi^2 E(x_n, x_{m-2}, x_{m-2}, t) \\ &\vdots \\ &\geq \psi^n E(x_n, x_{m-n}, x_{m-n}, t) \\ &\to 1 \text{ as } n \to \infty \end{aligned}$$

Hence (x_n) is a Cauchy sequence in $(X, E, *)$. Since X is a complete E-fuzzy metric space, (x_n) converges to some $x' \in X$.

Now to prove $T(x_n) \to T(x')$. For

$$\begin{aligned} E(T(x_n), T(x_n), T(x'), t) \geq \psi \min \{&E(x_n, x_n, x', t), E(x_n, T(x_n), T(x_n), t), \\ &E(x_n, T(x_n), T(x_n), t), E(T(x_n), x_n, x', t)\} \end{aligned}$$

Case 1:

$$\begin{aligned} \min \{&E(x_n, x_n, x', t), E(x_n, T(x_n), T(x_n), t), E(x_n, T(x_n), T(x_n), t), E(T(x_n), x_n, x', t)\} \\ &= E(x_n, x_n, x', t). \end{aligned}$$

$$\begin{aligned} \text{Then } E(T(x_n), T(x_n), T(x'), t) &\geq \psi E(x_n, x_n, x', t) \\ &> E(x_n, x_n, x', t) \\ &\to 1 \text{ as } n \to \infty \end{aligned}$$

Case 2:

$$\begin{aligned} \min \{&E(x_n, x_n, x', t), E(x_n, T(x_n), T(x_n), t), E(x_n, T(x_n), T(x_n), t), E(T(x_n), x_n, x', t)\} \\ &= E(x_n, T(x_n), T(x_n), t) \end{aligned}$$

$$\begin{aligned} \text{Then } E(T(x_n), T(x_n), T(x'), t) &\geq \psi E(x_n, T(x_n), T(x_n), t) \\ &\geq \psi E(x_n, x_{n+1}, x_{n+1}, t) \\ &> E(x_n, x_{n+1}, x_{n+1}, t) \\ &\to 1 \text{ as } n \to \infty \end{aligned}$$

Case 3:

$$\min\{E(x_n,x_n,x',t),E(x_n,T(x_n),T(x_n),t),E(x_n,T(x_n),T(x_n),t),E(T(x_n),x_n,x',t)\} = E(T(x_n),x_n,x',t)$$

$$\begin{aligned} \text{Then } E(T(x_n),T(x_n),T(x'),t) &\geq \psi E(T(x_n),x_n,x',t) \\ &\geq \psi E(x_{n+1},x_n,x',t) \\ &> E(x_{n+1},x_n,x',t) \\ &\to 1 \text{ as } n\to\infty \end{aligned}$$

Hence $T(x_n)\to T(x')$ in each case.

Now to prove x' is a fixed point of T, that is $T(x')=x'$. For

$$\begin{aligned} E(x_n,T(x_n),T(x_n),t) &= E(T(x_{n-1}),T(x_n),T(x_n),t) \\ &\geq \psi \min\{E(x_{n-1},x_n,x_n,t),E(x_{n-1},T(x_{n-1}),T(x_{n-1}),t), \\ &\quad E(x_n,T(x_n),T(x_n),t),E(T(x_{n-1}),x_n,x_n,t)\} \end{aligned}$$

Case 1:

$$\min\{E(x_{n-1},x_n,x_n,t),E(x_{n-1},T(x_{n-1}),T(x_{n-1}),t),E(x_n,T(x_n),T(x_n),t), E(T(x_{n-1}),x_n,x_n,t)\} = E(x_{n-1},x_n,x_n,t).$$

Then

$$\begin{aligned} E(x_n,T(x_n),T(x_n),t) &\geq \psi E(x_{n-1},x_n,x_n,t) \\ &> E(x_{n-1},x_n,x_n,t) \end{aligned}$$

As $n\to\infty$, $E(x',T(x'),T(x'),t)\to 1$

Case 2:

$$\min\{E(x_{n-1},x_n,x_n,t),E(x_{n-1},T(x_{n-1}),T(x_{n-1}),t),E(x_n,T(x_n),T(x_n),t), E(T(x_{n-1}),x_n,x_n,t)\} = E(x_{n-1},T(x_{n-1}),T(x_{n-1}),t)$$

