Available online at http://scik.org
Algebra Letters, 2018, 2018:2
https://doi.org/10.28919/al/3890
ISSN: 2051-5502

SOFT UNI-GROUP AND ITS APPLICATIONS TO GROUP THEORY

ASLIHAN SEZGIN[1,*], NAIM ÇAĞMAN[2], ZEYNEP KAYA TÜRK[1], EMRAH MUŞTUOĞLU[1]

[1]Department of Elementary Education, Faculty of Education, Amasya University, 05100 Amasya, Turkey

[2]Department of Mathematics, Faculty of Arts and Science, Gaziosmanpaşa University, 60250 Tokat, Turkey

Abstract. In this paper, we define soft union group, serving as a bridge among soft set theory, set theory and group theory and showing how a soft set effects on a group structure in the mean of union and inclusion of sets. We then derive its basic properties and relation with the notion of soft intersection group and obtain some analog of classical group theoretic concepts for soft union group. Moreover, we give some applications of soft union groups to group theory.

Keywords: soft sets; soft uni-group; soft uni-subgroup; normal soft uni-subgroup; soft anti image.

2010 AMS Subject Classification: 03G25, 20M17, 03H05, 03C55, 03E75.

1. Introduction

Molodtsov [21] introduced soft set theory for modeling vagueness and uncertainty. Works on soft set theory has been progressing rapidly since Maji et al. [18] presented some definitions on soft sets and Ali et al. [3] introduced several operations of soft sets. Sezgin and Atagün [23] studied on soft set operations as well. Based on these operations, the theory of soft sets has developed in many directions and is finding applications in a wide variety

*Corresponding author
E-mail address: aslihan.sezgin@amasya.edu.tr
Received September 13, 2018

of fields. Some of the recent papers [5,12,13,20] are extending the soft set theory, especially in decision making [6,7,19,22,30].

Soft groups and its related properties were first introduced and studied by Aktaş and Çağman [2]. Some authors [1,9,14,15,16,17,24,25,26,27,28,29] have then studied the soft algebraic structures in detail. Applying the definition of soft set, Atagün and Sezgin [4] studied the algebraic soft substructures of rings, fields and modules. In [8], Çağman et al. introduced soft intersection group (soft int-group) and studied its basic properties with respect to soft set operations. To develop the soft set theory, Çağman and Enginoğlu [7] redefined the operations of soft sets. By using their definitions, in this paper, we define *soft union group* (abbreviated as "*soft uni-group*"). It is based on the inclusion relation and union of sets. This new concept brings together the soft set theory, set theory and the group theory and therefore is very functional in the mean of improving the soft set theory with respect to group structure. Furthermore, it serves as a bridge among soft set theory, set theory and group theory and shows how a soft set effects on a group in the mean of union and inclusion of sets. Based on the definition of soft uni-group, we define the concepts of soft uni-subgroup and normal soft uni-subgroup. We also define soft anti image, e-left coset of a soft set and investigate these notions with respect to soft uni-group. Moreover, we obtain a significant relation between the concept of soft uni-group and soft uni-group and derive some analog of classical group theoretic concepts for soft uni-group. Finally, we give some applications of soft uni-group to group theory.

2. Preliminaries

In this section, we recall some basic notions relevant to soft sets. For further details related to this section may be found in earlier studies [7,10,11,18,21].

Throughout this paper, U refers to an initial universe, E is a set of parameters, $P(U)$ is the power set of U and $A,B,C \subseteq E$.

Definition 2.1. ([21]) A soft set f_A over U is a set defined by

$$f_A : E \to P(U) \text{ such that} f_A(x) = \emptyset \text{ if } x \notin A.$$

Here f_A is also called *approximate function*. A soft set over U can be represented by the set of ordered pairs

$$f_A = \{(x, f_A(x)) : x \in E, f_A(x) \in P(U)\}.$$

It is clear to see that a soft set is a parametrized family of subsets of the set U. It is worth noting that the sets $f_A(x)$ may be arbitrary. Some of them may be empty, some may have nonempty intersection.

Note that the set of all soft sets over U will be denoted by $S(U)$. If we define more then one soft set in a subset A of the set of parameters E, then the soft sets will be denoted by f_A, g_A, h_A etc. If we define more then one soft set in some subsets A, B, C etc. of parameters E, then the soft sets will be denoted by f_A, f_B, f_C etc., respectively.

Definition 2.2. [7] Let $f_A \in S(U)$. f $f_A(x) = \emptyset$ for all $x \in E$, then f_A is called an *empty soft set* and is denoted by f_Φ.

If $f_A(x) = U$ for all $x \in A$, then f_A is called *A-universal soft set* and is denoted by $f_{\tilde{A}}$.

Definition 2.3. [7] Let $f_A, f_B \in S(U)$. Then, f_A is called a soft subset of f_B and denoted by $f_A \tilde{\subseteq} f_B$, if $f_A(x) \subseteq f_B(x)$ for all $x \in E$.If $f_A(x) = f_B(x)$ for all $x \in E$, then f_A and f_B is called *soft equal* and is denoted by $f_A = f_B$.

Definition 2.4. [7] Let f_A, $f_B \in S(U)$. Then, *union* of f_A and f_B, denoted by $f_A \tilde{\cup} f_B$, is defined as $f_A \tilde{\cup} f_B = f_{A\tilde{\cup}B}$, where $f_{A\tilde{\cup}B}(x) = f_A(x) \cup f_B(x)$ for all $x \in E$.

Intersection of f_A and f_B, denoted by $f_A \tilde{\cap} f_B$, is defined as $f_A \tilde{\cap} f_B = f_{A\tilde{\cap}B}$, where $f_{A\tilde{\cap}B}(x) = f_A(x) \cap f_B(x)$ for all $x \in E$.

Complement of the soft set f_A over U, denoted by f_A^c, is defined as $f_A^c(\alpha) = U \setminus f_A(\alpha)$ for all $\alpha \in E$.

Definition 2.5. [7] Let f_A, $f_B \in S(U)$. Then, $\vee$-*product* of f_A and f_B, denoted by $f_A \vee f_B$, is defined as $f_A \vee f_B = f_{A\vee B}$, where $f_{A\vee B}(x,y) = f_A(x) \cup f_B(y)$ for all $(x,y) \in E \times E$.

$\wedge$-*product* of f_A and f_B, denoted by $f_A \wedge f_B$, is defined as $f_A \wedge f_B = f_{A\wedge B}$, where $f_{A\wedge B}(x,y) = f_A(x) \cap f_B(y)$ for all $(x,y) \in E \times E$.

