Available online at http://scik.org

Algebra Letters, 2017, 2017:7

ISSN: 2051-5502

ON SOME PROPERTIES OF AB-ALGEBRAS

AREEJ TAWFEEQ HAMEED* AND BANEEN NAJAH ABBAS

Department of Mathematics, College of Education for Girls, University of Kufa, Iraq

Abstract: We introduce the notion of fuzzy AB-ideal of AB-algebra, several theorems and properties are stated and proved. The fuzzy relations on AB-algebras are also studied as homomorphic, isomorphic and normal.

Keywords: t-derivation on AB-algebras; characterized Kerd by t-derivations; weak fuzzy AB-subalgebras; weak fuzzy AB-ideal.

2010 Mathematics Subject Classification: 06F35, 03G25, 03B52.

1.Introduction:

The notion of BCC-algebras was proposed by W.A. Dudek in ([6], [8]). S.S. Ahn and H.S. Kim have introduced the notion of QS-algebras. A.T. Hameed and et al [3] introduced the notions of QS-ideal and fuzzy QS-ideal of QS-algebra. A.T. Hameed introduce new of abstract algebras: is called KUS-algebras and define its ideals as KUS-algebras in ([4]). In 2017, A.T. Hameed and B.N. Abbas. introduced the notion of AB-ideals in AB-algebras and described connections between such ideal and congruences [1]. A.T. Hameed and B.N. Abbas., considered the fuzzification of AB-ideals in

AB-algebras [2]. In this paper, we introduce the notion of t-derivation on AB-algebras and obtain some of related properties and we characterized Kerd by t-derivations

2 Basic definitions of AB-algebra:

We review some definitions and properties that will be useful in our results.

Definition 2.1 ([1]) Let X be a set with a binary operation ”$*$” and a constant 0.Then$(X, *, 0)$

*Corresponding author

E-mail address: areej.tawfeeq@uokufa.edu.iq

Received June 12, 2017

is called an AB-algebra if the following axioms satisfied for all $x,y,z \in X$:

(i) $((x * y) * (z * y)) * (x * z) = 0$,

(ii) $0 * x = 0$,

(iii) $x * 0 = x$,

Note that: Define a binary relation ($\leq$) on X by letting $x * y = 0$ if and only if, $x \leq y$.

Then $(X, \leq)$ is a partially ordered set.

Proposition 2.2 ([1]) In any AB-algebra X, for all $x, y, z \in X$, the following properties hold:

(1) $(x * y) * x = 0$.

(2) $x \leq y$ implies $x * z \leq y * z$.

(3) $x \leq y$ implies $z * y \leq z * x$.

Remark2.3([1]) An AB-algebra is satisfies for all $x,y,z \in X$

(2) $(x * (x * y)) * y = 0$.

Definition 2.4 ([1]) A nonempty subset S of anAB-algebra X is called AB-subalgebras of X if $x * y \in S$ whenever $x, y \in S$.

Definition 2.5([1]) Let X be an AB-algebra and $\varphi = I \subseteq X$. I is called an AB-ideal of X if it satisfies the following conditions:

(i) $0 \in I$,

(ii) $(x * y) * z \in I$ and $y \in I$ imply $x * z \in I$.

Definition 2.6 For an AB-algebra X, we denote $x \wedge y = y * (y * x)$, for all $x,y \in X$, $x \wedge y \leq x, y$.

Definition 2.7 An AB-algebra is said to be commutative if and only if, satisfies for all $x, y \in X$, $x * (x * y) = y * (y * x)$, i.e, $x \wedge y = y \wedge x$.

Definition 2.8 LetX be an AB-algebra. A map $d: X \rightarrow X$ is a left-right derivation (briefly,(l,r)-derivation) of X,

if it satisfies theidentity$d(x * y) = (d(x) * y) \wedge (x * d(y))$, for all $x, y \in X$,

if d satisfies the identity$d(x * y) = (x * d(y)) \wedge (d(x) * y)$, for all $x, y \in X$.

