



Available online at <http://scik.org>

Algebra Lett. 2026, 2026:2

<https://doi.org/10.28919/al/10050>

ISSN: 2051-5502

SEMIGROUP-THEORETIC MATRIX MODEL FOR CORRUPTION IN GOVERNMENT PROCUREMENT

CHIDIEBERE G. HOLGET¹, R. U. NDUBUISI^{1,*}, M. C. OBI¹, IGNATIUS PHILIP NGWONGWO¹, ISAAC
OGAZIE NWAGWU¹, C. NWUTARA¹, VICTORY ONYEKACHI OBI¹, MILETUS OBINNA DURU²

¹Department of Mathematics, Federal University of Technology, Owerri, Nigeria

²Department of Physics, Federal University of Technology, Owerri, Nigeria

Copyright © 2026 the author(s). This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

ABSTRACT. Corruption in government procurement remains a persistent global challenge, undermining economic efficiency, transparency, and public trust. Contemporary studies emphasize statistical, machine learning, and network-based detection approaches; however, there is limited exploration of algebraic structures in modelling corruption dynamics. This paper presents a semigroup theoretic matrix model to formalize corruption interactions within procurement systems. Consequently, we give some analysis arising from our matrix representation. This interdisciplinary study offers a novel algebraic perspective to complement existing models of corruption.

Keywords: Semigroup theory; government procurement; corruption-mitigation matrix; monitoring.

2020 AMS Subject Classification: 20M10, 15A18, 20M30.

1. INTRODUCTION

Government procurement is responsible for large shares of public spending. Because of the volume of funds, the necessity of discretion, interactions between public and private actions, and frequent information asymmetries, procurement is highly susceptible to corruption. Corruption in procurement wastes resources, leads to poor quality public works, undermines public trust, distorts markets, and can entrench inefficiencies.

Many studies have characterized corruption risks factors (lack of transparency, weak oversight, specifications manipulation, collusion), for example, Gnaldi and Del [4] analysed how corruption

*Corresponding author

E-mail address: u_ndubuisi@yahoo.com

Received June 12, 2026

rise increase during emergency procurement situation when oversight is weaker and finding level rise. The authors develop indicators to measure corruption vulnerability across procurement stages and emphasise the importance of monitoring system during crises. Waxenecker and Prell [12] applied stochastic network modeling to examine corruption patterns in public procurement contracts. This study showed that collusion, political influence, and spending concentration play key roles in sustaining corruption networks in procurement systems. Common forms of procurement corruption such as bid-rigging, market sharing, and favouritism were studied by Zhu [13] while the most corruption-prone stages in public procurement systems using the analytic hierarchy process (AHP) were studied by Adjorlolo et al in [1]. In 2025, Santos et al [10] analysed modern data- driven methods used to detect fraud and corruption in public procurement. It highlights the growing use of machine learning, data analytics, and predictive models for identifying suspicious procurement activities.

There is less work that models how corrupt acts compose or interact in chains or enables one another, which is essential to understanding why some procurement systems become deeply corruption (system capture), while others remain relatively resilient. Valverde et al [11] developed an agent-based mathematical model to analyse corruption dynamics between government officials and contractors. Their results showed that institutional controls and salary structures can significantly influence corruption levels in public contract systems. Medina et al [7] used machine learning and network analysis to identify fraudulent suppliers in government procurement systems. Their model significantly improved the detection of corruption contracts compared with traditional risk indicators.

However, mathematical abstractions especially from algebraic structures offer underutilized yet powerful tools to model complex, interacting systems such as corruption networks.

Semigroup theory studies set with associative binary operations. It offers tools for modeling structure – how elements, in particular, corrupt acts or states compose, how some combinations become self-reinforcing, how subsystems emerge.

Such tools can help identify persistent and dangerous corruption patterns (e.g., acts which once they occur, lead inexorably towards system capture), model the enabling relationships among acts (which acts tend to follow/reinforce others), understand how interventions (reforms, transparency, oversight) might block particular composition paths, or “neutralize” certain corrupt enabling acts, provide complementary qualitative structural analysis to probabilistic/dynamic/graph/network

models. There has been recent mathematical of corruption and semigroup application in modeling, for example, see [1], [2], [8] and [9].

This paper presents a semigroup theoretic matrix framework, where procurement processes are modelled as transformations acting on a state space.

