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THE FISCHER-GRIESS MONSTER GROUP ($\mathbb{M}$) AND QUADRATIC PRIME GENERATING POLYNOMIALS

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Abstract: In this paper we show that, the quadratic prime-generating polynomials are connected to integer values of exactly 43 McKay- Thompson series of the conjugacy classes for the monster group. We have found that for p a prime divisor of $|\mathbb{M}|$, $T_p(\tau)$ is an algebraic number of degree one-half the h(-d). For p a product of two distinct prime divisors of $|\mathbb{M}|$ (except p=57 and p=93), the $T_p(\tau)$ of the appropriate conjugscy class is an algebraic number of degree one-fourth the h(-d). For p a product of three distinct prime divisors of $|\mathbb{M}|$, the $T_p(\tau)$ of the appropriate conjugacy class is an algebraic number of degree one-eighth the h(-d).

Keywords: Fischer-Griess monster group, Monstrous moonshine, McKay- Thompson series, prime-generating polynomial, Class number of fundamental discriminant.

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1. Introduction

The Fischer-Griess monster group $\mathbb{M}$ is the largest and the most popular among the twenty six sporadic finite simple groups of order

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$$2^{46}\cdot 3^{20}\cdot 5^{9}\cdot 7^{6}\cdot 9^{7}\cdot 11^{2}\cdot 13^{3}\cdot 17\cdot 19\cdot 23\cdot 29\cdot 31\cdot 41\cdot 47\cdot 59\cdot 71$$
$$=8080,17424,79451,28758,86459,90496,17107,57005,75436,80000,00000\approx 8\times 10^{53}.$$

In late 1973 Bernd Fischer and Robert Griess independently produced evidence for its existence. Conway and Norton [6] proposed to call this group the *Monster* and conjectured that it had a representation of dimension $196,883=47\cdot 59\cdot 71$.

In a remarkable work, Fischer et. al [8] computed the entire character table of $\mathbb{M}$ in 1974 under this assumption. It has 194 conjugacy classes and irreducible characters.

The Monster has not yet been proved to exist, but Thompson [14] has proved its uniqueness on similar assumptions. In 1982 Griess [10] constructed $\mathbb{M}$ as the automorphism group of his 196884-dimensional algebra thus proving existence. The monster contains all but six of the other sporadic groups as subquotients though their discoveries were largely independent of it.

Although the monster group was discovered within the context of finite simple groups, but hints later began to emerge that it might be strongly related to other branches of mathematics. One of these is the theory of modular functions and modular forms.

The elliptic modular function has Fourier series expansion as:

$$j(\tau)=\frac{1}{q}+744+196884q+21493760q^{2}+864299970q^{3}+20245856256q^{4}+333202640600q^{5}+$$
$$+4252023300096q^{6}+\ldots;\ \text{where}\ q=e^{2\pi i\tau},\quad \tau\in\mathbb{H}.$$

In 1978, Mc Kay [17] noticed that the coefficient of q (196884) in the j-function is $196883+1$ and Thompson [15] found that the later coefficients are linear combinations of the representations of $\mathbb{M}$ as given in [2, 3] as follows:

$$1=1 \qquad (1.1a)$$
$$196884=196883+1 \qquad (1.1b)$$
$$21493760=21296876+196883+1 \qquad (1.1c)$$
$$864299970=842609326+21296876+2.196883+2.1 \qquad (1.1d)$$

The numbers on the left sides of (1.1) are the first few coefficients of the j-function; whereas the numbers on the right are the dimensions of the smallest irreducible representations of the Fischer-Griess monster group $\mathbb{M}$.

Based on these observations, Mc Kay and Thompson [17] found further numerology suggesting that, the explanation for (1.1) should lie in the existence of a natural infinite dimensional graded $\mathbb{M}$-module V_n (later called head representation of $\mathbb{M}$) for the monster $V = \bigoplus_{n\geq 0} V_n$. The dimension of V_n is equal to the coefficient c_n of the elliptic modular function.

2. The Monstrous Moonshine

It is vital to mention the work of Conway and Norton [6], which marked as the starting point in the theory of moonshine, proposing a completely unexpected relationship between finite simple groups and modular functions, which relates the monster to the theory of modular forms. Conway and Norton conjectured in this paper that there is a close connection between conjugacy classes of $\mathbb{M}$ and action of certain subgroups of $SL_2(\mathbb{R})$ on the upper half plane .

