Available online at http://scik.org
Algebra Lett. 2020, 2020:1
https://doi.org/10.28919/al/4334
ISSN: 2051-5502

A NEW METHOD FOR THE INVERSE OF THE SQUARE MATRICES

NECATİ OLGUN* AND HUSEYIN TAMER

Department of Mathematics, Gaziantep University, Gaziantep 27310, Turkey

Abstract: In this paper, we introduce a new method to find the inverse of the square matrices.

Keywords: matrix; square matrix; inverse matrix; cross product.

2010 AMS Subject Classification: Primary 15A09.

1. INTRODUCTION

A matrix is a rectangular arrangement of scalar numbers in the form of rows and columns. The size of the matrices is determined by the number of rows and columns, and a matrix with m rows and n columns is named mxn. Matrices are generally used to solve systems of equations with n unknowns and m equations. In addition, matrices are used for mathematical transformations. Therefore, for computer programmers, the use of matrices in program writing provides a much easier way to solve the problem. In order to solve the systems of equations with n unknown equations and having m equations, the reduction system (Gaus Reduction Method or Gaus Jordan Reduction Method) or the inverse of the matrices, if any, are used on the coefficient matrix and the attached matrix of the equation system. For these methods, in general, the matrix can be reduced by using elementary row operations on the attached matrix of the equation

*Corresponding author
E-mail address: olgun@gantep.edu.tr
Received October 14, 2019

system or vice versa. In this study, inverse matrix finding method which is developed for the matrix of coefficients matrix in systems with n equations with n unknowns and used in the solution of linear equation systems is discussed. For this purpose, the methods developed to find the inverse of a matrix are examined.

We know that there is a unique inverse matrix A^{-1} for any square matrix A if the determinant of A is different from zero. In the literature [1,2,3,4,5,6] there are some algorithms for constructing A^{-1}. Nowadays, 3 methods are used to find the inverse of a square matrix. The first is the Montante's Method (Bareiss algorithm) [1], the second is the Gauss Jordan Elimination method and the third is the use of the adjoint matrix. It is generally used elementary row operations or the formula $A^{-1}=\frac{\mathrm{Adj(A)}}{\det(\mathrm{A})}$.

In this paper we introduced a new method and algorithm to find the inverse of a square matrix A if $|A| \neq 0$. Our method is new and it is more easy than others.

2. MAIN RESULTS

A New Method For A^{-1}

Theorem 2.1: *Let A=* $\begin{bmatrix} a_{11} & a_{21} & a_{31} \\ a_{21} & a_{22} & a_{32} \\ a_{31} & a_{23} & a_{33} \end{bmatrix}$ *be a 3x3 matrix and det(A)$\neq 0$. We know that there is only one unique inverse matrix* $A^{-1} = \begin{bmatrix} b_{11} & b_{21} & b_{31} \\ b_{21} & b_{22} & b_{32} \\ b_{31} & b_{23} & b_{33} \end{bmatrix}$.

If i-th row of the matrix A is R_i and i-th column of the matrix A^{-1} is C_i (R_i and C_i are vectors in R^3, R: Real or complex vector spaces) . Then

$$C_1= \frac{1}{detA}\,(R_2 x R_3)$$

$$C_2= \frac{1}{detA}\,(R_3 x R_1)$$

$$C_3= \frac{1}{detA}\,(R_1 x R_2)$$

where $R_i x R_j$ is the cross product of two vectors in R^3. That is

$$A^{-1} = [C_1 \quad C_2 \quad C_3] \ .$$

Proof: Let A= $\begin{bmatrix} R_1 \\ R_2 \\ R_3 \end{bmatrix}$ and $A^{-1} = [C_1 \quad C_2 \quad C_3]$. We must show that A.$A^{-1}$= A^{-1}.A= I_{3x3}.

$$\text{A.}A^{-1} = \begin{bmatrix} R_1 \\ R_2 \\ R_3 \end{bmatrix} [C_1 \quad C_2 \quad C_3] = \begin{bmatrix} R_1C_1 & R_1C_2 & R_1C_3 \\ R_2C_1 & R_2C_2 & R_2C_3 \\ R_3C_1 & R_3C_2 & R_3C_3 \end{bmatrix}$$

$R_1C_1 = R_1\left[\frac{1}{|A|}((R_2xR_3))\right] = \frac{1}{|A|}R_1(R_2xR_3) = \frac{|A|}{|A|} = 1$ since $R_1(R_2xR_3)$ is triple product.

