Available online at http://scik.org
Algebra Letters, 2019, 2019:2
https://doi.org/10.28919/al/4145
ISSN: 2051-5502

α-TRANSLATIONS OF INTUITIONISTIC FUZZY (AT-SUBALGEBRAS) AT-IDEALS ON AT-ALGEBRAS

AREEJ TAWFEEQ HAMEED* AND ESRAA KAREEM KADHIM

Department of Mathematics, Faculty of Education for Girls, University of Kufa, Iraq

Abstract: In this paper, the concepts of α-translation of intuitionistic fuzzy AT-subalgebras and α-translation of intuitionistic fuzzy AT-ideals on AT-algebras are introduced. The notion of intuitionistic fuzzy extensions and intuitionistic fuzzy multiplications of fuzzy AT-ideals with several related properties are investigated. Also the relationships between α-translation of intuitionistic fuzzy, intuitionistic fuzzy extensions and intuitionistic fuzzy multiplications of fuzzy AT-ideals are investigated.

Keywords: fuzzy AT-ideal; intuitionistic fuzzy AT-subalgebra; intuitionistic fuzzy AT-ideal; α-translation of intuitionistic fuzzy; intuitionistic fuzzy extension; intuitionistic fuzzy multiplication.

2010 AMS Subject Classification: 06F35, 03G25, 08A72.

1. INTRODUCTION

After the introduction of fuzzy sets by Zadeh [26], there have been a number of generalizations of this fundamental concept. The notion of intuitionistic fuzzy sets introduced by Atanassov [2-3] is one among them. Fuzzy sets give a degree of membership of an element in a given set, while intuitionistic fuzzy sets give both degrees of membership and of nonmembership. Both degrees belong to the interval [0; 1], and their sum should not exceed 1. BCK-algebras and BCI-algebras are two important classes of logical algebras introduced by Imai and Iseki [12,13]. It is known that the

---

*Corresponding author
E-mail addresses: areej.tawfeeq@uokufa.edu.iq, areej238@gmail.com
Received May 29, 2019

class of BCK-algebra is a proper subclass of the class of BCI-algebras. In 1991, Xi [25] applied the concept of fuzzy sets to BCK-algebras. In 1993, Jun [14] and Ahmad [1] applied it to BCI-algebras. After that Jun, Meng, Liu and several researchers investigated further properties of fuzzy subalgebras and ideals in BCK/BCI-algebras (see [15-18]). In [27], Zhan and Tan discussed characterization of fuzzy H-ideals and doubt fuzzy H-ideals in BCK-algebras. Recently, Satyanarayana et al. [21-22] introduced intuitionistic fuzzy H-ideals in BCK-algebras. The concept of fuzzy translations in fuzzy subalgebras and ideals in BCK/BCI-algebras has been discussed respectively. They investigated relations among fuzzy translations, fuzzy extensions and fuzzy multiplications. Motivated by this, in [19], the authors have studied fuzzy translations of fuzzy H-ideals in BCK/BCI-algebras. They also extend this study from fuzzy translations to intuitionistic fuzzy translations in BCK/BCI-algebras.

AT-ideals and fuzzy AT-ideals on AT-algebras was defined by A. T. Hameed. She introduced the notion of fuzzy AT-ideal, intuitionistic fuzzy AT-ideal and intuitionistic fuzzy AT-ideal on AT-algebras in [4-11] and a lot of properties are investigated of its.

In this paper, α-translation of intuitionistic fuzzy AT-ideals, intuitionistic fuzzy extensions and intuitionistic fuzzy multiplications of fuzzy AT-ideals in AT-algebras are discussed. Relations among intuitionistic fuzzy AT-algebras are also investigated.

2. PRELIMINARIES

In this section, some elementary aspects that are necessary for this paper are included.

Definition 2.1. (**[4,5]**). Let $(X;*,0)$ be an algebra of type (2,0) with a single binary for any x, y, z $\in X$,

(AT_1): $(x*y)*((y*z)(x*z)) = 0$,

(AT_2): $0*x = x$,

(AT_3): $x*0 = 0$.

In X we can define a binary relation " $\le$ " by : $x \le y$ if and only if, $y*x = 0$.

Lemma 2.2. (**[4,5]**). In any AT-algebra $(X;*,0)$, the following properties hold: for all x, y, z $\in X$;

a) $x \le y$ implies that $y*z \le x*z$,

b) $x \le y$ implies that $z*x \le z*y$,

c) $x*y \le z$ imply $z*y \le x$,

d) $(y*z)*(x*z) \le x*y$,

e) $z * x \le z * y$ implies that $x \le y$(left cancellation law).

Definition 2.3. (**[4,5]**). Let $(X;*,0)$ be an AT-algebra and let S be a nonempty of X. S is called an AT-subalgebra of X , if $x * y \in S$ whenever $x \in S$ and $y \in S$.

Definition 2.4. (**[4,5]**). A nonempty subset I of anAT-algebra$(X;*,0)$ is called an AT-ideal of X if it satisfies: for x , y, z ∈ X,

(ATI_1) $0 \in I$,

(ATI_2) $x * (y * z) \in I$ and $y \in I$ imply $x * z \in I$.

Proposition 2.5.([4,5]). Every AT-ideal of AT-algebra$(X;*,0)$ is an AT-subalgebra.

Definition 2.6.[10]. Let $(X;*,0)$ be a nonempty set, a fuzzy subset μ in X is a function $\mu: X \to [0,1]$.

Definition 2.7. (**[4,5]).** Let $(X;*,0)$ be an AT-algebra, a fuzzy subset μ in X is called a fuzzy AT-subalgebra of X if for all x , y ∈X, $\mu(x * y) \ge \min\{\mu(x), \mu(y)\}$.

Definition 2.8. (**[4,5]**). Let $(X;*,0)$ be an AT-algebra, a fuzzy subset μ in X is called a fuzzy AT-ideal of X if it satisfies the following conditions: for all x , y, z ∈X ,

(FAT_1) $\mu(0) \ge \mu(x)$,

(FAT_2) $\mu(x * z) \ge \min\{\mu(x * (y * z)), \mu(y)\}$.

Proposition 2.9.([4,5]). Every fuzzy AT-ideal of AT-algebra X is a fuzzy AT-subalgebra.

Definition 2.10. [2]. Let $(X;*,0)$ be an AT-algebra, a fuzzy subset μ in X is called an anti-fuzzy AT-subalgebra of X if for all x , y ∈X, $\mu(x * y) \le \max\{\mu(x), \mu(y)\}$.

Definition 2.11. [2]. Let $(X;*,0)$ be an AT-algebra, a fuzzy subset μ in X is called an anti-fuzzy AT-ideal of X if it satisfies the following conditions: , for all x , y, z ∈X ,

(FAT_1) $\mu(0) \le \mu(x)$,

(FAT_2) $\mu(x * z) \le \max\{\mu(x * (y * z)), \mu(y)\}$.

Proposition 2.12. [2]. Every of anti-fuzzy AT-ideal AT-algebra X is an anti-fuzzy AT-subalgebra.

Definition 2.13.([3,6]). An intuitionistic fuzzy subset A in a nonempty set X is an object having the form $A = \{(x, \mu_A(x), \nu_A(x)) \mid x \in X\}$ where the functions $\mu_A: X \to [0,1]$ and $\nu_A: X \to [0,1]$ denote the degree of membership and the degree of non-membership respectively, and $0 \le \mu_A(x) + \nu_A(x) \le 1$ for all x∈X .

Remark 2.14.([3,6]). If an intuitionistic fuzzy subset A in a nonempty set X , then $\mu_A(x) + \nu_A(x) = 1$, i.e., when $\nu_A(x) = 1 - \mu_A(x) = \mu_A^c(x)$ for all that x ∈X . Now μ_A is named fuzzy subset while $\nu_A = \mu_A^c$ is the complement of μ_A.

Definition 2.15.([3,6]). Let $A = \{(x, \mu_A(x), \nu_A(x)) \mid x \in X\}$ be an intuitionistic fuzzy subset of AT-algebra X . A is said to be an intuitionistic fuzzy AT-subalgebra of X if: for all x, y ∈X,

(IFS$_1$) $\mu_A(x * y) \geq \min\{\mu_A(x), \mu_A(y)\}$.

(IFS$_2$) $\nu_A(x * y) \leq \max\{\nu_A(x), \nu_A(y)\}$.

That mean μ_A is a fuzzy AT-subalgebra and ν_A is an anti-fuzzy AT-subalgebra.

Proposition 2.16.([3,6]). Every intuitionistic fuzzy AT- subalgebra $\{(x, \mu_A(x), \nu_A(x)) | x \in X\}$ of AT-algebra X satisfies the inequalities $\mu_A(0) \geq \mu_A(x)$ and $\nu_A(0) \leq \nu_A(x)$, for all $x \in X$.

Definition 2.17.([3,6]). For any $t \in [0,1]$ and a fuzzy subset μ in a nonempty set X, the set $U(\mu, t) = \{x \in X \mid \mu(x) \geq t\}$ is called an upper t-level cut of μ, and the set $L(\mu, t) = \{x \in X \mid \mu(x) \leq t\}$ is called a lower t-level cut of μ.

Theorem 2.18.([3,6]). *An intuitionistic fuzzy subset $A = \{(x, \mu_A(x), \nu_A(x)) \mid x \in X\}$ is an intuitionistic fuzzy AT-subalgebra of AT-algebra X if and only if, for all that, $t \in [0,1]$, the set $U(\mu_A, t)$ and $L(\nu_A, s)$ are AT-subalgebras of X.*

Definition 2.19.([3,6]). Let $A = \{(x, \mu_A(x), \nu_A(x)) \mid x \in X\}$ be an intuitionistic fuzzy subset of AT-algebra X . A is said to be an intuitionistic fuzzy AT-ideal of X if : for all that x, y, z ∈X,

(IFI$_1$) $\mu_A(0) \geq \mu_A(x)$ and $\nu_A(0) \leq \nu_A(x)$.

(IFI$_2$) $\mu_A(x * z) \geq \min\{\mu_A(x * (y * z), \mu_A(y)\}$ and $\nu_A(z * x) \leq \max\{\nu_A(x * (y * z)), \nu_A(y)\}$

That mean μ_A is a fuzzy AT-ideal and ν_A is an anti-fuzzy AT-ideal.

Theorem 2.20. ([3, 6]). *An intuitionistic fuzzy subset $A = \{(x, \mu_A(x), \nu_A(x)) \mid x \in X\}$ is an intuitionistic fuzzy AT-ideal of AT-algebra X if and only if , for all , t ∈[0,1] , the set $U(\mu_A, t)$ is an AT-ideal and $L(\nu_A, s)$ is an anti-fuzzy AT-ideal of X.*

Proposition 2.21.[3,6]. Let $A = \{(x, \mu_A(x), \nu_A(x)) \mid x \in X\}$ be an intuitionistic fuzzy AT-ideal of AT-algebra X, then A is an intuitionistic fuzzy AT-subalgebra of X.

Theorem 2.22.([3,6]). *An intuitionistic fuzzy subset A=(μ_A, ν_A) an intuitionistic fuzzy AT-ideal of AT-algebra X if and only if, the fuzzy sets μ_A fuzzy AT-ideal and ν_A anti-fuzzy AT-ideal of X.*

3. A-TRANSLATION OF INTUITIONISTIC FUZZY AT-SUBALGEBRA ON AT-ALGEBRA

For the sake of simplicity, we shall use the symbol $A = (\mu_A, \nu_A)$ for the intuitionistic fuzzy subset $A = \{< x, \mu_A(x), \nu_A(x) >: x \in X\}$. Throughout this paper, we take $\xi = \inf\{\nu_A(x) | x \in X\}$ for any intuitionistic fuzzy set $A = (\mu_A, \nu_A)$ of X.

Definition 3.1 [9] Let $A = (\mu_A, \nu_A)$ be an intuitionistic fuzzy subset of X and let $\alpha \in [0, \xi]$ an object having the form $A_\alpha^T = \{(x, (\mu_A)_\alpha^T, (\nu_A)_\alpha^T)\}$ is called a α-translation of intuitionistic fuzzy of A if $(\mu_A)_\alpha^T(x) = \mu_A(x) + \alpha$ and $(\nu_A)_\alpha^T(x) = \nu_A(x) - \alpha$ for all x∈ X.