This conjecture implies that extensive information on the representations of the monster is contained in the classical picture describing the action of $SL_2(\mathbb{R})$ on . Monstrous moonshine is the collection of questions (and few answers) that these observations had directly inspired.

With the emergence of [6], many researchers had presented more results on connections between modular forms and monster group, and most of the other finite simple sporadic groups have been discovered; they are collectively referred to as *Moonshine*. Significant progress was made in the 1990's, and Borcherds won a Field's medal in 1998 for his work in proving Conway and Norton's original conjectures. The proof opened up connections between number theory and representation theory with mathematical physics. For survey see [2, 3, 9].

3. The McKay-Thompson Series

The central structure in the attempt to understand (1.1) is an infinite-dimensional $\mathbb{Z}$-graded module for the monster .

$$\mathrm{V} = \mathrm{V}_0 \oplus \mathrm{V}_1 \oplus \mathrm{V}_2 \oplus \mathrm{V}_3 \oplus \ldots$$

Let ρ_{d} denote the d-dimensional irreducible representation of $\mathbb{M}$ ordered by dimension. The First subspaces will be:

$\mathrm{V}_0 = \rho_1$, $\mathrm{V}_1 = \{0\}$, $\mathrm{V}_2 = \rho_1 \oplus \rho_{196883}$, $\mathrm{V}_3 = \rho_{21296876} \oplus \rho_{196883} \oplus \rho_1$ and so on

The J-function is essentially its graded dimension

$$\mathrm{J}(\tau) = j(\tau) - 744 = \dim(\mathrm{V}_0)q^{-1} + \sum_{n=1}^{\infty} \dim(\mathrm{V}_n)\, q^n$$

From representation theory for finite groups, a dimension can be replaced with a character. This gives the graded traces

$$\mathrm{T}_{\mathrm{g}}(\tau) = \sum_{n \in \mathbb{Z}} \mathrm{tr}\,(\mathrm{g}|\mathrm{V}_n)\, q^n$$

Generally, Thompson [16] further suggested that, we consider the series (known as the Mc Kay-Thompson series for this module V)

$$\mathrm{T}_g(\tau) = q^{-1} \sum_{n=1}^{\infty} \mathrm{ch}_{\mathrm{V}n}(g)\, q^n$$

for each element $\mathrm{g} \in \mathbb{M}$, where $\mathrm{ch}_{\mathrm{V}_n}$ are characters.

For example, the smallest non-trivial representation of $\mathbb{M}$ is given by almost 10^{54} Complex matrices, each of size 196883 x 196883, while the corresponding character is completely specified by 194 integers (194 being the number of conjugacy classes in 𝕄). Taking $g=1$, we have $T_1(\tau)=J(\tau)$

We write c_n to be the coefficient of q^n in Mc Kay-Thompson series T_g, that is

$$T_g(\tau)=q^{-1}+\sum_{n=1}^{\infty}c_n q^n,\ q=e^{2\pi i\tau}.$$

Moreover, there are 8×10^{53} elements in the monster, so we expect about 8×10^{53} different Mc Kay-Thompson series T_g. However, a character evaluated at g and any of its conjugate elements hgh^{-1} have identical character values and hence have identical McKay-Thompson series $T_g=T_{hgh^{-1}}$. This implies that $T_g=T_h$ whenever the cyclic subgroups $\langle g\rangle$ and $\langle h\rangle$ are equal. Hence there can be at most 194 T_g, one for each conjugacy class. In fact there are 172 distinct McKay-Thompson series T_g and the first eleven coefficients of each are given in [6, Table 4].

Influenced by Ogg's observation, Thompson, Conway and Norton conjectured that for each element g of 𝕄, the McKay-Thompson series T_g is the hauptmodul $J_{G_g}(\tau)$ for a genus zero group $G_g\subseteq\Gamma$ of moonshine type. So for each n the coefficient $g\mapsto c_n$ defines a character $ch_{V_n}(g)$ of 𝕄. They explicitly identify each of the groups G_g; these groups each contain subgroup $\Gamma_0(N)$ as a normal subgroup, for some N dividing $\circ(g)\cdot(24,\circ(g))$. G_g corresponding to a McKay-Thompson series $T_g(\tau)$ is specified by giving the positive integer N and a subset of Hall divisors of n/h, n arises as the order of g. Then n divides N and the quotient $h=N/n$ divides 24. In fact h^2 divides N.

The full correspondence can be found in [6, Table 2]. The first 50 coefficients c_n of each T_g are given in [11].