2. PRELIMINARIES

In this section, we recall some definitions as well as some known results which will be useful in this paper. For notation and terminologies not mentioned in this paper, the reader is referred to [3], [5] and [8].

Definition 2.1. A semigroup is an algebraic structure $(S, \circ)$ where S is a non-empty set and $\circ$ is an associative binary operation : $\forall a, b, c \in S, (a \circ b) \circ c = a \circ (b \circ c)$.

Definition 2.2. An element $a \in S$ is called an idempotent if $a \circ a = a^2 = a$. Viewing elements as corrupt acts, self-sustaining corruption (e.g. institutional bribery) can be seen as idempotents.

Definition 2.3. An element $a \in S$ is called an absorbing or zero element if $a \circ b = a, \forall b \in S$. In our study, absorbing or zero elements are irreversible corruption nodes (e.g. godfatherism).

Definition 2.4. An element $1 \in S$ is called an identity element if $a \circ 1 = 1 \circ a = a$. Thus we call a semigroup with identity a monoid. In our study, identity element can be seen as anti-corruption acts that do not change the system state.

Definition 2.5. Let $(S_1, \circ)$ and $(S_2, *)$ be two semigroups. A mapping $\theta : S_1 \rightarrow S_2$ is a homomorphism if $\forall a, b \in S_1$,

$$\theta(a \circ b) = \theta(a) * \theta(b). \quad (1)$$

Suppose corruption in different agencies (say health and education) can be modelled as semigroups S_1 and S_2 , then a homomorphism $\theta : S_1 \rightarrow S_2$ can model how corruption patterns transfer between departments.

Definition 2.6 [8] Let S be a finite or countable set of corruption states or acts on procurement. Consider a simplified procurement system with ten elements namely;

b – bribe , c – collusion among bidders, s – specification manipulation/unfair technical requirements,

f – fraudulent misrepresentation in contract execution, m – monitoring/over sight failure, v – vendor favouritism, n – nepotism, w – whistle blower/auditing/transparency/reform act, z – system capture (absorbing state), 1 – reform identity element.

Define a binary operation $*$: $S \times S \rightarrow S$ by the rule; $a * b = \text{state/ act corresponding to "after act/state } a, \text{ act/state } b \text{ is either enabled, facilitated, or its effect is amplified or continues in presence of } a"$.

Definition 2.7 [5]. A non-empty subset A of S is called a left ideal if $SA \subseteq A$, a right ideal if $AS \subseteq A$ and two sided or an ideal if it is both a left and a right ideal.

Definition 2.8 [5]. Let S be a semigroup and define

$$S^1 = \begin{cases} S & \text{if } S \text{ has an identity element} \\ S \cup \{1\} & \text{otherwise} \end{cases}$$

We refer to S^1 as the monoid obtained from S by adjoining an identity if necessary. Let S be a semigroup and let $a \in S$. The principal right ideal generated by ' a ' is the set $aS' = aS \cup \{a\}$. Also, the left principal ideal generated by ' a ' is $S'a = Sa \cup \{a\}$. Now suppose S is a semigroup and $a, b \in S$. Then a and b are $\mathcal{R}$ -related written as $a \mathcal{R} b$ if a and b generate the same principal right ideal. Similarly, a and b are $\mathcal{L}$ -related if they generate the same principal left ideal, in which case we write $a \mathcal{L} b$. More so, a and b are said to be $\mathcal{J}$ -related if they generate the same principal two-sided ideal ($S'aS' = aS \cup Sa \cup SaS \cup \{a\}$). By the above definition, $a \mathcal{L} b$ if and only if for some $x, y \in S', a = xb$ and $b = ya$, $a \mathcal{R} b$ if and only if for some $x, y \in S', a = bx$ and $b = ay$,

$$a \mathcal{J} b \text{ if and only if for some } x, y, u, v \in S', xay = b \text{ and } ubv = a.$$

Theorem 2.9 [3]. Let $A = (a_{ij}) \in \mathbb{R}^{n \times n}$ be a real square matrix. For each $i = 1, 2, \dots, n$, define Gershgorin disc (in the complex plane) by $D_i = \{z \in \mathbb{C} : |z - a_{ii}| \leq R_i\}$, where $R_i = \sum_{j=1, j \neq i}^n |a_{ij}|$. Then every eigenvalue of A (which may be real or complex) lies in at least one of the discs D_i .