Definition 3.2 Let $A_\alpha^T = \{(x, (\mu_A)_\alpha^T, (\nu_A)_\alpha^T)\}$ be a α-translation of intuitionistic fuzzy of A. and $\alpha \in [0, \xi]$, A_α^T is said to be α-translation of intuitionistic fuzzy AT-subalgebra of X if : for all x, y ∈X,

(IFS1) $\mu_A(x * y) + \alpha \geq \min\{\mu_A(x) + \alpha, \mu_A(y) + \alpha\}$.

(IFS2) $\nu_A(x * y) - \alpha \leq \max\{\nu_A(x) - \alpha, \nu_A(y) - \alpha\}$.

That mean μ_A is a fuzzy AT-subalgebra and ν_A is an anti-fuzzy AT-subalgebra.

Theorem 3.3 *If $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy AT-subalgebra of X, then $A_\alpha^T = ((\mu_A)_\alpha^T, (\nu_A)_\alpha^T)$ is a α-translation of intuitionistic fuzzy AT-subalgebra of X, for all $\alpha \in [0, \xi]$.*

Proof.

Let $A = (\mu_A, \nu_A)$ be an intuitionistic fuzzy AT-subalgebra of X and $\alpha \in [0, \xi]$. Then for all x, y ∈ X.

$$(\mu_A)_\alpha^T(x * y) = \mu_A(x * y) + \alpha \geq \min\{\mu_A(x), \mu_A(y)\} + \alpha$$
$$= \min\{\mu_A(x) + \alpha, \mu_A(y) + \alpha\} = \min\{(\mu_A)_\alpha^T(x), (\mu_A)_\alpha^T(y)\}$$

and $(\nu_A)_\alpha^T(x * y) = \nu_A(x * y) - \alpha \leq \max\{\nu_A(x), \nu_A(y)\} - \alpha$

$$= \max\{\nu_A(x) - \alpha, \nu_A(y) - \alpha\} = \max\{(\nu_A)_\alpha^T(x), (\nu_A)_\alpha^T(y)\}$$

Hence, $A_\alpha^T = ((\mu_A)_\alpha^T, (\nu_A)_\alpha^T)$ of A is a α-translation of intuitionistic fuzzy AT-subalgebra of X.

Proposition 3.4. Every $A_\alpha^T = \{(x, (\mu_A)_\alpha^T, (\nu_A)_\alpha^T)\}$ is α-translation of intuitionistic fuzzy AT-subalgebra of AT-algebra X satisfies the inequalities, for all x∈ X

$\mu_A(0) \geq \mu_A(x)$ and $\nu_A(0) \leq \nu_A(x)$.

Proof.

$\mu_A(0) + \alpha = \mu_A(x * x) + \alpha \geq \min\{\mu_A(x) + \alpha, \mu_A(x) + \alpha\} = \mu_A(x) + \alpha$, then $\mu_A(0) \geq \mu_A(x)$ and $\nu_A(0)_\alpha = \nu_A(x * x) - \alpha \leq \max\{\nu_A(x) - \alpha, \nu_A(x) - \alpha\} = \nu_A(x) - \alpha$, then $\nu_A(0) \leq \nu_A(x)$

Theorem 3.5 *Let $A = (\mu_A, \nu_A)$ be an intuitionistic fuzzy subset of X such that the α-translation of intuitionistic fuzzy $A_\alpha^T = ((\mu_A)_\alpha^T, (\nu_A)_\alpha^T)$ of A is fuzzy AT-subalgebra of X. for some $\alpha \in [0, \xi]$. Then $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy AT-subalgebra of X.*

Proof.

Assume that $A_\alpha^T = ((\mu_A)_\alpha^T, (\nu_A)_\alpha^T)$ is a α-translation of intuitionistic fuzzy AT-subalgebra , for some $\alpha \in [0, \xi]$. Let x, y∈ X,

$$\mu_A(x * y) + \alpha = (\mu_A)_\alpha^T(x * y) \geq \min\{(\mu_A)_\alpha^T(x), (\mu_A)_\alpha^T(y)\} = \min\{\mu_A(x) + \alpha, \mu_A(y) + \alpha\}$$

$= \min\{\mu_A(x), \mu_A(y)\} + \alpha\}$ and

$\nu_A(x * y) - \alpha = (\nu_A)^T_\alpha(x * y) \leq \max\{(\nu_A)^T_\alpha(x), (\nu_A)^T_\alpha(y)\} = \max\{\nu_A(x) - \alpha, \nu_A(y) - \alpha\}$

$= \max\{\nu_A(x), \nu_A(y)\} - \alpha$

which implies that $\mu_A(x * y) \geq \min\{\mu_A(x), \mu_A(y)\}$ and $\nu_A(x * y) \leq \max\{\nu_A(x), \nu_A(y)\}$, then $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy AT-subalgebra of X.

Definition 3. 6.[6]. For any $t \in [0,1], \alpha \in [0, \xi]$ and a fuzzy subset μ in a nonempty set X, the set $U_\alpha(\mu, t) = \{x \in X \mid \mu(x) + \alpha \geq t\}$ is called α-translation of upper t-level cut of μ, and the set $L_\alpha(\mu, t) = \{x \in X \mid \mu(x) - \alpha \leq t\}$ is called α-translation of lower t-level cut of μ.

Theorem 3.7. *A α-translation of intuitionistic fuzzy $A^T_\alpha = \{(x, (\mu_A)^T_\alpha, (\nu_A)^T_\alpha)\}$ is α-translation of intuitionistic fuzzy AT-subalgebra of AT-algebra X if and only if, for all that, $t \in [0,1]$, the set $U_\alpha(\mu_A, t)$ is an AT-subalgebras of X and $L_\alpha(\nu_A, s)$ is an anti -AT-subalgebras of X.*

Proof.

Let $A^T_\alpha = \{(x, (\mu_A)^T_\alpha, (\nu_A)^T_\alpha)\}$ be a α-translation of intuitionistic fuzzy AT-subalgebra of X and $U_\alpha(\mu_A, t) \neq \emptyset \neq L_\alpha(\nu_A, s)$. and follow for every $x, y \in X$ such as $x, y \in U_\alpha(\mu_A, t)$, $x, y \in L_\alpha(\nu_A, s)$, then $\mu_A(x) + \alpha \geq t$ and $\mu_A(y) + \alpha \geq t$, so therefore $\mu_A(x * y) + \alpha \geq \min\{\mu_A(x) + \alpha, \mu_A(y) + \alpha\} = \min\{\mu_A(x), \mu_A(y)\} + \alpha \geq t$, so $(x * y) \in U_\alpha(\mu_A, t)$. thus $U_\alpha(\mu_A, t)$ this an AT-subalgebra from X.

In a similar way, we can prove that $L_\alpha(\nu_A, s)$ is an anti -AT-subalgebra of X.

Conversely, assume that for each , $t \in [0,1]$, the sets $U_\alpha(\mu_A, t)$is an AT-subalgebras of X and $L_\alpha(\nu_A, s)$ is an anti -AT-subalgebras of X. Let $x', y' \in U_\alpha(\mu_A, t)$ be such that $\mu_A(x' * y') + \alpha < \min\{\mu_A(x') + \alpha, \mu_A(y') + \alpha\}$. Then by taking $t_0 = \frac{1}{2}\{(\mu_A(x' * y') + \alpha) + \min\{\mu_A(x') + \alpha, \mu_A(y') + \alpha\}\}$, we get $\mu_A(x' * y') + \alpha < t_0 < \min\{\mu_A(x') + \alpha, \mu_A(y') + \alpha\}$ and hence $(x' * y') \notin U_\alpha(\mu_A, t_0)$, $x' \in U_\alpha(\mu_A, t_0)$, $y' \in U_\alpha(\mu_A, t_0)$, i.e. $U_\alpha(\mu_A, t_0)$, is not an AT-subalgebra of X, which make a contradiction. Hence $U_\alpha(\mu_A, t_0)$ is an AT-subalgebra of X.

Finally, assume $x', y' \in L_\alpha(\mu_A, t)$ be such that $\nu_A(x' * y') - \alpha > \max\{\nu_A(x') - \alpha, \nu_A(y') - \alpha\}$. Then by taking $s_0 = \frac{1}{2}\{(\nu_A(x' * y') - \alpha) + \max\{\nu_A(x') - \alpha, \nu_A(y') - \alpha\}\}$, we get $\max\{\nu_A(x') - \alpha, \nu_A(y') - \alpha\} > s_0 > (\nu_A(x' * y') - \alpha)$ and hence $(x' * y') \notin L_\alpha(\nu_A, s_0)$, $x' \in L_\alpha(\nu_A, s_0)$, $y' \in L_\alpha(\nu_A, s_0)$, i.e. $L_\alpha(\nu_A, s_0)$, is not an anti -AT-subalgebra of AT-algebra X, which make a contradiction. Therefore, $L_\alpha(\nu_A, s_0)$ is an anti -AT-subalgebra of X. Hence $A^T_\alpha = \{(x, (\mu_A)^T_\alpha, (\nu_A)^T_\alpha)\}$ is a α-translation of intuitionistic fuzzy AT-subalgebra of X.

Theorem 3.8 . *Let $A^T_\alpha = \{(x, (\mu_A)^T_\alpha, (\nu_A)^T_\alpha)\}$ is a α-translation of intuitionistic fuzzy of A. If there exists as sequence $\{x_n\}$ in X such that $\mu_A(x_n) + \alpha$ =1 and $\nu_A(x) - \alpha$ =0 , then $\mu_A(0) + \alpha$=1 and $\nu_A(0) - \alpha$=0.*

Proof .

By proposition (3.5), $\mu_A(0) \geq \mu_A(x)$, for all x∈ X, therefore, $\mu_A(0) + \alpha \geq \mu_A(x_n) + \alpha$ for every positive integer n. Consider, $1 \geq \mu_A(0) + \alpha \geq \mu_A(x_n) + \alpha = 1$. Hence $\mu_A(0) + \alpha = 1$.

Again by proposition (3.5), $\nu_A(0) \leq \nu_A(x)$, thus x∈ X, thus $\nu_A(0) - \alpha \leq \nu_A(x_n) - \alpha$, for every positive integer n. Now, $0 \leq \nu_A(0) - \alpha \leq \nu_A(x_n) - \alpha = 0$. Hence $\nu_A(0) - \alpha = 0$.

Definition 3.9. Let $A^T_\alpha = ((\mu_A)^T_\alpha, (\nu_A)^T_\alpha)$ and $B^T_\alpha = ((\mu_B)^T_\alpha, (\nu_B)^T_\alpha)$ be two α-translation of intuitionistic fuzzy sets of A, then the intersection of A^T_α and B^T_α are denoted by $A^T_\alpha \cap B^T_\alpha$ and is given by $A^T_\alpha \cap B^T_\alpha = \{\min\{(\mu_A)^T_\alpha, (\mu_B)^T_\alpha\}, \max\{(\nu_A)^T_\alpha, (\nu_B)^T_\alpha\}\}$.

Also, the complement of A denoted by $\overline{A}^T_\alpha$ and is defined by $\overline{A}^T_\alpha = ((\nu_A)^T_\alpha, (\mu_A)^T_\alpha)$.

Theorem 3.10. *Let $A_1$and A_2be two α-translation of intuitionistic fuzzy AT-subalgebras of AT-algebra X. Then $A_1 \cap A_2$is α-translation of intuitionistic fuzzy AT-subalgebra of X.*

Proof.