$R_1C_2 = R_1\left[\frac{1}{|A|}((R_3xR_1))\right] = \frac{1}{|A|}R_1(R_3xR_1) = \frac{0}{|A|} = 0$ since R_3xR_1 is perpendicular to both R_3 *and* R_1.

$R_1C_3 = R_1\left[\frac{1}{|A|}((R_1xR_2))\right] = \frac{1}{|A|}R_1(R_1xR_2) = \frac{0}{|A|} = 0$ since R_1xR_2 is perpendicular to both R_1 *and* R_2.

$R_2C_1 = R_2\left[\frac{1}{|A|}((R_2xR_3))\right] = \frac{1}{|A|}R_2(R_2xR_3) = \frac{0}{|A|} = 0$ since R_2xR_3 is perpendicular to both R_2 *and* R_3.

$R_2C_2 = R_2\left[\frac{1}{|A|}((R_3xR_1))\right] = \frac{1}{|A|}R_2(R_3xR_1) = \frac{|A|}{|A|} = 1$ since $R_2(R_3xR_1)$ is triple product.

$R_2C_3 = R_2\left[\frac{1}{|A|}((R_1xR_2))\right] = \frac{1}{|A|}R_2(R_1xR_2) = \frac{0}{|A|} = 0$ since R_1xR_2 is perpendicular to both R_1 *and* R_2.

$R_3C_1 = R_3\left[\frac{1}{|A|}((R_2xR_3))\right] = \frac{1}{|A|}R_3(R_2xR_3) = \frac{0}{|A|} = 0$ since R_2xR_3 is perpendicular to both R_2 *and* R_3.

$R_3C_2 = R_3\left[\frac{1}{|A|}((R_3xR_1))\right] = \frac{1}{|A|}R_3(R_3xR_1) = \frac{0}{|A|} = 0$ since R_3xR_1 is perpendicular to both R_3 *and* R_1.

$R_3C_3 = R_3\left[\frac{1}{|A|}((R_1xR_2))\right] = \frac{1}{|A|}R_3(R_1xR_2) = \frac{|A|}{|A|} = 1$ since $R_3(R_1xR_2)$ is triple product.

So we obtain A.$A^{-1} = I_{3x3}$.

As a result the inverse of a 3x3 matrix A= $\begin{bmatrix} R_1 \\ R_2 \\ R_3 \end{bmatrix}$ is also $A^{-1} = \frac{1}{|A|}\begin{bmatrix} R_2xR_3 \\ R_3xR_1 \\ R_1xR_2 \end{bmatrix}$.

(it is also AdjA= $\begin{bmatrix} R_2xR_3 \\ R_3xR_1 \\ R_1xR_2 \end{bmatrix}$).

Example 2.2: Let A= $\begin{bmatrix} 1 & -1 & 2 \\ 3 & 1 & 4 \\ 1 & 5 & 7 \end{bmatrix}$ be 3x3 matrix. We can find the inverse of A by applying the new algorithm.

$$|A|=\begin{vmatrix}1 & -1 & 2\\3 & 1 & 4\\1 & 5 & 7\end{vmatrix}=1\begin{vmatrix}1 & 4\\5 & 7\end{vmatrix}+1\begin{vmatrix}3 & 4\\1 & 7\end{vmatrix}+2\begin{vmatrix}3 & 1\\1 & 5\end{vmatrix} = (7-20) + (21-4) + 2(15-1)= -13+17+28 =32.$$