Definition 2.6. [3] Let f_A, $f_B \in S(U)$. Then, *restricted union* of f_A and f_B, denoted by $f_A \cup_{\mathscr{R}} f_B$, is defined as $f_A \cup_{\mathscr{R}} f_B = f_{A\cup_{\mathscr{R}}B}$, where $f_{A\cup_{\mathscr{R}}B}(x) = f_A(x) \cup f_B(x)$ for all $x \in A \cap B \neq \emptyset$.

Restricted intersection of f_A and f_B, denoted by $f_A \Cap f_B$, is defined as $f_A \Cap f_B = f_{A\Cap B}$, where $f_{A\Cap B}(x) = f_A(x) \cap f_B(x)$ for all $x \in A \cap B \neq \emptyset$.

Definition 2.7. [8] Let G be a group and $f_G \in S(U)$. Then, f_G is called a *soft intersection group* over U if if it satisfies the following properties:

i) $f_G(xy) \supseteq f_G(x) \cap f_G(y)$ for all $x, y \in G$,
ii) $f_G(x^{-1}) = f_G(x)$ for all $x \in G$.

For the sake of brevity, soft intersection group is abbreviated by soft int-group throughout this paper.

3. Soft uni-group

In this section, we first define *soft union group* that is abbreviated as *soft uni-group*. We then define soft uni-subgroup, normal soft uni-subgroup and investigate their basic properties. Throughout this section, G denotes an arbitrary group with identity e and if H is a subgroup of G, then it is denoted by $H \leq G$.

Definition 3.1. Let G be a group and $f_G \in S(U)$. Then, f_G is called a *soft uni-group* if it satisfies the following properties:

i) $f_G(xy) \subseteq f_G(x) \cup f_G(y)$ for all $x, y \in G$,
ii) $f_G(x^{-1}) = f_G(x)$ for all $x \in G$.

Example 3.2. Assume that $U = S_3$, symmetric group, is the universal set and $G = D_2 = \{<x,y>: x^2 = y^2 = e, xy = yx\} = \{e,x,y,yx\}$, dihedral group, is the subset of set of parameters. The group table of D_2 is known as

.	e	x	y	yx
e	e	x	y	yx
x	x	e	yx	y
y	y	yx	e	x
yx	yx	y	x	e

Now, we can construct a soft set f_G by

$$\begin{aligned} f_G(e) &= \{(13)\} \\ f_G(x) &= \{e,(12),(13)\} \\ f_G(y) &= \{e,(13),(23)\} \\ f_G(yx) &= \{e,(12),(13),(23)\}. \end{aligned}$$

Then, one can easily show that the soft set f_G is a soft uni-group over S_3.

Example 3.3. Consider $U = \mathbb{N}_{10}$ as the universal set and the group $G = \mathbb{Z}_{10}$ as the subset of set of parameters. We can define a soft set f_G as

$$f_G(x) = \{y \in \mathbb{Z}_{10} : y \in <x>\}$$

for all $x \in \mathbb{Z}_{10}$. Here, $f_G(\overline{0}) = \{0\}$, $f_G(\overline{1}) = f_G(\overline{3}) = f_G(\overline{5}) = f_G(\overline{7}) = f_G(\overline{9}) = \mathbb{Z}_{10}$, $f_G(\overline{2}) = f_G(\overline{4}) = f_G(\overline{6}) = f_G(\overline{8}) = \{\overline{0},\overline{2},\overline{4},\overline{6},\overline{8}\}$, $f_G(5) = \{\overline{0},\overline{5}\}$. Since $f_G(\overline{4}+\overline{5}) = f_G(\overline{9}) = \mathbb{Z}_{10} \nsubseteq f_G(\overline{4}) \cup f_G(\overline{5}) = \{\overline{0},\overline{2},\overline{4},\overline{5},\overline{6},\overline{8}\}$, f_G is not a soft uni-group over $\mathbb{Z}_{10}$.

It is easy to see that if we take the group as $G = \{e\}$, then f_G is a soft uni-group over U no matter how f_G is defined and no matter U is.

Proposition 3.4. *If f_G is a soft uni-group over U, then $f_G(e) \subseteq f_G(x)$ for all $x \in G$.*

Proof. Since f_G is a soft uni-group over U, then

$$f_G(e) = f_G(xx^{-1}) \subseteq f_G(x) \cup f_G(x) = f_G(x)$$

for all $x \in G$.

Theorem 3.5. *A soft set f_G over U is a soft uni-group over U if and only if $f_G(xy^{-1}) \subseteq f_G(x) \cup f_G(y)$ for all $x,y \in G$.*

Proof. Suppose that f_G is a soft uni-group over U. Then,

$$f_G(xy^{-1}) \subseteq f_G(x) \cup f_G(y^{-1}) = f_G(x) \cup f_G(y)$$

for all $x,y \in G$. Conversely, assume that $f_G(xy^{-1}) \subseteq f_G(x) \cup f_G(y)$ for all $x,y \in G$. If we choose $x = e$, then

$$f_G(ey^{-1}) = f_G(y^{-1}) \subseteq f_G(e) \cup f_G(y) = f_G(y),$$

since $f_G(yy^{-1}) = f_G(e) \subseteq f_G(y) \cup f_G(y) = f_G(y)$ for all $x \in G$. Similarly,

$$f_G(y) = f_G((y^{-1})^{-1}) \subseteq f_G(y^{-1}),$$

thus $f_G(y^{-1}) = f_G(y)$ for all $y \in G$. Moreover, by assumption

$$f_G(xy) \subseteq f_G(x) \cup f_G(y^{-1}) = f_G(x) \cup f_G(y)$$

Thus f_G is a soft uni-group over U.

Theorem 3.6. *Let f_G be a soft uni-group over U and $x \in G$. Then, for all $y \in G$*

$$f_G(xy) = f_G(y) \Leftrightarrow f_G(x) = f_G(e)$$

Proof. Suppose that $f_G(xy) = f_G(y)$ for all $y \in G$. Then by choosing $y = e$, we obtain that $f_G(x) = f_G(e)$. Conversely, assume that $f_G(x) = f_G(e)$. Then, we have

$$f_G(e) = f_G(x) \subseteq f_G(y), \quad \forall y \in G. \tag{1}$$

Since f_G is a soft uni-group over U, it follows that

$$f_G(xy) \subseteq f_G(x) \cup f_G(y) = f_G(y), \quad \forall y \in G.$$

Moreover, for all $y \in G$

$$\begin{aligned} f_G(y) &= f_G((x^{-1}x)y) \\ &= f_G(x^{-1}(xy)) \\ &\subseteq f_G(x^{-1}) \cup f_G(xy) \\ &= f_G(x) \cup f_G(xy) \\ &= f_G(xy) \end{aligned}$$

It follows that $f_G(xy) = f_G(y)$ for all $y \in G$, as required.

