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EXTINCTION OF A REACTION-DIFFUSION MODEL OF PLANKTON ALLELOPATHY WITH NONLOCAL DELAYS

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Abstract. In this paper, we consider a reaction-diffusion model of plankton allelopathy with nonlocal delays and one phytoplankton producing toxin. By using an iterative technique, the global stability of the semi-trivial steady states of the system are investigated. The results presented in this paper improve the corresponding results announced by many authors.

Keywords: Global stability; Plankton allelopathy; Extinction; Diffusion; Nonlocal delays.

2010 AMS Subject Classification: 34D23, 34D20, 34D40.

1. Introduction

Recently, competitive system with the effect of toxic substances become one of the important research topic, and many excellent results have been obtained (see [1]-[21] and the references cited therein). Tian Zhang and Ling [1] and Li, Chen and He [2] proposed and studied the

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following reaction-diffusion model of plankton allelopathy with nonlocal delays

$$\begin{aligned}
&\frac{\partial u_1}{\partial t} - D_1\Delta u_1 = u_1(r_1 - a_{11}u_1 - a_{12}u_2 - e_1u_1u_2),\\
&\frac{\partial u_2}{\partial t} - D_2\Delta u_2 = u_2\left(r_2 - a_{21}u_1 - a_{22}u_2 - e_2u_2\int_{-\infty}^{t}\int_{\Omega}K(x,y,t-s)u_1(s,y)\mathrm{d}y\mathrm{d}s\right),
\end{aligned} \tag{1.1}$$

for $t>0,\ \ x\in\Omega$, under the homogeneous Neumann boundary conditions

$$\frac{\partial u_1}{\partial v} = \frac{\partial u_2}{\partial v} = 0,\ \ t>0,\ \ x\in\partial\Omega \tag{1.2}$$

and initial conditions

$$u_1(\theta,x) = \phi_1(\theta,x)\geq 0,\ (\theta,x)\in(-\infty,0]\times\overline{\Omega},\ \ u_2(0,x) = \phi_2(0,x)\geq 0,\ x\in\overline{\Omega}\,, \tag{1.3}$$

where $\phi_i,\ i=1,2$, are non-negative, Höder continuous and satisfy $\dfrac{\partial u_i}{\partial v}=0$ on the boundary; $u_i(t,x)$ denotes the population density of the i-th species at time t and location x; Ω is a bounded domain in R^n with smooth boundary $\partial\Omega$; v is the outward normal vector; all of the coefficients presented in the above system are positive constants. They also assume that:

$$\begin{aligned}
&K(x,y,t) = G(x,y,t)k(t),\ x,y\in\Omega,\quad k(t)\geq 0;\\
&\int_{\Omega}G(x,y,t)\mathrm{d}x = \int_{\Omega}G(x,y,t)\mathrm{d}y = 1,\quad t\geq 0;\\
&\int_0^{+\infty}k(t)\mathrm{d}t = 1,\quad tk(t)\in L^1((0,+\infty);R),
\end{aligned} \tag{1.4}$$

where $G(x,y,t)$ is a non-negative function, which is continuous in $(x,y)\in\overline{\Omega}\times\overline{\Omega}$ for each $t\in[0,+\infty)$ and measurable in $t\in[0,+\infty)$ for each pair $(x,y)\in\overline{\Omega}\times\overline{\Omega}$.

Obviously system (1.1) admits two semi-trivial steady states $E_1(\dfrac{r_1}{a_{11}},0)$ and $E_2(0,\dfrac{r_2}{a_{22}})$. Let $\Omega=[0,\pi]$, concerned with the asymptotic behavior of these two steady states, Tian, Zhang and Ling [1] obtained the following theorem.

Theorem A[1] *Let $(u_1(t,x),u_2(t,x))$ be any solution of system (1.1) with boundary conditions (1.2) and initial conditions (1.3) satisfying $\phi_1(0,x)\not\equiv 0$ and $\phi_2(0,x)\not\equiv 0$.*

(i) *If*

$$\frac{r_1}{r_2}>\frac{e_1}{e_2}>\frac{a_{12}}{a_{22}}\quad\text{and}\quad\frac{r_1}{r_2}>\frac{e_1}{e_2}>\frac{a_{11}}{a_{21}}, \tag{1.5}$$

then $\lim\limits_{t\to+\infty}(u_1(t,x),u_2(t,x)) = \left(\dfrac{r_1}{a_{11}},0\right)$ uniformly for $x\in\overline{\Omega}$.

(ii) *If*

$$\frac{r_1}{r_2} < \frac{e_1}{e_2} < \frac{a_{12}}{a_{22}} \quad \text{and} \quad \frac{r_1}{r_2} < \frac{e_1}{e_2} < \frac{a_{11}}{a_{21}}, \tag{1.6}$$

then $\lim_{t\to+\infty}(u_1(t,x),u_2(t,x)) = \left(0,\frac{r_2}{a_{22}}\right)$ *uniformly for* $x \in \overline{\Omega}$.

With the aim of improving above results, by using an iterative technique, Li, Chen and He[2] obtained the following results:

Theorem B *Let* $(u_1(t,x),u_2(t,x))$ *be any solution of system (1.1) with boundary conditions (1.2) and initial conditions (1.3) satisfying* $\phi_1(0,x) \not\equiv 0$ *and* $\phi_2(0,x) \not\equiv 0$.

(i) *If*

$$\frac{r_1}{r_2} > \frac{a_{12}}{a_{22}}, \quad \frac{r_1}{r_2} \geq \frac{a_{11}}{a_{21}} \quad \text{and} \quad \frac{r_1}{r_2} \geq \frac{e_1}{e_2}, \tag{1.7}$$

then $\lim_{t\to+\infty}(u_1(t,x),u_2(t,x)) = \left(\frac{r_1}{a_{11}},0\right)$ *uniformly for* $x \in \overline{\Omega}$.

(ii) *If*

$$\frac{r_1}{r_2} \leq \frac{a_{12}}{a_{22}}, \quad \frac{r_1}{r_2} < \frac{a_{11}}{a_{21}} \quad \text{and} \quad \frac{r_1}{r_2} \leq \frac{e_1}{e_2}, \tag{1.8}$$

then $\lim_{t\to+\infty}(u_1(t,x),u_2(t,x)) = \left(0,\frac{r_2}{a_{22}}\right)$ *uniformly for* $x \in \overline{\Omega}$.