Let x, y ∈ $A_1 \cap A_2$, then x, y ∈ $A_1$and x, y ∈ A_2, then

$$(\mu_{A_1\cap A_2})^T_\alpha(x*y) = \min\{(\mu_{A_1})^T_\alpha(x*y), (\mu_{A_2})^T_\alpha(x*y)\}$$
$$\geq \min\{\min\{(\mu_{A_1})^T_\alpha(x), (\mu_{A_1})^T_\alpha(y)\}, \min\{(\mu_{A_2})^T_\alpha(x), (\mu_{A_2})^T_\alpha(y)\}\}$$
$$= \min\{\min\{(\mu_{A_1})^T_\alpha(x), (\mu_{A_2})^T_\alpha(x)\}, \min\{(\mu_{A_1})^T_\alpha(y), (\mu_{A_2})^T_\alpha(y)\}\}$$
$$= \min\{(\mu_{A_1\cap A_2})^T_\alpha(x), (\mu_{A_1\cap A_2})^T_\alpha(y)\} \quad \text{and}$$
$$(\nu_{A_1\cap A_2})^T_\alpha(x*y) = \max\{(\nu_{A_1})^T_\alpha(x*y), (\nu_{A_2})^T_\alpha(x*y)\}$$
$$\leq \max\{\max\{(\nu_{A_1})^T_\alpha(x), (\nu_{A_1})^T_\alpha(y)\}, \max\{(\nu_{A_2})^T_\alpha(x), (\nu_{A_2})^T_\alpha(y)\}\}$$
$$= \max\{\max\{(\nu_{A_1})^T_\alpha(x), (\nu_{A_2})^T_\alpha(x)\}, \max\{(\nu_{A_1})^T_\alpha(y),, (\nu_{A_2})^T_\alpha(y)\}\}$$
$$= \max\{(\nu_{A_1\cap A_2})^T_\alpha(x), (\nu_{A_1\cap A_2})^T_\alpha(y)\}$$

Hence $A_1 \cap A_2$is α-translation of intuitionistic fuzzy AT-subalgebra of X.

Corollary 3.11. Let$\{A_i|i=1,2,3,\ldots\}$be a family of α-translation of intuitionistic fuzzy AT-subalgebras of X, where $\cap A_i = (\min(\mu_{Ai})^T_\alpha(x), \max(v_{Ai})^T_\alpha(x))$. Then $\cap A_i$ is α-translation of intuitionistic fuzzy AT-subalgebra of X.

Theorem 3.12. *$A^T_\alpha = ((\mu_A)^T_\alpha, (v_A)^T_\alpha)$ is α-translation of intuitionistic fuzzy AT-subalgebra of X if and only if, the fuzzy sets $A_1=\{(\mu_A)^T_\alpha(x)|x\in A\}$ is fuzzy AT-subalgebras of X and $A_2=\{(\bar{v})^T_\alpha(x)|x\in A\}$ is anti-fuzzy AT-subalgebras of X.*

Proof .

Let $A^T_\alpha = ((\mu_A)^T_\alpha, (v_A)^T_\alpha)$ be α-translation of intuitionistic fuzzy AT-subalgebra of X. Cleary $A_1$is a fuzzy AT-subalgebra of X. For every x, y ∈ X, we have

$(\bar{v}_A)^T_\alpha(x * y) = 1 - (v_A)^T_\alpha(x * y) \geq 1 - \max\{(v_A)^T_\alpha(x), (v_A)^T_\alpha(y)\} = \max\{1 - (v_A)^T_\alpha(x), 1 - (v_A)^T_\alpha(y)\}$

$= \max\{(\bar{v}_A)^T_\alpha(x), (\bar{v}_A)^T_\alpha(y)\}$.

Hence $A_2$is an aniy-fuzzy AT-subalgebra of X.

Conversely, assume that A_1 is fuzzy AT-subalgebras of X and A_2 is anti-fuzzy AT-subalgebras of X. For every x, y ∈ X,

$(\mu_A)^T_\alpha(x * y) \geq \min\{(\mu_A)^T_\alpha(x), (\mu_A)^T_\alpha(y)\}$ and $(v_A)^T_\alpha(x * y) \leq \max\{(v_A)^T_\alpha(x), (v_A)^T_\alpha(y)\}$.

Hence $A^T_\alpha = ((\mu_A)^T_\alpha, (v_A)^T_\alpha)$ be α-translation of intuitionistic fuzzy AT-subalgebra of X.

For any element x and y of X, let us write $\Pi^n x * y$ for $x*(\cdots*(x* y)))$, where x occurs n times.

Theorem 3.13. *Let $A^T_\alpha = ((\mu_A)^T_\alpha, (v_A)^T_\alpha)$ be α-translation of intuitionistic fuzzy AT-subalgebra of X and let n∈ N (the set of natural numbers) then for all x∈ X.*

(i) $(\mu_A)^T_\alpha(\Pi^n x * x) \geq (\mu_A)^T_\alpha(x)$, *for any add number n,*

(ii) $(v_A)^T_\alpha(\Pi^n x * x) \leq (\mu_A)^T_\alpha(x)$, *for any add number n,*

(iii) $(\mu_A)^T_\alpha(\Pi^n x * x) = (\mu_A)^T_\alpha(x)$, *for any even number n,*

(iv) $(v_A)^T_\alpha(\Pi^n x * x) = (\mu_A)^T_\alpha(x)$, *for any even number n.*

Proof.

Let $x \in X$ and assum that n is odd, then n=2p-1 for some positive integer p. We prove the theorem by induction.

Now, $(\mu_A)^T_\alpha(x * x) = (\mu_A)^T_\alpha(0) \geq (\mu_A)^T_\alpha(x)$ and $(v_A)^T_\alpha(x * x) = (v_A)^T_\alpha(0) \leq (v_A)^T_\alpha(x)$.

Suppose that $(\mu_A)^T_\alpha(\Pi^{2p-1}x*x) \geq (\mu_A)^T_\alpha(x)$ and$(v_A)^T_\alpha(\Pi^{2p-1}x*x) \leq (v_A)^T_\alpha(x)$, then by assumption,

$(\mu_A)^T_\alpha(\Pi^{2(p+1)-1}x*x) = (\mu_A)^T_\alpha(\Pi^{2p+1}x*x) = (\mu_A)^T_\alpha(\Pi^{2p-1}x*(x*(x*x))) = (\mu_A)^T_\alpha(\Pi^{2p-1}x*x) \geq (\mu_A)^T_\alpha(x)$ and

$(v_A)^T_\alpha(\Pi^{2(p+1)-1}x*x) = (v_A)^T_\alpha(\Pi^{2p+1}x*x) = (v_A)^T_\alpha(\Pi^{2p-1}x*(x*(x*x))) = (v_A)^T_\alpha(\Pi^{2p-1}x*x) \geq (v_A)^T_\alpha(x)$, which proves (*i*)and(*ii*). Proofs are similar to the cases (*iii*) and (*iv*).

Definition 3.14.

The sets $\{x\in X|(\mu_A)^T_\alpha(x)=(\mu_A)^T_\alpha(0)\}$ is denoted by I_{μ_A} AT- subalgebra of X and $\{x\in X|(v_A)^T_\alpha(x)=(v_A)^T_\alpha(0)\}$is denoted by I_{v_A} AT-subalgebra of X.

Theorem 3.15. *Let* $A^T_\alpha = ((\mu_A)^T_\alpha, (v_A)^T_\alpha)$ *be α-translation of intuitionistic fuzzy AT-subalgebra of X, then the sets* I_{μ_A} *and* I_{v_A} *are AT-subalgebras of X.*

Proof.

Let x, y∈ I_{μ_A}, then $(\mu_A)^T_\alpha(x)=\mu_A(x) + \alpha = \mu_A(0) + \alpha = (\mu_A)^T_\alpha(0)$ and $(\mu_A)^T_\alpha(y)=\mu_A(y) + \alpha = \mu_A(0) + \alpha = (\mu_A)^T_\alpha(0)$ and so, $(\mu_A)^T_\alpha(x* y)= \mu_A(x*y) + \alpha \geq \min\{\mu_A(x)+\alpha, \mu_A(y)+\alpha\} = \min\{\mu_A(x), \mu_A(y)\} + \alpha \leq \mu_A(0)+\alpha$ by proposition (3.5), we know that$(\mu_A)^T_\alpha(x* y) \leq (\mu_A)^T_\alpha(0)$ or equivalently $x* y \in I_{\mu_A}$. Hence I_{μ_A} is AT-subalgebra of X.

Again, let x, y∈ I_{v_A}, then$(v_A)^T_\alpha(x)= v_A(x) - \alpha = v_A(0) - \alpha = (v_A)^T_\alpha(0)=v_A(y) - \alpha = (v_A)^T_\alpha(y)$ and so,$(v_A)^T_\alpha(x* y) = v_A(x*y) - \alpha \leq \max\{v_A(x) -\alpha, v_A(y) -\alpha\}= v_A(0) -\alpha$. Again by proposition (3.5), we know that $(v_A)^T_\alpha(x* y)=(v_A)^T_\alpha(0)$ or equivalently $x* y \in I_{v_A}$. Hence I_{v_A} is AT-subalgebra of X.

Theorem 3.16. *Let B a nonempty subset of AT-algebra X and* $A^T_\alpha = ((\mu_A)^T_\alpha, (v_A)^T_\alpha)$ *be α-translation of intuitionistic fuzzy subset on X defined by*

$$(\mu_A)^T_\alpha(x) = \begin{cases} \lambda & if\ x \in B, \\ \tau & otherwise \end{cases} \quad and \quad (v_A)^T_\alpha = \begin{cases} \gamma & if\ x \in B, \\ \delta & otherwise \end{cases}$$

for all λ, τ, γ *and* $\delta \in D[0,1]$, *with* $\lambda \geq \tau$ *and* $\gamma \leq \delta$*and* $\lambda + \gamma \leq 1; \tau + \delta \leq 1$. *An other* A^T_α *is α-translation of intuitionistic fuzzy AT-subalgebra of X if and only if, B is AT-subalgebra of X. Furthermore it,* $I_{\mu_A} = B = I_{v_A}$.

Proof.

Let *A* be α-translation of intuitionistic fuzzy of AT-subalgebra of X and x, y ∈X, be such that x, y ∈B. then $(\mu_A)^T_\alpha(x* y) \geq \min\{(\mu_A)^T_\alpha(x), (\mu_A)^T_\alpha\} = \min\{\lambda, \lambda\}=\lambda$ and $(v_A)^T_\alpha(x* y) \leq \max\{(v_A)^T_\alpha(x),(v_A)^T_\alpha(y)\}= \max\{\gamma, \gamma\}=\gamma$. Thus x∗ y ∈B, therefore B is an AT-subalgebra from *X* .

Conversely, assume that B is AT-subalgebra from X and let x, y ∈X. study two cases.

Case1:-If $x, y \in B$, then $x * y \in B$, so $(\mu_A)^T_\alpha(x* y)= \mu_A(x * y) + \alpha=\lambda$ =min{μ_A(x)+α,μ_A(y) +α}and $(v_A)^T_\alpha(x* y)=v_A(x * y) - \alpha = \gamma$=max{$v_A$(x) −α,$v_A$(y) −α}.

Case 2:-If x∉B or y∉B, then $(\mu_A)^T_\alpha(x* y)= \mu_A(x * y) + \alpha \geq \tau$ =min{μ_A(x) +α,μ_A(y) +α} and $(v_A)^T_\alpha(x* y)= v_A(x * y) - \alpha \leq \delta$ =max{v_A(x) −α,v_A(y) −α}.

Hence, A^T_α is α-translation of intuitionistic fuzzy AT-subalgebra of X. Also,

I_{μ_A}={x∈X|$(\mu_A)^T_\alpha$(x)=$(\mu_A)^T_\alpha$(0)}={x∈X|$(\mu_A)^T_\alpha$(x)= λ}=B and

I_{v_A}={x∈X|$(v_A)^T_\alpha$(x)=$(v_A)^T_\alpha$(0)}=={x∈X|$(v_A)^T_\alpha$(x)=γ}=B.

Theorem 3.17. *Any AT-subalgebra of AT-algebra X can be realized as both the α-translation of upper Level and α-translation of Lower Level of some α-translation of intuitionistic fuzzy AT-subalgebra of X.*

Proof.