Theorem 3.7. *Let f_G be a soft uni-group over U and $x \in G$. Then,*

$$f_G(x) = f_G(e) \Rightarrow f_G(xy) = f_G(yx)$$

for all $y \in G$.

Proof: If $f_G(x) = f_G(e)$, then $f_G(xy) = f_G(y)$ for all $y \in G$. Therefore, it is enough to show that for all $y \in G$, $f_G(yx) = f_G(y)$ to complete the proof. Let $x \in G$, then

$$\begin{aligned} f_G(yx) &= f_G(yx(yy^{-1})) \\ &= f_G(y(xy)y^{-1}) \\ &\subseteq f_G(y) \cup f_G(xy) \cup f_G(y) \\ &= f_G(y) \cup f_G(xy) \\ &= f_G(y) \cup f_G(y) \\ &= f_G(y) \end{aligned}$$

for all $y \in G$. Moreover,

$$\begin{aligned} f_G(y) &= f_G(y(xx^{-1})) \\ &= f_G((yx)x^{-1}) \\ &\subseteq f_G(yx) \cup f_G(x) \\ &= f_G(yx) \end{aligned}$$

for all $y \in G$. It follows that $f_G(yx) = f_G(y)$ and $f_G(xy) = f_G(yx)$, $\forall y \in G$.

Remark 3.8. Let f_G be a soft uni-group over U and $x \in G$. Then,

$$f_G(x) = f_G(e) \Leftrightarrow f_G(xy) = f_G(yx) = f_G(y)$$

for all $y \in G$.

Theorem 3.9. *Let f_G be a soft uni-group over U such that the image of f_G is ordered by inclusion for all $x \in G$. If $f_G(y) \subsetneq f_G(x)$ for $x, y \in G$, where $f_G(y) \subsetneq f_G(x)$ means that $f_G(y) \subseteq f_G(x)$ but $f_G(y) \neq f_G(x)$, then*

$$f_G(xy) = f_G(x) = f_G(yx).$$

Proof: Since f_G is a soft uni-group over U, it follows that

$$f_G(xy) \subseteq f_G(x) \cup f_G(y) = f_G(x).$$

Moreover,

$$\begin{aligned} f_G(x) &= f_G(x(yy^{-1})) \\ &= f_G((xy)y^{-1}) \\ &\subseteq f_G(xy) \cup f_G(y) \end{aligned}$$

Since $f_G(y) \subsetneq f_G(x)$ and $f_G(x) \subseteq f_G(xy) \cup f_G(y)$, then $f_G(y) \subseteq f_G(xy)$ under assumption. Thus, $f_G(x) \subseteq f_G(xy)$, so $f_G(xy) = f_G(x)$ for all $y \in G$. One can similarly show that $f_G(yx) = f_G(x)$ is satisfied under assumption, so this completes the proof.

Remark 3.10. Theorem 3.9 fails, if in the hypothesis we replace $f_G(y) \subsetneq f_G(x)$ by $f_G(y) \subseteq f_G(x)$.

Example 3.11. Assume that universal set is the group $G = \{1, -1, i, -i\}$ and let G be the subset of set of parameters. If we define a soft set f_G by

$$f_G(x) = \{y \in G : y = x^n, n \in \mathbb{N}\}$$

for all $x \in G$, then $f_G(1) = \{1\}$, $f_G(-1) = \{-1, 1\}$, $f_G(i) = f_G(-i) = \{-1, 1, i, -i\} = G$. One can easily show that f_G is a soft uni-group over U. Here, $f_G(i) \subseteq f_G(-i)$, but $f_G(i.-i) = f_G(-i.i) \neq f_G(-i)$. In [8], Çağman et al. showed that the $\wedge$-product of two soft int-groups over U is a soft int-group over U. Here, we show that $\vee$-product of two soft uni-groups over U is a soft uni-group over U with the following theorem:

Theorem 3.12. *Let f_G and f_H be soft uni-groups over U. Then, $f_G \vee f_H$ is a soft uni-group over U.*

Proof: Let $f_G \vee f_H = f_{G \vee H}$, where $f_{G \vee H}(x, y) = f_G(x) \cup f_H(y)$ for all $(x, y) \in E \times E$. Since G and H are groups, then so is $G \times H$. Let $(x_1, y_1), (x_2, y_2) \in G \times H$. Then,

$$\begin{aligned} f_{G\vee H}((x_1,y_1)(x_2,y_2)^{-1}) &= f_{G\vee H}(x_1{x_2}^{-1},y_1{y_2}^{-1}) \\ &= f_G(x_1{x_2}^{-1})\cup f_H(y_1{y_2}^{-1}) \\ &\subseteq (f_G(x_1)\cup f_G(x_2))\cup(f_H(y_1)\cup f_H(y_2)) \\ &= (f_G(x_1)\cup f_H(y_1))\cup(f_G(x_2)\cup f_H(y_2)) \\ &= f_{G\vee H}(x_1,y_1)\cup f_{G\vee H}(x_2,y_2) \end{aligned}$$

Thus, $f_G \vee f_H$ is a soft uni-group over U.