Theorem 2.10 [3]. For a nonnegative irreducible matrix $A = (a_{ij}) \in \mathbb{R}^{n \times n}$, there exists a real positive eigenvalue $\rho(A)$ (spectral radius) and a corresponding eigenvector such that this eigenvalue dominates all others in modulus.

2.11 Semigroup Construction [8]

Suppose $S = \{b, c, s, f, m, v, n, w, z, 1\}$ be a set representing the procurement system element. Define a binary operation $*$: $S \times S \rightarrow S$ that models the intersection between these elements such that $\forall a, b, c \in S, (a * b) * c = a * (b * c)$.

To define the Cayley table, we make the following assumption:

- i. Corrupt acts reinforce each other: combination of corrupt elements lead to inverse corruption.
- ii. Whistle-blowers (w) can reduce or nullify certain corrupt acts.
- iii. Reform identity (1) acts as a right and left identity; $a * 1 = 1 * a = a \quad \forall a \in S$.
- iv. System capture (z) absorbs other corrupt acts; $a * z = z * a = z, \forall a \in S$, modeling how deep corruption overtake others.
- v. $w * a = 1$ for $a \in S$, meaning whistle-blowing can neutralize individual corruption types.

$*$	b	c	s	f	m	v	n	w	z	1
b	z	z	z	z	z	z	z	z	z	b
c	z	z	z	z	z	z	z	z	z	c
s	z	z	z	z	z	z	z	z	z	s
f	z	z	z	z	z	z	z	z	z	f
m	z	z	z	z	z	z	z	z	z	m
v	z	z	z	z	z	z	z	z	z	v
n	z	z	z	z	z	z	z	z	z	n
w	1	1	1	1	1	1	1	1	1	w
z	z	z	z	z	z	z	z	z	z	z
1	b	c	s	f	m	v	n	w	z	1

The absorbing nature of the system capture element (z) indicates that once corruption has fully infiltrated the system, no further reforms or interventions can alter the state, highlighting the importance of early intervention. The whistle-blower resets corrupt elements to reform (1) while reform elements act neutrally.

Lemma 2.11.1 [8] $(S, *)$ is a semigroup.

Theorem 2.11.2 [8] z is $\mathcal{L}, \mathcal{R}, \mathcal{H}, \mathcal{J}$ – minimal.

Lemma 2.11.3 [8] The whistle-blower w forms its own $\mathcal{H}$ -class.

Theorem 2.11.4 [8] The semigroup $(S, *)$ admits a faithful representation.

3. MAIN RESULTS

In this section, we present our results. Consequently, we give some analysis arising from our matrix representation.

Let S be defined as in section 2 with reform identity now denoted as r and define the state vector:

$$X_t \in \mathbb{R}^{10}, X_t = \begin{bmatrix} x_b \\ x_c \\ x_s \\ x_f \\ x_m \\ x_v \\ x_n \\ x_w \\ x_z \\ x_r \end{bmatrix},$$

where each component represents the level (intensity) of each activity.

We define a linear operator: $A : \mathbb{R}^{10} \rightarrow \mathbb{R}^{10}$ such that $X_{t+1} = AX_t$.

Thus, $X_t = A^t X_0$. Let the corruption-mitigation matrix $A \in \mathbb{R}^{10 \times 10}$ be defined by

$$\begin{bmatrix} 1 & \alpha & \beta & \gamma & -\mu & \delta & \delta & -\omega & \eta & 0 \\ \alpha & 1 & \beta & \gamma & -\mu & \delta & \delta & -\omega & \eta & 0 \\ \beta & \beta & 1 & \gamma & -\mu & \delta & \delta & -\omega & \eta & 0 \\ \gamma & \gamma & \gamma & 1 & -\mu & \delta & \delta & -\omega & \eta & 0 \\ -\mu & -\mu & -\mu & -\mu & 1 & -\mu & -\mu & -\omega & -\mu & \tau \\ \delta & \delta & \delta & \delta & -\mu & 1 & \delta & -\omega & \eta & 0 \\ \delta & \delta & \delta & \delta & -\mu & \delta & 1 & -\omega & \eta & 0 \\ -\omega & -\omega & -\omega & -\omega & -\omega & -\omega & -\omega & 1 & -\omega & \tau \\ \eta & \eta & \eta & \eta & -\mu & \eta & \eta & -\omega & 1 & 0 \\ 0 & 0 & 0 & 0 & \tau & 0 & 0 & \tau & -\eta & 1 \end{bmatrix}$$