Then

$$E(x_n,T(x_n),T(x_n),t) \geq \psi E(x_{n-1},T(x_{n-1}),T(x_{n-1}),t)$$

$$\begin{aligned} &\geq \psi E(x_{n-1},x_n,x_n,t) \\ &> E(x_{n-1},x_n,x_n,t) \end{aligned}$$

As $n\to\infty$, $E(x',T(x'),T(x'),t)\to 1$

Case 3:

$$\min\{E(x_{n-1},x_n,x_n,t),E(x_{n-1},T(x_{n-1}),T(x_{n-1}),t),E(x_n,T(x_n),T(x_n),t),$$
$$E(T(x_{n-1}),x_n,x_n,t)\} = E(x_n,T(x_n),T(x_n),t)$$

Then

$$\begin{aligned} E(x_n,T(x_n),T(x_n),t) &\geq \psi E(x_n,x_{n+1},x_{n+1},t) \\ &> E(x_n,x_{n+1},x_{n+1},t) \end{aligned}$$

As $n\to\infty, E(x',T(x'),T(x'),t)\to 1$

Case 4:

$$\min\{E(x_{n-1},x_n,x_n,t),E(x_{n-1},T(x_{n-1}),T(x_{n-1}),t),E(x_n,T(x_n),T(x_n),t),$$
$$E(T(x_{n-1}),x_n,x_n,t)\} = E(T(x_{n-1}),x_n,x_n,t)$$

Then

$$\begin{aligned} E(x_n,T(x_n),T(x_n),t) &\geq \psi E(T(x_{n-1}),x_n,x_n,t) \\ &\geq \psi E(x_n,x_n,x_n,t). \end{aligned}$$

As $n\to\infty, E(x',T(x'),T(x'),t)\to 1.$

Hence $T(x')=x'$ for each case. That is x' is a fixed point of T.

To prove uniqueness, let x_1 be another fixed point of T with $x'\neq x_1$. Then

$$\begin{aligned} E(x_1,x_1,x',t) &= E(T(x_1),T(x_1),T(x'),t) \\ &\geq \min\{E(x_1,x_1,x',t),E(x_1,T(x_1),T(x_1),t), \\ &\quad E(x_1,T(x_1),T(x_1),t),E(T(x_1),x_1,x',t)\} \\ &\geq \psi E(x_1,x_1,x',t) \\ &> E(x_1,x_1,x',t). \end{aligned}$$

Which is impossible. Hence $x_1 = x'$.

Since $x_n \to x \implies T(x_n) \to T(x')$, T is continuous at x'.

□

4. Fixed Point Theorems for Contractive Mapping Satisfying ϕ - Maps

Let $\phi : (0,\infty) \to (0,\infty)$ be the function such that $\phi(t) < t$ and $lim_{n\to\infty}\ \phi^n(t) = 0$ for any $t > 0$. Using this ϕ function, some fixed point theorem for ϕ contraction in complete $E-$ fuzzy metric spaces are obtained in this section.

Theorem 4.1. Let $(X,E,*)$ be a E-fuzzy metric space. If $\forall x,y,z \in X$ and $t > 0$, $E(x,y,z,\phi(t)) \geq E(x,y,z,t)$ then $x = y = z$.

Proof. Since $\phi(t) < t$ for any $t > 0$, implies $E(x,y,z,\phi(t)) \leq E(x,y,z,t)$ and also from theorem statement, we get $E(x,y,z,\phi(t)) = E(x,y,z,t)\ \forall x,y,z \in X$ and $t > 0$. From this for $n \geq 1$, $E(x,y,z,\phi^n(t)) = E(x,y,z,t)\ \forall x,y,z \in X$ and $t > 0$. To show $E(x,y,z,t) = 1\ \forall t > 0$. Suppose there exists $t_0 > 0$ such that $E(x,y,z,t_0) < 1$. Since $lim_{n\to\infty}E(x,y,z,t) = 1$, there exists some $t_1 > t_0$ such that $E(x,y,z,t_1) > E(x,y,z,t_0)\ \forall x,y,z \in X$. Since $t_0 > 0$ and $lim_{n\to\infty}\phi^n(t_1) = 0$, there exists $n_0 \in \mathcal{N}$ such that $\phi^{n_0}(t_1) < t_0$. Hence $E(x,y,z,t_0) \geq E(x,y,z,\phi^{n_0}(t_1)) = E(x,y,z,t_1)$, which is a contradiction. Hence $E(x,y,z,t) = 1\ \forall x,y,z \in X$ and $t > 0$. Thus $x = y = z$. □

