Available online at http://scik.org
Advances in Fixed Point Theory, 3 (2013), No. 2, 443-450
ISSN: 1927-6303

ASYMPTOTICALLY STABLE SETS AND ATTRACTORS OF INVERSE LIMIT DYNAMICAL SYSTEMS

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Abstract. In this paper we study asymptotically stable sets and attractors of inverse limit dynamical system which is induced from dynamical system on a compact metric space. We give the implication of asymptotically stable sets and attractors between inverse limit dynamical systems and original systems. More precisely, the inverse limit system has asymptotically stable sets implies original system has asymptotically stable sets. Also, we prove that the inverse limit system has attractors implies original system has attractors.

Keywords: Inverse limit dynamical system; Lyapnov stable set; Asymptotically stable set; Attractor.

2000 AMS Subject Classification: 54H20, 37B20

1. Introduction

The concept of asymptotically stable set for discrete dynamical system was introduced by Block and Coppel [2] and Robinson [13] and the concept of attractors for flows on metric space was introduced by Temam [14] and Hale [7]. In recent years, Marzocchi and Necca [9] studied ω-limit sets and attractors in regular topological space, Mimna and Steele [10] discussed ω-limit sets and asymptotically stable sets for semi-homeomorphisms,

†Supported by the Natural Science Foundation of Henan Province(122300410427), PR China.

Received April 18, 2013

Aniello and Steele [1] discussed the stability of ω-limit sets, Oprocha [11] studied asymptotically stable sets in continuous dynamical systems and Braga and Souza [4] studied attractors for semigroup actions.

Along with the deep research on the properties of topological dynamical systems, many people also considered dynamical properties in some induced dynamical systems such as inverse limit dynamical systems. Li [8] studied Devaney chaos of inverse limit dynamical systems and proved that an inverse limit dynamical system is Devaney chaos if and only if its original system is Devaney chaos. Chen and Li [6] discussed shadowing property for inverse limit spaces, Ye [15] studied topological entropy of inverse limit dynamical system, Block, Jakimovik, Keesling and Kailhofer [3], Bruin [5] and Raines and Stimac [12] discussed the properties of inverse limit spaces of tent maps.

In this paper we discuss asymptotically stable sets and attractors of inverse limit dynamical systems on the basis of [9]. Our purpose is to discuss implication of asymptotically stable sets and attractors between inverse limit systems and original system. We prove that the inverse limit system has asymptotically stable sets implies original system has asymptotically stable sets. Also, we prove that the inverse limit system has attractors implies original system has attractors.

2. Preliminaries

Throughout this paper a topological dynamical system is a pair (X, f), where X is a compact metric space with metric d and $f : X \to X$ is a continuous map.

Let (X, d) be a compact metric space and let $f : X \to X$ be continuous map. The inverse limit space of f is a metric space defined by the sequence

$$X \overset{f}{\leftarrow} X \overset{f}{\leftarrow} X \overset{f}{\leftarrow} \cdots$$

whose elements $\underline{x} = (x_0, x_1, x_2, \cdots)$ satisfy $f(x_{i+1}) = x_i$, $i = 0, 1, 2 \cdots$ and the metric is defined by

$$\underline{d}(\underline{x}, \underline{y}) = \sum_{i=0}^{\infty} \frac{d(x_i, y_i)}{2^i}.$$

The inverse limit space of (X, f) is denoted by $\lim\limits_{\leftarrow}(X, f)$.

The inverse limit space $\lim\limits_{\leftarrow}(X, f)$ is a compact subspace of product space $\prod\limits_{i=1}^{\infty} X_i$, $(X_i = X, i = 1, 2, \cdots)$, the shift map $\sigma_f : \lim\limits_{\leftarrow}(X, f) \to \lim\limits_{\leftarrow}(X, f)$ is defined by $\sigma_f(x_0, x_1, \cdots) = (f(x_0), x_0, x_1, \cdots)$. Furthermore, $\sigma_f^k(x_0, x_1, \cdots) = (f^k(x_0),$

$f^k(x_1), \cdots)$, where k is a non-negative integer. σ_f is a homeomorphism and $\sigma_f^{-1}(x_0, x_1, x_2, \cdots) = (x_1, x_2, \cdots)$. The inverse limit dynamical system is denoted by $(\varprojlim(X, f), \sigma_f)$.