Then d is a right-left derivation (briefly, (r, l)-derivation) of X.

Moreover, if d is both a (l,r) and (r,l)-derivation, then d is a derivation of X.

Example 2.9. Let $X = \{0,1,2,3\}$ be an AB-algebra in which the operation (*) is defined as follows:

*	0	1	2	3
0	0	0	0	0
1	1	0	1	0
2	2	2	0	0
3	3	3	1	0

Define a map d : G → G by $d(x) = \begin{cases} 0 & if\ x = 0,1,3, \\ 2 & if\ x = 2. \end{cases}$

And define a map d^* : G → G by $d^*(x) \begin{cases} 0 & if\ x = 0,1, \\ 2 & if\ x = 2,3. \end{cases}$

Then it is easily checked that d is both a (l,r) and (r,l)-derivation of G and d^* is a (r,l)- derivation but not a (l,r)-derivation of G.

Definition2.10 A derivation of a AB-algebra is said to be regular if d(0) = 0.

3.Main results

3.1. Basic definitions of weak AB-algebra:

We review the definition of weak AB-algebra and study some properties of it.

Definition 3.1.1 An algebra (G; *, 0) of type (2, 0) with a relation defined by (1) is a weak AB-algebra if and only if, for all x, y, z ∈ G the following conditions are satisfied:

(i′) $((x * y) * (z * y)) \leq (x * z)$,

(ii′) $x \leq x$,

(iii′) $x*0 = x$,

From (i′) it follows that in weak AB-algebras implications

(1) $x \leq y \Rightarrow x*z \leq y*z$

(2) $x \leq y \Rightarrow z*y \leq z*x$ are valid, for all x, y, z ∈ G.

Definition 3.1.2. Let (G; *, 0) be a weak AB-algebras, a subset A weak AB-algebra is called a **AB-ideal** if

a) 0 ∈ A,
b) For all y ∈ A, (x*y)* z ∈ A implies x*z∈ A.

Note that: A special case of an AB-ideal is an AB-ideal, i.e., a subset A such that 0 ∈ A, and y,

$x*y \in A$ imply $x \in A$.

The main properties of this map are collected in the following theorem proved.

Remark 3.1.3. In the investigations of algebras connected with various types of logics an important role plays the self map $\phi(x) = 0*x$, for all $x \in G$.

Theorem 3.1.4. Let G be a weak AB-algebra. Then for all $x, y \in G$.

(1) $\phi^2(x) \leq x$,

(2) $x \leq y \Rightarrow \phi(x) = \phi(y)$,

(3) $\phi^3(x) = \phi(x)$,

(4) $\phi^2(x*y) = \phi^2(x)* \phi^2(y)$

Definition 3.1.5. A weak AB-algebra in which $\mathrm{Ker}\phi(x) = \{0\}$ is called group-like or anti-grouped.

Remark 3.1.6.

1- A weak AB-algebra $(G; \cdot, 0)$ is group-like if and only if, there exists a group $(G; *, 0)$ such that $x.y = x * y^{-1}$.
2- The set of all minimal (with respect to $\leq$) elements of G will be denoted by I (G). It is a AB-subalgebras of G. Moreover, $I(G) = \phi(G) = \{a \in G : \phi^2(a) = a\}$.
3- The set $B(a) = \{x \in G : a \leq x\}$,where $a \in I(G)$ is called the branch initiated by a.
4- Branches initiated by different elements are disjoint. Comparable elements are in the same branch, but there are weak AB-algebras containing branches in which not all elements are comparable.

Lemma 2.1.7.

1- Elements x and y are in the same branch if and only if, $x*y\in B(0)$.
2- B(0) is a AB-subalgebras of G. It is a maximal AB-algebra contained in G.
3- In a weak AB-algebra G, for all $a, b \in I(G)$ we have $B(a)*B(b) = B(a*b)$.
4- The identity $(x*y)* z = (x*z)* y$. plays an important role in the theory of BCI-algebras.

Definition 3.1.8. A weak AB-algebra G is called solid if the above condition is valid for all x, y belonging to the same branch and arbitrary $z \in G$.