Recently, Sol*é* et al [2] considered a Lotka-Volterra type of model for two interacting phytoplankton species, where one species could produce toxic, while the other one is non-toxic produce. The model is described as follow:

$$\begin{aligned} \dot{u}_1(t) &= u_1(t)[r_1 - a_1u_1(t) - c_1u_1(t)u_2^2(t)], \\ \dot{u}_2(t) &= u_2(t)[r_2 - b_2u_2(t)]. \end{aligned} \tag{1.9}$$

Their experiment is based on the batch cultures and they have neglected the interspecific competition terms since the growth curve of the toxic alga was similar for the monocultures and all the mixed cultures for different initial concentrations. Their experiment also indicated that the effect of the competition term is significant after a time period longer than the experimental time.

Stimulated by the works of [1, 2, 3], we propose and study the following reaction-diffusion

model of plankton allelopathy with nonlocal delays

$$\begin{aligned}
&\frac{\partial u_1}{\partial t} - D_1\Delta u_1 = u_1(r_1 - a_{11}u_1 - a_{12}u_2),\\
&\frac{\partial u_2}{\partial t} - D_2\Delta u_2 = u_2\left(r_2 - a_{21}u_1 - a_{22}u_2 - e_2u_2\int_{-\infty}^{t}\int_{\Omega}K(x,y,t-s)u_1(s,y)\mathrm{d}y\mathrm{d}s\right),
\end{aligned} \tag{1.10}$$

where e_2 denotes the rate of toxic inhibition for the species u_2 released by the first species and all other parameters have the same biological interpretations as system (1.1). We will consider system (1.10) with boundary conditions (1.2) and initial conditions (1.3) satisfying $\phi_1(0,x) \not\equiv 0$ and $\phi_2(0,x) \not\equiv 0$. Let us take $\Omega = \pi$, system (1.1) admits two semi-trivial steady states $E_1(\frac{r_1}{a_{11}},0)$ and $E_2(0,\frac{r_2}{a_{22}})$. Concerning with the asymptotic behavior of these two semi-trivial steady states, one may conjecture that above Theorem A and Theorem B could be applied directly to system (1.10) since the system we considered here is the special case of system (1.1). However, noting that the inequality $\frac{r_1}{r_2} < \frac{e_1}{e_2}$ in (1.6) or $\frac{r_1}{r_2} \leq \frac{e_1}{e_2}$ in (1.8) no longer hold due to $e_1 = 0$, Theorem A and B could not be applied to system (1.10). Therefore, it become an interesting and challenging task to study the extinction property of system (1.10).

The aim of this paper is, by developing the analysis technique of Li, Chen and He[2], to obtain some sufficient conditions which ensure the extinction of species in system (1.10). The organization of this paper is as follows. Main results are stated in Section 2; In Section 3, we introduce some useful lemmas, while the extinction of system (1.10) are investigated in Section 4. We end this paper by a brief discussion.

2. Main results

Our main results are the following Theorem 2.1-2.4.

Theorem 2.1. *Let $(u_1(t,x),u_2(t,x))$ be any solution of system (1.10) with boundary conditions (1.2) and initial conditions (1.3) satisfying $\phi_1(0,x) \not\equiv 0$ and $\phi_2(0,x) \not\equiv 0$. If*

$$\frac{r_1}{r_2} < \frac{a_{12}}{a_{22}}, \quad \frac{r_1}{r_2} < \frac{a_{11}}{a_{21}} \tag{2.1}$$

and

$$e_2 < \frac{a_{11}a_{22}}{r_1r_2}\min\left\{r_2 - r_1\frac{a_{22}}{a_{12}}, r_2 - r_1\frac{a_{21}}{a_{11}}\right\}, \tag{2.2}$$

then $\lim_{t\to+\infty}(u_1(t,x),u_2(t,x)) = \left(0,\frac{r_2}{a_{22}}\right)$ *uniformly for* $x\in\overline{\Omega}$.

Theorem 2.2. *Let* $(u_1(t,x),u_2(t,x))$ *be any solution of system (1.10) with boundary conditions (1.2) and initial conditions (1.3) satisfying* $\phi_1(0,x)\not\equiv 0$ *and* $\phi_2(0,x)\not\equiv 0$. *In addition to (2.1), further assume that*

$$e_2 < \frac{a_{22}}{r_2}\left(a_{11}\frac{r_2}{r_1} - a_{21}\right), \tag{2.3}$$

then $\lim_{t\to+\infty}(u_1(t,x),u_2(t,x)) = \left(0,\frac{r_2}{a_{22}}\right)$ *uniformly for* $x\in\overline{\Omega}$.

Theorem 2.3. *Let* $(u_1(t,x),u_2(t,x))$ *be any solution of system (1.10) with boundary conditions (1.2) and initial conditions (1.3) satisfying* $\phi_1(0,x)\not\equiv 0$ *and* $\phi_2(0,x)\not\equiv 0$. *In addition to (2.1), further assume that*

$$e_2 < \frac{a_{11}}{r_1}\left(a_{12}\frac{r_2}{r_1} - a_{22}\right), \tag{2.4}$$

then $\lim_{t\to+\infty}(u_1(t,x),u_2(t,x)) = \left(0,\frac{r_2}{a_{22}}\right)$ *uniformly for* $x\in\overline{\Omega}$.

It seems interesting to investigate the extinction property of the second species in system (1.10), since the system has no symmetrical characteristic on toxic substance term, one could not expect the similar results as Theorem A, Theorem B and Theorem 2.1-2.3 can be established. For this case, we have

Theorem 2.4. *Let* $(u_1(t,x),u_2(t,x))$ *be any solution of system (1.10) with boundary conditions (1.2) and initial conditions (1.3) satisfying* $\phi_1(0,x)\not\equiv 0$ *and* $\phi_2(0,x)\not\equiv 0$. *If*

$$\frac{r_1}{r_2} > \frac{a_{12}}{a_{22}}, \quad \frac{r_1}{r_2} > \frac{a_{11}}{a_{21}}, \tag{2.5}$$

then $\lim_{t\to+\infty}(u_1(t,x),u_2(t,x)) = \left(\frac{r_1}{a_{11}},0\right)$ *uniformly for* $x\in\overline{\Omega}$.