Let P be α-translation of intuitionistic fuzzy AT-subalgebra of X and A be α-translation of intuitionistic fuzzy subset on X defined by:

$$(\mu_A)^T_\alpha(x)=\begin{cases}\lambda, & \text{if } x \in X\\ 0, & \text{otherwise}\end{cases} \quad \text{and} \quad (v_A)^T_\alpha(x)=\begin{cases}\tau, & \text{if } x \in X\\ 1, & \text{othrwise}\end{cases}$$

for all $\lambda, \tau \in [0,1]$ and $\lambda + \tau \leq 1$.We consider the following cases

Case 1:- If x, y ∈ P, then $(\mu_A)^T_\alpha(x)=\lambda$, $(v_A)^T_\alpha(x)= \tau$ and $(\mu_A)^T_\alpha(y)= \lambda$,$(v_A)^T_\alpha(y)= \tau$.

Thus, $(\mu_A)^T_\alpha(x* y)=\mu_A + \alpha = \lambda$ =min{λ, λ}=min{μ_A(x) +α,μ_A(y) +α}and

$(v_A)^T_\alpha(x* y) =v_A - \alpha = \tau$= max{τ, τ}= max{$v_A$(x)−α, v_A(y) −α}

Case 2:- If x ∈ P and y∉ P, then $(\mu_A)^T_\alpha(x)= \lambda$, $(v_A)^T_\alpha(x)=\tau$ and $(\mu_A)^T_\alpha(y)=0$,$(v_A)^T_\alpha(y)=1$.

Thus, $(\mu_A)^T_\alpha(x* y)= \mu_A(x * y) +\alpha \geq 0$ = min{λ, 0}=min{μ_A(x) +α,μ_A(y) +α}and

$(v_A)^T_\alpha(x* y) = v_A(x * y) - \alpha \leq 1$ =max{τ, 1}=max{v_A(x)−α, v_A(y) −α}.

Case3:- If x∉ P and y∈ P, then $(\mu_A)^T_\alpha(x) = 0$, $(v_A)^T_\alpha(x)=1$and $(\mu_A)^T_\alpha(y)=0$,$(v_A)^T_\alpha(y)=1$

Thus, $(\mu_A)^T_\alpha(x* y) = \mu_A + \alpha \geq 0$= min{0,λ}=min{$\mu_A$(x) +α,$\mu_A$(y) +α}and

$(v_A)^T_\alpha(x* y) = v_A - \alpha \leq 1$=max{1, τ}=max{$v_A$(x)−α, v_A(y) −α}.

Case4 If x∉ P, y ∉ P, then $(\mu_A)^T_\alpha(x) = 0$, $(v_A)^T_\alpha(x)=1$and $(\mu_A)^T_\alpha(y)=0$,$(v_A)^T_\alpha(y)= 1$

Now, $(\mu_A)^T_\alpha(x* y) = \mu_A + \alpha \geq 0$ =min{0,0}=min{μ_A(x) +α,μ_A(y) +α}and

$(v_A)^T_\alpha(x* y) = v_A - \alpha \leq 1$ =max{1,1}=max{v_A(x)−α, v_A(y) −α}.

Therefore, A is α-translation of intuitionistic fuzzy AT-subalgebra of X.

Theorem 3.18. *Let X be AT-algebra, another any offered chain of AT-subalgebras $p_0 \subset p_1 \subset p_2 \subset \ldots \subset p_r = X$, exists α-translation of intuitionistic fuzzy AT-subalgebra A of X, whose upper level AT-subalgebras are exactly the AT-subalgebras and lower level anti -AT-subalgebras are exactly the AT-subalgebras of the chain.*

Proof.

Consider set of numbers $s_0 > s_1 > \ldots > s_r$ and $t_0 < t_1 \ldots < t_r$, where each $s_i, t_i \in [0,1]$ and $s_i + t_i \leq 1$. Let $A_\alpha^T = ((\mu_A)_\alpha^T, (v_A)_\alpha^T)$ be α-translation of intuitionistic fuzzy A defined by

$(\mu_A)_\alpha^T(x) = \begin{cases} s_0, & \text{if } x \in p_0 \\ s_i, & \text{if } x \in p_i - p_{i-1} \end{cases}$ (1) and $(v_A)_\alpha^T(x) = \begin{cases} t_0, & \text{if } x \in p_0\ t_i \\ t_i, & \text{if } x \in p_i - p_{i-1} \end{cases}$ (2)

$0 < i \leq r$ We consider the following two cases:

Case 1:- Let x, y$\in p_i - p_{i-1}$. Therefore, by (1) and(2), $(\mu_A)_\alpha^T(x) = (\mu_A)_\alpha^T(y) = s_i$ and $(v_A)_\alpha^T(x) = (v_A)_\alpha^T(y) = t_i$. Since p_i is an AT-subalgebra, we have $x * y \in p_i$, and so either $x * y \in p_i - p_{i-1}$ or $x * y \in p_{i-1}$. In any case, we conclude that

$(\mu_A)_\alpha^T(x * y) = \mu_A + \alpha \geq s_i = \min\{\mu_A(x) + \alpha, \ \mu_A(y) + \alpha\}$ and

$(v_A)_\alpha^T(x * y) = v_A - \alpha \leq t_i = \max\{v_A(x) - \alpha, \ v_A(y) - \alpha\}$.

Case 2:- Let $x \in p_i - p_{i-1}$ and $y \in p_i - p_{i-1}$, for $i > j$, therefore, by (1) and (2), $(\mu_A)_\alpha^T(x) = s_i$, $(v_A)_\alpha^T(x) = t_i$ and $(\mu_A)_\alpha^T(y) = s_j$, $(v_A)_\alpha^T(y) = t_j$. Then $x * y \in p_i$ since p_i an AT-subalgebra of X and $p_j \subset p_i$. and the following

$(\mu_A)_\alpha^T(x * y) = \mu_A + \alpha \geq s_j = \min\{\mu_A(x) + \alpha, \mu_A(y) + \alpha\}$ and

$(v_A)_\alpha^T(x * y) = v_A - \alpha \leq t_j = \max\{v_A(x) - \alpha, v_A(y) - \alpha\}$.

Thus A_α^T is α-translation of intuitionistic fuzzy AT-subalgebra of X.

From (1) and (2), it follows that, $Im(\mu_A) = \{s_0, s_1, \ldots, s_r\}$ and $Im(v_A) = \{t_0, t_1, \ldots, t_r\}$.

Hence, the upper level AT-subalgebras of A are given by the chain of AT-subalgebras and the lower level anti -AT-subalgebras of A are given by the chain of AT-subalgebras.

$U_\alpha(\mu_A|s_0) \subset U_\alpha(\mu_A|s_1) \subset \ldots \subset U_\alpha(\mu_A|s_r) = X$ (3)

and

$L_\alpha(v_A|t_0) \subset L_\alpha(v_A|t_1) \subset \ldots \subset L_\alpha(v_A|t_0) = X$ (4)

Now, $U_\alpha(\mu_A|s_0) = \{x \mid x \in X \ \& \ \mu_A \geq s_0\} = P_0 = \{x \mid x \in X \ \& \ \mu_A \leq t_0\} = L_\alpha(\mu_A|t_0)$.

Finally, we prove that U_α $(\mu_A|s_i)= P_i=L\alpha$ $(\mu_A|t_i)$ for $0< i \leq r$. Clearly, $p_i \subseteq U_\alpha$ $(\mu_A|s_i)$ and $L\alpha$ $(\mu_A|t_i)$. If $x\in U_\alpha$ $(\mu_A|s_i)$ and $L_\alpha(\mu_A|t_i)$, then $\mu_A(x)\geq s_i$ and $\nu_A(x)\leq t_i$ which implies that $x\notin P_j$, for $j>i$. Hence, $\mu_A(x)\in\{s_0, s_1,\ldots, s_i\}$ and $\nu_A(x)\in \{t_0, t_1,\ldots, t_r\}$.and so $x\in p_A$, for some $k\leq i$. as $p_k \subseteq p_i$, Which follows this $x\in p_i$ and result, U_α $(\mu_A|s_i)= P_i=L_\alpha$ $(\nu_A|t_i)$, for $0 <i\leq r$.

4. A-TRANSLATION OF INTUITIONISTIC FUZZY AT-IDEAL IN AT-ALGEBRA

Definition 4,1. Let $A_\alpha^T = \{(x, (\mu_A)_\alpha^T, (\nu_A)_\alpha^T)\}$ be α-translation of intuitionistic fuzzy of AT-algebra X. A_α^T is said to be α-translation of intuitionistic fuzzy AT-ideal of X if : for all x, y, z ∈X,

(IFAT$_1$) $(\mu_A)_\alpha^T(0) = \mu_A(0) + \alpha \geq (\mu_A)_\alpha^T(x) = \mu_A(x) + \alpha$ and

$$(\nu_A)_\alpha^T(0) = \nu_A(0) - \alpha \leq (\nu_A)_\alpha^T(x) = \nu_A(x) - \alpha.$$

(IFAT$_2$) $(\mu_A)_\alpha^T(x * z) = \mu_A(x * z) + \alpha\geq \min\{\mu_A(x * (y * z) + \alpha, \mu_A(y) + \alpha\}$and

$$(\nu_{A_1})_\alpha^T(x * z) = \nu_A(x * z) - \alpha \leq \max\{ \nu_A (x * (y * z)) - \alpha, \nu_A (y) -\alpha\}$$

That mean μ_A is a fuzzy AT-ideal and ν_A is an anti-fuzzy AT-ideal.

Theorem 4.2 *If $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy AT-ideal of X, then the α-translation of intuitionistic fuzzy $A_\alpha^T = ((\mu_A)_\alpha^T, (\nu_A)_\alpha^T)$ is an intuitionistic fuzzy AT-ideal of X for all $\alpha \in [0, \xi]$.*

Proof.

Let $A = (\mu_A, \nu_A)$ be an intuitionistic fuzzy AT-ideal of X and $\alpha \in [0, \xi]$. Then

$(\mu_A)_\alpha^T(0) = \mu_A(0) + \alpha \geq \mu_A(x) + \alpha = (\mu_A)_\alpha^T(x)$ and

$(\nu_A)_\alpha^T(0) = \nu_A(0) - \alpha \leq \nu_A(x) - \alpha=(\nu_A)_\alpha^T(x)$, for all x, y, z ∈ X.

$$\begin{aligned}(\mu_A)_\alpha^T(x * z) &= \mu_A(x * z) + \alpha \geq \min\{\mu_A(x * (y * z)), \mu_A(y)\} + \alpha\\ &= \min\{\mu_A(x * (y * z)) + \alpha, \mu_A(y) + \alpha\}\\ &= \min\{(\mu_A)_\alpha^T(x * (y * z)), (\mu_A)_\alpha^T(y)\} \text{ and}\end{aligned}$$

$$\begin{aligned}(\nu_A)_\alpha^T(x * z) &= \nu_A(x * z) - \alpha \leq \max\{\nu_A(x * (y * z)), \nu_A(y)\} - \alpha\\ &= \max\{\nu_A(x * (y * z)) - \alpha, \nu_A(y) - \alpha\}\\ &= \max\{(\nu_A)_\alpha^T(x * (y * z)), (\nu_A)_\alpha^T(y)\}.\end{aligned}$$

Hence, the α-translation of fuzzy intuitionistic fuzzy A_α^T is an intuitionistic fuzzy AT-ideal of X.

Theorem 4.3 *Let $A = (\mu_A, \nu_A)$ be an intuitionistic fuzzy subset of X such that the α-translation of intuitionistic fuzzy $A_\alpha^T = ((\mu_A)_\alpha^T, (\nu_A)_\alpha^T)$ is an intuitionistic fuzzy AT- ideal of X. for some $\alpha \in [0, \xi]$. Then $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy AT-ideal of X.*

Proof.