Note that if f_G and f_H are two soft uni-groups over U, then $f_G \wedge f_H$ is not a soft uni-group over U as shown with the following example:

Example 3.13. Let f_G be the soft uni-group over S_3 in Example 3.2 and consider $H=\{\overline{0},\overline{3}\}\leq \mathbb{Z}_6$ as the subset of set of parameters of the soft set f_H defined by $f_H(\overline{0})=\{(13)\}$ and $f_H(\overline{3})=\{(13),(23),(132)\}$. One can easily show that f_H is a soft uni-group over S_3. We now consider the soft set $f_{G\wedge H}$ over S_3. Then,

$$\begin{aligned} f_{G\wedge H}((x,\overline{3})(yx,\overline{0})) &= f_{G\wedge H}(y,\overline{3}) \\ &= \{(13),(23)\}, \end{aligned}$$

$$\begin{aligned} f_{G\wedge H}(x,\overline{3})\cup f_{G\wedge H}(yx,\overline{0}) &= (f_G(x)\cap f_H(\overline{3}))\cup(f_G(yx)\cap f_H(\overline{0})) \\ &= \{(13)\}\cup\{(13)\} \\ &= \{(13)\}. \end{aligned}$$

It is obvious that $f_{G\wedge H}((x,\overline{3})(yx,\overline{0}))\nsubseteq f_{G\wedge H}(x,\overline{3})\cup f_{G\wedge H}(yx,\overline{0})$. Thus, $f_{G\wedge H}$ is not a soft uni-group over U. However, we have the following:

Theorem 3.14. *Let f_G and f_H be soft uni-groups over U. Then,*

$$f_{G\wedge H}((x_1,y_1)(x_2,y_2)^{-1})\subseteq f_{G\vee H}(x_1,y_1)\cup f_{G\vee H}(x_2,y_2)$$

for all $(x_1,y_1),(x_2,y_2)\in G\times H$.

Proof: Let $(x_1,y_1),(x_2,y_2) \in G\times H$. We have

$$\begin{aligned}
f_{G\wedge H}((x_1,y_1)(x_2,y_2)^{-1}) &= f_{G\wedge H}(x_1{x_2}^{-1}, y_1{y_2}^{-1})\\
&= f_G(x_1{x_2}^{-1}) \cap f_H(y_1{y_2}^{-1})\\
&\subseteq (f_G(x_1)\cup f_G(x_2)) \cap (f_H(y_1)\cup f_H(y_2))\\
&\subseteq (f_G(x_1)\cup f_G(x_2)) \cup (f_H(y_1)\cup f_H(y_2))\\
&= (f_G(x_1)\cup f_H(y_1)) \cup (f_G(x_2)\cup f_H(y_2))\\
&= f_{G\vee H}(x_1,y_1) \cup f_{G\vee H}(x_2,y_2)
\end{aligned}$$

Theorem 3.15. *If f_G and h_G are two soft uni-groups over U, then so is $f_G\widetilde{\cup}h_G$.*

Proof: Let $x,y \in G$. Then,

$$\begin{aligned}
(f_G\widetilde{\cup}h_G)(xy^{-1}) &= f_G(xy^{-1}) \cup h_G(xy^{-1})\\
&\subseteq (f_G(x)\cup f_G(y)) \cup (h_G(x)\cup h_G(y))\\
&= (f_G(x)\cup h_G(x)) \cup (f_G(y)\cup h_G(y))\\
&= (f_G\widetilde{\cup}h_G)(x) \cup (f_G\widetilde{\cup}h_G)(y),
\end{aligned}$$

Therefore, $f_G\widetilde{\cup}h_G$ is a soft uni-group over U.

The following theorem gives the relation between soft int-group and soft uni-group over U.

Theorem 3.16. *Let f_G be a soft set over U. Then, f_G is a soft uni-group over U if and only if f_G^c is a soft int-group over U.*

Proof: Let f_G be a soft uni-group over U. Then,

$$\begin{aligned}
f_G^c(xy^{-1}) &= U \setminus f_G(xy^{-1})\\
&\supseteq U \setminus ((f_G(x)\cup f_G(y))\\
&= (U\setminus f_G(x)) \cap (U \setminus f_G(y))\\
&= f_G^c(x) \cap f_G^c(y)
\end{aligned}$$

for all $x, y \in G$, which shows that f_G^c is a soft int-group over U.

Conversely, let f_G^c be a soft int-group over U. Then,

$$\begin{aligned} f_G(xy^{-1}) &= U \setminus f_G^c(xy^{-1}) \\ &\subseteq U \setminus (f_G^c(x) \cap f_G^c(y)) \\ &= (U \setminus f_G^c(x)) \cup (U \setminus f_G^c(y)) \\ &= f_G(x) \cup f_G(y) \end{aligned}$$

for all $x, y \in G$. Thus, f_G is a soft uni-group over U.

Above theorem shows that if a soft set is a soft uni-group over U, then its complement is a soft int-group over U and vice versa.

Definition 3.17. Let G be a group, H be a subgroup of G and f_G be a soft uni-group over U. If f_H, the soft subset of f_G, itself is a soft uni-group over U, then f_H is said to be a *soft uni-subgroup* of f_G over U and denoted by $f_H \widetilde{\leq}_u f_G$.

Example 3.18. Consider $U = \mathbb{Z}$ as the universal set and the additive group $G = \mathbb{Z}_4$ as the subset of set of parameters. Let us define a soft set f_G by $f_G(\overline{0}) = \{0, 2\}$, $f_G(\overline{1}) = f_G(\overline{3}) = \{0, 1, 2, 3\}$ and $f_G(\overline{2}) = \{0, 2, 3\}$. Then it is obvious that f_G is a soft uni-group over U. Let $H = \{\overline{0}, \overline{2}\} \leq \mathbb{Z}_4$ and f_H over $\mathbb{Z}$ be defined by $f_H(\overline{0}) = \{0\}$ and $f_H(\overline{2}) = \{0, 2\}$. Since f_H is a soft subset of f_G and itself is a soft uni-group over U, it follows that $f_H \widetilde{\leq}_u f_G$.

In [8], Çağman et al. showed that the intersection of two soft int-groups of f_G over U is a soft int-group of f_G over U. Here, we have a similar theorem for the restricted union of two soft uni-groups of f_G over U:

Theorem 3.19. *Let $f_H \widetilde{\leq}_u f_G$ and $f_K \widetilde{\leq}_u f_G$ over U. Then, $f_H \cup_{\mathscr{R}} f_K \widetilde{\leq}_u f_G$ over U.*

Proof: Let $f_H \cup_{\mathscr{R}} f_K = f_{H \cup_{\mathscr{R}} K}$, where $f_{H \cup_{\mathscr{R}} K}(x) = f_H(x) \cup f_K(x)$ for all $x \in H \cap K \neq \emptyset$. Let $x \in H \cap K$, then

$$\begin{aligned} f_{H \cup_{\mathscr{R}} K}(xy^{-1}) &= f_H(xy^{-1}) \cup f_K(xy^{-1}) \\ &\subseteq (f_H(x) \cup f_H(y)) \cup (f_K(x) \cup f_K(y)) \end{aligned}$$

(2)

$$
\begin{aligned}
&= (f_H(x) \cup f_K(x)) \cup (f_H(y) \cup f_K(y)) \\
&= f_{H \cup_{\mathscr{R}} K}(x) \cup f_{H \cup_{\mathscr{R}} K}(y)
\end{aligned}
$$

Therefore, $f_H \cup_{\mathscr{R}} f_K \widetilde{\leq}_u f_G$ over U.