3. Preliminaries

Now let us state several lemmas which will be useful in the proof of our main results.

It can be easily seen that $(0,0)$ and (M_1,M_2) are a pair of coupled upper and lower solutions of problem system (1.10) with boundary conditions (1.2) and initial conditions (1.3) satisfying $\phi_1(0,x)\not\equiv 0$ and $\phi_2(0,x)\not\equiv 0$, where

$$M_i = \max\left\{\frac{r_i}{a_{ii}}, ||\phi_i||\right\}, \tag{3.1}$$

where $||\phi_1|| = \sup_{(t,x)\in(-\infty,0]\times\overline{\Omega}}|\phi_1(\theta,x)|$, $||\phi_2|| = \sup_{x\in\overline{\Omega}}|\phi_2(0,x)|$. Hence, the global existence of solutions $(u_1(t,x),u_2(t,x))$ of problem system (1.10) with boundary conditions (1.2) and initial conditions (1.3) can be derived based on the theory of upper-lower solution pairs (see, for example,Pao [22]). It follows that $0 \le u_i(t,x) \le M_i$ $(i=1,2)$ for $(t,x)\in R\times\overline{\Omega}$. In addition, if $\phi_i(0,x)\not\equiv 0$ $(i=1,2)$, then it follows from the strong maximum principle that $u_i(t,x)>0,(i=1,2)$ for all $t>0, x\in\overline{\Omega}$.

Lemma 3.1([2]*), Let $(u_1(t,x),u_2(t,x))$ be any solution of system (1.10) with boundary conditions (1.2) and initial conditions (1.3) satisfying $\phi_1(0,x)\not\equiv 0$ and $\phi_2(0,x)\not\equiv 0$. Then for $i=1,2$*

$$\limsup_{t\to+\infty}\max_{x\in\overline{\Omega}} u_i(t,x) \le \frac{r_i}{a_{ii}},\ \ i=1,2. \tag{3.2}$$

Lemma 3.2. *Let $(u_1(t,x),u_2(t,x))$ be any solution of system (1.10) with boundary conditions (1.2) and initial conditions (1.3) satisfying $\phi_1(0,x)\not\equiv 0$ and $\phi_2(0,x)\not\equiv 0$. Assume that* (2.5) *holds, then there exists an $\alpha>0$ such that*

$$\liminf_{t\to+\infty}\min_{x\in\overline{\Omega}} u_1(t,x) \ge \alpha. \tag{3.3}$$

Lemma 3.3. *Let $(u_1(t,x),u_2(t,x))$ be any solution of system (1.10) with boundary conditions (1.2) and initial conditions (1.3) satisfying $\phi_1(0,x)\not\equiv 0$ and $\phi_2(0,x)\not\equiv 0$. Assume that* (2.1) *holds, then there exists a $\beta>0$ such that*

$$\liminf_{t\to+\infty}\min_{x\in\overline{\Omega}} u_2(t,x) \ge \beta. \tag{3.4}$$

Proof. The proof of Lemma 2.2 and 2.3 is similar to that of the proof of Lemma 2.3 in [2], we omit the detail here.

Lemma 3.4.[2] *If $u(t,x)$ is a bounded nonnegative function, where $(t,x)\in(0,+\infty)\times\Omega$, and $K(x,y,t)$ is defined by (1.4), then*

$$\begin{aligned}\liminf_{t\to+\infty}\min_{x\in\overline{\Omega}} u(t,x) &\le \liminf_{t\to+\infty}\int_{-\infty}^{t}\int_{\Omega} K(x,y,t-s)u(s,y)\mathrm{d}y\mathrm{d}s\\ &\le \limsup_{t\to+\infty}\int_{-\infty}^{t}\int_{\Omega} K(x,y,t-s)u(s,y)\mathrm{d}y\mathrm{d}s \le \limsup_{t\to+\infty}\max_{x\in\overline{\Omega}} u(t,x),\end{aligned}$$

uniformly for $x\in\overline{\Omega}$.

4. Proof of the main results

We will adopt the idea of Li, Chen and He[2] to prove our results.

Proof of Theorem 2.1. Condition (2.1) and (2.2) is equivalent to

$$a_{11}\frac{r_2-e_2\frac{r_1r_2}{a_1a_2}}{r_1}>a_{21},\ a_{22}<a_{12}\frac{r_2-e_2\frac{r_1r_2}{a_1a_2}}{r_1}.$$

And so, there exists a enough small $\varepsilon_1>0$ such that

$$a_{11}\frac{r_2-e_2(\frac{r_1}{a_{11}}+\varepsilon_1)(\frac{r_2}{a_{22}}+\varepsilon_1)}{r_1}>a_{21},\ a_{22}<a_{12}\frac{r_2-e_2(\frac{r_1}{a_{11}}+\varepsilon_1)(\frac{r_2}{a_{22}}+\varepsilon_1)}{r_1}. \tag{4.1}$$

It follows from Lemma 2.1 that $u_i(t,x)$, $i=1,2$, are bounded. Let $\overline{u}_1=\limsup_{t\to+\infty}\max_{x\in\overline{\Omega}}u_1(t,x)$ and $\underline{u}_2=\liminf_{t\to+\infty}\min_{x\in\overline{\Omega}}u_2(t,x)$. From Lemma 2.3 we know that $\underline{u}_2\geq\beta>0$. Obviously $\overline{u}_1\geq 0$. To prove $\lim_{t\to+\infty}u_1(t,x)=0$ uniformly for $x\in\overline{\Omega}$, it suffices to show that $\overline{u}_1=0$. Otherwise, we suppose that $\overline{u}_1>0$.