Assume that $A_\alpha^T = ((\mu_A)_\alpha^T, (\nu_A)_\alpha^T)$ is an intuitionistic fuzzy AT-ideal of X for some $\alpha \in [0, \xi]$. Let x, y, z∈ X,

$\mu_A(0) + \alpha = (\mu_A)_\alpha^T(0) \geq (\mu_A)_\alpha^T(x) = \mu_A(x) + \alpha$

$\nu_A(0) - \alpha = (\nu_A)_\alpha^T(o) \leq (\nu_A)_\alpha^T(x) = \nu_A(x) - \alpha$

which implies $\mu_A(0) \geq \mu_A(x)$ and $\nu_A(0) \leq \nu_A(x)$. Now, we have

$\mu_A(x * z) + \alpha = (\mu_A)_\alpha^T(x * z) \geq \min\{(\mu_A)_\alpha^T(x * (y * z)), (\mu_A)_\alpha^T(y)\}$

$= \min\{\mu_A(x * (y * z)) + \alpha, \mu_A(y) + \alpha\} = \min\{\mu_A(x * (y * z)), \mu_A(y)\} + \alpha\}$ and

$\nu_A(x * z) - \alpha = (\nu_A)_\alpha^T(x * z) \leq \max\{(\nu_A)_\alpha^T(x * (y * z)), (\nu_A)_\alpha^T(y)\}$

$= \max\{\nu_A(x * (y * z)) - \alpha, \nu_A(y) - \alpha\} = \max\{\nu_A(x * (y * z)), \nu_A(y)\} - \alpha$

which implies that $\mu_A(x * z) \geq \min\{\mu_A(x * (y * z)), \mu_A(y)$ and

$\nu_A(x * z) \leq \max\{\nu_A(x * (y * z))$, then $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy AT-ideal of X.

Theorem 4.4 *If the α-translation of intuitionistic fuzzy $A_\alpha^T = ((\mu_A)_\alpha^T, (\nu_A)_\alpha^T)$ of A is an intuitionistic fuzzy AT-ideal of X for all $\alpha \in [0, \xi]$, then it must be an intuitionistic fuzzy subalgebra of X.*

Proof.

Let the α-translation of intuitionistic fuzzy $A_\alpha^T = ((\mu_A)_\alpha^T, (\nu_A)_\alpha^T)$ of A be an intuitionistic fuzzy AT-ideal of X . We have, for all $x, y, z \in X$

$(\mu_A)_\alpha^T(x * z) \geq \min\{(\mu_A)_\alpha^T(x * (y * z)), (\mu_A)_\alpha^T(y)\}$ and

$(\nu_A)_\alpha^T(x * z) \leq \max\{(\nu_A)_\alpha^T(x * (y * z)), (\nu_A)_\alpha^T(y)\}$, then

$(\mu_A)_\alpha^T(x * z) \geq \min\{(\mu_A)_\alpha^T(x * (y * z)), (\mu_A)_\alpha^T(y)\}$

$= \min\{(\mu_A)_\alpha^T(x * 0), (\mu_A)_\alpha^T(y)\} = \min\{(\mu_A)_\alpha^T(x), (\mu_A)_\alpha^T(y)\}$ and

$(\nu_A)_\alpha^T(x * z) \leq \max\{(\nu_A)_\alpha^T(x * (y * z)), (\nu_A)_\alpha^T)(y)\}$

$= \max\{(\nu_A)_\alpha^T(x * 0), (\nu_A)_\alpha^T(y)\} = \max\{(\nu_A)_\alpha^T(x), (\nu_A)_\alpha^T(y)$

then A_α^T is intuitionistic fuzzy AT- subalgebra of X .

Theorem 4.5. *If $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy subset of X such that the α-translation of intuitionistic fuzzy $A_\alpha^T = ((\mu_A)_\alpha^T, (\nu_A)_\alpha^T)$ of A is an intuitionistic fuzzy AT-ideal of X for $\alpha \in [0, \xi]$, then the sets $I_{\mu_A} = \{x | x \in X \text{ and } (\mu_A)_\alpha^T(x) = (\mu_A)_\alpha^T(0)\}$ and $I_{\nu_A} = \{x | x \in X \text{ and } (\nu_A)_\alpha^T(x) = (\nu_A)_\alpha^T(0)\}$ are AT-ideals of X.*

Proof.

Let $A_\alpha^T = ((\mu_A)_\alpha^T, (\nu_A)_\alpha^T)$ is an intuitionistic fuzzy ideal of X. so $(\mu_A)_\alpha^T$ and $(\nu_A)_\alpha^T$ are fuzzy AT-ideal of X. clearly $0 \in I_{\mu_A}, I_{\nu_A}$. Suppose x, y, z ∈ X such that $(x * (y * z)) \in I_{\mu_A}$ and $y \in I_{\nu_A}$. hence $(\mu_A)_\alpha^T (x * (y * z)) = (\mu_A)_\alpha^T (0) = (\mu_A)_\alpha^T (y)$ and $(\mu_A)_\alpha^T (x * z) \geq \min\{(\mu_A)_\alpha^T (x * (y * z)), (\mu_A)_\alpha^T (y)\} = (\mu_A)_\alpha^T (0)$. Since, $(\mu_A)_\alpha^T$ is fuzzy AT-ideal of X, then $(\mu_A)_\alpha^T (x * z) = (\mu_A)_\alpha^T (0)$. Hence $\mu_A(x * z) + \alpha = \mu_A(0) + \alpha$,
$\Rightarrow \mu_A(x * z) = \mu_A(0)$ and $(x * z) \in I_{\mu_A}$ then I_{μ_A} is AT-ideal of X.

Also, suppose that u, v, w, ∈ X such that $(u * (v * w)) \in I_{\nu_A}$ and $v \in I_{\nu_A}$. Hence $(\nu_A)_\alpha^T (u * (v * w)) = (\nu_A)_\alpha^T (0) = (\nu_A)_\alpha^T (v)$ and $(\nu_A)_\alpha^T (u * w) \leq \max\{(\nu_A)_\alpha^T (u * (v * w)), (\nu_A)_\alpha^T (v)\} = (\nu_A)_\alpha^T (0)$. Since, $(\nu_A)_\alpha^T$ is anti-fuzzy AT-ideal of X, then $(\nu_A)_\alpha^T (u * w) = (\nu_A)_\alpha^T (0)$. Hence $\nu_A(u * w) + \alpha = \nu_A(0) + \alpha$,
$\nu_A(u * v) = \nu_A(0)$ and $(u * v) \in I_{\nu_A}$, then I_{ν_A} is AT-ideal of X.

Proposition 4.6 Let the α-translation of intuitionistic fuzzy $A_\alpha^T = ((\mu_A)_\alpha^T, (\nu_A)_\alpha^T)$ of A be intuitionistic fuzzy AT-ideal of X for $\alpha \in [0, \xi]$. If $x \leq y$, then $(\mu_A)_\alpha^T (x) \geq (\mu_A)_\alpha^T (y)$ and $(\nu_A)_\alpha^T(x) \leq (\nu_A)_\alpha^T(y)$, that is, $(\mu_A)_\alpha^T$ is order-reversing and $(\nu_A)_\alpha^T$ is order-preserving.

Proof.

Suppose that x, y ∈ X and $x \leq y$. Then $x * y = 0$ and

$$(\mu_A)_\alpha^T (x) = (\mu_A)_\alpha^T (x * 0) \geq \min\{(\mu_A)_\alpha^T (x * (y * 0)), (\mu_A)_\alpha^T (y)\}$$
$$= \min\{(\mu_A)_\alpha^T (x * y), (\mu_A)_\alpha^T (y)\} = \min\{(\mu_A)_\alpha^T (0), (\mu_A)_\alpha^T (y)\} = (\mu_A)_\alpha^T (y) \quad \text{and}$$
$$(\nu_A)_\alpha^T(x) = (\nu_A)_\alpha^T(x * 0) \leq \max\{(\nu_A)_\alpha^T(x * (y * 0)), (\nu_A)_\alpha^T(y)\}$$
$$= \max\{(\nu_A)_\alpha^T(x * y), (\nu_A)_\alpha^T(y) = \max\{(\nu_A)_\alpha^T(0), (\nu_A)_\alpha^T(y)\} = (\nu_A)_\alpha^T(y).$$

Theorem 4.7. *The α-translation of intuitionistic fuzzy $A_\alpha^T = \{(x, (\mu_A)_\alpha^T, (\nu_A)_\alpha^T)\}$ of A is an α-translation of intuitionistic fuzzy AT-ideal of AT-algebra X if and only if , for all $t \in [0,1]$, the set $U_\alpha(\mu_A, t)$ is an AT-ideal and $L_\alpha(\nu_A, s)$ is an anti-AT-ideal of X.*

Proof.

Let $A_\alpha^T = \{(x, (\mu_A)_\alpha^T, (\nu_A)_\alpha^T)\}$ be α-translation of intuitionistic fuzzy AT-ideal of X and $U_\alpha(\mu_A, t) \neq \emptyset \neq L_\alpha(\nu_A, s)$. Since $\mu_A(0) + \alpha \geq t$ and $\nu_A(0) - \alpha \leq s$,
let $x, y, z \in X$ be such that $(x * (y * z)) \in U_\alpha(\mu_A, t)$, $y \in U_\alpha(\mu_A, t)$, then $\mu_A(x * (y * z)) + \alpha \geq t$ and $\mu_A(y) + \alpha \geq t$, it follows that

$\mu_A(x*z)+\alpha \geq \min\{\mu_A(x*(y*z)+\alpha, \mu_A(y)+\alpha\} \geq t$, so that $(x*z) \in U_\alpha(\mu_A, t)$.
Hence $U_\alpha(\mu_A, t)$ is an AT-ideal of X.

In a similar way, we can prove that $L_\alpha(\nu_A, s)$ is an anti- AT-ideal of X.

Conversely, assume that for each , $t \in [0,1]$, the sets $U_\alpha(\mu_A, t)$ is an AT-ideal and $L_\alpha(\nu_A, s)$ is an anti- AT-ideal of X. For any $x \in X$, let $\mu_A(x)+\alpha = t$ and $\nu_A(x)-\alpha = s$. Then $x \in U_\alpha(\mu_A, t) \cap L_\alpha(\nu_A, s)$ and so $U_\alpha(\mu_A, t) \neq \emptyset \neq L_\alpha(\nu_A, s)$. Since $U_\alpha(\mu_A, t)$and $L_\alpha(\nu_A, s)$ are AT-ideals of X, therefore $0 \in U_\alpha(\mu_A, t) \cap L_\alpha(\nu_A, s)$. Hence $\mu_A(0)+\alpha \geq t = \mu_A(x)$ and $\nu_A(0)-\alpha \leq s = \nu_A(x)$.
If there exist $x', y', z' \in X$ be such that $\mu_A(x'*z')+\alpha < \min\{\mu_A(x'*'(y'*z')+\alpha, \mu_A(y')+\alpha\}$. Then by taking $t_0 = \frac{1}{2}\{\mu_A(x'*z')+\alpha + \min\{\mu_A(x'*'(y'*z')+\alpha, \mu_A(y')+\alpha\}\}$, we get $\mu_A(x'*z')+\alpha < t_0 < \min\{\mu_A(x'*'(y'*z')+\alpha, \mu_A(y')+\alpha\}$ and hence $(x'*z') \notin U_\alpha(\mu_A, t_0)$, $(x'*'(y'*z') \in U_\alpha(\mu_A, t_0)$, $y' \in U_\alpha(\mu_A, t_0)$, i.e. $U_\alpha(\mu_A, t_0)$, is not an AT-ideal of X, Leading to contradiction.

Finally assume that there exist $x', y', z' \in X$ such that $\nu_A(x'*z')-\alpha > \max\{\nu_A(x'*'(y'*z')-\alpha, \mu_A(y')-\alpha\}$. Then by taking $s_0 = \frac{1}{2}\{\nu_A(x'*z')-\alpha + \max\{\nu_A(x'*'(y'*z')-\alpha, \mu_A(y')-\alpha\}\}$, we get $\max\{\nu_A(x'*'(y'*z')-\alpha, \mu_A(y')-\alpha\} > s_0 > \nu_A(x'*z')$ and hence $(x'*z') \notin L_\alpha(\nu_A, s_0)$, $(x'*'(y'*z') \in L_\alpha(\nu_A, s_0)$, $y' \in L_\alpha(\nu_A, s_0)$, i.e. $L_\alpha(\nu_A, s_0)$, is not an anti-AT-ideal of AT-algebra X, Leading to contradiction.
As a result $A_\alpha^T = \{(x, (\mu_A)_\alpha^T, (\nu_A)_\alpha^T)\}$ is α-translation of intuitionistic fuzzy AT-ideal of X.