4. PRIME-GENERATING POLYNOMIALS AND MONSTER GROUP

Piezas [12] Showed for any τ in the quadratic field $\mathbb{Q}([\sqrt{-d(n)}])$. $j(\tau)$ is an algebraic integer . He also showed how prime generating polynomials are connected to integer values of some moonshine functions for small order p.

Here we consider the McKay-Thompson series of class 1A and some conjugacy classes for monster group and establish relationship to quadratic prime-generating polynomials; by showing that the value $\mathrm{T_p}(\tau)$ for an appropriate τ is an algebraic number.

Definition 4.1 (Fundamental discriminant): An integer $d \neq 1$ is a fundamental discriminant, if for any odd prime $k, k^2 \nmid d$ and satisfies $d \equiv 1 (\mathrm{mod}\, 4)$ or $d \equiv 8,12 (\mathrm{mod}\, 16)$.

The function NumberFieldFundamentalDiscriminantQ[d] in the mathematica version 8.0 add-on package Number Theory 'Number Theory Function' tests if an integer d is a fundamental discriminant.

See [18] for details on fundamental discriminants

Definition 4.2 (Class number): Is the order of the ideal class group of the number fields with discriminant d .When the class number of a ring of integers in a number field is 1, the ring corresponding to a given ideal has unique factorization and in a sense, the class number is a measure of the failure of unique factorization in that ring. Finding the class number is a computationally difficult problem.

The mathematica function: NumberFieldClassNumber[sqrt[d]] gives the class number h(-d) for d a fundamental discriminant.

The complete set of the negative discriminants having class numbers 1-5 and odd 7-23 are known. Buell [4] gives the smallest and largest class numbers for fundamental discriminants with $d < 4000000$, partitioned into even discriminants, discriminants $1 (\mathrm{mod}\, 8)$ and discriminants $5 (\mathrm{mod}\, 8)$. Arno et al [1] give complete lists of values of fundamental discriminants d with $h(-d) = k$ for odd $k = 5, 7, 9, \ldots, 23$, Wagner [18] gives complete lists of values for $k = 5, 6$ and 7. List of negative fundamental discriminants corresponding to imaginary guadratic fields $\mathbb{Q}([\sqrt{-d(n)}])$ having small class numbers $h(-d)$ for class numbers $h \le 25$ is given in [5] and [7]

As cited in [13] Kronecker (1857) discovered that j-function detects the class number of $\mathbb{Q}([\sqrt{-d(n)}])$ for any imaginary quadratic integer $\sqrt{-d}$. He showed that for any τ in the quadratic field $\mathbb{Q}([\sqrt{-d(n)}])$. $j(\tau)$ is an algebraic integer.

I. $\mathbf{T_{1A}(\tau)}$

The Mckay-Thompson series for the monster class 1A is defined by

$$\mathrm{T}_{1\mathrm{A}}(\tau) = \frac{1}{q} + 196884q + 21493760q^2 + 864299970q^3 + \ldots \text{where } q = e^{2\pi i\tau}$$

This series detects class number **h(-d)=1** of negative fundamental discriminants.

Two forms of τ:

Case 1: $\tau = (1+\sqrt{-d})/2$ (Associated with odd discriminant d).

Case 2: $\tau = \sqrt{-m}$ (Associated with even discriminant $d = 4m$).

For case 1, $q = -e^{-\pi\sqrt{d}}$ and $\mathrm{T}_{1\mathrm{A}}(\tau)$ is negative.

For case 2, $q = e^{-2\pi\sqrt{m}}$ and $\mathrm{T}_{1\mathrm{A}}(\tau)$ is positive.

Table 1 below shows their associated quadratic prime-generating polynomial $p(k) = ak^2 + bk + c$, its discriminant d, and $\mathrm{T}_{1\mathrm{A}}(\tau)$ using a root τ of $p(k) = 0$, which is given by the quadratic formula $k = \tau = \dfrac{-b + \sqrt{b^2 - 4ac}}{2a}$.