Theorem 4.2. Let $(X,E,*)$ be a complete E-fuzzy metric space. Suppose that the map $T : X \to X$ satisfies

$$E(T(x),T(y),T(z),\phi(t)) \geq E(x,y,z,t), \forall x,y,z \in X \text{ and } t > 0.$$

Then T has a unique fixed point and T is continuous at the fixed point.

Proof. Let $x_0 \in X$ be an arbitrary point and let $x_n = T(x_{n-1}), n \in \mathcal{N}$. To prove (x_n) is a Cauchy sequence in $(X,E,*)$. For $n \in \mathcal{N}$,

$$\begin{aligned} E(x_0,x_1,x_1,t) &\leq E(T(x_0),T(x_1),T(x_1),\phi(t)) \\ &\leq E(x_1,x_2,x_2,\phi(t)) \\ &\leq E(T(x_1),T(x_2),T(x_2),\phi^2(t)) \\ &\leq E(x_2,x_3,x_3,\phi^2(t)) \end{aligned}$$

$$\begin{aligned} &\vdots \\ &\leq \quad E(x_n, x_{n+1}, x_{n+1}, \phi^n(t)) \end{aligned}$$

For any $0 < \delta < 1$ there exists $0 < \varepsilon < 1$ such that

$$\underbrace{(1-\varepsilon) * (1-\varepsilon) * \ldots * (1-\varepsilon)}_{n} > (1-\delta) \quad \forall n \in \mathscr{N}. \tag{1}$$

We have $sup\{E(x_0, x_1, x_1, t), t > 0\} = 1$. For any $t > 0$ and $0 < \varepsilon < 1$, there exists $t_0 > 0$ such that $E(x_0, x_1, x_1, t_0) > 1 - \varepsilon$. Since $\phi^n(t_0) \to 0$ there exists $k_0 \in \mathscr{N}$ such that $\phi^n(t_0) < t \ \forall n \geq k_0$. By monotonicity,

$$\begin{aligned} E(x_n, x_{n+1}, x_{n+1}, t) &\geq E(x_n, x_{n+1}, x_{n+1}, \phi^n(t_0)) \quad \forall n \geq k_0 \\ &\geq E(x_0, x_1, x_1, t_0) \\ &> 1 - \varepsilon. \end{aligned}$$

Thus for any $t > 0$ there exists $0 < \varepsilon < 1$ such that

$$E(x_n, x_{n+1}, x_{n+1}, t) > 1 - \varepsilon. \tag{2}$$

Similarly, for any $t > 0$ there exists $0 < \varepsilon < 1$ such that

$$E(x_n, x_n, x_{n+1}, t) > 1 - \varepsilon. \tag{3}$$

For $m, n \in \mathscr{N}$ with $m > n$.

$$E(x_n, x_m, x_m, t) \geq E(x_n, x_{n+1}, x_m, \frac{t}{2}) * E(x_{n+1}, x_m, x_m, \frac{t}{2}) \tag{4}$$

Now, using the properties of E-fuzzy metric space and applying (2) and (3), we get

$$\begin{aligned} E(x_n, x_{n+1}, x_m, \frac{t}{2}) &= E(x_m, x_n, x_{n+1}, \frac{t}{2}) \\ &\geq E(x_n, x_{n+1}, x_{n+1}, \frac{t}{4}) * E(x_{n+1}, x_{n+2}, x_{n+2}, \frac{t}{16}) \\ &\quad * E(x_{n+2}, x_{n+3}, x_{n+3}, \frac{t}{64}) * \ldots * E(x_{m-1}, x_m, x_m, \frac{t}{2^{2(m-n)}}) \\ &\quad * E(x_{n+1}, x_{n+1}, x_{n+2}, \frac{t}{8}) * E(x_{n+2}, x_{n+2}, x_{n+3}, \frac{t}{32}) \\ &\quad * E(x_{n+3}, x_{n+3}, x_{n+4}, \frac{t}{128}) * \ldots * E(x_{m-1}, x_{m-1}, x_m, \frac{t}{2^{2(m-n)-1}}) \end{aligned}$$