From (2.1) one could choose sufficiently small $\varepsilon>0$ such that

$$\frac{r_2-a_{21}(\frac{r_1}{a_{11}}+\varepsilon)}{a_{22}+e_2(\frac{r_1}{a_{11}}+\varepsilon)}-\varepsilon>0. \tag{4.2}$$

Let $(u_1(t,x),u_2(t,x))$ be any solution of system (1.10) with boundary conditions (1.2) and initial conditions (1.3) satisfying $\phi_1(0,x)\not\equiv 0$ and $\phi_2(0,x)\not\equiv 0$. It follows from Lemma 2.1 that

$$\limsup_{t\to+\infty}\max_{x\in\overline{\Omega}}u_1(t,x)\leq\frac{r_1}{a_{11}}. \tag{4.3}$$

According to Lemma 2.4, from (4.3), we have

$$\begin{aligned}&\limsup_{t\to+\infty}\int_{-\infty}^{t}\int_{\Omega}K(x,y,t-s)u_1(s,y)\mathrm{d}y\mathrm{d}s\leq\limsup_{t\to+\infty}\max_{x\in\overline{\Omega}}u_1(t,x)\leq\frac{r_1}{a_{11}},\\ &\text{uniformly for } x\in\overline{\Omega}.\end{aligned} \tag{4.4}$$

Hence, for $\varepsilon>0$ defined by (4.2), there exists a $T_1'>0$ such that for any $x\in\overline{\Omega}$ and $t\geq T_1'$,

$$u_1(t,x)\leq M_1^{(1)},\ \ \int_{-\infty}^{t}\int_{\Omega}K(x,y,t-s)u_1(s,y)\mathrm{d}y\mathrm{d}s\leq M_1^{(1)}, \tag{4.5}$$

where

$$M_1^{(1)}\overset{\text{def}}{=}\frac{r_1}{a_{11}}+\varepsilon.$$

Similarly, it follows from Lemma 2.1 that there exists a $T_1 \geq T_1'$ such that

$$u_2(t,x) \leq \frac{r_2}{a_{22}} + \varepsilon \stackrel{\text{def}}{=} M_2^{(1)}, \text{ for any } x \in \overline{\Omega}, \ t \geq T_1. \tag{4.6}$$

(4.5) and the second equation of system (1.10) imply

$$\frac{\partial u_2}{\partial t} - D_2 \Delta u_2 \geq u_2 \left(r_2 - a_{21} M_1^{(1)} - (a_{22} + e_2 M_1^{(1)}) u_2 \right), \quad \text{for } t \geq T_1.$$

Let $w_1(t)$ be a solution of the following ordinary differential equation

$$w_2'(t) = w_2 \left(r_2 - a_{21} M_1^{(1)} - (a_{22} + e_2 M_1^{(1)}) w_2 \right), \ w_2(T_1) = \min_{x \in \overline{\Omega}} u_2(T_1, x), \ t \geq T_1.$$

then

$$\lim_{t \to +\infty} w_2(t) = \frac{r_2 - a_{21} M_1^{(1)}}{a_{22} + e_2 M_1^{(1)}}.$$

From the comparison principle, we obtain $u_2(t,x) \geq w_1(t)$, hence

$$\liminf_{t \to +\infty} \min_{x \in \overline{\Omega}} u_2(t,x) \geq \frac{r_2 - a_{21} M_1^{(1)}}{a_{22} + e_2 M_1^{(1)}}. \tag{4.7}$$

and so, from (4.2) and (4.7) there exists a $T_2 \geq T_1$ such that for $t \geq T_2$

$$u_2(t,x) \geq \frac{r_2 - a_{21} M_1^{(1)}}{a_{22} + e_2 M_1^{(1)}} - \varepsilon \stackrel{\text{def}}{=} m_2^{(1)} > 0. \tag{4.8}$$

From (4.8) and the first equation of system (1.10),

$$\frac{\partial u_1}{\partial t} - D_1 \Delta u_1 \leq u_1 \left(r_1 - a_{12} m_2^{(1)} - a_{11} u_1 \right), \quad \text{for } t \geq T_2.$$

Let $z_1(t)$ be a solution of the following ordinary differential equation

$$z_1'(t) = z_1 \left(r_1 - a_{12} m_2^{(1)} - a_{11} z_1 \right), \ z_1(T_2) = \max_{x \in \overline{\Omega}} u_1(T_2, x), \ t \geq T_2.$$

If $r_1 - a_{12} m_2^{(1)} \leq 0$, then $\lim_{t \to +\infty} z_1(t) = 0$, by the comparison principle, we have

$$\limsup_{t \to +\infty} \max_{x \in \overline{\Omega}} u_1(t,x) = 0 \text{ uniformly for } x \in \overline{\Omega}.$$

Therefore, without loss of generality, we consider

$$r_1 - a_{12} m_2^{(1)} > 0. \tag{4.9}$$

From the comparison principle implies $u_1(t,x) \le z_1(t)$, hence

$$\limsup_{t\to+\infty} \max_{x\in\overline{\Omega}} u_1(t,x) \le \frac{r_1 - a_{12}m_2^{(1)}}{a_{11}}. \tag{4.10}$$

According to Lemma 2.4, from (4.10), we have

$$\limsup_{t\to+\infty} \int_{-\infty}^{t}\int_{\Omega} K(x,y,t-s)u_1(s,y)\mathrm{d}y\mathrm{d}s \le \limsup_{t\to+\infty} \max_{x\in\overline{\Omega}} u_1(t,x) \le \frac{r_1 - a_{12}m_2^{(1)}}{a_{11}}, \tag{4.11}$$

uniformly for $x \in \overline{\Omega}$.

Hence, for $\varepsilon > 0$ be defined by (4.2), there exists a $T_3 > T_2$ such that for any $x \in \overline{\Omega}$ and $t \ge T_3$,

$$u_1(t,x) \le M_1^{(2)}, \quad \int_{-\infty}^{t}\int_{\Omega} K(x,y,t-s)u_1(s,y)\mathrm{d}y\mathrm{d}s \le M_1^{(2)}, \tag{4.12}$$

where

$$M_1^{(2)} \overset{\text{def}}{=} \frac{r_1 - a_{12}m_2^{(1)}}{a_{11}} + \frac{\varepsilon}{2}.$$

From (4.12) and the second equation of system (1.10), we have

$$\frac{\partial u_2}{\partial t} - D_2\Delta u_2 \ge u_2\left(r_2 - a_{21}M_1^{(2)} - (a_{22} + e_2M_1^{(2)})u_2\right), \quad \text{for } t \ge T_3.$$

Let $p_1(t)$ be a solution of the following ordinary differential equation

$$p_1'(t) = p_1\left(r_2 - a_{21}M_1^{(2)} - (a_{22} + e_2M_1^{(2)})p_1\right), \ \ p_1(T_3) = \min_{x\in\overline{\Omega}} u_2(T_3,x), \ \ t \ge T_3.$$