The projection map $\pi_i : \varprojlim(X, f) \to X$ is defined by $\pi_i(x_0, x_1, \cdots x_i, \cdots) = x_i$ for each $i = 0, 1, \cdots$. Clearly, π_i is a continuous mapping, and $f \circ \pi_i = \pi_i \circ \sigma_f$ for $i = 0, 1, \cdots$. If f is a surjective map, then π_i is an open surjective mapping for each $i = 0, 1, \cdots$. The metric $\underline{d}$ induces the inverse limit topology. This topology has a basis

$$\mathcal{B} = \{V : V = \pi_i^{-1}(U) \text{ for some } i \geq 0 \text{ and some open set } U \text{ in } X\}.$$

Let (X, f) be a dynamical system, $\gamma(x, f)$ denotes the orbit of x under f for some $x \in X$, i.e., $\gamma(x, f) = \{x, f(x), f^2(x), \cdots, f^n(x), \cdots\}$ where $f^n = f \circ f^{n-1}$ and f^0 denotes the identity map on X. A point is called an ω-limit point if it is an accumulation point of the forward orbit of some point in X. The collection of the ω-limit points of one point x is denoted by $\omega(x, f)$.

In the present paper, a dynamical system we mean a pair (X, f), where X is a compact metric space and $f : X \to X$ is a surjective continuous map. Let $\mathbb{N}$ denotes natural number set and $\mathbb{Z}_+ = \mathbb{N} \cup \{0\}$.

Definition 0.1. [9] *Let (X, f) be a dynamical system. For every $B \subseteq X$ and $m \in \mathbb{Z}_+$, the set $\gamma_m(B, f) = \bigcup_{x \in B} \{f^n(x) : n \geq m\}$ is called positive orbit through B starting at time m. If $B = \{x\}$, we will write $\gamma_m(x, f)$ instead of $\gamma_m(\{x\}, f)$. If $m = 0$, we will omit time index.*

Definition 0.2. [9] *Let (X, f) be a dynamical system. Let $B \subseteq X$ and define $\omega(B, f)$ as the set of limit points of the orbit $\gamma(B, f)$, i.e., $\omega(B, f) = \bigcap_{m \in \mathbb{Z}_+} \overline{\gamma_m(B, f)}$. If $B = \{x\}$, we will write $\omega(x, f)$ instead of $\omega(\{x\}, f)$.*

In fact, $\omega(B, f)$ is a nonempty closed set of X and strongly invariant, i.e., $f(\omega(B, f)) = \omega(B, f)$.

Definition 0.3. [2] *Let (X, f) be a dynamical system, where (X, f) be a compact metric space. A is a nonempty closed set in X.*

- **(1):** *A is said to be Lyapunov stable if for each open set U containing A there exists an open set V containing A such that $\gamma(x, f) \subseteq U$ for every $x \in V$.*
- **(2):** *A is said to be asymptotically stable if A is Lyapunov stable and there exists an open set U_0 containing A such that $\omega(x, f) \subseteq A$ for every $x \in U_0$.*

Definition 0.4. [9] *Let (X, f) be a dynamical system, A and B are two subsets in X. A attracts B if for any open set U containing A there exists $m \in \mathbb{Z}_+$ such that $\gamma_m(B, f) \subseteq U$.*

In [9], $A \subseteq X$ and $\mathcal{B} \subseteq \mathcal{P}(X)$ be a family of subsets of X. A is said to be $\mathcal{B}-$attracting if it attracts every $B \in \mathcal{B}$. For simplicity, in the sequel we will say that A is attracting if it is $\mathcal{B}-$attracting for some $\mathcal{B} \subseteq \mathcal{P}(X)$.