Parameter Assumptions:

- Positive $\rightarrow$ corruption reinforcement (e.g., bribery increases corruption)
- Negative $\rightarrow$ suppression or correction/ mitigation effects
- $0 < \alpha, \beta, \gamma, \delta, \eta < 1, \quad -1 < \mu, \omega < 0$

With the model above, we now have the following results;

Lemma 3.1. The set $T = \{A^t : t \in \mathbb{N}_0\}$ is a semigroup under the operation $*$.

Proof. Let $A^m, A^n \in T$ such that $A^m A^n = A^{m+n}$. Since $m + n \in \mathbb{N}_0$, closure holds.

Matrix multiplication is associative: $(A^m A^n) A^k = A^m (A^n A^k)$ for some $A^t \in T$.

Thus, $(T, *)$ is a semigroup.

Lemma 3.2. Let $A \in \mathbb{R}^{10 \times 10}$ be the corruption-mitigation matrix and let $M \in \mathbb{R}^{10 \times 10}$ represent the monitoring operator acting on the state space $\mathbb{R}^{10}$, suppose that M is a projection operator, i.e. $M^2 = M, MA = AM = MAM$. Then M is an idempotent stabilizer of the system, i.e. for every state vector $X_t \in \mathbb{R}^{10}, M(MX_t) = MX_t, X_{t+1} = MAX_t$.

Proof. By assumption, $M^2 = M$. This immediately implies that for any state vector X_t ,

$$M(MX_t) = M^2X_t = MX_t. \quad (2)$$

Hence, once monitoring is applied, reapplying monitoring has no further effect. This is the defining property of an idempotent operator.

Let $Y_t = MX_t$. We show that the evolution keeps Y_t inside the image of M .

Consider the monitored dynamics:

$$X_{t+1} = MAX_t. \quad (3)$$

Apply M to both sides:

$$MX_{t+1} = M(MAX_t).$$

Using associativity and $M^2 = M$, $MX_{t+1} = MAX_t = X_{t+1}$.

Thus, $X_{t+1} \in \text{Im}(M)$.

Hence, the subspace $\text{Im}(M)$ is invariant under the monitored dynamics.

Using the commutation condition:

$$MA = AM = MAM,$$

we obtain:

$MAX_t = AMX_t$. Thus, the monitored evolution can be written equivalently as:

$$X_{t+1} = A(MX_t). \quad (4)$$

This shows that monitoring can be applied before or after the system evolution and the dynamics are consistent within the monitored regime.

Since M projects onto a subspace (desirable/controlled state), repeated application does not change the further state and the evolution remains inside this subspace. It follows that M filters out unstable or corrupt components and keeps the system within a stable invariant structure.

Thus, M acts as a stabilizer.

Lemma 3.3. Let $A \in \mathbb{R}^{10 \times 10}$ be a corruption-mitigation matrix and let $z = \rho(A)$ be the system capture level. Let r and ω be such that $\tau + \omega > z$, then the system capture is not stable.

Proof. Let A be a real 10×10 matrix and let $z = \rho(A)$ denote its spectral radius, that is

$$\rho(A) = \max\{|\lambda_1|, |\lambda_2|, \dots, |\lambda_{10}|\} \quad (5)$$

where $\lambda_1, \lambda_2, \dots, \lambda_{10}$ are all the eigenvalues of A .

This means that z is the largest magnitude among all eigenvalues of A . Thus, for any eigenvalue λ of A , we must have

$$|\lambda| \leq z. \quad (6)$$

Also, it is known that for any complex number λ , we have that

$$\operatorname{Re}(\lambda) \leq |\lambda|. \quad (7)$$

Combining (6) and (7) we have that

$$\operatorname{Re}(\lambda) \leq |\lambda| \leq z. \quad (8)$$

To incorporate the effects of reform and whistle-blowing, consider the modified matrix

$$\bar{A} = A - (\tau + \omega)I, \quad (9)$$

where I is the identity matrix. This represents a uniform reduction in the influence of corruption across all sectors. A standard property of eigenvalues shows that if λ is an eigenvalue of A , then the corresponding eigenvalue of $\bar{A}$ is given by

$$\bar{\lambda} = \lambda - (\tau + \omega). \quad (10)$$

Thus, every eigenvalue of A is shifted by the same amount $(\tau + \omega)$ to the left in the complex plane. Taking real parts, we obtain

$$\operatorname{Re}(\bar{\lambda}) = \operatorname{Re}(\lambda) - (\tau + \omega).$$