Let $A^{-1} = [C_1 \quad C_2 \quad C_3]$.
where

$$C_1= \frac{1}{|A|}(R_2xR_3) = \frac{1}{32}\begin{vmatrix}i & j & k\\3 & 1 & 4\\1 & 5 & 7\end{vmatrix} = \frac{1}{32}(-13i-17j+14k)$$

$$C_2= \frac{1}{|A|}(R_3xR_1)= \frac{1}{32}\begin{vmatrix}i & j & k\\1 & 5 & 7\\1 & -1 & 2\end{vmatrix}= \frac{1}{32}(17i+5j-6k)$$

$$C_3=\frac{1}{|A|}(R_1xR_2)= \frac{1}{32}\begin{vmatrix}i & j & k\\1 & -1 & 2\\3 & 1 & 4\end{vmatrix}=\frac{1}{32}(-6i+2j+4k)$$

Then $$A^{-1}=\frac{1}{32}\begin{bmatrix}-13 & 17 & -6\\-17 & 5 & 2\\14 & -6 & 4\end{bmatrix}.$$

$$A.A^{-1} = \frac{1}{32}\begin{bmatrix}1 & -1 & 2\\3 & 1 & 4\\1 & 5 & 7\end{bmatrix}\begin{bmatrix}-13 & 17 & -6\\-17 & 5 & 2\\14 & -6 & 4\end{bmatrix}$$

$$= \frac{1}{32}\begin{bmatrix}-13+17+28 & 17-5-12 & -6-2+8\\-39-17+56 & 51+5-24 & -18+2+16\\-13-65+98 & 17+25-42 & -6+10+28\end{bmatrix}$$

$$= \begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$$

We cannot use the above method to generalize the new method since the cross product of two vectors is not defined for n>3. We need the following lemma.

Lemma 2.3: *Let* R^n *be vector spaces with dimension n*≥ 3. *Let* $v_1, v_2, v_3, \dots, v_n \in R^n$.

Define $\Lambda: R^n \mathrm{x} R^n x \dots x R^n \to R^n$

$$v_1 \Lambda\, v_2 \Lambda\, v_3 \Lambda\, \dots \Lambda \widetilde{v_i} \Lambda\, \dots \Lambda\, v_n = \begin{vmatrix}k_1 & k_2 & \dots & k_n\\ & & v_1 & \\ & & v_2 & \\ & & & \\ & & v_n & \end{vmatrix}$$

where $k_1=(1,0,\ldots,0)$, $k_2=(0,1,0\ldots,0)$, $k_3=(0,0,1,0,\ldots,0),\ldots, k_n=(0,0,\ldots,0,1)$ *are standart basis vectors in* R^n. *Then*

$$v_j \ . \ (v_1\Lambda\, v_2\Lambda\, v_{3i}\Lambda\, \ldots\Lambda\widetilde{v_i}\Lambda\, \ldots\Lambda\, v_n) = \begin{cases} \begin{vmatrix} v_j \\ v_1 \\ \vdots \\ v_n \end{vmatrix} & \text{if} \quad i=j \\ 0 & \text{if} \quad i \neq j \end{cases}$$

That is v_j *is orthogonal to* $v_1\Lambda\, v_2\Lambda\, v_3\Lambda\, \ldots\Lambda\widetilde{v_i}\Lambda\, \ldots\Lambda\, v_n \quad if \quad i \neq j.$

Proof: It is clear that for n=3 by Theorem 1.1. We obtain

$$v_1\Lambda\, v_2\Lambda\, v_3\Lambda\, \ldots\Lambda\widetilde{v_i}\Lambda\, \ldots\Lambda\, v_n = \begin{vmatrix} k_1 & k_2 & \ldots & k_n \\ & & v_1 & \\ & & v_2 & \\ & & & \\ & & v_n & \end{vmatrix}$$

$$= k_1 M_{k_1} - k_2 M_{k_2} + \cdots \widetilde{k_i M_{k_i}} \ldots . + (-1)^n k_n M_{k_n}$$

from the definition of the mapping.