It follows from (4.2), (4.5) and (4.12) that

$$r_2 - a_{21}M_1^{(2)} > r_2 - a_{21}M_1^{(1)} > 0. \tag{4.13}$$

Then

$$\lim_{t\to+\infty} p_1(t) = \frac{r_2 - a_{21}M_1^{(2)}}{a_{22} + e_2M_1^{(2)}}.$$

It follows from the comparison principle that $u_2(t,x) \ge p_1(t)$, hence

$$\liminf_{t\to+\infty} \min_{x\in\overline{\Omega}} u_2(t,x) \ge \frac{r_2 - a_{21}M_1^{(2)}}{a_{22} + e_2M_1^{(2)}}. \tag{4.14}$$

Hence, for above $\varepsilon > 0$, it follows from (4.14) that there exists a $T_4 \ge T_3$ such that

$$u_2(t,x) \ge \frac{r_2 - a_{21}M_1^{(2)}}{a_{22} + e_2M_1^{(2)}} - \frac{\varepsilon}{2} \overset{\text{def}}{=} m_2^{(2)}, \ \ \text{for any } \ x \in \overline{\Omega}, \ \ t \ge T_4. \tag{4.15}$$

From (4.5), (4.8), (4.12) and (4.15), for any $x \in [0,\pi]$ and $t \geq T_4$, we have

$$0 < u_1(t,x) < M_1^{(2)} < M_1^{(1)},\ 0 < m_2^{(1)} < m_2^{(2)} < u_2(t,x). \tag{4.16}$$

Repeating the above procedure, we get two sequences $M_1^{(n)}, m_2^{(n)}, i = 1,2, n = 1,2,\cdots$, such that for $n \geq 2$

$$\begin{aligned} M_1^{(n)} &= \frac{r_1 - a_{12} m_2^{(n-1)}}{a_{11}} + \frac{\varepsilon}{n}; \\ m_2^{(n)} &= \frac{r_2 - a_{21} M_1^{(n)}}{a_{22} + e_2 M_1^{(n)}} - \frac{\varepsilon}{n}. \end{aligned} \tag{4.17}$$

Without loss of generality, we assume $r_1 - a_{12} m_2^{(n-1)} > 0, n = 2,3,...$ Then

$$0 < u_1(t,x) < M_1^{(n)},\ 0 < m_2^{(n)} < u_2(t,x) \text{ for any } x \in [0,\pi],\ t \geq T_{2n}. \tag{4.18}$$

We claim that sequences $M_1^{(n)}$ is monotonically decreasing, and the sequence $m_2^{(n)}$ is monotonically increasing. To proof this claim, we will carry out by induction. Firstly, we immediately get $M_1^{(2)} < M_1^{(1)}$ and $m_2^{(1)} < m_2^{(2)}$. Let us assume now that our claim is true for n, that is, $M_1^{(n)} < M_1^{(n-1)}$ and $m_2^{(n-1)} < m_2^{(n)}$. By computation, we have

$$M_1^{(n+1)} = \frac{r_1 - a_{12} m_2^{(n)}}{a_{11}} + \frac{\varepsilon}{n+1} < \frac{r_1 - a_{12} m_2^{(n-1)}}{a_{11}} + \frac{\varepsilon}{n} = M_1^{(n)},$$

$$\begin{aligned} m_2^{(n+1)} &= \frac{r_2 - a_{21} M_1^{(n+1)}}{a_{22} + e_2 M_1^{(n+1)}} - \frac{\varepsilon}{n+1} \\ &> \frac{r_2 - a_{21} M_1^{(n)}}{a_{22} + e_2 M_1^{(n)}} - \frac{\varepsilon}{n} = m_2^{(n)} \end{aligned}$$

Therefore, the limits of $M_1^{(n)}$ and $m_2^{(n)}$ exist. Denote that

$$\lim_{t \to +\infty} M_1^{(n)} = \overline{u}_1,\ \lim_{t \to +\infty} m_2^{(n)} = \underline{u}_2. \tag{4.19}$$

Letting $n \to +\infty$ in (4.17) yields

$$r_1 - a_{11}\overline{u}_1 - a_{12}\underline{u}_2 = 0, \tag{4.20}$$

$$r_2 - a_{21}\overline{u}_1 - a_{22}\underline{u}_2 - e_2\overline{u}_1\underline{u}_2 = 0. \tag{4.21}$$

It follows from Lemma 2.1 that for ε_1 be defined by (4.1)

$$\overline{u}_1 < \frac{r_1}{a_{11}} + \varepsilon_1,\ \underline{u}_2 < \frac{r_2}{a_{22}} + \varepsilon_1. \tag{4.22}$$

Substituting (4.22) into (4.21) leads to

$$r_2 - e_2\left(\tfrac{r_1}{a_{11}}+\varepsilon_1\right)\left(\tfrac{r_2}{a_{22}}+\varepsilon_1\right) - a_{21}\overline{u}_1 - a_{22}\underline{u}_2 \le 0. \tag{4.23}$$

Multiplying (4.20) by $-\dfrac{r_2 - e_2\left(\frac{r_1}{a_{11}}+\varepsilon_1\right)\left(\frac{r_2}{a_{22}}+\varepsilon_1\right)}{r_1}$ and adding it to (4.23), we obtain

$$\begin{aligned}&\left(a_{11}\frac{r_2 - e_2\left(\frac{r_1}{a_{11}}+\varepsilon_1\right)\left(\frac{r_2}{a_{22}}+\varepsilon_1\right)}{r_1} - a_{21}\right)\overline{u}_1\\ &+\left(a_{12}\frac{r_2 - e_2\left(\frac{r_1}{a_{11}}+\varepsilon_1\right)\left(\frac{r_2}{a_{22}}+\varepsilon_1\right)}{r_1} - a_{22}\right)\underline{u}_2 \le 0,\end{aligned} \tag{4.24}$$

that is

$$\begin{aligned}&\left(a_{21} - a_{11}\frac{r_2 - e_2\left(\frac{r_1}{a_{11}}+\varepsilon_1\right)\left(\frac{r_2}{a_{22}}+\varepsilon_1\right)}{r_1}\right)\overline{u}_1\\ \ge\ &\left(a_{12}\frac{r_2 - e_2\left(\frac{r_1}{a_{11}}+\varepsilon_1\right)\left(\frac{r_2}{a_{22}}+\varepsilon_1\right)}{r_1} - a_{22}\right)\underline{u}_2.\end{aligned} \tag{4.25}$$