$$> \underbrace{(1-\varepsilon)*(1-\varepsilon)*.....(1-\varepsilon)}_{(m-n)} * \underbrace{(1-\varepsilon)*(1-\varepsilon)*.....(1-\varepsilon)}_{(m-n-1)} \tag{5}$$

and,

$$\begin{aligned} E(x_{n+1},x_m,x_m,\frac{t}{2}) &= E(x_m,x_m,x_{n+1},\frac{t}{2}) \\ &\geq E(x_{n+1},x_m,x_{n+1},\frac{t}{4})*E(x_m,x_{n+1},x_{n+1},\frac{t}{4}) \\ &\geq E(x_m,x_{n+1},x_{n+1},\frac{t}{4})*E(x_m,x_{n+1},x_{n+1},\frac{t}{4}) \end{aligned} \tag{6}$$

Now, by using (2) and (3), and by the definition of $E-$ fuzzy metric space,

$$\begin{aligned} E(x_m,x_{n+1},x_{n+1},\frac{t}{4}) &\geq E(x_{n+1},x_{n+1},x_{n+2},\frac{t}{8})*E(x_{n+2},x_{n+2},x_{n+3},\frac{t}{32}) \\ &\quad *E(x_{n+3},x_{n+3},x_{n+4},\frac{t}{128})*...*E(x_{m-1},x_m,x_m,\frac{t}{2^{2(m-n)-1}}) \\ &\quad *E(x_{n+2},x_{n+2},x_{n+1},\frac{t}{16})*E(x_{n+3},x_{n+3},x_{n+2},\frac{t}{64}) \\ &\quad *...*E(x_m,x_m,x_{m-1},\frac{t}{2^{2(m-n)}}) \end{aligned}$$

$$> \underbrace{(1-\varepsilon)*(1-\varepsilon)*.....(1-\varepsilon)}_{(m-n-1)} * \underbrace{(1-\varepsilon)*(1-\varepsilon)*.....(1-\varepsilon)}_{(m-n-1)}. \tag{7}$$

By using (1), (5),(6) and (7), (4) becomes $E(x_n,x_m,x_m,t) > (1-\delta)$. So (x_n) is a Cauchy sequence in $(X,E,*)$ and convergent to some x' in X. Now

$$\begin{aligned} E(x',T(x'),T(x'),t) &\geq E(x',x_{n+1},T(x'),t-\phi(t))*E(x_{n+1},T(x'),T(x'),\phi(t)) \\ &\geq E(T(x'),x',x_{n+1},t-\phi(t))*E(x_n,x',x',t) \\ &\geq E(T(x'),x_{n+1},x_{n+1},\frac{t-\phi(t)}{2})*E(x_{n+1},x',x_{n+1},\frac{t-\phi(t)}{2}) \\ &\quad *E(x_n,x',x',t) \\ &> E(T(x'),T(x_n),T(x_n),\phi(\frac{t-\phi(t)}{2}))*E(x_{n+1},x',x_{n+1},\frac{t-\phi(t)}{2}) \\ &\quad *E(x_n,x',x',t) \end{aligned}$$

$$> E(x',x_n,x_n,\frac{t-\phi(t)}{2})*E(x_{n+1},x',x_{n+1},\frac{t-\phi(t)}{2})*E(x',x',x_n,t)$$

As $n \to \infty$ and since E is continuous on its variable, we get $E(x', T(x'), T(x'), t) = 1$. Hence $T(x') = x'$. Hence x' is a fixed point of T. Now to prove uniqueness, let x_1 be another fixed point of T with $x' \neq x_1$.

$$\begin{aligned} E(x', x', x_1, t) &= E(T(x'), T(x'), T(x_1), t) \\ &> E(T(x'), T(x'), T(x_1), \phi(t)) \\ &> E(x', x', x_1, t) \end{aligned}$$

It is a contradiction, hence $x' = x_1$. To prove T is continuous,

$$\begin{aligned} E(T(x'), T(x_n), T(x_n), t) &> E(T(x'), T(x_n), T(x_n), \phi(t)) \\ &> E(x', x_n, x_n, t) \end{aligned}$$

Since the sequence (x_n) converges to x', $E(T(x'), T(x_n), T(x_n), t) \to 1$, as $n \to \infty$. Hence $T(x_n)$ convergent to $T(x')$. So T is continuous at the fixed point x'.