So the following statement

$$v_j \ . \ (v_1\Lambda\, v_2\Lambda\, v_3\Lambda\, \ldots\Lambda\widetilde{v_i}\Lambda\, \ldots\Lambda\, v_n) = v_j \ . \begin{vmatrix} k_1 & k_2 & \ldots & k_n \\ & & v_1 & \\ & & v_2 & \\ & & \ldots & \\ & & v_n & \end{vmatrix}$$

$$= v_j \ .(k_1 M_{k_1} - k_2 M_{k_2} + \cdots \widetilde{k_i M_{k_i}} \ldots . + (-1)^n k_n M_{k_n})$$

$$= v_{j_1} M_{k_1} - v_{j_2} M_{k_2} + \cdots \widetilde{v_{j_i} M_{k_i}} \ldots . + (-1)^n v_{j_n} M_{k_n})$$

$$= \begin{vmatrix} v_j \\ v_1 \\ v_2 \\ \ldots \\ v_n \end{vmatrix}$$

is true for n.

Finally we obtain

$$v_j \ . \ (v_1 \wedge v_2 \wedge v_3 \wedge \ldots \wedge \widetilde{v_i} \wedge \ldots \wedge v_n) = \begin{cases} \begin{vmatrix} v_j \\ v_1 \\ \\ , \\ v_n \end{vmatrix} & \text{if} \quad i=j \\ 0 & \text{if} \quad i \neq j \end{cases}$$

by the definition of the determinant function.

Corollary 2.4: $$v_1 \ .(v_2 \wedge \ v_3 \wedge \ \ldots \wedge \widetilde{v_1} \wedge \ \ldots \wedge v_n) = \begin{vmatrix} v_1 \\ v_2 \\ v_3 \\ \ldots \\ v_n \end{vmatrix} = |A|$$

Theorem 2.5: *Let A be nxn matrix and* $|A| \neq 0$ *with n≥4. If i-th row of the matrix A is* R_i *and j-th column of the matrix* A^{-1} *is* C_j (R_i *and* C_j *are vectors in* R^n) *then*

$$A^{-1} = [C_1 \quad C_2 \quad \ldots C_n]$$

where $$C_j = \frac{(-1)^{j+1}}{|A|} (R_1 \wedge R_2 \wedge \ \ldots \wedge \widetilde{R_j} \wedge \ \ldots \wedge R_n) \ .$$

Proof: Let $A = \begin{bmatrix} R_1 \\ R_2 \\ \vdots \\ R_n \end{bmatrix}$ and $A^{-1} = [C_1 \quad C_2 \quad \ldots C_n]$ be matrices .

$$A.A^{-1} = \begin{bmatrix} R_1 \\ R_2 \\ \vdots \\ R_n \end{bmatrix} [C_1 \quad C_2 \quad \ldots \quad C_n] = \begin{bmatrix} R_1C_1 & R_1C_2 & \ldots & R_1C_n \\ .. & . & & . \\ R_nC_1 & R_nC_2 & \ldots & R_nC_n \end{bmatrix}$$

$$R_i \ . \ C_j = R_i \ . \left[\frac{(-1)^{j+1}}{|A|} (R_1 \wedge R_2 \wedge \ \ldots \wedge \widetilde{R_j} \wedge \ \ldots \wedge R_n) \right]$$

$$= \frac{(-1)^{j+1}}{|A|} R_i . (R_1 \wedge R_2 \wedge \ \ldots \wedge \widetilde{R_j} \wedge \ \ldots \wedge R_n) \qquad if \quad i = j.$$

$$= \frac{(-1)^{j+1}}{|A|} (-1)^{j+1} |A| = 1$$

$R_i . \left(R_1 \wedge R_2 \wedge \ \ldots \wedge \widetilde{R_j} \wedge \ \ldots \wedge R_n\right) = 0$ (R_i is orthogonal to $R_1 \wedge R_2 \wedge \ \ldots \wedge \widetilde{R_j} \wedge \ \ldots \wedge R_n$)

$if \quad i \neq j$ by Lemma 1.2.

We obtain $R_i \,.\, C_j = \begin{cases} 1 & if \;\; i = j \\ 0 & if \;\; i \neq j \end{cases}$

That is

$$A^{-1} = [C_1 \quad C_2 \quad \ldots \quad C_n]$$
$$= \frac{(-1)^{j+1}}{|A|} [\, R_2 \Lambda\, R_3 \Lambda\, \ldots \Lambda\, R_n \quad R_1 \Lambda\, R_3 \Lambda\, \ldots \Lambda\, R_n \quad \ldots \quad R_1 \Lambda\, R_2 \Lambda\, \ldots \Lambda\, R_{n-1} \,]$$

Now we can use the new algorithm for constructing A^{-1} for the given some examples.