Remark 3.20. If f_G is a soft uni-group over U, $f_N \widetilde{\leq}_u f_G$ and $f_H \widetilde{\leq}_u f_G$, then $f_N \vee f_H$ is not a soft uni-subgroup of f_G over U. Because if N and H are subgroups of G, then $N \times H$ is not a subgroup of G. Moreover, if f_N and f_H are two soft uni-groups of f_G over U, then the restricted intersection and intersection of f_N and f_H needs not be a soft uni-subgroup of f_G over U. However, we have the following:

Theorem 3.21. *Let $f_H \widetilde{\leq}_u f_G$ and $f_K \widetilde{\leq}_u f_G$ over U. Then,*

$$
f_{H \Cap K}(xy^{-1}) \subseteq f_{H \cup_{\mathscr{R}} K}(x) \cup f_{H \cup_{\mathscr{R}} K}(y)
$$

for all $x, y \in H \cap K$.

Proof: Assume that $x \in H \cap K$, then

$$
\begin{aligned}
f_{H \Cap K}(xy^{-1}) &= f_H(xy^{-1}) \cap f_K(xy^{-1}) \\
&\subseteq (f_H(x) \cup f_K(y)) \cap (f_H(x) \cup f_K(y)) \\
&\subseteq (f_H(x) \cup f_K(y)) \cup (f_H(x) \cup f_K(y)) \\
&= (f_H(x) \cup f_K(x)) \cup (f_H(y) \cup f_K(y)) \\
&= f_{H \cup_{\mathscr{R}} K}(x) \cup f_{H \cup_{\mathscr{R}} K}(y)
\end{aligned}
$$

Definition 3.22. Let f_G be a soft uni-group over U. Then, f_G is called an *soft abelian uni-group* over U, if $f_G(xy) = f_G(yx)$ for all $x, y \in G$.

It is obvious that if G is abelian, then f_G is a soft abelian int-group over U.

Definition 3.23. Let f_G be a soft uni-group over U and $f_N \widetilde{\leq}_u f_G$. Then, f_N is called a *normal soft uni-subgroup* of f_G over U if f_N is a soft abelian uni-group over U and denoted by $f_N \widetilde{\lhd}_u f_G$. It is clear that if G is an abelian group, then f_N is a normal soft uni-subgroup of f_G over U. It shows the analogy with the fact that a subgroup H of a group G is normal in G if G is an abelian group.

Example 3.24. Assume that $U = \mathbb{Z}_8$ is the universal set and let $G = S_3$ be the set of parameters. We construct a soft group f_G by $f_G(e) = \{\overline{2},\overline{3}\}$, $f_G(12) = f_G(23) = f_G(13) = \{\overline{1},\overline{2},\overline{3},\overline{5},\overline{7}\}$ and $f_G(123) = f_G(132) = \{\overline{1},\overline{2},\overline{3},\overline{5}\}$. One can easily show that f_G is a soft uni-group over U. Let $N = A_3 = \{e,(123),(132)\} \leq S_3$ be the alternating group and the soft set f_N over S_3 such that $f_N(e) = \{\overline{3}\}$ and $f_N(123) = F(132) = \{\overline{2},\overline{3},\overline{5}\}$. One can easily show that $f_N \widetilde{\leq}_u f_G$, moreover $f_N \widetilde{\lhd}_u f_G$.

Theorem 3.25. *Let $f_H \widetilde{\lhd}_u f_G$ and $f_K \widetilde{\lhd}_u f_G$ over U. Then, $f_H \cup_{\mathscr{R}} f_K \widetilde{\lhd}_u f_G$ over U.*

Proof: It has already been shown that union of two soft uni-subgroups of f_G is a soft uni-subgroup of f_G over U. Let $f_H \cup_{\mathscr{R}} f_K = f_{H\cup_{\mathscr{R}}K}$, where $f_{H\cup_{\mathscr{R}}K}(x) = f_H(x) \cup f_K(x)$ for all $x \in H \cap K \neq \emptyset$. Let $x \in H \cap K$, then

$$\begin{aligned} f_{H\cup_{\mathscr{R}}K}(xy) &= f_H(xy) \cup f_K(xy) \\ &= f_H(yx) \cup f_K(yx), \ \ since \ \ f_H, f_K \widetilde{\lhd}_u f_G, \\ &= f_{H\cup_{\mathscr{R}}K}(yx) \end{aligned}$$

Therefore $f_H \cup_{\mathscr{R}} f_K \widetilde{\lhd}_u f_G$ over U.

Theorem 3.26. *Let $f_H \widetilde{\leq}_u f_G$ over U such that $f_H(e) = f_H(x)$ for all $x \in H$ and $f_K \widetilde{\lhd}_u f_G$ over U. Then, $f_H \cup_{\mathscr{R}} f_K \widetilde{\lhd}_u f_G$ over U.*

Proof: The fact that union of two soft uni-subgroups of f_G is a soft uni-subgroup of f_G over U has already been shown, we now show that $f_H \cup_{\mathscr{R}} f_K = f_{H\cup_{\mathscr{R}}K}$ is a normal subsoft uni-group of f_G. We have that $f_H(xy) = f_H(yx)$ for all $y \in H$. Now,

$$\begin{aligned} f_{H\cup_{\mathscr{R}}K}(xy) &= f_H(xy) \cup f_K(xy) \\ &= f_H(yx) \cup f_K(yx) \\ &= f_{H\cup_{\mathscr{R}}K}(yx) \end{aligned}$$

Therefore, $f_H \cup_{\mathscr{R}} f_K \widetilde{\lhd}_u f_G$ over U.

4. Applications of soft uni-groups

In this section, we define soft anti image, e-left coset of a soft set and investigate these notions with respect to soft uni-group. Moreover, we give some applications of soft uni-group to group theory.