□

Example 4.1. Let $X = [0, \infty)$. Define $a * b = \min(a, b)$. Define $G(x, y, z) = |x - y| + |y - z| + |z - x|$ and $E(x, y, z, t) = \begin{cases} \frac{t}{t + G(x,y,z)} & , \quad t \leq G(x, y, z) \\ 1 & , \quad t > G(x, y, z) \end{cases}$.
Then $(X, E, *)$ be a complete E-fuzzy metric space. Define $T : X \to X$ by $T(x) = \frac{x}{3}$ and $\phi : (0, \infty) \to (0, \infty)$ by $\phi(t) = \frac{t}{3}$.

If $\phi(t) > G(T(x), T(y), T(z))$, then $E(T(x), T(y), T(z), \phi(t)) = 1$.

Hence $E(T(x), T(y), T(z), \phi(t)) \geq E(x, y, z, t)$.

If $\phi(t) \leq G(T(x), T(y), T(z))$, then

$$\begin{aligned} E(T(x), T(y), T(z), \phi(t)) &= \frac{\phi(t)}{\phi(t) + G(T(x), T(y), T(z))} \\ &= \frac{\frac{t}{3}}{\frac{t}{3} + \frac{G(x,y,z)}{3}} \\ &= \frac{t}{t + G(x, y, z)} \\ &= E(x, y, z, t). \end{aligned}$$

Then by Theorem 4.2 T has a fixed point and the fixed point is $x = 0$.

Corollary 4.1. Let $(X,E,*)$ be a complete E-fuzzy metric space. Suppose that the map $T : X \to X$ satisfies for $n \in \mathcal{N}$,

$$E(T^n(x),T^n(y),T^n(z),\phi(t)) \geq E(x,y,z,t), \forall x,y,z \in X \text{ and } t > 0.$$

Then T has a unique fixed point.

Proof. By Theorem 4.2 we get T^n has unique fixed point say x' that is $T^n(x') = x'$.
$T(x') = T(T^n(x')) = T^{n+1}(x') = T^n(T(x'))$. Hence $T(x')$ is a fixed point of T^n. Since the uniqueness of fixed point $T(x') = x'$. □

Corollary 4.2. Let $(X,E,*)$ be a complete E-fuzzy metric space. Suppose there is $k \in (0,1)$ such that the map $T : X \to X$ satisfies

$$E(T(x),T(y),T(z),kt) \geq E(x,y,z,t), \forall x,y,z \in X \text{ and } t > 0.$$

Then T has a unique fixed point and T is continuous at this fixed point.

Proof. Define $\phi : (0,1) \to (0,1)$ as $\phi(t) = kt$. Then by Theorem 4.2 we get the result. □

Theorem 4.3. Let X be a complete E-fuzzy metric space and $T : X \to X$ be the map satisfies

$$E(T(x),T(y),T(z),\phi(t)) \geq \min\{E(x,T(x),T(x),t),E(y,T(y),T(y),t),E(x,y,z,t), E(T(x),y,z,t)\},$$

$\forall x,y,z \in X$ and $t > 0$. Then T has a unique fixed point and T is continuous at this fixed point.

Proof. let $x_0 \in X$ and $x_n = T(x_{n-1}), n \in \mathcal{N}$. Assume $x_n \neq x_{n-1}$ for $n \in \mathcal{N}$. Since $t > \phi(t)$ and E is non decreasing, we have

$$\begin{aligned} E(x_n,x_{n+1},x_{n+1},t) &> E(x_n,x_{n+1},x_{n+1},\phi(t)) \\ &> E(T(x_{n-1}),T(x_n),T(x_n),\phi(t)) \\ &> \min\{E(x_{n-1},x_n,x_n,t),E(x_n,x_{n+1},x_{n+1},t),E(x_{n-1},x_n,x_n,t), E(x_n,x_n,x_n,t)\} \\ &> \min\{E(x_{n-1},x_n,x_n,t),E(x_n,x_{n+1},x_{n+1},t)\}. \end{aligned}$$

If $\min\{E(x_{n-1},x_n,x_n,t),E(x_n,x_{n+1},x_{n+1},t)\} = E(x_n,x_{n+1},x_{n+1},t)$, which is impossible.