Example 2.6: Let A $= \begin{bmatrix} -1 & 3 & 2 & 0 \\ 2 & 1 & 1 & 0 \\ 1 & 4 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}$ be a 4x4 matrix.

$$|A| = -1 \begin{vmatrix} -1 & 3 & 2 \\ 2 & 1 & 1 \\ 0 & 1 & 1 \end{vmatrix} + 1 \begin{vmatrix} -1 & 3 & 2 \\ 2 & 1 & 1 \\ 1 & 4 & 0 \end{vmatrix} = 23$$

Let $A^{-1} = [C_1 \quad C_2 \quad C_3 \quad C_4]$

$$C_1 = \frac{1}{|A|}(R_2 \Lambda\, R_3 \Lambda\, R_4) = \frac{1}{23} \begin{vmatrix} i & j & k & t \\ 2 & 1 & 1 & 0 \\ 1 & 4 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{vmatrix} = \frac{1}{23}(-4i+3j+5k-8t)$$

$$C_2 = \frac{-1}{|A|}(R_1 \Lambda\, R_3 \Lambda\, R_4) = -\frac{1}{23} \begin{vmatrix} i & j & k & t \\ -1 & 3 & 2 & 0 \\ 1 & 4 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{vmatrix} = -\frac{-1}{23}(-9i+j-6k+5t)$$

$$C_3 = \frac{1}{|A|}(R_1 \Lambda\, R_2 \Lambda\, R_4) = \frac{1}{23} \begin{vmatrix} i & j & k & t \\ -1 & 3 & 2 & 0 \\ 2 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{vmatrix} = \frac{1}{23}(i+5j-7k+2t)$$

$$C_4 = \frac{-1}{|A|}(R_1 \Lambda\, R_2 \Lambda\, R_3) = -\frac{1}{23} \begin{vmatrix} i & j & k & t \\ -1 & 3 & 2 & 0 \\ 2 & 1 & 1 & 0 \\ 1 & 4 & 0 & 1 \end{vmatrix} = -\frac{-1}{23}(i+15j-7k-21t)$$

We find $A^{-1} = \frac{1}{23}\begin{bmatrix} -4 & 9 & 1 & -1 \\ 3 & -1 & 5 & -5 \\ 5 & 6 & -7 & 7 \\ -8 & -5 & 2 & 21 \end{bmatrix}$ if the new method is applied.

The result satisfies the following equality

$$A.A^{-1} = \frac{1}{23}\begin{bmatrix} 4+9+10+0 & -9-3+12+0 & -1+15-14+0 & 0 \\ -8+3+5 & 18-1+6 & 2+5-7 & 0 \\ -4+12+8 & 9-4-5 & 1+20+2 & -1-20+21 \\ 3+5-8 & -1+6-5 & 5-7+2 & -5+7+21 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Example 2.6: Let $A = \begin{bmatrix} 2 & 4 & -3 & 2 & 1 & 2 & -1 \\ 1 & 3 & -1 & -3 & 0 & 3 & 2 \\ -4 & 2 & 0 & 2 & -3 & 4 & 3 \\ -2 & -1 & 2 & 1 & 3 & 2 & 2 \\ 0 & 2 & -1 & 3 & 5 & 2 & 1 \\ -3 & 1 & 2 & -2 & -1 & 1 & 0 \\ 2 & 3 & 6 & 5 & 4 & 2 & -1 \end{bmatrix}$ be a 7x7 matrix. Then $|A| = 22740$.