Table 1. Prime-generating Polynomial for Discriminants with h(-d) = 1 and $T_{1A}(\tau)$

$p(k) = ak^2 + bk + c$	$d = b^2 - 4ac$	$T_{1A}(\tau)$
$k^2 + 1$	-4	$2^3 \cdot 3 \cdot 41$
$k^2 + 2$	-8	$2^3 \cdot 907$
$k^2 - k + 1$	-3	-751
$k^2 - k + 2$	-7	$-3 \cdot 1373$
$k^2 - k + 3$	-11	$-2^3 \cdot 59 \cdot 71$
$k^2 - k + 5$	-19	$-2^3 \cdot 3 \cdot 5 \cdot 47 \cdot 157$
$k^2 - k + 11$	-43	$-2^3 \cdot 3 \cdot 36864031$
$k^2 - k + 17$	-67	$-2^3 \cdot 3^1 \cdot 6133248031$
$k^2 - k + 41$	-163	$-2^{18} \cdot 3^3 \cdot 5^3 \cdot 23^3 \cdot 29^3$

II. $T_p(\tau)$

The Mckay-Thompson series for the monster is of the form:

$$T_p(\tau) = \frac{1}{q} + c_1 q + c_2 q^2 + c_3 q^3 + \ldots \qquad \text{where } q = e^{2\pi i \tau} \ .$$

Two forms of τ:

Case 1: $\tau = (1 + \sqrt{-r})/2;\ r = \dfrac{4c}{a} - 1.$

Case 2: $\tau = \sqrt{-s}\ ;\ s = \dfrac{c}{a}.$

For case 1, $q = -e^{-\pi\sqrt{r}}$ and $T_p(\tau)$ is negative.

For case 2, $q = e^{-2\pi\sqrt{s}}$ and $T_p(\tau)$ is positive.

A. p as prime divisor of $|\mathbb{M}|$

$p \in \{2, 3, 5, 7, 9, 11, 13, 17, 19, 23, 29, 31, 41, 47, 59, 71\}$

There is Mckay-Thompson series $T_p(\tau)$ for each of the 15 prime divisors of $|\mathbb{M}|$.

Table 2 shows their associated quadratic prime-generating polynomials $p(k) = ak^2 + bk + c$,

and value $\mathrm{T_p}(\tau)$ using the root τ of $p(k) = 0$

Table 2. Prime-generating Polynomial for Discriminants with h(-d)=2n and $\mathrm{T_{pA}}(\tau)$

p	$\mathbf{p(k) = ak^2 + bk + c}$	$\mathbf{d = b^2 - 4ac}$	**h(-d)**	$\mathbf{T_{pA}(\tau)}$
2	$2k^2 - 2k + 3$	$-20 = -2 \cdot 10$	2	$-2^3 \cdot 3 \cdot 47$
2	$2k^2 - 2k + 7$	$-52 = -2 \cdot 26$	2	$-2^3 \cdot 7 \cdot 1483$
2	$2k^2 - 2k + 19$	$-148 = -2 \cdot 74$	2	$-2^3 \cdot 56$
2	$2k^2 + 3$	$-24 = -2 \cdot 12$	2	$2^3 \cdot 5^2 \cdot 11$
2	$2k^2 + 5$	$-40 = -2 \cdot 20$	2	$2^3 \cdot 2579$
2	$2k^2 + 11$	$-88 = -2 \cdot 44$	2	$2^3 \cdot 313619$
2	$2k^2 + 29$	$-232 = -2 \cdot 116$	2	$-2^3 \cdot 3073907219$
3	$3k^2 - 3k + 2$	$-15 = -3 \cdot 5$	2	$-3 \cdot 23$
3	$3k^2 - 3k + 5$	$-51 = -3 \cdot 17$	2	$-2 \cdot 3 \cdot 5 \cdot 59$
3	$3k^2 - 3k + 11$	$-123 = -3 \cdot 41$	2	$-2 \cdot 3 \cdot 18439$
3	$3k^2 - 3k + 23$	$-267 = -3 \cdot 89$	2	$-2^3 \cdot 5 \cdot 233 \cdot 2897$
3	$3k^2 + 2$	$-24 = -3 \cdot 8$	2	$2 \cdot 3 \cdot 29$
5	$5k^2 - 5k + 2$	$-15 = -5 \cdot 3$	2	$-2^2 \cdot 5$
5	$5k^2 - 5k + 3$	$-35 = -5 \cdot 7$	2	$-5 \cdot 11$
5	$5k^2 - 5k + 7$	$-115 = -5 \cdot 23$	2	$-3^2 \cdot 5 \cdot 19$
5	$5k^2 - 5k + 13$	$-235 = -5 \cdot 47$	2	$-5 \cdot 7 \cdot 1493$
5	$5k^2 + 2$	$-40 = -5 \cdot 8$	2	$5 \cdot 11$
7	$7k^2 - 7k + 3$	$-35 = -7 \cdot 5$	2	$-3 \cdot 5$