If $\min\{E(x_{n-1},x_n,x_n,t),E(x_n,x_{n+1},x_{n+1},t)\} = E(x_{n-1},x_n,x_n,t)$.

Then $E(x_n,x_{n+1},x_{n+1},t) > E(T(x_{n-1}),T(x_n),T(x_n),\phi(t)) > E(x_{n-1},x_n,x_n,t)$.

Then from proof of Theorem 4.2, we get (x_n) is a Cauchy sequence in $(X,E,*)$. Since X is a complete E-fuzzy metric space, (x_n) converges to some $x' \in X$. Now, for $n \in \mathscr{N}$,

$$\begin{aligned}
E(T(x'),x',x',t) &\geq E(x_n,x',x',t/2) * E(T(x'),x_n,x',t/2) \\
&\geq E(x_n,x',x',t^1) * E(x',T(x'),x_n,t^1-\phi(t^1)+\phi(t^1)), \text{ put } t/2 = t^1 \\
&\geq E(x_n,x',x',t^1) * E(x',x_n,x_n,t^1-\phi(t^1)) * E(x_n,T(x'),x_n,\phi(t^1)) \\
&\geq E(x_n,x',x',t^1) * E(x',x_n,x_n,t^1-\phi(t^1)) \\
&\quad * E(T(x_{n-1}),T(x_{n-1}),T(x'),\phi(t)) \\
&\geq E(x_n,x',x',t^1) * E(x',x_n,x_n,t^1-\phi(t^1)) * \min\{E(x_{n-1},x_n,x_n,t), \\
&\quad E(x_{n-1},x_n,x_n,t),E(x_{n-1},x_{n-1},x',t),E(x_n,x_{n-1},x',t)\}
\end{aligned}$$

In each cases of minimum, as $n \to \infty$, we get $E(T(x'),x',x',t) = 1$. That is $T(x') = x'$. Hence x' is a fixed point of T. Let x_1 be another fixed point of T with $x' \neq x_1$.

$$\begin{aligned}
E(x',x',x_1,t) &> E(x',x',x_1,\phi(t)) \\
&> E(T(x'),T(x'),T(x_1),\phi(t)) \\
&> \min\{E(x',x',x',t),E(x',x',x',t),E(x',x',x_1,t),E(x',x',x_1,t)\} \\
&> E(x',x',x_1,t)
\end{aligned}$$

which is a contradiction. Hence $x' = x_1$.

To prove T is continuous at x', let (x_n) is a sequence in X converges to x'.

$$\begin{aligned}
E(x',x',T(x_n),t) &= E(T(x'),T(x'),T(x_n),t) \\
&> E(T(x'),T(x'),T(x_n),\phi(t)) \\
&> \min\{E(x',x',x',t),E(x',x',x',t),E(x',x',x_n,t),E(x',x',x_n,t)\} \\
&> E(x',x',x_n,t).
\end{aligned}$$

As $n \to \infty$, $T(x_n)$ convergent to $T(x') = x'$. Thus T is continuous at x'.

□

Corollary 4.3. Let $(X, E, *)$ be a complete E-fuzzy metric space. Suppose $k \in (0,1)$ such that the map $T : X \to X$ satisfies

$$E(T(x), T(y), T(z), kt) \geq \min\{E(x, T(x), T(x), t), E(y, T(y), T(y), t), E(x, y, z, t), E(T(x), y, z, t)\},$$

$\forall x, y, z \in X$ and $t > 0$. Then T has a unique fixed point and T is continuous at this fixed point.

Proof. Define $\phi : (0,1) \to (0,1)$ as $\phi(t) = kt$. Using Theorem 4.3 we obtained the result. □

5. APPLICATIONS

Integral equations have applications in many fields such as Chemistry, Physics, Engineering and Biology. The aim of this section is to give the existence and uniqueness for a solution of the following integral equations using Theorem 4.2.

(1) Let $X = C[0,1]$ be the set of all continuous functions defined on $[0,1]$.