From the second inequality of (4.1), $\underline{u}_2 \ge \beta > 0$ and (4.25), we have

$$\left(a_{21} - a_{11}\frac{r_2 - e_2\left(\frac{r_1}{a_{11}}+\varepsilon_1\right)\left(\frac{r_2}{a_{22}}+\varepsilon_1\right)}{r_1}\right)\overline{u}_1 > 0. \tag{4.26}$$

It follows from the first inequality in (4.1) and (4.26) that $\overline{u}_1 < 0$, which is a contradiction, then we obtain

$$\lim_{t\to+\infty} u_1(t,x) = 0, \ \text{ uniformly for } \ x \in \overline{\Omega}. \tag{4.27}$$

According to Lemma 2.4, from (4.27), we have

$$\lim_{t\to+\infty}\int_{-\infty}^{t}\int_{\Omega} K(x,y,t-s)u_1(s,y)\mathrm{d}y\mathrm{d}s = 0, \ \text{ uniformly for } \ x \in \overline{\Omega}.$$

Hence, for any $\varepsilon > 0$ sufficiently small, there exists a $T' > 0$ such that for any $x \in \overline{\Omega}$ and $t \ge T'$,

$$u_1(t,x) \le \varepsilon; \ \int_{-\infty}^{t}\int_{\Omega} K(x,y,t-s)u_1(s,y)\mathrm{d}y \le \varepsilon.$$

This and the second equation of system (1.10) imply

$$\frac{\partial u_2}{\partial t} - D_2\Delta u_2 \ge u_2\left(r_2 - a_{21}\varepsilon - (a_{22}+e_2\varepsilon)u_2\right), \quad \text{for } \ t \ge T'.$$

From the comparison principle, we have

$$\liminf_{t\to+\infty}\min_{x\in\overline{\Omega}} u_2(t,x) \ge \frac{r_2 - a_{21}\varepsilon}{a_{22}+e_2\varepsilon}. \tag{4.28}$$

Setting $\varepsilon \to 0$ in (4.28), we obtain

$$\liminf_{t\to+\infty} \min_{x\in\overline{\Omega}} u_2(t,x) \geq \frac{r_2}{a_{22}}.$$

This together with Lemma 2.1 deduces

$$\frac{r_2}{a_{22}} \leq \liminf_{t\to+\infty} \min_{x\in\overline{\Omega}} u_2(t,x) \leq \limsup_{t\to+\infty} \max_{x\in\overline{\Omega}} u_2(t,x) \leq \frac{r_2}{a_{22}},$$

that is

$$\lim_{t\to+\infty} u_2(t,x) = \frac{r_2}{a_{22}}, \text{ uniformly for } x \in \overline{\Omega}. \tag{4.29}$$

It follows from (4.27) and (4.29) that

$$\lim_{t\to+\infty} (u_1(t,x), u_2(t,x) = \Big(0, \frac{r_2}{a_{22}}\Big), \text{ uniformly for } x \in \overline{\Omega}.$$

This ends the proof of Theorem 2.1.

Proof of Theorem 2.2. One could easily verify that (2.1) and (2.3) is equivalent to the following inequalities

$$\frac{r_1}{r_2} < \frac{a_{12}}{a_{22}}, \ \frac{r_1}{r_2} < \frac{a_{11}}{a_{21} + \frac{e_2 r_2}{a_{22}}} \tag{4.30}$$

Thus, we could choose enough small positive constant $\varepsilon_2 > 0$ such that

$$\frac{r_1}{r_2} < \frac{a_{12}}{a_{22}}, \ \frac{r_1}{r_2} < \frac{a_{11}}{a_{21} + e_2(\frac{r_2}{a_{22}} + \varepsilon_2)} \tag{4.31}$$

holds, and so

$$\big(a_{21} + e_2(\tfrac{r_2}{a_{22}} + \varepsilon_2)\big)\frac{r_1}{r_2} - a_{11} < 0, \ a_{12} - a_{22}\frac{r_1}{r_2} > 0. \tag{4.32}$$

Let $\overline{u}_1$ and $\underline{u}_2$ be defined as in the proof of Theorem 2.1. For above $\varepsilon_2 > 0$, it follows from Lemma 2.3 that

$$\underline{u}_2 < \frac{r_2}{a_{22}} + \varepsilon_2. \tag{4.33}$$

Obviously $\overline{u}_1 \geq 0$. To prove $\lim_{t\to+\infty} u_1(t,x) = 0$ uniformly for $x \in \overline{\Omega}$, it suffices to show that $\overline{u}_1 = 0$. Otherwise, we suppose that $\overline{u}_1 > 0$.

With the same argument as that of (4.2)-(4.20), we can obtain the following equalities

$$r_1 - a_{11}\overline{u}_1 - a_{12}\underline{u}_2 = 0, \tag{4.34}$$

$$r_2 - a_{21}\overline{u}_1 - a_{22}\underline{u}_2 - e_2\overline{u}_1\underline{u}_2 = 0. \tag{4.35}$$

Substituting (4.33) into (4.35), we have

$$r_2 - \big(a_{21} + e_2(\frac{r_2}{a_{22}} + \varepsilon_2)\big)\overline{u}_1 - a_{22}\underline{u}_2 \le 0. \tag{4.36}$$

Multiplying (4.36) by $-\frac{r_1}{r_2}$, we obtain

$$-r_1 + \big(a_{21} + e_2(\tfrac{r_2}{a_{22}} + \varepsilon_2)\big)\tfrac{r_1}{r_2}\overline{u}_1 + a_{22}\tfrac{r_1}{r_2}\underline{u}_2 \ge 0. \tag{4.37}$$

Now adding (4.34) to (4.37) leads to

$$\Big[\big(a_{21} + e_2(\tfrac{r_2}{a_{22}} + \varepsilon_2)\big)\tfrac{r_1}{r_2} - a_{11}\Big]\overline{u}_1 + \Big[a_{22}\tfrac{r_1}{r_2} - a_{12}\Big]\underline{u}_2 \ge 0. \tag{4.38}$$