Definition 0.5. [9] *Let (X, f) be a dynamical system where (X, d) is a compact metric space. $A \subseteq X$ is said to be an attractor if it is an attracting, strictly invariant and compact set.*

3. Asymptotically stable sets of the inverse limit spaces

In this section, we will discuss asymptotically stable sets in the inverse limit spaces and obtain that the inverse limit dynamical system $(\underleftarrow{\lim}(X, f), \sigma_f)$ has asymptotically stable sets implies (X, f) has asymptotically stable sets.

It follows at once from the definition that any Lyapunov stable set, and hence also any asymptotically stable set, is invariant, i.e., if A is an asymptotically stable set in X, then $f(A) \subseteq A$.

Lemma 0.1. [8] *Let (X, f) be a dynamical system, $x_0 \in X$ and $\underline{x} \in (\underleftarrow{\lim}(X, f), \sigma_f)$ satisfy $\underline{x} = (x_0, x_1, x_2, \cdots)$. Then $\omega(\underline{x}, \sigma_f) = \underleftarrow{\lim}(\omega(x_0, f), f)$.*

Theorem 0.1. *Let (X, f) be a dynamical system and $f : X \to X$ be a surjective map. If $\widetilde{A}$ is an asymptotically stable set in $(\underleftarrow{\lim}(X, f), \sigma_f)$, then $\pi_0(\widetilde{A})$ is an asymptotically stable set in (X, f).*

Proof. Suppose that $\widetilde{A}$ is an asymptotically stable set in $(\underleftarrow{\lim}(X, f), \sigma_f)$. Then $\widetilde{A}$ is closed set in $\underleftarrow{\lim}(X, f)$. Since f is a surjective map, which implies $\pi_0 : \underleftarrow{\lim}(X, f) \to X$ is an open surjective continuous map. Furthermore, $\pi_0(\underleftarrow{\lim}(X, f)\backslash\widetilde{A}) = \pi_0(\underleftarrow{\lim}(X, f))\backslash\pi_0(\widetilde{A}) = X \setminus \pi_0(\widetilde{A})$ is an open set in X. Hence, $\pi_0(\widetilde{A})$ is a closed set in X.

Firstly, we show that $\pi_0(\widetilde{A})$ is Lyapunov stable in X. Let U is an open set in X and $U \supseteq \pi_0(\widetilde{A})$. As π_0 is a continuous map, $\pi_0^{-1}(U)$ is an open set in $\underleftarrow{\lim}(X, f)$ and $\widetilde{A} \subseteq \pi_0^{-1}(U)$. Since $\widetilde{A}$ is Lyapunov stable in $(\underleftarrow{\lim}(X, f), \sigma_f)$, there exists an open set $\widetilde{W} \supseteq \widetilde{A}$ such that $\gamma(\underline{x}, \sigma_f) \subseteq \pi_0^{-1}(U)$ for every $\underline{x} \in \widetilde{W}$. Moreover, π_0 is an open map, $\pi_0(\widetilde{W})$ is an open set in X and $\pi_0(\widetilde{A}) \subseteq \pi_0(\widetilde{W})$. For any $x \in \pi_0(\widetilde{W})$, we have $\pi_0^{-1}(x) \cap \widetilde{W} \neq \emptyset$. Furthermore, let $\underline{x} = (x, x_1, x_2, \cdots) \in \pi_0^{-1}(x) \cap \widetilde{W}$, we have $\gamma(\underline{x}, \sigma_f) \subseteq \pi_0^{-1}(U)$. Since $f^n \circ \pi_0 = \pi_0 \circ \sigma_f^n$ for any $n \geq 0$, it follows that $\pi_0(\gamma(\underline{x}, \sigma_f)) = \{\pi_0(\underline{x}), \pi_0 \circ \sigma_f(\underline{x}), \pi_0 \circ \sigma_f^2(\underline{x}), \cdots\} = \{\pi_0(\underline{x}), f \circ \pi_0(\underline{x}), f^2 \circ \pi_0(\underline{x}), \cdots\} = \{x, f(x), f^2(x), \cdots\} = \gamma(x, f)$. Hence, $\gamma(x, f) \subseteq \pi_0(\pi_0^{-1}(U) = U$. This shows $\pi_0(\widetilde{A})$ is Lyapunov stable in (X, f).