Define $E : X^3 \times (0,\infty) \to [0,1]$ by

$$E(x, y, z, t) = \frac{t}{t + G(x, y, z)}$$

where $G(x, y, z) = \sup_{t' \in [0,1]} |x(t') - y(t')| + \sup_{t' \in [0,1]} |y(t') - z(t')| + \sup_{t' \in [0,1]} |z(t') - x(t')|$, $\forall x, y, z \in X$.

Then $(X, E, *)$ be a complete E-fuzzy metric space. Let $\phi : (0,\infty) \to (0,\infty)$ be defined as $\phi(t) = rt$, $0 < r < 1$. Consider the following integral equation

$$T(x(t')) = \int_0^1 k(t', s) f(s, x(s)) ds \text{ for } t', s \in [0,1],$$

where

(a) $k : [0,1] \times [0,1] \to (0,\infty)$ is continuous

(b) $f : [0,1] \times R \to R$

(c) there exists a constant $\lambda > 0$ such that $|f(t', x) - f(t', y)| \leq \lambda |x - y|$

(d) $\lambda \parallel k \parallel_\infty \leq r$ where $\parallel k \parallel_\infty = \sup \{k(t', s) : t', s \in [0,1]\}$.

Then the integral equation has unique solution in $C[0,1]$ by Theorem 4.2.

$E(T(x(t')),T(y(t')),T(z(t')),\phi(t)) = \frac{\phi(t)}{\phi(t)+G(T(x(t')),T(y(t')),T(z(t')))}$

$$= \phi(t)/(\phi(t)+\sup_{t'\in[0,1]}|T(x(t'))-T(y(t'))|+\sup_{t'\in[0,1]}|T(y(t'))-T(z(t'))| + \sup_{t'\in[0,1]}|T(z(t'))-T(x(t'))|$$

$$= \phi(t)/(\phi(t)+\sup_{t'\in[0,1]}|\int_0^1 k(t',s)f(s,x(s))ds-\int_0^1 k(t',s)f(s,y(s))ds| + \sup_{t'\in[0,1]}|\int_0^1 k(t',s)f(s,y(s))ds-\int_0^1 k(t',s)f(s,z(s))ds| + \sup_{t'\in[0,1]}|\int_0^1 k(t',s)f(s,z(s))ds-\int_0^1 k(t',s)f(s,x(s))ds|)$$

$$= \phi(t)/(\phi(t)+\sup_{t'\in[0,1]}|\int_0^1 k(t',s)(f(s,x(s))-f(s,y(s)))ds| + \sup_{t'\in[0,1]}|\int_0^1 k(t',s)(f(s,y(s))-f(s,z(s)))ds| + \sup_{t'\in[0,1]}|\int_0^1 k(t',s)(f(s,z(s))-f(s,x(s)))ds|)$$

$$\geq \phi(t)/(\phi(t)+\lambda\sup_{t'\in[0,1]}\int_0^1 k(t',s)|x(s)-y(s)|ds + \lambda\sup_{t'\in[0,1]}\int_0^1 k(t',s)|y(s)-z(s)|ds + \lambda\sup_{t'\in[0,1]}\int_0^1 k(t',s)|z(s))-x(s)|ds)$$

Using Cauchy Schwarz inequality, we have

$$\begin{aligned}\int_0^1 k(t',s)|x(s)-y(s)|ds &\leq (\int_0^1 k^2(t',s)ds)^{1/2}(\int_0^1 |x(s)-y(s)|^2ds)^{1/2}\\ &\leq \| k \|_\infty sup_{t'\in[0,1]}|x(t')-y(t')|\end{aligned}$$

Thus

$$\begin{aligned}E(T(x(t')),T(y(t')),T(z(t')),\phi(t)) &\geq \phi(t)/(\phi(t)+\lambda\| k \|_\infty \sup_{t'\in[0,1]}|x(t')-y(t')| + \lambda\| k \|_\infty \sup_{t'\in[0,1]}|y(t')-z(t')| + \lambda\| k \|_\infty \sup_{t'\in[0,1]}|z(t')-x(t')|)\\ &\geq \frac{\phi(t)}{\phi(t)+\lambda\| k \|_\infty (G(x(t'),y(t')z(t'))}\end{aligned}$$

$$\begin{aligned} &\geq \frac{rt}{rt + r(G(x(t'),y(t'),z(t'))} \\ &\geq \frac{t}{t + (G(x(t'),y(t'),z(t'))} \\ &\geq E(x,y,z,t). \end{aligned}$$

It satisfies Theorem 4.2.