Remark 3.1.9.

1- A simple example of a solid weak AB-algebra is a BCI-algebra.
2- BCK-algebra is a solid weak AB-algebra.
3- A solid AB-algebra is a BCK-algebra.

4- There are solid weak AB-algebras which are not BCI-algebras.

5- The smallest such weak AB-algebra has 5 elements.

Example 3.1.10. Consider the set X = {0, 1, 2, 3, 4, 5} with the operation defined by the following table:

*	0	1	2	3	4	5
0	0	0	4	4	2	2
1	1	0	4	4	2	2
2	2	2	0	0	4	4
3	3	2	1	0	4	4
4	4	4	2	2	0	0
5	5	4	3	3	1	0

Since (S; *, 0), where S = {0, 1, 2, 3, 4}, is an AB-algebra, it is not difficult to verify that (X ; *, 0) is a weak AB-algebra. It is proper because $(5*3)*2 \neq (5*2)*3$. Simple calculations show that this weak AB-algebra is solid.

Proposition 3.1.11. In any solid weak AB-algebra we have

(a) $x*(x*y)*y$,

(b) $x*(x*(x*y)) = x*y$, for all x, y belonging to the same branch.

Corollary 3.1.12. In a solid weak AB-algebra from $x, y \in B(a)$ it follows

$x*(x*y), y*(y*x) \in B(a)$.

Proposition 3.1.13. The map $\phi(x) = 0*x$ is an endomorphism of each solid weak AB-algebra.

Proposition 3.1.14. In each solid weak AB-algebra for $x*y$, $x*z$ (or $x*y$ and $z*y$) belonging to the same branch we have $(x*y)*(x*z) \leq (z*y)$.

Proof.

Indeed, $((x*y)*(x*z))*(z*y) = ((x*y)*(z*y))*(x*z) = 0$.

3.2.t-derivations in AB-algebras

The following definitions introduce the notion of t-derivation for a AB-algebra.

Definition 3.2.1 Let X be a AB-algebra. Then for any $t \in X$, the self map $d_t: X \to X$is called a right translation by t and is denoted by R_t if $d_t(x)=x*t$, for all $x \in X$.

Example3.2.2 LetG={0,1,2,3,4}bean AB-algebra with the following Cayley table:

*	0	1	2	3	4
0	0	0	0	0	0
1	1	0	0	0	1
2	2	2	0	0	1
3	3	2	1	0	1
4	4	4	4	4	0

For any t ∈ X, define a self map d_t : G → G by

$$d_t(x) = x * t = \begin{cases} 0 & if \quad x = 0,1,2,3 \\ 4 & if \quad x = 4 \end{cases}$$

Then it is easy to check that d_t is a t-derivation on G.(i.e. d_t is (l, r) and(r, l)-t-derivations)

Example 3.2.3 Let G = {0, a, b} be anAB-algebra with the following Cayley table:

∗	0	a	b
0	0	0	0
a	a	0	a
b	b	b	0

For any t ∈ X, define a self map

d_t : G →Gby$d_t(x) = x * t = \begin{cases} 0 & if\ x = 0, a \\ b & if\ x = b \end{cases}$

Then it is easily checked that d_t is both (l, r) and (r, l)-t-derivation of G.

Definition 3.2.4A self map d_t of an AB-algebra X is said to be t-regular if $d_t(0) = 0$.

Theorem 3.2.5A (l, r)-t-derivation of an AB-algebra X is t-regular.

Proof.

$d_t(0) = d_t(0 * x) = (d_t(0) * x) \wedge (0 * d_t(x)) = ((0 * t) * x) \wedge (0 * (x * t))$

$= ((0 * t) * x) \wedge 0 = 0 * (0 * ((0 * t) * x)) = 0.$

Theorem 3.2.6 A (r, l)-t-derivation of an AB-algebra X is t-regular.

Proof.