Definition 4.1. Let f_G be a soft uni-group over U. Then, *e-set* of f_G, denoted by G_{f_G}, is defined as

$$G_{f_G} = \{x \in G : f_G(x) = f_G(e)\}.$$

Theorem 4.2. *Let f_G be a soft uni-group over U. Then, G_{f_G} is a subgroup of G.*

Proof: It is obvious that $e \in G_{f_G}$ and $\emptyset \neq G_{f_G} \subseteq G$. We need to show that $xy^{-1} \in G_{f_G}$ for all $x,y \in G_{f_G}$. Since $x,y \in G_{f_G}$, then $f_G(x) = f_G(y) = f_G(e)$. $f_G(e) \subseteq f_G(xy^{-1})$ for all $x,y \in G_{f_G}$. Since f_G is a soft uni-group over U, then $f_G(xy^{-1}) \subseteq f_G(x) \cup f_G(y) = f_G(e)$ for all $x,y \in G_{f_G}$. It follows that G_{f_G} is a subgroup of G.

Note that if G is abelian, then G_{f_G} is a normal subgroup of G. Moreover, we have the following:

Proposition 4.3. *Let f_G be a soft abelian uni-group over U. Then, G_{f_G} is a normal subgroup of G.*

Proof: Let $g \in G$ and $x \in G_{f_G}$. Then,

$$\begin{aligned} f_G(gxg^{-1}) &= f_G(gg^{-1}x) \\ &= f_G(x) \\ &= f_G(e) \end{aligned}$$

Thus, $gxg^{-1} \in G_{f_G}$. Hence, G_{f_G} is normal in G.

Definition 4.4. Let f_G be a soft uni-group over U and $x \in G$. We define a map

$$x\hat{f}_G : G \to U$$

$x\hat{f}_G(g) = f_G(gx^{-1})$ for all $g \in G$. $\hat{f}_G$ is called the *left coset* of f_G determined by x and f_G. It is obvious that $e\hat{f}_G = f_G$.

Definition 4.5. Let f_G be a soft uni-group over U. Then *e-left coset set* of f_G, denoted by $G_{\hat{f}_G}$, is defined as

$$G_{\hat{f}_G} = \{x \in G : x\hat{f}_G = e\hat{f}_G\}$$

Theorem 4.6. *Let f_G be a soft uni-group over U. Then, $G_{\hat{f}_G}$ is a subgroup of G.*

Proof: It is obvious that $e \in G_{\hat{f}_G}$ and $\emptyset \neq G_{\hat{f}_G} \subseteq G$. We need to show that $xy^{-1} \in G_{\hat{f}_G}$, that is $f_G(g(xy^{-1})^{-1}) = f_G(ge^{-1}) = f_G(g)$ for all $g \in G$ and $x, y \in G_{\hat{f}_G}$. Since $x, y \in G_{\hat{f}_G}$ and f_G is a soft uni-group over U, it follows that $f_G(gx^{-1}) = f_G(gy^{-1}) = f_G(ge^{-1}) = f_G(g)$ for all $g \in G$ and $x, y \in G_{\hat{f}_G}$. Thus,

$$\begin{aligned} f_G(g(xy^{-1})^{-1}) &= f_G(g(yx^{-1})) \\ &= f_G(gy(g^{-1}g)x^{-1}) \\ &= f_G(g(yg^{-1})(gx^{-1})) \\ &\subseteq f_G(g) \cup f_G(yg^{-1}) \cup f_G(gx^{-1}) \\ &= f_G(g) \cup f_G((gy^{-1})^{-1}) \cup f_G(g) \\ &= f_G(g) \cup f_G(gy^{-1}) \cup f_G(g) \\ &= f_G(g) \cup f_G(g) \cup f_G(g) \\ &= f_G(g) \end{aligned}$$

for all $g \in G$ and $x, y \in G_{\hat{f}_G}$. Similarly, one can show that $f_G(g) \subseteq f_G(g(xy^{-1})^{-1})$, thus $f_G(g(xy^{-1})^{-1}) = f_G(g)$ for all $g \in G$ and $x, y \in G_{\hat{f}_G}$, implying that $xy^{-1} \in G_{\hat{f}_G}$ and $G_{\hat{f}_G}$ is a subgroup of G.

Theorem 4.7. *Let f_G be a soft uni-group over U. Then, $G_{f_G} = G_{\hat{f}_G}$.*

Proof: Let $a \in G_{\hat{f}_G}$. Then we have $a\hat{f}_G = e\hat{f}_G$. That is,

$$f_G(ga^{-1}) = f_G(g)$$

for all $g \in G$. If we choose $g = e$, then

$$f_G(a^{-1}) = f_G(e),$$

which means that $a^{-1} \in G_{f_G}$ and so $a \in G_{f_G}$, since G_{f_G} is a subgroup of G. Thus, we have that

$$G_{\hat{f}_G} \subseteq G_{f_G}.$$

On the other hand, let $b \in G_{f_G}$. Then,

$$f_G(b) = f_G(e).$$

In order to show that $b \in \hat{f}_G$, we need to prove $b\hat{f}_G = e\hat{f}_G$, that is

$$f_G(gb^{-1}) = f_G(g), \forall g \in G.$$

Let $g \in G$, then

$$\begin{aligned} f_G(gb^{-1}) &\subseteq f_G(g) \cup f_G(b^{-1}) \\ &= f_G(g) \cup f_G(b) \\ &= f_G(g) \cup f_G(e) \\ &= f_G(g) \end{aligned}$$

Again,

$$\begin{aligned} f_G(g) &= f_G(g(b^{-1}b)) \\ &= f_G(gb^{-1})b) \\ &\subseteq f_G(gb^{-1}) \cup f_G(b) \\ &= f_G(gb^{-1}) \cup f_G(e) \\ &= f_G(gb^{-1}) \end{aligned}$$

for all $g \in G$. It follows that

$$G_{f_G} \subseteq G_{\hat{f}_G}, \text{ thus } G_{\hat{f}_G} = G_{f_G}.$$

Definition 4.8. [8] Let f_A and f_B be soft sets over the common universe U and Ψ be a function from A to B. Then, *soft image* of f_A under Ψ, denoted by $\Psi(f_A)$, is a soft set over U defined as

$$(\Psi(f_A))(b) = \begin{cases} \bigcup\{f_A(a) \mid a \in A \ \ and \ \ \Psi(a) = b\}, & \text{if } \ \Psi^{-1}(b) \neq \emptyset, \\ \emptyset, & otherwise \end{cases}$$

for all $b \in B$. And *soft preimage (or soft inverse image*) of f_B under Ψ, denoted by $\Psi^{-1}(f_B)$, is a soft set over U by $(\Psi^{-1}(f_B))(a) = f_B(\Psi(a))$ for all $a \in A$.