That is,

$$\Big[\big(a_{21} + e_2(\tfrac{r_2}{a_{22}} + \varepsilon_2)\big)\tfrac{r_1}{r_2} - a_{11}\Big]\overline{u}_1 \ge \Big[a_{12} - a_{22}\tfrac{r_1}{r_2}\Big]\underline{u}_2. \tag{4.39}$$

From Lemma 2.3 we have $\underline{u}_2 \ge \beta > 0$, this together with the second inequality of (4.32) leads to

$$\Big[\big(a_{21} + e_2(\frac{r_2}{a_{22}} + \varepsilon_2)\big)\frac{r_1}{r_2} - a_{11}\Big]\overline{u}_1 > 0. \tag{4.40}$$

(4.40) combines with the first inequality of (4.32) leads to $\overline{u}_1 < 0$, which is a contradiction, then we obtain

$$\lim_{t\to+\infty} u_1(t,x) = 0, \ \text{ uniformly for } \ x \in \overline{\Omega}. \tag{4.41}$$

With the help of (4.41), the rest of the proof is the same as that of the analysis of (4.27)-(4.29) in Theorem 2.1. The proof of Theorem 2.2 is completed.

Proof of Theorem 2.3. One could easily verify that (2.1) and (2.4) is equivalent to the following inequalities

$$\frac{r_1}{r_2} < \frac{a_{12}}{a_{22} + \frac{e_2 r_1}{a_{11}}}, \ \frac{r_1}{r_2} < \frac{a_{11}}{a_{21}} \tag{4.42}$$

Thus, we could choose enough small positive constant $\varepsilon_3 > 0$ such that

$$\frac{r_1}{r_2} < \frac{a_{12}}{a_{22} + e_2(\frac{r_1}{a_{11}} + \varepsilon_3)}, \ \frac{r_1}{r_2} < \frac{a_{11}}{a_{21}} \tag{4.43}$$

holds, and so

$$a_{12} - \big(a_{22} + e_2(\tfrac{r_1}{a_{11}} + \varepsilon_3)\big)\frac{r_1}{r_2} > 0, \ a_{21}\frac{r_1}{r_2} - a_{11} < 0. \tag{4.44}$$

Let $\overline{u}_1$ and $\underline{u}_2$ be defined as in the proof of Theorem 2.1. For above $\varepsilon_3 > 0$, it follows from Lemma 2.3 that

$$\overline{u}_1 < \frac{r_1}{a_{11}} + \varepsilon_3. \tag{4.45}$$

Obviously $\overline{u}_1 \geq 0$. To prove $\lim\limits_{t\to+\infty} u_1(t,x) = 0$ uniformly for $x \in \overline{\Omega}$, it suffices to show that $\overline{u}_1 = 0$. Otherwise, we suppose that $\overline{u}_1 > 0$.

With the same argument as that of (4.2)-(4.20), we can obtain the following equalities

$$r_1 - a_{11}\overline{u}_1 - a_{12}\underline{u}_2 = 0, \tag{4.46}$$

$$r_2 - a_{21}\overline{u}_1 - a_{22}\underline{u}_2 - e_2\overline{u}_1\underline{u}_2 = 0. \tag{4.47}$$

Substituting (4.45) into (4.47), we have

$$r_2 - a_{21}\overline{u}_1 - \big(a_{22} + e_2(\frac{r_1}{a_{11}} + \varepsilon_3)\big)\underline{u}_2 \leq 0. \tag{4.48}$$

Multiplying (4.48) by $-\frac{r_1}{r_2}$, we obtain

$$-r_1 + a_{21}\tfrac{r_1}{r_2}\overline{u}_1 + \big(a_{22} + e_2(\tfrac{r_1}{a_{11}} + \varepsilon_3)\big)\tfrac{r_1}{r_2}\underline{u}_2 \geq 0. \tag{4.49}$$

Now adding (4.34) to (4.37) leads to

$$\Big[a_{21}\tfrac{r_1}{r_2} - a_{11}\Big]\overline{u}_1 + \Big[\big(a_{22} + e_2(\tfrac{r_1}{a_{11}} + \varepsilon_3)\big) - a_{12}\Big]\underline{u}_2 \geq 0. \tag{4.50}$$

That is,

$$\Big[a_{21}\tfrac{r_1}{r_2} - a_{11}\Big]\overline{u}_1 \geq \Big[a_{12} - \big(a_{22} + e_2(\tfrac{r_1}{a_{11}} + \varepsilon_3)\big)\Big]\underline{u}_2. \tag{4.51}$$

From Lemma 2.3 we have $\underline{u}_2 \geq \beta > 0$, this together with the first inequality of (4.44) leads to

$$\Big[a_{21}\frac{r_1}{r_2} - a_{11}\Big]\overline{u}_1 > 0. \tag{4.52}$$

(4.52) combines with the second inequality of (4.44) implies that $\overline{u}_1 < 0$, which is a contradiction, then we obtain

$$\lim_{t\to+\infty} u_1(t,x) = 0, \ \text{ uniformly for } \ x \in \overline{\Omega}. \tag{4.53}$$

The rest of the proof is the same as that of the analysis of (4.27)-(4.29) in Theorem 2.1. We omit the detail here.

Proof of Theorem 2.4. Condition (2.5) is equivalent to

$$a_{12} < a_{22}\frac{r_1}{r_2},\ a_{21}\frac{r_1}{r_2} > a_{11}. \tag{4.54}$$

It follows from Lemma 2.1 that $u_i(t,x), i = 1,2$, are bounded. Let $\underline{u}_1 = \liminf_{t\to+\infty} \min_{x\in\overline{\Omega}} u_1(t,x)$ and $\overline{u}_2 = \limsup_{t\to+\infty} \max_{x\in\overline{\Omega}} u_2(t,x)$. From Lemma 2.2 we know that $\underline{u}_1 \geq \alpha > 0$. Obviously $\overline{u}_2 \geq 0$. To prove $\lim_{t\to+\infty} u_2(t,x) = 0$ uniformly for $x \in \overline{\Omega}$, it suffices to show that $\overline{u}_2 = 0$. Otherwise, we suppose that $\overline{u}_2 > 0$.