$d_t(0) = d_t(0 * x) = (0 * d_t(x)) \wedge (d_t(0) * x)$

$= 0 \wedge ((0 * t) * x) = ((0 * t) * x) * (((0 * t) * x) * 0)$

$= ((0 * t) * x) * ((0 * t) * x) = 0.$

Corollary 3.2.7 A t-derivation of an AB-algebra X is t-regular.

Proposition 3.2.8 Let d_t be a self map of an AB-algebra X. Then

i) If d_t is a (l, r)-t-derivation of X, then $d_t(x) = d_t(x) \wedge x$, for all $x \in X$.

ii) If d_t is a (r, l)-t-derivation of X, then $d_t(x) = x \wedge d_t(x)$ for all $x \in X$.

Proof.

(i). Let d_t be a (l, r)-t-derivation of X, then

$d_t(x) = d_t(x * 0) = (d_t(x) * 0) \wedge (x * d_t(0)) = d_t(x) \wedge (x * 0) = d_t(x) \wedge x.$

(ii). Let d_t be a (r, l)-t-derivation of X, Then

$d_t(x) = d_t(x * 0) = (x * d_t(0)) \wedge (d_t(x) * 0) = (x * 0) \wedge d_t(x) = x \wedge d_t(x).$

Theorem 3.2.9 Let d_t be a t-regular (r, l)-t-derivation of an AB-algebra X. Then, the following hold:

i) $d_t(x) \leq x$, for all $x \in X$.

ii) $d_t(x) * y \leq x * d_t(y)$, for all $x, y \in X$.

iii) $d_t(x * y) = d_t(x) * y \leq d_t(x) * d_t(y)$, for all $x, y \in X$.

iv) $d_t^{-1}(0) = Ker(d_t) = \{x \in X : d_t(x) = 0\}$ is a AB-subalgebras of X.

v) $d_t(d_t(x)) \leq x$.

vi) $d_t(x * d_t(x)) = 0$.

Proof.

(i). For any $x \in X$, we have:

$d_t(x) = d_t(x * 0) = (x * d_t(0)) \wedge (d_t(x) * 0) = (x * 0) \wedge d_t(x) = x \wedge d_t(x) \leq x.$

(ii). Since $d_t(x) \leq x$ for all $x \in X$, then $d_t(x) * y \leq x * y \leq x * d_t(y)$ and hence the proof follows.

(iii). For any $x, y \in X$, we have

$d_t(x * y) = (x * d_t(y)) \wedge (d_t(x) * y) = (d_t(x) * y) * ((d_t(x) * y) * (x * d_t(y)))$

$= (d_t(x) * y) * 0 = (d_t(x) * y) \leq d_t(x) * d_t(y)$.

(iv). Let $x, y \in Ker(d_t) \Rightarrow d_t(x) = 0 = d_t(y)$. From (iii) , we have $d_t(x*y) \leq d_t(x) * d_t(y)$

$= 0 * 0 = 0$. implying $d_t(x* y) \leq 0$ and so $d_t(x* y) = 0$.

Consequently $Ker(d_t)$ is a AB-subalgebras of X.

(v). $d_t(d_t(x)) = d_t(x* t) \leq d_t(x) * d_t(t) = (x* t) * (t * t) = (x* t) * 0 = (x*t) \leq x$.

(iv) $d_t(x* d_t(x)) = d_t(x) * d_t(x) = 0$.

Proposition 3.2.10 Let X be an AB-algebra and d_t a t-derivation. If $y \in Ker(d_t)$ and $x \in X$, then $x \wedge y \in Ker(d_t)$.

Proof.

Let d_t be a (l, r)-t-derivation and $y \in Ker(d_t)$. Then we get $d_t(y) = 0$, and so

$$\begin{aligned} d_t(x \wedge y) &= d_t(y * (y * x)) \\ &= (d_t(y) * (y * x)) \wedge (y * d_t(y * x)) \\ &= (0 * (y * x)) \wedge (y * d_t(y * x)) \\ &= 0 \wedge (y * d_t(y * x)) \\ &= (y * d_t(y * x)) * ((y * d_t(y * x)) * 0) \\ &= (y * d_t(y * x)) * (y * d_t(y * x)) = 0 \end{aligned}$$

Hence we have $x \wedge y \in Ker(d_t)$. Similarly, we can prove in case of (r, l)-t-derivation.