Theorem 4.8. *Let $A_1$and A_2be two α-translation of intuitionistic fuzzy AT-ideals of AT-algebra X. Then $A_1 \cap A_2$is α-translation of intuitionistic fuzzy AT-ideal of X.*

Proof.

Let $x, y, z \in A_1 \cap A_2$, then $x, y, z \in A_1$and A_2. Now,

$(\mu_{A_1\cap A_2})_\alpha^T(0) = (\mu_{A_1\cap A_2})_\alpha^T(x*x) \geq \min\{(\mu_{A_1\cap A_2})_\alpha^T(x), (\mu_{A_1\cap A_2})_\alpha^T(x)\} = (\mu_{A_1\cap A_2})_\alpha^T(x)$ and

$(v_{A_1\cap A_2})_\alpha^T(0) = (v_{A_1\cap A_2})_\alpha^T(x*x) \leq \max\{(v_{A_1\cap A_2})_\alpha^T(x), (v_{A_1\cap A_2})_\alpha^T(x)\} = (v_{A_1\cap A_2})_\alpha^T(x)$. Also,

$$(\mu_{A_1\cap A_2})_\alpha^T(x*z) = \min\{(\mu_{A_1})_\alpha^T(x*z), (\mu_{A_2})_\alpha^T(x*z)\}$$

$$\geq \min\{\min\{(\mu_{A_1})_\alpha^T(x*(y*z)), (\mu_{A_1})_\alpha^T(y)\}, \min\{(\mu_{A_2})_\alpha^T(x*(y*z)), (\mu_{A_2})_\alpha^T(y)\}\}$$

$$= \min\{\min\{(\mu_{A_1})_\alpha^T(x*(y*z)), (\mu_{A_2})_\alpha^T(x*(y*z))\}, \min\{(\mu_{A_1})_\alpha^T(y), (\mu_{A_2})_\alpha^T(y)\}\}$$

$= \min\{\left(\mu_{A_1\cap A_2}\right)_\alpha^T(x*(y*z)), \left(\mu_{A_1\cap A_2}\right)_\alpha^T(y)\}$

And

$\left(v_{A_1\cap A_2}\right)_\alpha^T(x*y) = \max\{\left(v_{A_1}\right)_\alpha^T(x*z), \left(v_{A_2}\right)_\alpha^T(x*z)\}$

$\leq \max\{\max\{\left(v_{A_1}\right)_\alpha^T(x*(y*z)), \left(v_{A_1}\right)_\alpha^T(y)\}, \max\{\left(v_{A_2}\right)_\alpha^T(x*(y*z)), \left(v_{A_2}\right)_\alpha^T(y)\}\}$

$=\max\{\max\{\left(v_{A_1}\right)_\alpha^T(x*(y*z)), \left(v_{A_2}\right)_\alpha^T(x*(y*z))\}, \max\{\left(v_{A_1}\right)_\alpha^T(y), \left(v_{A_2}\right)_\alpha^T(y)\}\}$

$= \max\{\left(v_{A_1\cap A_2}\right)_\alpha^T(x*(y*z)), \left(v_{A_1\cap A_2}\right)_\alpha^T(y)\}$

Hence, $A_1 \cap A_2$ is α-translation of intuitionistic fuzzy AT-ideal of X.

Corollary 4.9. Let$\{A_i|i=1,2,3,\ldots\}$be a family of intuitionistic fuzzy AT-ideals of AT-algebra X, then $\cap A_i$ is α-translation of intuitionistic fuzzy AT-ideal of AT-algebra X, where

$\cap A_i = (\min(\mu_{Ai})_\alpha^T(x), \max(v_{Ai})_\alpha^T(x))$

Theorem 4.10. the α-translation of intuitionistic fuzzy $A_\alpha^T = ((\mu_A)_\alpha^T, (\nu_A)_\alpha^T)$ of A is α-translation of intuitionistic fuzzy AT-ideal of AT-algebra X if and only if, the fuzzy sets $(\mu_A)_\alpha^T$ and $(\bar{\nu}_A)_\alpha^T$ are fuzzy AT-ideals of X.

Proof.

Let $A_\alpha^T = ((\mu_A)_\alpha^T, (\nu_A)_\alpha^T)$ be α-translation of intuitionistic fuzzy AT-ideal of X. Cleary, μ_A is a fuzzy AT-ideal of X. For every x, y, z ∈X,we have

$(\bar{\nu}_A)_\alpha^T(0)=1-(\nu_A)_\alpha^T(0) \geq 1-(\nu_A)_\alpha^T(x)=(\bar{\nu}_A)_\alpha^T(x)$ and

$(\bar{\nu}_A)_\alpha^T(x*z)=1-(\nu_A)_\alpha^T(x*z) \geq 1-\max\{(\nu_A)_\alpha^T(x*(y*z)), (\nu_A)_\alpha^T(y)\}$

$=\min\{1-(\nu_A)_\alpha^T(x*(y*z)), 1-(\nu_A)_\alpha^T(y)\}=\min\{(\bar{\nu}_A)_\alpha^T(x*(y*z)), (\bar{\nu}_A)_\alpha^T(y)\}$.

Hence,$(\bar{\nu}_A)_\alpha^T$ is a fuzzy AT-ideal of X.

Conversely, assume that$(\mu_A)_\alpha^T$ and $(\bar{\nu}_A)_\alpha^T$are two intuitionistic fuzzy AT-ideals of X. For every x, y, z ∈X, we get $(\mu_A)_\alpha^T(0) \geq (\mu_A)_\alpha^T(x)$ and $(\bar{\nu}_A)_\alpha^T(0) \geq (\bar{\nu}_A)_\alpha^T(x)$, this implies,$1-(\nu_A)_\alpha^T(0) \geq 1-(\nu_A)_\alpha^T(x)$. That is ,$(\nu_A)_\alpha^T(0) \leq (\nu_A)_\alpha^T(x)$.

Also, $(\bar{\nu}_A)_\alpha^T(x*z) \geq \min\{(\bar{\nu}_A)_\alpha^T(x*(y*z)), (\bar{\nu}_A)_\alpha^T(y)\}$and

$1-(\nu_A)_\alpha^T(x*z) = (\bar{\nu}_A)_\alpha^T(x*z) \geq \min\{(\bar{\nu}_A)_\alpha^T(x*(y*z)),(\bar{\nu}_A)_\alpha^T(y)$

$=\min\{1-(\nu_A)_\alpha^T(x*(y*z)), 1-(\nu_A)_\alpha^T(y)\} = 1-\max\{(\nu_A)_\alpha^T(x*(y*z)), (\nu_A)_\alpha^T(y)\}$. That is,

$(\nu_A)_\alpha^T(x*z) \leq \max\{(\nu_A)_\alpha^T(x*(y*z)), (\nu_A)_\alpha^T(y)\}$.

Hence $A_\alpha^T = ((\mu_A)_\alpha^T, (\nu_A)_\alpha^T)$ be α-translation of intuitionistic fuzzy AT-ideal of X.

Theorem 4.11. *Let* $A_{\alpha}^{T} = ((\mu_A)_{\alpha}^{T}, (\nu_A)_{\alpha}^{T})$ *be α-translation of intuitionistic fuzzy of A. Then* $A_{\alpha}^{T} = ((\mu_A)_{\alpha}^{T}, (\nu_A)_{\alpha}^{T})$ *) is α-translation of intuitionistic fuzzy AT-ideal of X if and only if,* □*A=* $((\mu_A)_{\alpha}^{T}, (\bar{\mu}_A)_{\alpha}^{T})$ *and* △*A=* $((\nu_A)_{\alpha}^{T}, (\bar{\nu}_A)_{\alpha}^{T})$ *are α-translation of intuitionistic fuzzy AT-ideals of X.*

Proof.

If $A_{\alpha}^{T} = ((\mu_A)_{\alpha}^{T}, (\nu_A)_{\alpha}^{T})$ is α-translation of intuitionistic fuzzy AT-ideal of X, then $(\mu_A)_{\alpha}^{T}$ and $(\bar{\nu}_A)_{\alpha}^{T}$ are intuitionistic fuzzy AT-ideals of X, and $((\nu_A)_{\alpha}^{T}$ and $(\bar{\mu}_A)_{\alpha}^{T})$) are intuitionistic anti-fuzzy AT-ideals of X, by Theorem (4.3).

Hence, □A= $((\mu_A)_{\alpha}^{T}, (\bar{\mu}_A)_{\alpha}^{T})$ and △A= $((\nu_A)_{\alpha}^{T}, (\bar{\nu}_A)_{\alpha}^{T})$) are α-translation of intuitionistic fuzzy AT-ideals of X.

Conversely,if □A= $((\mu_A)_{\alpha}^{T}, (\bar{\mu}_A)_{\alpha}^{T})$ and △A= $((\nu_A)_{\alpha}^{T}, (\bar{\nu}_A)_{\alpha}^{T})$) are α-translation of intuitionistic fuzzy AT-ideals of X, then $(\mu_A)_{\alpha}^{T}$ and $(\bar{\nu}_A)_{\alpha}^{T}$ are intuitionistic fuzzy AT-ideals of X, and $((\nu_A)_{\alpha}^{T}$ and $(\bar{\mu}_A)_{\alpha}^{T})$) are intuitionistic anti-fuzzy AT-ideals of X, hence $A_{\alpha}^{T} = ((\mu_A)_{\alpha}^{T}, (\nu_A)_{\alpha}^{T})$) is α-translation of intuitionistic fuzzy AT-ideal of X.

Theorem 4.12. *Let* $\{I_t | t \in \Lambda\}$ *be collection of AT-ideals of AT-algebra X such that*

(i) $X = \bigcup_{t \in \Lambda} I_t$

(ii) $s > t$ *if and only if,* $I_s \subset I_t$, *for all* $s, t \in \Lambda$.

Then α-translation of intuitionistic fuzzy $A_{\alpha}^{T} = ((\mu_A)_{\alpha}^{T}, (\nu_A)_{\alpha}^{T})$ *) of A defined by* $(\mu_A)_{\alpha}^{T} := \sup\{t \in \Lambda | x \in I_t\}$, $(\nu_A)_{\alpha}^{T} := \inf\{t \in \Lambda | x \in I_t\}$. , *for all* $x \in X$ *is α-translation of intuitionistic fuzzy AT-ideal of X.*

Proof.

According to Theorem(4.7) ,it is sufficient to show that $U_{\alpha}(\mu_A, t)$ and $L_{\alpha}(\nu_A, s)$ are AT-ideals of X, for every $t \in [0, (\mu_A)_{\alpha}^{T}(0)]$ and $s \in [(\nu_A)_{\alpha}^{T}(0), 1]$. In order to prove that $U_{\alpha}(\mu_A, t)$ is an AT-ideal of X, we divide the proof into the following two cases:

(i) $t = \sup\{q \in \Lambda | q < t\}$

(ii) $t \neq \sup\{q \in \Lambda | q < t\}$

For the case(i),implies that $x \in U_{\alpha}(\mu_A, t) \Leftrightarrow x \in I_q, \ \forall q < t \Leftrightarrow x \in \bigcap_{q<t} I_q$, so that $U_{\alpha}(\mu_A, t) = \bigcap_{q<t} I_q$, which is an AT-ideal of X.

For the case (ii),we claim that $U_{\alpha}(\mu_A, t) = \bigcup_{q<t} I_q$. If $x \in \bigcup_{q<t} I_q$,then $x \in I_q$, for some $q \geq t$. It follows that $(\mu_A)_{\alpha}^{T}(x) \geq q \geq t$, so that $x \in U_{\alpha}(\mu_A, t)$. This shows that $\bigcup_{q \geq t} I_q \subseteq U_{\alpha}(\mu_A, t)$.

Now, assume that $x \notin \bigcup_{q\geq t} I_q$. Then $x \notin I_q$, for all that $q \geq t$. Since $t \neq \sup\{q \in \Lambda | q<t\}$, there exists $\varepsilon > 0$ such that $(t - \varepsilon, t) \cap \Lambda = \emptyset$. Hence $x \notin I_q$ for all that $q \geq t- \varepsilon$, which means that if $x \in I_q$, then $q \geq t- \varepsilon$. Thus $(\mu_A)^T_\alpha(x) = q \leq t- \varepsilon < t$, $x \notin U_\alpha(\mu_A, t)$. Therefore $U_\alpha(\mu_A, t) \subseteq \bigcup_{q\geq t} I_q$, and this $U_\alpha(\mu_A, t) = \bigcup_{q\geq t} I_q$, which is an AT-ideal of X.