(2) Consider the following integral equation,

$$T(x(t)) = \int_0^1 k(t,s,x(s))ds + f(t) \text{ for } s,t \in [0,1].$$

Let $X = C[0,1]$ be the set of all continuous functions defined on $[0,1]$.

Define $E : X^3 \times (0,\infty) \to [0,1]$ by $E(x,y,z,t') = \frac{t'}{t'+G(x,y,z)}$,

where $G(x,y,z) = \sup_{t\in[0,1]} |x(t)-y(t)| + \sup_{t\in[0,1]} |y(t)-z(t)| + \sup_{t\in[0,1]} |z(t)-x(t)|$

$\forall x,y,z \in X$. Then $(X,E,*)$ is a complete E-fuzzy metric space. Let $\phi : (0,\infty) \to (0,\infty)$ be defined as $\phi(t') = rt'$, $0 < r < 1$. Suppose the following hypotheses hold,

(a) $k : [0,1] \times [0,1] \times R^+ \to R^+$ and $f : [0,1] \to R^+$ are continuous.

(b) There exists a continuous function $k' : [0,1] \times [0,1] \to [0,\infty)$ such that $|k(t,s,x) - k(t,s,y)| \leq k'(t,s)|x-y|$ for each $x,y \in R^+$ and $t,s \in [0,1]$.

(c) $\sup_{t\in[0,1]} \int_0^1 k'(t,s)ds \leq r$ for some $0 < r < 1$.

Then the integral equation has a solution $x \in C[0,1]$ by Theorem 4.2. For

$E(T(x(t)),T(y(t)),T(z(t)),\phi(t')) = \frac{\phi(t')}{\phi(t')+G(T(x(t)),T(y(t)),T(z(t))}$

$$\begin{aligned} &= \phi(t')/(\phi(t') + \sup_{t\in[0,1]} |T(x(t)) - T(y(t))| + \sup_{t\in[0,1]} |T(y(t)) - T(z(t))| \\ &\qquad + \sup_{t\in[0,1]} |T(z(t)) - T(x(t))|) \end{aligned}$$

$$\begin{aligned} &= \phi(t')/(\phi(t') + sup_{t\in[0,1]} | \int_0^1 k(t,s,x(s)) - k(t,s,y(s))ds| + sup_{t\in[0,1]} | \int_0^1 k(t,s,y(s)) - \\ &\qquad k(t,s,z(s))ds| + sup_{t\in[0,1]} | \int_0^1 k(t,s,z(s)) - k(t,s,x(s))ds|) \end{aligned}$$

$$\begin{aligned} &\geq \phi(t')/(\phi(t') + sup_{t\in[0,1]} \int_0^1 k'(t,s)|x(s)-y(s)|ds + sup_{t\in[0,1]} \int_0^1 k'(t,s)|y(s) \\ &\qquad - z(s)|ds + sup_{t\in[0,1]} \int_0^1 k'(t,s)|z(s)-x(s)|ds) \end{aligned}$$

$$\geq \phi(t')/(\phi(t') + sup_{t\in[0,1]} \int_0^1 k'(t,s)ds\{sup_{t\in[0,1]}|x(t)-y(t)| + sup_{t\in[0,1]}|y(t)-z(t)| + sup_{t\in[0,1]}|z(t)-x(t)|\})$$

$$\geq rt'/(rt' + rG(x,y,z))$$

$$\geq t'/(t' + G(x,y,z))$$

$$\geq E(x,y,z,t').$$

Thus all conditions of Theorem 4.2 are satisfied. Hence there exists a solution $x \in C[0,1]$ of the given integral equation.

6. CONCLUSION

Fixed point theory finds applications in many fields of mathematics, such as mathematical economics, nonlinear programming, mathematical modeling, theory of integral equations and so on. In this paper some results concerning the existence and uniqueness of fixed points via $\phi-$ contraction and $\psi-$ contraction in complete generalized $E-$ fuzzy metric spaces are proved. These results generalized some fixed point results existing in the literature for fuzzy metric space. Some applications in the existence and uniqueness of solutions to integral equations are presented.

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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