Proposition 3.2.11 Let X be a commutative AB-algebra and d_t a t-derivation. If $x \leq y$ and $y \in Ker(d_t)$, Then $x \in Ker(d_t)$.

Proof.

Let $x \leq y$ and $y \in Ker(d_t)$. Then we get $x * y = 0$ and $d_t(y) = 0$, and so

$d_t(x) = d_t(x* 0) = d_t(x*(x*y)) = d_t(y * (y * x)) = d_t(y) * (y * x) = 0 * (y * x) = 0$

Hence we have $x \in Ker(d_t)$.

Proposition 3.2.12 Let X be an AB-algebra and d_t a t-derivation. If $x \in Ker(d_t)$, we have $x* y \in Ker(d_t)$, for all $y \in X$.

Proof.

Let $x \in Ker(d_t)$. Then $d_t(x) = 0$. Thus we have $d_t(x * y) = d_t(x) * y = 0 * y = 0$

which implies $x* y \in Ker(d_t)$.

Proposition 3.2.13 Let X be a commutative AB-algebra and d_t a t-derivation. Then $Ker(d_t)$ is an ideal of X.

Proof.

Clearly, $d_t(0) = 0$. Let $x * y \in Ker(d_t)$ and $y \in Ker(d_t)$. Then we get $d_t(x * y) = 0$ and $d_t(y) = 0$, and so

$d_t(x) = d_t(x * 0) = d_t(x*(x*y)) = d_t(y * (y * x)) = d_t(y) * (y * x) = 0 * (y * x) = 0 \Rightarrow x \in Ker(d_t)$.

Definition 3.2.14 Let d_t be a t-derivation of an AB-algebra X. An AB-ideal A of X is said to be d_t-invariant if $d_t(A) \subseteq A$, where $d_t(A) = \{d_t(x) | x \in A\}$.

Theorem 3.2.15 Let d_t be a t-derivation of an AB-algebra X. Then every ideal A of X is d_t-invariant.

Proof.

Let A be an AB-ideal of an AB-algebra X. Let $y \in d_t(A)$. Then $y = d_t(x)$, for some $x \in A$.

It follows that $y * x = d_t(x) * x = 0 \in A$, which implies $y \in A$.

Thus $d_t(A) \subseteq A$. Hence A is d_t-invariant.

3.3. Commutative solid weak AB-algebras

In any BCK-algebra G we can define a binary operation $\wedge$ by putting $x \wedge y = y *(y*x)$, for all x, $y \in G$. A BCK-algebra satisfying the identity

(1) $x *(x*y) = y*(y*x)$,i.e., $y \wedge x = x \wedge y$, is called commutative.

A commutative BCK-algebra is a lower semi lattice with respect to the operation $\wedge$.This definition cannot be extended to BCI-algebras, AB-algebras and weak AB-algebras since in any weak AB-algebra satisfying (1) we have $0 *(0*x) = x *(x*0) = 0$, i.e., $\phi^2 (x) = 0$, for every $x \in G$. Thus $\phi(x) = \phi^3(x) = 0$, by Theorem (3.1.7). Hence in this algebra $0 \leq x$ for every $x \in G$.

This means that this algebra is a commutative AB-algebra. But in any AB-algebra G we have $0 \leq y*x$ for all x, $y \in G$, which together with (3) implies $y *(y*x) \leq y$. Thus a commutative AB-algebra satisfies the inequality $(y*x)*(x*y) = y *(y*x) \leq y$.

Consequently, it satisfies the identity $(x *(x*y))*y = 0$, so it is a BCK-algebra. Hence a commutative (weak) AB-algebra is a commutative BCK-algebra. Analogously, a commutative BCI-algebra is a commutative BCK-algebra. But there are weak AB-algebras in which the condition (1) is satisfied only by elements belonging to the same branch.