Secondly, we show that there exists an open set $U_0 \supseteq \pi_0(\widetilde{A})$ such that $\omega(x,f) \subseteq \pi_0(\widetilde{A})$ for every $x \in U_0$. Since $\widetilde{A}$ is asymptotically stable in $(\lim\limits_{\leftarrow}(X,f),\sigma_f)$, there exists an open set $\widetilde{U_0} \supseteq \widetilde{A}$ such that $\omega(\overline{x},f) \subseteq \widetilde{A}$ for every $\underline{x} \in \widetilde{U_0}$. As π_0 is an open map, $\pi_0(\widetilde{U_0})$ is an open set in X and $\pi_0(\widetilde{A}) \subseteq \pi_0(\widetilde{U_0})$. For any $x \in \pi_0(\widetilde{U_0})$, we have $\pi_0^{-1}(x) \cap \widetilde{U_0} \neq \emptyset$. Furthermore, take $\underline{x} = (x, x_1, x_2, \cdots) \in \pi_0^{-1}(x) \cap \widetilde{U_0}$, $\omega(\underline{x}, \sigma_f) \subseteq \widetilde{A}$. According to Lemma 0.1, $\omega(\underline{x},\sigma_f) = \lim\limits_{\leftarrow}(\omega(x,f),f)$. Since $\widetilde{A}$ is invariant and $\omega(\underline{x},\sigma_f) \subseteq \widetilde{A}$, we have $\lim\limits_{\leftarrow}(\omega(x,f),f) \subseteq \widetilde{A}$. Furthermore, $\pi_0(\lim\limits_{\leftarrow}(\omega(x,f),f)) = \omega(x,f) \subseteq \pi_0(\widetilde{A})$. Hence, $\omega(x,f) \subseteq \pi_0(\widetilde{A})$ for every $x \in \pi_0(\widetilde{U_0})$. This shows $\pi_0(\widetilde{A})$ is an asymptotically stable set in (X,f).

4. Attractors of of inverse limit dynamical systems

In this section, let (X,d) be a compact metric space and $f: X \to X$ be a surjective continuous map. We will discuss the relationship between attractors of inverse limit dynamical system $(\lim\limits_{\leftarrow}(X,f),\sigma_f)$ and attractors of original system (X,f).

Lemma 0.2. [8] *Let W be a nonempty closed and strongly invariant subset of X. If there is a closed and strongly invariant subset $\widetilde{W} \subseteq \lim\limits_{\leftarrow}(X,f)$ such that $\pi_0(\widetilde{W}) = W$, then $\widetilde{W} = \lim\limits_{\leftarrow}(W,f)$.*

Theorem 0.2. *Let (X,f) be a dynamical system where (X,d) is a compact metric space and B is a nonempty subset of X. If $\widetilde{B} = \pi_0^{-1}(B) \in \lim\limits_{\leftarrow}(X,f)$, i.e., for any $\underline{x} \in \widetilde{B}$, $\pi_0(\underline{x}) \in B$. Then $\omega(\widetilde{B},\sigma_f) = \lim\limits_{\leftarrow}(\omega(B,f),f)$.*