Next, we prove that $L_\alpha(\nu_A, s)$ is an AT-ideal of X. We consider the following two case:

(iii) $s = \inf\{r \in \Lambda | s<r\}$

(iv) $s \neq \inf\{r \in \Lambda | s<r\}$

For the case *(iii)*, we have $x \in L_\alpha(\nu_A, t) \Leftrightarrow x \in I_r \forall s < r \Leftrightarrow x \in \bigcap_{s<r} I_r$, and hence $L_\alpha(\nu_A, t) = \bigcap_{s<r} I_r$, which is an anti-AT-ideal of X.

For the case (iv) there exists $\varepsilon > 0$ such that $(s, s + \varepsilon) \cap \Lambda = \emptyset$. We will show that $L_\alpha(\nu_A, t) = \bigcup_{s\geq r} I_r$, then $x \in I_r$, for some $r \leq s$. It $(\nu_A)^T_\alpha(x) \leq r \leq s$, so that $x \in L_\alpha(\nu_A, s)$. Hence, $\bigcup_{s<r} I_r \subseteq L_\alpha(\nu_A, s)$.

Now, assume that $x \notin \bigcup_{s\geq r} I_r$, then $x \notin I_r$, for all that $r \leq s$, which implies that $x \notin I_r$, for all that $r < s + \varepsilon$, that is, if $x \in I_r$, then $r \geq s+\varepsilon$. This $(\nu_A)^T_\alpha(x) \geq s + \varepsilon > s$, that is, $x \notin L_\alpha(\nu_A, s)$ Therefore, $L_\alpha(\nu_A, t) = \bigcup_{s\geq r} I_r$, and consequently $L_\alpha(\nu_A, s) = \bigcup_{s\geq r} I_r$ which is an anti-AT- ideal of X.

5. HOMOMORPHISM OF A-TRANSLATION OF INTUITIONISTIC FUZZY AT-IDEAL IN AT-ALGEBRA

Definition 5.1.[7]. Let $(X; *, 0)$ and $(Y; *', 0')$ be two nonempty sets. A mapping $f: X \to Y$ is said to be a homomorphism if $f(x * y) = f(x) *' f(y)$ for all. $x, y \in X$.

Note that:- If $f: X \to Y$ is a homomorphism of sets, then $f(0) = 0'$.

Definition 5. 2. Let $f : (X; *, 0) \to (Y; *', 0')$ be a homomorphism of AT-algebras, for any $A^T_\alpha = \{(y, (\mu_A)^T_\alpha(y), (\nu_A)^T_\alpha(y)) \mid y \in Y\}$ in Y, and $\alpha \in [0, \xi]$ we define new $(A^T_\alpha)^f = (x, ((\mu_A)^T_\alpha)^f, ((\nu_A)^T_\alpha)^f) | x \in X\}$ in X by $((\mu_A)^T_\alpha)^f = (\mu_A)^T_\alpha(f(x))$ and $((\nu_A)^T_\alpha)^f(x) = (\nu_A)^T_\alpha(f(x))$, for all $x \in X$.

Theorem 5.3. *Let $f : (X; *, 0) \to (Y; *', 0')$ be a homomorphism of AT-algebras. If $B^T_\alpha = ((\mu_B)^T_\alpha, (\nu_B)^T_\alpha)$ is α-translation of intuitionistic fuzzy AT-ideal of Y, then the pre-image $f^{-1}(B^T_\alpha) = (f^{-1}((\mu_B)^T_\alpha), f^{-1}((\nu_B)^T_\alpha))$ of B^T_α under f in X is α-translation of intuitionistic fuzzy AT-ideal of X.*

Proof.

For all x, y, z ∈ X , then

$f^{-1}((\mu_B)^T_\alpha)(x) = (\mu_B)^T_\alpha(f(x)) \leq (\mu_B)^T_\alpha(0) = (\mu_B)^T_\alpha(f(0)) = f^{-1}((\mu_B)^T_\alpha)(0)$ and

$f^{-1}((\nu_B)^T_\alpha)(x) = (\nu_B)^T_\alpha(f(x)) \geq (\nu_B)^T_\alpha(0) = (\nu_B)^T_\alpha(f(0)) = f^{-1}((\nu_B)^T_\alpha)(0)$.

$$f^{-1}((\mu_B)^T_\alpha)(x * z) = (\mu_B)^T_\alpha(f(x * z)) \geq \min\{(\mu_B)^T_\alpha\left(f(x) *'(f(y) *' f(z))\right), (\mu_B)^T_\alpha(f(y))\}$$

$$\geq \min\{(\mu_B)^T_\alpha(f(x * (y * z)), (\mu_B)^T_\alpha(f(y))\} = \min\{f^{-1}((\mu_B)^T_\alpha)(x * (y * z)), f^{-1}((\mu_B)^T_\alpha)(y)\}$$

And

$$f^{-1}((\nu_B)^T_\alpha)(x * z) = (\nu_B)^T_\alpha(f(x * z)) \leq \max\{(\nu_B)^T_\alpha\left(f(x) *'(f(y) *' f(z))\right), (\nu_B)^T_\alpha(f(y))\}$$

$$\geq \max\{(\nu_B)^T_\alpha(f(x * (y * z)), (\nu_B)^T_\alpha(f(y))\} = \max\{f^{-1}((\nu_B)^T_\alpha)(x * (y * z)), f^{-1}((\nu_B)^T_\alpha)(y)\}.$$

Then $f^{-1}(B)^T_\alpha = (f^{-1}((\mu_B)^T_\alpha), f^{-1}((\nu_B)^T_\alpha))$ is α-translation of intuitionistic fuzzy AT-ideal of X.

Corollary 5.4. Let $f : (X;*,0) \to (Y;*',0')$ be a homomorphism of AT-algebras. If $B^T_\alpha = ((\mu_B)^T_\alpha, (\nu_B)^T_\alpha)$ is α-translation of intuitionistic fuzzy AT-subalgebra of Y, then the pre-image $f^{-1}(B^T_\alpha) = (f^{-1}((\mu_B)^T_\alpha), f^{-1}((\nu_B)^T_\alpha))$ of B^T_α under f in X is α-translation of intuitionistic fuzzy AT- subalgebra of X.

Theorem 5. 5. *Let $f : (X;*,0) \to (Y;*',0')$ be an epimorphism of AT-algebras. If $A^T_\alpha = ((\mu_A)^T_\alpha, (\nu_A)^T_\alpha)$ is α-translation of intuitionistic fuzzy AT-ideal of X, then $f(A^T_\alpha)$=$(f((\mu_A)^T_\alpha), f((\nu_A)^T_\alpha))$ of A^T_α is α-translation of intuitionistic fuzzy AT-ideal of Y.*

Proof.

For any a∈ X, there exists y ∈ Y such that $f(a) = y$. Then

$f((\mu_A)^T_\alpha)(y) = f((\mu_A)^T_\alpha)(f(a)) = f^{-1}(f((\mu_A)^T_\alpha))(a) = (\mu_B)^T_\alpha(a) = (\mu_B)^T_\alpha(0)$

$=f^{-1}(f((\mu_A)^T_\alpha))(0) = f((\mu_A)^T_\alpha)(f(0)) = f((\mu_A)^T_\alpha)(0')$ and

$f((\nu_A)^T_\alpha)(y) = f((\nu_A)^T_\alpha)(f(a)) = f^{-1}(f((\nu_A)^T_\alpha))(a) = (\nu_A)^T_\alpha(a) \geq (\nu_A)^T_\alpha(0)$

$= f^{-1}(f((\nu_A)^T_\alpha))(0) = f((\nu_A)^T_\alpha)(f(0)) = f((\nu_A)^T_\alpha)(0')$.

Let x, y, z ∈ Y, then f (a)=x and f (b)=y and f (c) =z for some a, b, c ∈ X. Thus

$f((\mu_A)^T_\alpha)(x *' z) = f((\mu_A)^T_\alpha)(f(a) *' f(c)) = f^{-1}(f((\mu_A)^T_\alpha))(a * c) = (\mu_A)^T_\alpha(a * c)$

$\geq \{ (\mu_A)^T_\alpha(a * (b * c)), (\mu_A)^T_\alpha(b)\} = \{ f^{-1}(f((\mu_A)^T_\alpha))(a * (b * c)), f^{-1}(f((\mu_A)^T_\alpha))(b)\}$

$= \{ (f((\mu_A)^T_\alpha))\left(f(a) *'(f(b) *' f(c))\right), (f((\mu_A)^T_\alpha))(f(b))\}$

$= \{f((\mu_A)^T_\alpha)(x *'(y *' z)), f((\mu_A)^T_\alpha)(y)\}$ and

$f((\nu_A)^T_\alpha)(x *' z) = f((\nu_A)^T_\alpha)(f(a) *' f(c)) = f^{-1}(f((\nu_A)^T_\alpha))(a * c) = (\nu_A)^T_\alpha(a * c)$

$\leq \{ (\nu_A)^T_\alpha(a * (b * c)), (\nu_A)^T_\alpha(b)\} = \{ f^{-1}(f((\nu_A)^T_\alpha))(a * (b * c)), f^{-1}(f((\nu_A)^T_\alpha))(b)\}$

$= \{ f((\nu_A)^T_\alpha)\left(f(a) *' (f(b) *' f(c))\right), f((\nu_A)^T_\alpha)(f(b))\}$

$= \{ f((\nu_A)^T_\alpha)(x *' (y *' z)), f((\nu_A)^T_\alpha)(y)\}$.

Then $f(A^T_\alpha)$=$(f((\mu_A)^T_\alpha), f((\nu_A)^T_\alpha))$ is α-translation of intuitionistic fuzzy AT-ideal of Y.

Corollary 5. 6. Let $f : (X;*,0) \to (Y;*', 0')$ be an epimorphism of AT-algebras. If $A^T_\alpha = ((\mu_A)^T_\alpha, (\nu_A)^T_\alpha)$ is α-translation of intuitionistic fuzzy AT-subalgebra of X, then $f(A^T_\alpha)$=$(f((\mu_A)^T_\alpha), f((\nu_A)^T_\alpha))$ of A^T_α is α-translation of intuitionistic fuzzy AT-subalgebra of Y.

6. CARTESIAN PRODUCT OF Α-TRANSLATION OF INTUITIONISTIC FUZZY AT-IDEAL

In this section, we will discuss, investigate a new notion called Cartesian product of α-translation of intuitionistic fuzzy AT-ideals and we study several basic properties which related to α-translation of intuitionistic fuzzy AT-ideals .

Definition 6.1.[1]. Let β and λ be are two fuzzy subsets in the set X. the Cartesian product $\beta \times \lambda : X \times X \to [0,1]$ is defined by, $\beta \times \lambda(x, y) = \{\beta(x), \lambda(y)\}$,for all x, y ∈X.

Definition 6.2. Let $A^T_\alpha = \left\{\left(x, \left(\beta_A\right)^T_\alpha(x), (\lambda_A)^T_\alpha(x)\right) \middle| x \in X\right\}$ and $B^T_\alpha = \left\{\left(x, \left(\beta_B\right)^T_\alpha(x), (\lambda_B)^T_\alpha(x)\right) \middle| x \in X\right\}$ are two α-translation of intuitionistic fuzzy subsets of X, and $\alpha \in [0, \xi]$ the Cartesian product $A^T_\alpha \times B^T_\alpha = (X \times X, (\beta_A \times \beta_B)^T_\alpha, (\lambda_A \times \lambda_B)^T_\alpha)$ such that $(\beta_A \times \beta_B)^T_\alpha : X \times X \to [0,1]$ is defined by $(\beta_A \times \beta_B)^T_\alpha(x, y) = \{ (\beta_A)^T_\alpha(x), (\beta_B)^T_\alpha(y)\}$ $= \{\beta_A(x) + \alpha, \beta_B(x) + \alpha\}$ and $(\lambda_A \times \lambda_B)^T_\alpha : X \times X \to [0,1]$ is defined by $(\lambda_A \times \lambda_B)^T_\alpha(x, y) = \{ (\lambda_A)^T_\alpha(x), (\lambda_B)^T_\alpha(y)\} = \{\lambda_A(x) - \alpha, \lambda_B(x) - \alpha\}$, for all x, y ∈X.