Example 3.3.1A weak AB-algebra defined by the following table:

*	0	a	b	c	d
0	0	0	0	c	c
a	a	0	0	c	c
b	b	a	0	d	c
c	c	c	c	0	0
d	d	c	c	a	0

has two branches: B(0) = {0, a, b} and B(c) = {c, d}. It is not difficult to verify that in this weak AB-algebra (1) is satisfied only by elements belonging to the same branch.

Definition 3.3.2A weak AB-algebra in which Remark (3.1.9(5)) is satisfied by elements belonging to the same branch is called branch wise commutative.

Theorem 3.3.3 For a solid weak AB-algebra G the following conditions are equivalent:

(1) G is branch wise commutative,

(2) $x*y = x*(y*(y*x))$ for x, y from the same branch,

(3) $x = y*(y*x))$ for $x \leq y$,

(4) $x*(x*y)) = y*(y*(x *(x*y)))$, for x, y from the same branch,

(5) each branch of G is a semilattice with respect to the operation $x \wedge y = y *(y*x)$.

In the proof of the next theorem we will need the following well-known result from the theory of BCK-algebras.

Lemma 3.3.4 If p is the greatest element of a commutative BCK-algebra G, then $(G; \leq)$ is a distributive lattice with respect to the operations

$x \wedge y = y*(y*x))$ and $x \vee y = p*((p*x)\wedge(p*y))$.

Theorem 3.3.5 In a solid branch wise commutative weak AB-algebra G, for every $p \in G$, the set $A(p) = \{x \in G : x \leq p\}$ is a distributive lattice with respect to theoperations $x \wedge y = y*(y*x))$ and $x \vee y = p*((p*x)\wedge(p*y))$.

Proof.

(A). We prove that $(A_p; \leq)$, where$A_p= \{p*x : x \in A(p)\}$,is a distributive lattice.

First, we show that A_p is a AB-subalgebras of B(0). It is clear that $0 = p*p \in A_p$ and $A(p) \subseteq B(a)$ for some $a \in I(G)$. Thus $A_p \subset B(0)$. Obviously $a \leq x$ for every $x \in A(p)$.

Consequently, $p*x \leq p*a$. Hence $p*a$ is the greatest element of A_p.

Let $p*x$, $p*y$ be arbitrary elements of A_p. Then obviously $y*x \in B(0)$ and $z = p*(y*x) \in A_p$ because $z*p = (p*(y*x))*p = 0$.

Moreover, $z*x = (p*(y*x))*x = (p*x)*(y*x) \leq p*y$, by(i′). Since

$((p*x)*(p*z))*(z*x) = ((p*x)*(z*x))*(p*z) = 0$,we also have

$((p*x)*(p*z)) \leq z*x \leq p*y$.

Therefore, $0 = ((p*x)*(p*z))*(p*y) = ((p*x)*(p*z))*(p*z)$, i.e.,

(**2**) $((p*x)*(p*y)) \leq p*z$.

On the other hand, since a weak AB-algebra G is branch wise commutative, for every $y \in A(p)$, according to Theorem (3.3.3), we have $p*(p*y) = y$.

Hence $((p*x)*(p*y)) = (p*(p*y))*x = y*x$.

But $(p*z)*(y*x) = p*(p*(y*x))*(y*x) = (p*(y*x))*(p*(y*x)) = 0$.

Thus $p*z \leq y*x = (p*x)*(p*y)$,which together with (**2**) gives $(p*x)*(p*y) = p*z$. Hence A_p is a AB-subalgebras of B(0).

Obviously, B(0) as anAB-algebra contained in G is commutative, and consequently it is a commutative BCK-algebra. Thus A_p is a commutative BCK-algebra, too. By Lemma (3.3.4), $(A_p; \leq)$ is a distributive lattice.