Let $A^{-1} = [C_1 \quad C_2 \quad C_3 \quad C_4 \quad C_5 \quad C_6 \quad C_7]$

$$C_1 = \frac{1}{|A|}(R_2 \Lambda\, R_3 \Lambda\, R_4 \Lambda\, R_5 \Lambda\, R_6 \Lambda\, R_7)$$

$$= \begin{vmatrix} k_1 & k_2 & k_3 & k_4 & k_5 & k_6 & k_7 \\ 1 & 3 & -1 & -3 & 0 & 3 & 2 \\ -4 & 2 & 0 & 2 & -3 & 4 & 3 \\ -2 & -1 & 2 & 1 & 3 & 2 & 2 \\ 0 & 2 & -1 & 3 & 5 & 2 & 1 \\ -3 & 1 & 2 & -2 & -1 & 1 & 0 \\ 2 & 3 & 6 & 5 & 4 & 2 & -1 \end{vmatrix}$$

$$= \frac{1}{22740}(924k_1 - 7572k_2 - 3036k_3 - 508k_4 - 488k_5 + 14912\, k_6 - 13752\, k_7)$$

$C_2 = \frac{-1}{|A|}(R_1 \Lambda\, R_3 \Lambda\, R_4 \Lambda\, R_5 \Lambda\, R_6 \Lambda\, R_7)$

$$= \begin{vmatrix} k_1 & k_2 & k_3 & k_4 & k_5 & k_6 & k_7 \\ 2 & 4 & -3 & 2 & 1 & 2 & -1 \\ -4 & 2 & 0 & 2 & -3 & 4 & 3 \\ -2 & -1 & 2 & 1 & 3 & 2 & 2 \\ 0 & 2 & -1 & 3 & 5 & 2 & 1 \\ -3 & 1 & 2 & -2 & -1 & 1 & 0 \\ 2 & 3 & 6 & 5 & 4 & 2 & -1 \end{vmatrix}$$

$= \frac{-1}{22740}$ ($-2652\, k_1 - 2484\, k_2 - 1032\, k_3 + 2344\, k_4 - 76\, k_5 + 1204\, k_6 - 5124\, k_7$

$C_3 = \frac{1}{|A|}(R_1 \Lambda\, R_2 \Lambda\, R_4 \Lambda\, R_5 \Lambda\, R_6 \Lambda\, R_7)$

$$= \begin{vmatrix} k_1 & k_2 & k_3 & k_4 & k_5 & k_6 & k_7 \\ 2 & 4 & -3 & 2 & 1 & 2 & -1 \\ 1 & 3 & -1 & -3 & 0 & 3 & 2 \\ -2 & -1 & 2 & 1 & 3 & 2 & 2 \\ 0 & 2 & -1 & 3 & 5 & 2 & 1 \\ -3 & 1 & 2 & -2 & -1 & 1 & 0 \\ 2 & 3 & 6 & 5 & 4 & 2 & -1 \end{vmatrix}$$

$= \frac{1}{22740}$ ($-789\, k_1 + 1962\, k_2 + 156\, k_3 + 2378\, k_4 - 2192\, k_5 - 2077\, k_6 + 4212\, k_7$)

$C_4 = \frac{-1}{|A|}(R_1 \Lambda\, R_2 \Lambda\, R_3 \Lambda\, R_5 \Lambda\, R_6 \Lambda\, R_7)$

$$= \begin{vmatrix} k_1 & k_2 & k_3 & k_4 & k_5 & k_6 & k_7 \\ 2 & 4 & -3 & 2 & 1 & 2 & -1 \\ 1 & 3 & -1 & -3 & 0 & 3 & 2 \\ -4 & 2 & 0 & 2 & -3 & 4 & 3 \\ 0 & 2 & -1 & 3 & 5 & 2 & 1 \\ -3 & 1 & 2 & -2 & -1 & 1 & 0 \\ 2 & 3 & 6 & 5 & 4 & 2 & -1 \end{vmatrix}$$

$= \frac{-1}{22740}$ ($-1623\, k_1 + 13374\, k_2 + 1272\, k_3 + 1606\, k_4 - 964\, k_5 - 18539\, k_6 + 11604\, k_7$)

$C_5 = \frac{1}{|A|}(R_1 \Lambda\, R_2 \Lambda\, R_3 \Lambda\, R_4 \Lambda\, R_6 \Lambda\, R_7)$

$$= \begin{vmatrix} k_1 & k_2 & k_3 & k_4 & k_5 & k_6 & k_7 \\ 2 & 4 & -3 & 2 & 1 & 2 & -1 \\ 1 & 3 & -1 & -3 & 0 & 3 & 2 \\ -4 & 2 & 0 & 2 & -3 & 4 & 3 \\ -2 & -1 & 2 & 1 & 3 & 2 & 2 \\ -3 & 1 & 2 & -2 & -1 & 1 & 0 \\ 2 & 3 & 6 & 5 & 4 & 2 & -1 \end{vmatrix}$$