Remark 6.3.[3,4]. Let X and Y be AT-algebras, we define * on $X \times Y$ by: For every (x, y), $(u, v) \in X \times Y$, $(x, y) * (u, v) = (x * u, y * v)$, then clearly $(X \times Y, *, (0,0))$ is a AT-algebra .

Proposition 6.4.[3] Let $A^T_\alpha = \{(x, (\beta_A)^T_\alpha(x), (\lambda_A)^T_\alpha(x)) \mid x \in X\}$ and $B = \{(x, (\beta_B)^T_\alpha(x), (\lambda_B)^T_\alpha(x)) \mid x \in X\}$ are α-translation of intuitionistic fuzzy AT-ideals of X, then $A \times B$ is α-translation of intuitionistic fuzzy AT-ideal of $X \times X$.

Proof.

For all x, y , z ∈ X,

$(\beta_A \times \beta_B)^T_\alpha(0,0) = \{ (\beta_A)^T_\alpha(0), (\beta_B)^T_\alpha(0)\} \geq \{ (\beta_A)^T_\alpha(x), (\beta_B)^T_\alpha(y)\} = (\beta_A \times \beta_B)^T_\alpha(x, y)$ and

$(\lambda_A \times \lambda_B)^T_\alpha(0,0) = \{ (\lambda_A)^T_\alpha(0), (\lambda_B)^T_\alpha(0)\} \leq \{ (\lambda_A)^T_\alpha(x), (\lambda_B)^T_\alpha(y)\} = (\lambda_A \times \lambda_B)^T_\alpha(x, y)$.

Now, let$(x_1, x_2), (y_1, y_2), (z_1, z_2) \in X \times X$, then

$(\beta_A \times \beta_B)^T_\alpha((x_1, x_2) * (z_1, z_2)) = (\beta_A \times \beta_B)^T_\alpha(x_1 * z_1, x_2 * z_2) = \{(\beta_A)^T_\alpha(x_1 * z_1), (\beta_B)^T_\alpha(x_2 * z_2)\}$

$\geq \{\min\{(\beta_A \times \beta_A)^T_\alpha(x_1 * (y_1 * z_1)), (\beta_A \times \beta_A)^T_\alpha(y_1)\}, \min\{(\beta_B \times \beta_B)^T_\alpha(x_2 * (y_2 * z_2)), (\beta_B \times \beta_B)^T_\alpha(y_2)\}\}$

$= \{\min\{(\beta_A \times \beta_B)^T_\alpha(((x_1 * (y_1 * z_1)), (x_2 * (y_2 * z_2)))\}, \min\{(\beta_A \times \beta_B)^T_\alpha(y_1, y_2)\}\}$

$= \min\{(\beta_A \times \beta_B)^T_\alpha\left((x_1, x_2) * \left((y_1, y_2) * (z_1, z_2)\right)\right), (\beta_A \times \beta_B)^T_\alpha(y_1, y_2)\}$ and

$(\lambda_A \times \lambda_B)^T_\alpha((x_1, x_2) * (z_1, z_2)) = \lambda_A \times \lambda_B)^T_\alpha(x_1 * z_1, x_2 * z_2) = \{(\lambda_A)^T_\alpha(x_1 * z_1), (\lambda_B)^T_\alpha(x_2 * z_2)\}$

$\leq \{\max\{(\lambda_A \times \lambda_A)^T_\alpha(x_1 * (y_1 * z_1)), (\lambda_A \times \lambda_A)^T_\alpha(y_1)\}, \max\{(\lambda_B \times \lambda_B)^T_\alpha(x_2 * (y_2 * z_2)), (\lambda_B \times \lambda_B)^T_\alpha(y_2)\}\}$

$= \{\max\{(\lambda_A \times \lambda_B)^T_\alpha(((x_1 * (y_1 * z_1)), (x_2 * (y_2 * z_2)))\}, \min\{(\lambda_A \times \lambda_B)^T_\alpha(y_1, y_2)\}\}$

$= \max\{(\lambda_A \times \lambda_B)^T_\alpha\left((x_1, x_2) * \left((y_1, y_2) * (z_1, z_2)\right)\right), (\lambda_A \times \lambda_B)^T_\alpha(y_1, y_2)\}$.

Corollary 6.5. Let $A^T_\alpha = (x, (\beta_A)^T_\alpha(x), (\lambda_A)^T_\alpha(x))|x \in X\}$ and $B = \{(x, (\beta_B)^T_\alpha(x), (\lambda_B)^T_\alpha(x))|x \in X\}$ are α-translation of intuitionistic fuzzy AT-subalgebras of X, then $A \times B$ is α-translation of intuitionistic fuzzy AT-subalgebra of $X \times X$.

Definition 6.6. Let $A^T_\alpha = (x, (\beta_A)^T_\alpha(x), (\lambda_A)^T_\alpha(x))|x \in X\}$ and $B = \{(x, (\beta_B)^T_\alpha(x), (\lambda_B)^T_\alpha(x))|x \in X\}$ $B^T_\alpha = \{(x, (\beta_B)^T_\alpha(x), (\lambda_B)^T_\alpha(x)) \mid x \in X\}$ are α-translation of intuitionistic fuzzy subsets of AT-algebra X, for $s, t \in [0,1]$ and $\alpha \in [0, \xi]$ the set $U_\alpha(\beta_A \times \beta_B, s) = \{(x, y) \in X \times X \mid (\beta_A \times \beta_B(x, y)) + \alpha \geq s\}$ is called α-translation of upper level of $U_\alpha(\beta_A \times \beta_B, s)$ and
$L_\alpha(\lambda_A \times \lambda_B, t) = \{(x, y) \in X \times X \mid (\lambda_A \times \lambda_B(x, y)) - \alpha \leq t\}$ is called α-translation of lower level of $L_\alpha(\lambda_A \times \lambda_B, t)$.

Theorem 6.7. *$A^T_\alpha = \{(x, (\beta_A)^T_\alpha(x), (\lambda_A)^T_\alpha(x)) \mid x \in X\}$ and $B^T_\alpha = \{(x, (\beta_B)^T_\alpha(x), (\lambda_B)^T_\alpha(x)) \mid x \in X\}$ are α-translation of intuitionistic fuzzy AT-ideals of X if and only if the nonempty set α-translation of upper s-level cut $U_\alpha(\beta_A \times \beta_B, s)$ and the nonempty α-translation of t-level cut $L_\alpha(\lambda_A \times \lambda_B, t)$ are AT-ideals of $X \times X$ for any $s, t \in [0,1]$.*

Proof.

Let A^T_α and B^T_α are α-translation of intuitionistic fuzzy AT-ideals of X, therefore for any $(x, y) \in X \times X$, $\{(\beta_A \times \beta_B)(0,0)\} + \alpha = \{\beta_A(0) + \alpha, \beta_B(0) + \alpha \geq \{\beta_A(x) + \alpha, \beta_B(y) + \alpha\}$
$= \{(\beta_A \times \beta_B)(x, y)\} + \alpha$ and
$\{(\lambda_A \times \lambda_B)(0,0)\} - \alpha = \{\lambda_A(0) - \alpha, \lambda_B(0) - \alpha\} \leq \{\lambda_A(x) - \alpha, \lambda_B(y) - \alpha\}$
$= \{(\lambda_A \times \lambda_B)(x, y)\} - \alpha$, for all $s, t \in [0,1]$.

Let $(x_1, x_2), (y_1, y_2), (z_1, z_2) \in X \times X$, such that

$((x_1,x_2)*((y_1,y_2)*(z_1,z_2))) \in U_\alpha(\beta_A\times\beta_B,s)$ and $(y_1,y_2)\in(\beta_A\times\beta_B,s)$, then

$\{(\beta_A\times\beta_B)((x_1,x_2)*(z_1,z_2))\}+\alpha = \{(\beta_A\times\beta_B)(x_1*z_1,x_2*z_2)\}+\alpha$

$\geq \min\{\{(\beta_A\times\beta_B)\big((x_1,x_2)*((y_1,y_2)*(z_1,z_2))\big)\}+\alpha, \{(\beta_A\times\beta_B)\big((y_1,y_1)\big)\}+\alpha\} \geq \min\{s,s\}=s$

Therefore, $((x_1,x_2)*(z_1,z_2)) \in U_\alpha(\beta_A\times\beta_B,s)$ and

$\{(\lambda_A\times\lambda_B)((x_1,x_2)*(z_1,z_2))\}+\alpha = \{(\lambda_A\times\lambda_B)(x_1*z_1,x_2*z_2)\}+\alpha$

$\leq \max\{\{(\lambda_A\times\lambda_B)\big((x_1,x_2)*((y_1,y_2)*(z_1,z_2))\big)\}+\alpha, \{(\lambda_A\times\lambda_B)\big((y_1,y_1)\big)\}+\alpha\} \leq \min\{t,t\}=t$.

Therefore, $((x_1,x_2)*(z_1,z_2)) \in L_\alpha(\beta_A\times\beta_B,t)$.

Hence $U_\alpha(\beta_A\times\beta_B,s)$ is AT-ideal of $X\times X$ and $L_\alpha(\lambda_A\times\lambda_B,t)$ is anti-AT-ideal of $X\times X$.

Corollary 6.8. Let $A_\alpha^T = \{(x,(\beta_A)_\alpha^T(x),(\lambda_A)_\alpha^T(x)) \mid x\in X\}$ and $B_\alpha^T = \{(x,(\beta_B)_\alpha^T(x),(\lambda_B)_\alpha^T(x)) \mid x\in X\}$ are α-translation of intuitionistic fuzzy AT-subalgebras of X, if and only if the nonempty set α-translation of upper s-level cut $U_\alpha(\beta_A\times\beta_B,s)$ and the nonempty set α-translation of lower t-level cut $L_\alpha(\lambda_A\times\lambda_B,t)$ are AT-subalgebras of for any $s,t\in[0,1]$.

Conflict of Interests

The authors declare that there is no conflict of interests.

References

[1] A.T. Hameed, Fuzzy ideal of some algebras, PH.D.SC. Thesis, Faculty of Science, Ain Shams University, Egypt, 2015.

[2] A.T. Hameed, and B.H. Hadi, Anti-Fuzzy AT-Ideals on AT-algebras, J. Al-Qadisyah Computer Sci. Math. 10 (3) (2018), 63- 72.

[3] A.T. Hameed, and B.H. Hadi, Intuitionistic Fuzzy AT-Ideals on AT-algebras, J. Adv. Res. Dyn. Control Syst. 10 (2018), 1994-2006.

[4] A.T. Hameed, AT-ideals & Fuzzy AT-ideals of AT-algebras, J. Iraqi AL-Khwarizmi Soc. 1(2) (2018), 2521-2621.

[5] A.T. Hameed, AT-ideals and Fuzzy AT-ideals of AT-algebra, LAP LEMBRT Academic Publishing, Germany, 2016.

[6] A.T. Hameed, Intuitionistic Fuzzy AT-ideals of AT-algebras , LAP LEMBERT Academic Publishing , Germany, 2019.

[7] S.M. Mostafa, M.A. Abd-Elnaby and O.R. Elgendy, intuitionistic fuzzy KU-ideals in KU-algebras , Int. J. Math. Sci. Appl. 1(3) (2011), 1379-1384.

[8] T. Senapati, M. Bhowmik and M. Pol, Atanassov's intuitionistic fuzzy translations of intuitionistic fuzzy subalgebras and ideals in BCK/BCI-algebras, J. Eurasl. Math. 6(1) (2015), 96-114.

[9] T. Senapati, M. Bhowmik and M. Pol, Atanassov's intuitionistic fuzzy translations of intuitionistic fuzzy H-ideals in BCK/BCI-algebras, Notes IFS, 19 (1) (2013), 32-47.

[10] L.A. Zadeh, Fuzzy sets, Inform. Control, l (8) (1965), 338-353.