By a similar argument as that of (4.2)-(4.21), we can obtain the following inequalities

$$r_1 - a_{11}\underline{u}_1 - a_{12}\overline{u}_2 = 0, \tag{4.55}$$

$$r_2 - a_{21}\underline{u}_1 - a_{22}\overline{u}_2 - e_2\underline{u}_1\overline{u}_2 = 0. \tag{4.56}$$

Multiplying (4.56) by $-\frac{r_1}{r_2}$, we obtain

$$-r_1 + a_{21}\frac{r_1}{r_2}\underline{u}_1 + a_{22}\frac{r_1}{r_2}\overline{u}_2 + e_2\frac{r_1}{r_2}\underline{u}_1\overline{u}_2 = 0. \tag{4.57}$$

Now adding (4.55) to (4.57) leads to

$$\left[a_{21}\frac{r_1}{r_2} - a_{11}\right]\underline{u}_1 + \left[a_{22}\frac{r_1}{r_2} - a_{12}\right]\overline{u}_2 \leq 0. \tag{4.58}$$

That is,

$$\left[a_{12} - a_{22}\frac{r_1}{r_2}\right]\overline{u}_2 \geq \left[a_{21}\frac{r_1}{r_2} - a_{11}\right]\underline{u}_1. \tag{4.59}$$

From Lemma 2.2 we have $\underline{u}_1 \geq \alpha > 0$, this together with the second inequality of (4.54) leads to

$$\left[a_{12} - a_{22}\frac{r_1}{r_2}\right]\overline{u}_2 > 0. \tag{4.60}$$

(4.60) combines with the first inequality of (4.54) implies that $\overline{u}_2 < 0$, which is a contradiction, then we obtain

$$\lim_{t\to+\infty} u_2(t,x) = 0,\ \text{uniformly for}\ x \in \overline{\Omega}. \tag{4.61}$$

Hence, for any $\varepsilon > 0$ sufficiently small, there exists a $T'' > 0$ such that for any $x \in \overline{\Omega}$ and $t \geq T''$,

$$u_2(t,x) \leq \varepsilon.$$

This and the first equation of system (1.10) imply

$$\frac{\partial u_1}{\partial t} - D_1 \Delta u_1 \geq u_1 \left(r_1 - a_{12}\varepsilon - a_{11}u_1\right), \quad \text{for } t \geq T''.$$

From the comparison principle, we have

$$\liminf_{t\to+\infty} \min_{x\in\overline{\Omega}} u_1(t,x) \geq \frac{r_1 - a_{12}\varepsilon}{a_{11}}. \tag{4.62}$$

Setting $\varepsilon \to 0$ in (4.62), we obtain

$$\liminf_{t\to+\infty} \min_{x\in\overline{\Omega}} u_1(t,x) \geq \frac{r_1}{a_{11}}.$$

This together with Lemma 2.1 deduces

$$\frac{r_1}{a_{11}} \leq \liminf_{t\to+\infty} \min_{x\in\overline{\Omega}} u_1(t,x) \leq \limsup_{t\to+\infty} \max_{x\in\overline{\Omega}} u_1(t,x) \leq \frac{r_1}{a_{11}},$$

that is

$$\lim_{t\to+\infty} u_1(t,x) = \frac{r_1}{a_{11}}, \text{ uniformly for } x \in \overline{\Omega}. \tag{4.63}$$

It follows from (4.61) and (4.63) that

$$\lim_{t\to+\infty} (u_1(t,x), u_2(t,x) = \left(\frac{r_1}{a_{11}}, 0\right), \text{ uniformly for } x \in \overline{\Omega}.$$

This ends the proof of Theorem 2.4.

5. Examples

In this section, we give an example to show the feasibility of our results. Similar to [2], we take $\Omega = [0,\pi], K(x,y,t) = G(x,y,t)k(t)$, where $k(t) = \frac{1}{\tau}e^{-\frac{t}{\tau}}$, and

$$G(x,y,t) = \frac{1}{\pi} + \frac{2}{\pi}\sum_{n=1}^{\infty} e^{-D_2 n^2 t}\cos nx \sin ny,$$

Example 5.1. In system (1.10), set $r_1 = 1; r_2 = 3; a_{11} = 2; a_{21} = 1; a_{12} = 2; a_{22} = 1; e_2 = 1; \tau = 2; k(t) = \frac{1}{\tau}e^{-t/\tau}$. Obviously

$$\frac{r_1}{r_2} = \frac{1}{3} < \frac{a_{12}}{a_{22}} = 2, \ \frac{r_1}{r_2} = \frac{1}{3} < \frac{a_{11}}{a_{21}} = 2, \tag{5.1}$$

$$e_2 = 1 < \frac{5}{3} = \frac{a_{11}a_{22}}{r_1 r_2} \min\left\{r_2 - r_1\frac{a_{22}}{a_{12}}, r_2 - r_1\frac{a_{21}}{a_{11}}\right\}; \tag{5.2}$$

$$e_2 = 1 < \frac{5}{3} = \frac{a_{22}}{r_2}\left(a_{11}\frac{r_2}{r_1} - a_{21}\right); \tag{5.3}$$

$$e_2 = 1 < 10 = \frac{a_{11}}{r_1}\left(a_{12}\frac{r_2}{r_1} - a_{22}\right). \tag{5.4}$$

Clearly, inequalities (5.1), (5.2), (5.3) and (5.4) show that the coefficients of the system (5.1) satisfy the conditions of Theorem 2.1, 2.2 and 2.3. Then species 1 is extinct while species 2 is globally attractive.

6. Discussion

Tian, Zhang and Ling[1] proposed a reaction-diffusion model of plankton allelopathy with nonlocal delays, which is described by system (1.1). Using an iterative technique, the global stability of the positive steady state and the semi-trivial steady states of the system was revisited by Li, Chen and He[2], they showed that toxic substances and nonlocal delays are harmless for the stability of the positive steady state. In this paper, we assume that one species is nontoxic phytoplankton while the other one is toxic liberating phytoplankton, this results in model (1.10). Though system (1.10) is the special case of system (1.1), as we analysis in the introduction section, the results of [2] could not be applied to the system directly. By developing some analysis technique, we are able to establish some sufficient conditions which ensure the extinction of one species and the global attractivity of the other species. The main results of this paper show that if the toxic rate is enough low such that inequality (2.2) or (2.3) or (2.4) holds, then second species is still possible be driven to extinction. To draw the conclusion: we find that enough small toxic substances is harmless for the stability of the semi-steady state. It's in this sense that our results supplement the main results of [2].

Conflict of Interests

The author declares that there is no conflict of interests.

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