Definition 4.9. Let f_A and f_B be soft sets over the common universe U and Ψ be a function from A to B. Then, *soft anti image* of f_A under Ψ, denoted by $\Psi^\star(f_A)$, is a soft set over U defined as

$$(\Psi^\star(f_A))(b) = \begin{cases} \bigcap\{f_A(a) \mid a \in A \ \ and \ \ \Psi(a) = b\}, & \text{if } \Psi^{-1}(b) \neq \emptyset, \\ \emptyset, & otherwise \end{cases}$$

for all $b \in B$.

Theorem 4.10. *Let f_{A_1} and f_{A_2} be two soft sets over the common universe U and Ψ be a function from A to B, where $A_1, A_2 \subseteq A$. Then,*

a) $\Psi^\star(f_{A_1} \tilde{\cap} f_{A_2}) = \Psi^\star(f_{A_1}) \tilde{\cap} \Psi^\star(f_{A_2})$.
b) *If $f_{A_1} \tilde{\subseteq} f_{A_2}$, then $\Psi^\star(f_{A_1}) \tilde{\subseteq} \Psi^\star(f_{A_2})$.*

Proof: **a)** The proof is obvious.

b) Let $b \in B$, then

$$\begin{aligned} (\Psi^\star(f_{A_1}))(b) &= \bigcap\{f_{A_1}(a) : a \in A_1, \Psi(a) = b\} \\ &\subseteq \bigcap\{f_{A_2}(a) : a \in A_2, \Psi(a) = b\} \\ &= (\Psi^\star(f_{A_2}))(b) \end{aligned}$$

which completes the proof.

Theorem 4.11. *Let f_Φ be the null soft set, $f_{\tilde{A}}$ be the A-universal soft set and let Ψ be a function from A to A. Then,*

a) $\Psi(f_\Phi) = f_\Phi$, $\Psi^{-1}(f_\Phi) = f_\Phi$ *and* $\Psi^\star(f_\Phi) = f_\Phi$.
b) $\Psi^{-1}(f_{\tilde{A}}) = f_{\tilde{A}}$. *And if Ψ is a surjective function, then $\Psi(f_{\tilde{A}}) = f_{\tilde{A}}$ and $\Psi^\star(f_{\tilde{A}}) = f_{\tilde{A}}$.*

Proof: The proof is obvious.

Theorem 4.12. *Let f_A and f_B be soft sets over U, f_A^c, f_B^c be their relative soft sets, respectively and Ψ be a function from A to B. Then,*

a) $\Psi^{-1}(f_B^c) = (\Psi^{-1}(f_B))^c$.
b) $\Psi(f_A^c) = (\Psi^\star(f_A))^c$ *and* $\Psi^\star(f_A^c) = (\Psi(f_A))^c$.

Proof: **a)** Let $a \in A$, then

$$\begin{aligned}(\Psi^{-1}(f_B^c))(a) &= f_B^c(\Psi(a)) \\ &= U \setminus f_B(\Psi(a)) \\ &= U \setminus \Psi^{-1}(f_B(a)) \\ &= (\Psi^{-1}(f_B))^c(a).\end{aligned}$$

Thus, $\Psi^{-1}(f_B^c) = (\Psi^{-1}(f_B))^c$.

b) Let $b \in B$, then

$$\begin{aligned}(\Psi(f_A^c))(b) &= \bigcup\{f_A^c(a) : a \in A, \Psi(a) = b\} \\ &= \bigcup\{U \setminus f_A(a) : a \in A, \Psi(a) = b\} \\ &= U \setminus \{(\bigcap f_A(a)) : a \in A, \Psi(a) = b\} \\ &= U \setminus \Psi^\star(f_A)(b) \\ &= (\Psi^\star(f_A))^c(b)\end{aligned}$$

Thus, $\Psi(f_A^c) = (\Psi^\star(f_A))^c$. And similarly,

$$\begin{aligned}(\Psi^\star(f_A^c))(b) &= \bigcap\{f_A^c(a) : a \in A, \Psi(a) = b\} \\ &= \bigcap\{U \setminus f_A(a) : a \in A, \Psi(a) = b\} \\ &= U \setminus \{(\bigcup f_A(a)) : a \in A, \Psi(a) = b\} \\ &= U \setminus \Psi(f_A)(b) \\ &= (\Psi(f_A))^c(b)\end{aligned}$$

Therefore, $\Psi^\star(f_A^c) = (\Psi(f_A))^c$.

Theorem 4.13. [8] *Let f_G and f_H be soft sets over U and Ψ be a group isomorphism from G to H. If f_G is a soft int-group over U, then $\Psi(f_G)$ is a soft int-group over U.*

Theorem 4.14. [8] *Let f_G and f_H be soft sets over U and Ψ be a group homomorphism from G to H. If f_H is a soft int-group over U, then $\Psi^{-1}(f_H)$ is a soft int-group over U.*

Theorem 4.15. *Let f_G and f_H be soft sets over U and Ψ be a group homomorphism from G to H. If f_H is a soft uni-group over U, then so is $\Psi^{-1}(f_H)$.*

Proof: Let f_H be a soft uni-group over U. Then, f_H^c is a soft int-group over U and $\Psi^{-1}(f_H^c)$ is a soft int-group over U. Thus, $\Psi^{-1}(f_H^c) = (\Psi^{-1}(f_H))^c$ is a soft int-group over U. Therefore, $\Psi^{-1}(f_H)$ is a soft uni-group over U.

Theorem 4.16. *Let f_G and f_H be soft sets over U and Ψ be a group isomorphism from G to H. If f_G is a soft uni-group over U, then so is $\Psi^{\star}(f_G)$.*

Proof: Let f_G be a soft uni-group over U. Then, f_G^c is a soft int-group over U and $\Psi(f_G^c)$ is a soft int-group over U. Thus, $\Psi(f_G^c) = (\Psi^{\star}(f_G))^c$ is a soft int-group over U. Therefore, $\Psi^{\star}(f_G)$ is a soft uni-group over U.