Since $\operatorname{Re}|\lambda| \leq z$ for all eigenvalues λ of A , it follows that

$$\operatorname{Re}(\bar{\lambda}) \leq z - (\tau + \omega).$$

Now, under the assumption that $(\tau + \omega) > z$, we have $z - (\tau + \omega) < 0$, and therefore $\operatorname{Re}(\bar{\lambda}) < 0$ for every eigenvalue $\bar{\lambda}$ of $\bar{A}$. This shows that all eigenvalues of the modified matrix $\bar{A}$ lie strictly in the half of the complex plane.

Furthermore, the conclusion is consistent with Gershgorin's theorem. It can be shown that $a_{ii} + R_i \leq \rho(A) = z$ for all i . Indeed, every eigenvalue of $\bar{A}$ lies in at least one Gershgorin disc with center at $a_{ii} - (\tau + \omega)$ and radius $R_i = \sum_{j=1, j \neq i}^n |a_{ij}|$. Hence each eigenvalue satisfies

$$\operatorname{Re}(\bar{\lambda}) \leq a_{ii} - (\tau + \omega) + R_i. \quad (11)$$

Since $a_{ii} + R_i$ produces an upper bound compatible with the spectral radius, it follows that

$$\operatorname{Re}(\bar{\lambda}) \leq z - (\tau + \omega) < 0,$$

so all Gershgorin discs lie entirely in the left half-plane.

Because all eigenvalues of $\bar{A}$ is asymptotically stable in the sense that all modes decay over time. This implies that the reinforcing structure responsible for sustaining corruption cannot persist once the combined mitigation effect $(\tau + \omega)$ exceeds the intrinsic system capture level z .

Thus, the system capture is not stable.

This completes the proof.

Lemma 3.4. Suppose $\mu + \omega + \tau$ is sufficiently large, then $\rho(A) < 1$.

Proof. By Gershgorin theorem [3], each eigenvalue lies on:

$$|\lambda - a_{ii}| \leq R_i, \text{ so } |\lambda| \leq |a_{ii}| + R_i. \quad (12)$$

For mitigation rows we know that off-diagonal negative values increase radius control. That $R_i < 1$ follows from $\mu + \omega + \tau$ being large. Now, if $R_i < 1$ then $|\lambda| < 1$.

Thus, all eigenvalues lie inside unit disk.

Lemma 3.5. There exists a dominant eigenvalue $\lambda_{max} = \rho(A)$ such that

$$\lim_{t \rightarrow \infty} \frac{X_t}{\|X_t\|} = v, \quad (13)$$

where v is the dominant eigenvector.

Proof. Since A is a real matrix, by the Perron-Frobenius type argument (generalized) [6], the spectral radius $\rho(A)$ exists and there is an eigenvector v . So, we have that

$$A^t X_0 = \lambda_{max}^t v + O(\lambda_2^t), \text{ where } |\lambda_2| < \lambda_{max} \quad (14)$$

Thus, $\frac{X_t}{\|X_t\|} \rightarrow v$.

Remark 3.6. Lemma 3.5 gives the existence of dominant growth model. It is important to note that in the presence of negative interactions representing anti-corruption mechanisms, the classical Perron-Frobenius framework is extended using spectral radius analysis and eventually positive matrix theory to characterize system stability.

4. ANALYSIS OF RESULTS

From the results obtained in section 3 above, we have the following analysis;

4.1 Corruption grows (Reinforcing actions dominate)

Let us consider a scenario whereby a contract is being awarded such that we have the following;

- i) A contractor offers a bribe (b).
- ii) Officials collaborate secretly-collusion (c).
- iii) Requirements are tailored – specification manipulation (s).

iv) Fake documents are submitted – fraud (f),

Then each of these adds to the next, i.e.

$$b \rightarrow c \rightarrow s \rightarrow f \text{ so that } X_{t+1} > X_t .$$

It is obvious that corruption leads to even more corruption and eventually honest bidders drops out, contract become fixed and system becomes captured (z).