$= \frac{1}{22740}$ $(-3816\, k_1 + 10008\, k_2 - 456\, k_3 + 1212\, k_4 + 3492\, k_5 - 14628\, k_6 + 10428\, k_7)$

$C_6 = \frac{-1}{|A|}(R_1 \Lambda\, R_2 \Lambda\, R_3 \Lambda\, R_4 \Lambda\, R_5 \Lambda\, R_7)$

$$= \begin{vmatrix} k_1 & k_2 & k_3 & k_4 & k_5 & k_6 & k_7 \\ 2 & 4 & -3 & 2 & 1 & 2 & -1 \\ 1 & 3 & -1 & -3 & 0 & 3 & 2 \\ -4 & 2 & 0 & 2 & -3 & 4 & 3 \\ -2 & -1 & 2 & 1 & 3 & 2 & 2 \\ 0 & 2 & -1 & 3 & 5 & 2 & 1 \\ 2 & 3 & 6 & 5 & 4 & 2 & -1 \end{vmatrix}$$

$= \frac{1}{22740}$ $(5148\, k_1 - 3204\, k_2 - 672\, k_3 + 2584\, k_4 - 1636\, k_5 + 784\, k_6 + 4596\, k_7)$

$C_7 = \frac{1}{|A|}(R_1 \Lambda\, R_2 \Lambda\, R_3 \Lambda\, R_4 \Lambda\, R_5 \Lambda\, R_6)$

$$= \begin{vmatrix} k_1 & k_2 & k_3 & k_4 & k_5 & k_6 & k_7 \\ 2 & 4 & -3 & 2 & 1 & 2 & -1 \\ 1 & 3 & -1 & -3 & 0 & 3 & 2 \\ -4 & 2 & 0 & 2 & -3 & 4 & 3 \\ -2 & -1 & 2 & 1 & 3 & 2 & 2 \\ 0 & 2 & -1 & 3 & 5 & 2 & 1 \\ -3 & 1 & 2 & -2 & -1 & 1 & 0 \end{vmatrix}$$

$= \frac{1}{22740}$ $(1443\, k_1 + 1686\, k_2 + 2568\, k_3 + 954\, k_4 - 516\, k_5 - 1101\, k_6 + 1116\, k_7)$

We find

$$A^{-1} = \frac{1}{22740}\begin{bmatrix} 924 & 2652 & -789 & 1623 & -3816 & -5148 & 1443 \\ -7572 & 2484 & 1962 & -13374 & 10008 & 3204 & 1686 \\ -3036 & 1032 & 156 & -1272 & -456 & 672 & 2568 \\ -508 & -2344 & 2378 & -1606 & 1212 & -2584 & 954 \\ -488 & 76 & -2192 & 964 & 3492 & 1636 & -516 \\ 14912 & -1204 & -2077 & 18539 & -14628 & -784 & -1101 \\ -13752 & 5124 & 4212 & -11604 & 10428 & -4596 & 1116 \end{bmatrix}$$

CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests.

REFERENCES

[1] Bareiss, Erwin H. Sylvester's identity and multistep integer-preserving Gaussian elimination. Math. Comput. 22 (103) (1968), 565-578.

[2] Anton, Howard. Elementary linear algebra. John Wiley & Sons, 2010.

[3] Bernstein, Dennis S. Matrix mathematics: Theory, facts, and formulas with application to linear systems theory. Vol. 41. Princeton: Princeton University Press, 2005.

[4] Meyer, Carl D. Matrix analysis and applied linear algebra. Vol. 71. SIAM, 2000.

[5] Poole, David. Linear algebra: A modern introduction. Cengage Learning, 2014.

[6] Lipschutz, Seymour, and Marc Lipson. Schaum's Outline of Linear Algebra. McGraw Hill Professional, 2017.