Theorem 4.17. *Let f_G and f_H be soft sets over U and Ψ be a group homomorphism from G to H. If $f_H \widetilde{\lhd}_u f_G$ over U, then $\Psi^{-1}(f_H) \widetilde{\lhd}_u f_G$ over U.*

Proof: If $f_H \widetilde{\lhd}_u f_G$ over U, then f_H itself is an soft uni-group over U. $\Psi^{-1}(f_H)$ is a soft uni-group over U. Therefore, we only show that $\Psi^{-1}(f_H)(xy) = \Psi^{-1}(f_H)(yx)$ for all $x, y \in G$. Let $x, y \in G$, then

$$\begin{aligned}
\Psi^{-1}(f_H)(xy) &= f_H(\Psi(xy)) \\
&= f_H(\Psi(x)\Psi(y)) \\
&= f_H(\Psi(y)\Psi(x)), \ \ \textit{since} \ \ f_H \widetilde{\lhd}_u f_G. \\
&= f_H(\Psi(yx)) \\
&= \Psi^{-1}(f_H)(yx)
\end{aligned}$$

Hence, $\Psi^{-1}(f_H) \widetilde{\lhd}_u f_G$ over U.

5. Conclusion

In this paper, by using soft sets and union operation of sets we define soft uni-group, based on the inclusion relation and union of sets and thus more functional for obtaining results in the mean of soft set theory with respect to group structure. We have then introduced the concepts of soft uni-subgroup, normal soft uni-subgroup, e-left coset and soft anti image of a soft set and investigate these notions with respect to soft uni-groups. Furthermore, we have pointed out the relation between soft int-group defined in [8] and soft uni-group and give some applications of soft uni-group to group theory. To extend this study, one can further study the other algebraic properties of soft uni-groups.

Conflict of Interests

The authors declare that there is no conflict of interests.

Acknowledgement

This paper is a part of the MSc thesis of the third author at Gaziosmanpaşa University, Graduate School of Natural and Applied Sciences, Tokat, Turkey.The first and fourth authors acknowledge the financial support by the FMB-BAP 16-0227 Project, entitled ”Construction of normal soft union groups and quotient groups” carried out by Amasya University, Turkey.

REFERENCES

[1] U. Acar, F. Koyuncu and B. Tanay, Soft sets and soft rings, Comput. Math. Appl. 59 (2010), 3458-3463.

[2] H. Aktaṣ and N. Çağman, Soft sets and soft groups, Inform. Sci. 177 (2007), 2726-2735.

[3] M.I. Ali, F. Feng, X. Liu, W.K. Min and M. Shabir, On some new operations in soft set theory, Comput. Math. Appl. 57 (2009), 1547-1553.

[4] A.O. Atagün and A. Sezgin, Soft substructures of rings, fields and modules, Comput. Math. Appl. 61 (2011), 592-601

[5] K.V. Babitha and J.J. Sunil, Soft set relations and functions, Comput. Math. Appl. 607 (2010), 1840-1849.

[6] N. Çağman and S. Enginoğlu, Soft matrix theory and its decision making, Comput. Math. Appl. 59 (2010), 3308-3314.

[7] N. Çağman and S. Enginoğlu, Soft set theory and uni-int decision making, Eur. J. Oper. Res. 207 (2010), 848-855.

[8] N. Çağman, F. Çıtak and H. Aktaş, Soft int-groups and its applications to group theory, Neural Comput. Appl. 21 (1) (2012), 151-158.

[9] F. Feng, Y. B. Jun and X. Zhao, Soft semirings, Comput. Math. Appl. 56 (2008), 2621-2628.

[10] F. Feng, X.Y. Liu, V. Leoreanu-Fotea, Y.B. Jun, Soft sets and soft rough sets, Inform. Sci. 181 (2011), 1125-1137.

[11] F. Feng, C. Li, B. Davvaz and M. I. Ali, Soft sets combined with fuzzy sets and rough sets: a tentative approach, Soft Comput. 14 (2010), 899-911.

[12] K. Gong, Z. Xiao and X. Zhang, The bijective soft set with its operations, Comput. Math. Appl. 60 (2010), 2270-2278.

[13] Y. Jiang, Y. Tang, Q. Chen, J. Wang and S. Tang, Extending soft sets with description logics, Comput. Math. Appl. 59 (2010), 2087-2096.

[14] Y.B. Jun, Soft BCK/BCI-algebras, Comput. Math. Appl. 56 (2008), 1408-1413.

[15] Y.B. Jun and C.H. Park, Applications of soft sets in ideal theory of BCK/BCI-algebras, Inform. Sci. 178 (2008), 2466-2475.

[16] Y.B. Jun, K.J. Lee and J. Zhan, Soft p-ideals of soft BCI-algebras, Comput. Math. Appl. 58 (2009), 2060-2068.

[17] O. Kazancı, Ş. Yılmaz and S. Yamak, Soft sets and soft BCH-algebras, Hacet. J. Math. Stat. 39 (2010), 205-217.

[18] P.K. Maji, R. Biswas and A.R. Roy, Soft set theory, Comput. Math. Appl. 45 (2003), 555-562.

[19] P.K. Maji, A.R. Roy and R. Biswas, An application of soft sets in a decision making problem, Comput. Math. Appl. 44 (2002), 1077-1083.

[20] P. Majumdar and S.K. Samanta, On soft mappings, Comput. Math. Appl. 60 (2010), 2666-2672.

[21] D. Molodtsov, Soft set theory-first results, Comput. Math. Appl. 37 (1999), 19-31.

[22] D.A. Molodtsov, V. Yu. Leonov and D. V. Kovkov, Soft sets technique and its application, Nechetkie Sistemy i Myagkie Vychisleniya 1 (2006), 8-39.

[23] A. Sezgin and A.O. Atagün, On operations of soft sets, Comput. Math. Appl. 61 (2011), no. 5, 1457-1467.

[24] A. Sezgin, A.O. Atagün and E. Aygün, A note on soft near-rings and idealistic soft near-rings, Filomat 25 (1) (2011) 53-68.

[25] A. Sezgin and A.O. Atagün, Soft groups and normalistic soft groups, Comput. Math. Appl. 62 (2) (2011), 685-698.

[26] A. Sezgin, A.O. Atagün and N. Çağman, Union soft substructures of near-rings and N-groups, Neural Comput. Appl. 21 (1) (2012), 133-143.

[27] A. Sezgin, A.O. Atagün and N. Çağman, Soft intersection near-rings with applications, Neural Comput. Appl. 21 (1) (2012), 221-229.

[28] A. Sezgin Sezer, A new view to ring theory via soft union rings, ideals and bi-ideals, Knowl-Based Syst. 36 (2012), 300-314.

[29] A. Sezgin, A.O. Atagün and N. Çağman, N-group SU-action and its application to N-group theory, Inf. Sci. Lett. 5 (3) (2016), 63-69.

[30] Y. Zou and Z. Xiao, Data analysis approaches of soft sets under incomplete information, Knowl-Based Syst. 21 (2008), 941-945.