4.2 Corruption declines (control dominates)

Let us consider the same procurement process, but now;

- i) Strong monitoring (m) is in place.
- ii) An insider reports wrong doing $\rightarrow$ whistle blower (w).
- iii) Government enforces new rules – reform (r).

Then we have that control actions subtract from corruption:

$$X_{t+1} = AX_t \text{ (with negative effects),}$$

so that $X_{t+1} < X_t$.

It is obvious that bribes are detected, collusion is exposed and contracts are cancelled.

4.3 System capture (worst case)

Let us consider a scenario where corruption has been happening for years, oversight institutions are weak and whistleblowers are silenced.

Then the system reaches: z = system capture, i.e. corruption controls everything and rules exist only on paper.

Obviously, contracts always go to the same people, corruption becomes normal and reform becomes very difficult.

4.4 Recovery through reform

Now suppose a new administration introduces:

- i) Digital procurement system
- ii) independent anti-corruption agencies
- iii) Whistle-blower protection laws.

Then reform strengthens control:

$$m + w + r > b + c + f + v + n,$$

so that $X_t \rightarrow 0$ (low corruption).

Certainly, transparency increases, corruption becomes risky and honest competition returns.

5. CONCLUSION

This study developed a semigroup theoretical matrix model to analyse corruption in government procurement as a dynamic and structured process. The study demonstrates that the spectral properties of the transformation matrix determine whether corruption grows, stabilizes, or declines. These findings highlight that effective anti-corruption strategies must focus on altering the underlying system structure. More so, the model provides a mathematical framework for understanding, predicting, and controlling corruption dynamics, with potential for further extension through empirical validation and advanced algebraic techniques.

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

REFERENCES

- [1] G. Adjorlolo, Z. Tang, G. Wank, P.A. Sarfo, A.B. Bramah, R. Blankson Safo, B. Nyianyi, Evaluating corruption-prone public procurement stages for block chain integration using the AHP approach, *Systems* 13 (2025), 267.
- [2] J. Chen, K. Wu, Deep-OSG: Deep learning of operators in semigroup, *arXiv Preprint* (2023), 1-22.
- [3] S. Gershgorin, On the localization of the eigenvalues of a matrix, *Izv. Akad. Nauk SSSR, Fiz-Mat. Ser.* 6 (1931), 749-754.
- [4] M. Gnaldi, S. Del Sarto, Measuring corruption risk in public procurement over emergency periods, *Social Indicators Res.* 172 (2024), 859-877.
- [5] J.M. Howie, *Fundamentals of Semigroup Theory*, Oxford University Press, 1995.
- [6] G. Frobenius, On matrices with nonnegative elements, *Proc. Roy. Prussian Acad. Sci. (Berlin)* (1912), 456-477.
- [7] M. Medina-Hernández, J. Kertész, M. Fazekas, Learning from sanctioned government suppliers: A machine learning and network science approach to detecting fraud and corruption in Mexico, *arXiv Preprint* (2025), 1-25.
- [8] R.U. Ndubuisi, C.G. Holget, M.C. Obi, A.B. Panle, K.M. Patil, W.I. Osuji, Application of semigroup theory in modelling and mitigating corruption in government procurement, *Int. J. Appl. Math.* 39 (2026), 1075-1084.
- [9] U.K. Nwajeri, K.K.J. Asamoah, R.U. Ndubuisi, A. Oname, Z. Jin, A mathematical model of corruption dynamics endowed with fractal-fractional derivative, *Results Phys.* 52 (2023), 106894.
- [10] E.S. Santos, M.M. Santos, M. Castro, J.T. Carvalho, Detection of fraud in public procurement using data-driven methods: A systematic mapping study, *EPJ Data Sci.* 14 (2025), 52.
- [11] P. Valverde, J. Fernández, E. Buenaño, J.S. González-Avella, M. Cosenza, Controlling systematic corruption through group size and salary dispersion of public servants, *arXiv Preprint* (2023), 1-18.
- [12] H. Waxenecker, C. Prell, Corruption dynamics in public procurement: A longitudinal network analysis of local construction contracts in Guatemala, *Social Networks* 79 (2024), 154-167.

- [13] P. Zhu, Current state and countermeasures of procurement collusion: An exploratory study, *Adv. Econ. Manag. Polit. Sci.* 18954 (2024), 1-10.