Proof. Let $\underline{x} = (x_0, x_1, \cdots) \in \omega(\widetilde{B},\sigma_f)$. Then there exist $\underline{y} = (y_0, y_1, \cdots) \in \widetilde{B}$ and a sequence $\{n_i\}_{i=1}^{\infty}$ such that $\lim\limits_{i\to\infty} \sigma_f^{n_i}(\underline{y}) = \underline{x}$. Furthermore, $\pi_0(\lim\limits_{i\to\infty} \sigma_f^{n_i}(\underline{y})) = \lim\limits_{i\to\infty} \pi_0(\sigma_f^{n_i}(\underline{y})) = \lim\limits_{i\to\infty} f^{n_i}(\pi_0(\underline{y})) = \pi_0(\underline{x})$, thus $\lim\limits_{i\to\infty} f^{n_i}(y_0) = x_0$. But $\pi_0^{-1}(B) = \widetilde{B}$, we have $\pi_0(\omega(\widetilde{B},\sigma_f)) \subseteq \omega(B,f)$. On the other hand, for any $x \in \omega(B,f)$, there exist $y \in B$ and a sequence $\{m_i\}_{i=1}^{\infty}$ such that $\lim\limits_{i\to\infty} f^{m_i}(y) = x$. Let $\underline{y} = (y, y_1, \cdots) \in \pi_0^{-1}(B) = \widetilde{B}$. Since $\omega(\widetilde{B},\sigma_f)$ is compact subset of X, then $\{\sigma_f^{m_i}(\underline{y})\}_{i=1}^{\infty}$ has a convergence subsequence, without loss of generality, let $\lim\limits_{i\to\infty} \sigma_f^{m_i}(\underline{y}) = \underline{x}$. Furthermore, $\pi_0(\lim\limits_{i\to\infty} \sigma_f^{m_i}(\underline{y})) = \lim\limits_{i\to\infty} \pi_0(\sigma_f^{m_i}(\underline{y})) = \lim\limits_{i\to\infty} f^{m_i}(\pi_0(\underline{y})) = x$, thus $x = \pi_0(\underline{x})$. This shows that $x \in \pi_0(\omega(\widetilde{B},\sigma_f))$. Therefore, $\pi_0(\omega(\widetilde{B},\sigma_f)) = \omega(B,f)$. Note that both $\omega(B,f)$ and $\omega(\widetilde{B},\sigma_f)$ are closed and strongly invariant. By Lemma 0.2, we have $\omega(\widetilde{B},\sigma_f) = \lim\limits_{\leftarrow}(\omega(B,f),f)$.

Lemma 0.3. *Let (X, f) be a dynamical system where (X, d) is a compact metric space. Let B is a nonempty subset of X. If $\widetilde{B} = \pi_0^{-1}(B) \in \varprojlim(X, f)$, then $\pi_0(\gamma_m(\widetilde{B}, \sigma_f)) = \gamma_m(B, f)$.*

Proof. Let $\underline{x} \in \gamma_m(\widetilde{B}, \sigma_f)$. Then there exist $\underline{y} \in \widetilde{B}$ and $n \geq m$ such that $\underline{x} = \sigma_f^n(\underline{y})$. Furthermore, we have $\pi_0(\underline{x}) = \pi_0(\sigma_f^n(\underline{y}) = f^n(\pi_0(\underline{y})) \in \gamma_m(B, f)$, which implies $\pi_0(\gamma_m(\widetilde{B}, \sigma_f)) \subseteq \gamma_m(B, f)$.

On the other hand, if $x \in \gamma_m(B, f)$, then there exist $y \in B$ and $n \geq m$ such that $x = f^n(y)$. Let $\underline{x} = (x, x_1, x_2, \cdots) \in \widetilde{B}$. Then $\underline{x} \in \pi_0^{-1}(x) = \pi_0^{-1}(f^n(y))$. Furthermore, $\underline{x} \in \sigma_f^n(\pi_0^{-1}(y))$, there exists $\underline{y} = (y, y_1, y_2, \cdots) \in \pi_0^{-1}(y) \subseteq \widetilde{B}$ such that $\underline{x} = \sigma_f^n(\underline{y})$. Therefore, we have $x \in \pi_0(\gamma_m(\widetilde{B}, \sigma_f))$. This shows that $\pi_0(\gamma_m(\widetilde{B}, \sigma_f)) = \gamma_m(B, f)$.