7	$7k^2-7k+5$	$-91=-7\cdot 13$	2	-73
7	$7k^2-7k+17$	$-427=7\cdot 61$	2	$-2^5\cdot 3^2\cdot 37$
11	$11k^2-11k+7$	$-187=-11\cdot 17$	2	$-2\cdot 5^2$
13	$13k^2-13k+5$	$-91=-13\cdot 7$	2	-11
13	$13k^2-13k+11$	$-403=-13\cdot 31$	2	-2^7
17	$17k^2-17k+5$	$-51=-17\cdot 3$	2	-5
19	$19k^2-19k+29$	$-1843=-19\cdot 97$	6	$-\sqrt[3]{1771561000}$
23	$23k^2-23k+7$	$-115=-23\cdot 5$	2	-5
29	$29k^2+2$	$-232=-29\cdot 8$	2	5
31	$31k^2-31k+11$	$-403=-31\cdot 13$	2	-3^2
41	$41k^2-41k+11$	$-123=-41\cdot 3$	2	-3
47	$47k^2-47k+13$	$-235=-47\cdot 5$	2	-5
59	$59k^2-59k+19$	$-1003=-59\cdot 17$	4	$-(3+2\sqrt{2})$
71	$71k^2+2$	$-568=-71\cdot 8$	4	$\sqrt{17}$

B. **p as product of two distinct prime divisors of $|\mathbb{M}|$**

$2\cdot\{3,5,7,11,13,17,19,23,31,47\}=\{6,10,14,22,26,34,38,46,62,94\}$,

$3\cdot\{5,7,11,13,17,19,23,29,31\}=\{15,21,33,39,51,69,87,93\}$,

$5\cdot\{7,11,19\}=\{35,55,95\}$, $7\cdot 17=119$.

Therefore $\mathrm{p}\in\{6,10,14,15,21,22,26,33,34,35,38,39,46,51,55,57,62,69,87,93,94,95,119\}$

There are 23 such orders, though $\mathrm{p}=57$ and 93 are to be excluded. For the relevant McKay Thompson series in this family, these two are the only ones where the powers of q are not consecutive but are in the progression $3m+2$. So what remains are 21 series.

In addition, there is Mckay-Thompson series $T_p(\tau)$ for each p. Hence, we present them and their associated $p(k)$ and $T_{pA}(\tau)$ in the following table:

Table 3. Prime-generating Polynomials for Discriminants with h(-d) = 4, 8. 12 and $T_{pA}(\tau)$

p	$p(k)=ak^2+bk+c$	$d=b^2-4ac$	h(d)	$T_p(\tau)$
6	$6k^2+5$	$-120=-6\cdot 20$	4	$2\cdot 5\cdot 31$
6	$6k^2+7$	$-168=-6\cdot 28$	4	$2\cdot 443$
6	$6k^2+13$	$-312=-6\cdot 52$	4	$2\cdot 5\cdot 1039$
6	$6k^2+17$	$-408=-6\cdot 68$	4	$2\cdot 5\cdot 3919$
6	$6k^2-6k+5$	$-84=-6\cdot 14$	4	$-2\cdot 61$
6	$6k^2-6k+7$	$-132=6\cdot 22$	4	$-2\cdot 5\cdot 41$
6	$6k^2-6k+11$	$-228=6\cdot 38$	4	$-2\cdot 23\cdot 59$
6	$6k^2-6k+17$	$-372=-6\cdot 62$	4	$-2\cdot 12157$
6	$6k^2-6k+31$	$-708=-6\cdot 118$	4	$-2\cdot 3\cdot 401\cdot 467$
10	$10k^2+3$	$-120=-10\cdot 12$	4	2^5
10	$10k^2+7$	$-280=-10\cdot 28$	4	$2^6\cdot 3$
10	$10k^2+13$	$-520=-10\cdot 52$	4	$2^2\cdot 17\cdot 19$
10	$10k^2+19$	$-760=-10\cdot 76$	4	$2^2\cdot 3\cdot 13\cdot 37$
10	$10k^2-10k+11$	$-340=-10\cdot 34$	4	$-2^3\cdot 41$
14	$14k^2+3$	$-168=-28\cdot 6$	4	19
14	$14k^2+5$	$-280=-14\cdot 20$	4	43