(B). Now, we show that $(A(p); \leq)$ is a distributive lattice. Clearly, p is the greatestelement of A(p).Let $x, y \in A(p)$. Then $(p*x), (p*y) \in A_p$ and from the fact that $(A_p; \leq)$ is a lattice itfollows that there exists the last upper bound $p*z \in A_p$, i.e.,

(**1**) $(p*x) \vee_p (p*y) = p*z$.

Observe that for $x, y \in A(p)$ we have

(**2**) $p*y \leq p*x \Leftrightarrow x \leq y$.

Indeed, in view of (3), $x \leq y$ implies $p*y \leq p*x$. Similarly, $p*y \leq p*x$ implies

$p*(p*x) \leq p*(p*y)$.

But G is branchwise commutative, hence by Theorem (4.3)., for every $v \in A(p)$, we have

$p*(p*v) = v$. Therefore $x = p*(p*x) \leq p*(p*y) = y$.

From (3) and (4) it follows that z is the greatest lower bound for x and y. Hence $x \wedge y = z$.

Moreover, we have $p*(x \wedge y) = p*z$ and $(p*x) \vee_p (p*y) = p*z$, which implies

(**3**) $p*(x \wedge y) = (p*x)\vee_p(p*y)$.

Analogously, we can prove that for all x, y ∈ A(p) there exists $x \vee_p y$ and

(**4**) $p*(x \vee_p y) = (p*x)\wedge (p*y)$.

Therefore (A(p); ≤) is a lattice.

Since (5) and (6) are satisfied in $A_p$and (A_p ; ≤) is a distributive lattice, we have

$(p*x) \vee_p ((p*y)\wedge(p*z)) = ((p*x)\vee_p(p*y)) \wedge ((p*x)\vee_p(p*z))$This, in view of (5) and (6), gives

$p*(x \wedge (y \vee_p z)) = p*((x \wedge y) \vee_p (x \wedge z))$.

Consequently,$p* p*(x \wedge (y \vee_p z)) = p * p*((x \wedge y) \vee_p (x \wedge z))$and

$x \wedge (y \vee_p z) = (x \wedge y) \vee_p (x \wedge z)$,because $x \wedge (y \vee_p z)$, $(x \wedge y) \vee_p (x \wedge z) \in A(p)$. This means that (A(p); ≤) is adistributive lattice.In this lattice

$x \vee_p y = p*(p*(x \vee_p y)) = p*((p*x)\wedge(p*y))$.

This completes the proof.

Conflict of Interests

The authors declare that there is no conflict of interests.

REFERENCES

[1] A.T.Hameed and B.N.Abbas, AB-ideals of AB-algebra, Appl. Math. Sci. 11(35)(2017), 1715-1723

[2] A.T.Hameed and B.N.Abbas, Fuzzy AB-ideals of AB-algebra, in press.

[3] A.T.Hameed, Fuzzy ideals of QS-algebras, Int. J. Algebra, 11(1)(2007), 43-52.

[4] A.T.Hameed, Fuzzy ideals of some algebras, PH. Sc. Thesis, Ain Shams University, Faculty of Sciences, Egypt , 2005.

[5] S.S. Ahn and H.S.Kim, On QS-algebras, J. Chungcheong Math. Soc., 12 (1999), 33-41.

[6] W. A.Dudek and Y. B. Jun, Fuzzy BCC-ideals in BCC-algebras, Math. Montisnigri ,10 (1999), 21 − 30.

[7] W.A.Dudek and J.Thomys, On some generalizations of BCC-algebras, Intern. J. Comp. Math. 89(2012), 1596-1606.

[8] W.A.Dudek, On BCC-algebras, Logique et Analyse ,129–131(1991),103–111.

[9] Y.Komori, The class of BCC-algebras is not a variety, Math. Japon., 29(1984),390−394.

[10] Mostafa S. M., Abdel Naby M. A. Abdel-Halim F. and Hameed A .T. , On KUS-algebras, Int. J. Algebra 7(3)(2003), 131 – 144.

[11] Mostafa S. M., Hameed A. T., and Abed A.H., Fuzzy KUS-ideals of KUS-algebra, Basra J. Sci. (A), 34 (2)(2006), 73-84.