14	$14k^2-14k+5$	$-84=-14\cdot 6$	4	$-3\cdot 59$
14	$14k^2-14k+13$	$-532=-14\cdot 38$	4	$-5\cdot 7$
15	$15k^2-15k+7$	$-195=-15\cdot 13$	4	-19
15	$15k^2+2$	$-120=-15\cdot 8$	4	11
21	$21k^2-21k+11$	$-483=-21\cdot 23$	4	-3^3
22	$22k^2-22k+7$	$-132=-22\cdot 6$	4	-11
22	$22k^2-22k+17$	$-1012=-22\cdot 46$	4	$-11\cdot 3^2$
26	$26k^2+3$	$-312=-26\cdot 12$	4	3^2
26	$26k^2+5$	$-520=-26\cdot 20$	4	4^2
33	$33k^2-33k+17$	$-1155=-33\cdot 35$	8	$-(20+2\sqrt{5})$
34	$34k^2+7$	$-952=-34\cdot 28$	8	$21-\sqrt{10}$
35	$35k^2+2$	$-280=-35\cdot 8$	4	5
38	$38k^2-38k+13$	$-532=-38\cdot 14$	4	-7
39	$39k^2+2$	$-312=-39\cdot 8$	8	$3\sqrt{11}-5$
46	$46k^2-46k+17$	$-1012=-46\cdot 22$	4	$-2^2\cdot 7$
51	$51k^2+2$	$-408=-51\cdot 8$	4	2^2
55	$55k^2-55k+17$	$-715=-55\cdot 13$	4	-5
62	$62k^2+5$	$-1240=-62\cdot 20$	8	$1+2\sqrt{6}$
69	$69k^2-69k+19$	$-483=-69\cdot 7$	4	-3
87	$87k^2+2$	$-696=-87\cdot 8$	12	$\sqrt[3]{18}$
94	$94k^2+7$	$-2632=-94\cdot 28$	8	$5+\sqrt{5}$
95	$95k^2+2$	$-760=-95\cdot 8$	4	3
119	$119k^2-119k+43$	$-6307=-119\cdot 53$	8	$-(4+\sqrt{17})$

C. p as product of three distinct prime divisors of $|\mathbb{M}|$.

$2\cdot 3\cdot\{5,7,11,13\}=\{30,42,66,78\}$, $2\cdot 5\cdot\{7,11\}=\{70,110\}, 3\cdot 5\cdot 7=105.$

Therefore $p\in\{30, 42, 66, 70, 78, 105, 110\}$

Table 4 gives their associated quadratic prime-generating polynomial $p(k)$ and $\mathrm{T}_{\mathrm{pA}}(\tau)$.

Table 4. Prime-generating Polynomials for Discriminants with h(-d) = 8 and $\mathbf{T_p(\tau)}$

p	$p(k) = ak^2 + bk + c$	$d = b^2 - 4ac$	$\mathrm{T}_{\mathrm{p}}(\tau)$
30	$30k^2 + 7$	$-840 = -30 \cdot 28$	$3 \cdot 7$
42	$42k^2 + 11$	$-1848 = -42 \cdot 44$	5^2
66	$66k^2 + 7$	$-1848 = -66 \cdot 28$	2^3
70	$70k^2 - 70k + 23$	$-1540 = -70 \cdot 22$	$-2 \cdot 3$
78	$78k^2 - 78k + 23$	$-1092 = -78 \cdot 14$	-2^2
105	$105k^2 - 105k + 31$	$-1995 = -105 \cdot 19$	-2^2
110	$110k^2 + 3$	$-1320 = -110 \cdot 12$	3

Note: All the series are from T_{pA} except $\mathrm{T}_{30\mathrm{B}}$, $\mathrm{T}_{33\mathrm{B}}$, $\mathrm{T}_{46\mathrm{C}}$.

5. Conclusion

1. For p a prime divisor of $|\mathbb{M}|$, $\mathrm{T}_{\mathrm{p}}(\tau)$ is an algebraic number of degree one-half the h(-d). There are 15 such series.

2. For p a product of two distinct prime divisors of $|\mathbb{M}|$ (except p=57 and p=93), the $\mathrm{T}_{\mathrm{p}}(\tau)$ of the appropriate conjugscy class is an algebraic number of degree one-fourth the h(-d). There are 21 such series.

3. For p a product of three distinct prime divisors of $|\mathbb{M}|$, the $\mathrm{T}_{\mathrm{p}}(\tau)$ of the appropriate conjugacy class is an algebraic number of degree one-eighth the h(-d). There are 7 such series.

Thus, there is a total of 15 +21 + 7 = 43 such series, which is equivalent to the least number of conjugacy classes of the maximal subgroups of the monster group.

Conflict of Interests

The author declares that there is no conflict of interests.

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