Theorem 0.3. *Let (X, f) be a dynamical system where (X, d) is a compact metric space. Let $\widetilde{A}$ and $\widetilde{B}$ are two subsets in $\varprojlim(X, f)$. If $\widetilde{A}$ attracts $\widetilde{B}$, then $\pi_0(\widetilde{A})$ attracts $\pi_0(\widetilde{B})$.*

Proof. Let $A = \pi_0(\widetilde{A})$, $B = \pi_0(\widetilde{B})$ and U is any open set of X with U containing A, then $\pi_0^{-1}(U)$ is open set of $\varprojlim(X, f)$ and $\widetilde{A} \subseteq \pi_0^{-1}(U)$. Since $\widetilde{A}$ attracts $\widetilde{B}$, it follows that there exists $m \in \mathbb{Z}_+$ such that $\gamma_m(\widetilde{B}, \sigma_f) \subseteq \pi_0^{-1}(U)$. Furthermore,

$$\pi_0(\gamma_m(\widetilde{B}, \sigma_f)) = \pi_0(\bigcup_{\underline{x} \in \widetilde{B}} \{\sigma_f^n(\underline{x}) : n \geq m\}) = \bigcup_{\underline{x} \in \widetilde{B}} \{\pi_0(\sigma_f^n(\underline{x})) : n \geq m\}.$$

Since $\pi_0(\sigma_f^n(\underline{x})) = f^n(\pi_0(\underline{x}))$ and $\pi_0(\widetilde{B}) = B$, it follows that

$$\pi_0(\gamma_m(\widetilde{B}, \sigma_f)) = \bigcup_{x \in B} \{f^n(x) : n \geq m\} = \gamma_m(B, f).$$

Note that $\gamma_m(\widetilde{B}, \sigma_f) \subseteq \pi_0^{-1}(U)$ and π_0 is an open surjective map, thus $\pi_0(\gamma_m(\widetilde{B}, \sigma_f)) \subseteq U$. Therefore, we have $\gamma_m(B, f) \subseteq U$. This shows $\pi_0(\widetilde{A})$ attracts $\pi_0(\widetilde{B})$.

Theorem 0.4. *Let (X, f) be a dynamical system where (X, d) is a compact metric space. If $\widetilde{A} \subseteq \varprojlim(X, f)$ is an attractor of inverse limit dynamical system $(\varprojlim(X, f), \sigma_f)$, then $\pi_0(\widetilde{A})$ is an attractor of (X, f).*

Proof. Let $A = \pi_0(\widetilde{A})$. Since $\widetilde{A}$ is an attractor of $(\varprojlim(X, f), \sigma_f)$, then $\widetilde{A}$ is an attracting, strictly invariant and compact set. Note that π_0 is a continuous map, we have A is a compact set of X. Furthermore, $\widetilde{A}$ is attracting, there exists $\underline{\mathcal{B}} \in \mathcal{P}(\varprojlim(X, f))$ such that $\widetilde{A}$ attracts $\widetilde{B}$ for any $\widetilde{B} \in \underline{\mathcal{B}}$. Hence, $\pi_0(\underline{\mathcal{B}}) = \{\pi_0(\widetilde{B}) : \widetilde{B} \in \underline{\mathcal{B}}\} \subseteq \mathcal{P}(X)$. Since $\widetilde{A}$ is strictly invariant set of (X, f), then $\sigma_f(\widetilde{A}) = \widetilde{A}$, thus we have $\pi_0(\sigma_f(\widetilde{A})) = f(\pi_0(\widetilde{A})) = \pi_0(\widetilde{A})$. This shows that $f(A) = A$, i.e., A is strictly invariant set of (X, f).

Finally, we prove that A is attracting. Since $\widetilde{A}$ attracts $\widetilde{B}$ for any $\widetilde{B} \in \underline{\mathcal{B}}$, by the Theorem 0.3, it follows that $A = \pi_0(\widetilde{A})$ attracts $\pi_0(\widetilde{B})$ for any $\pi_0(\widetilde{B}) \in \pi_0(\underline{\mathcal{B}})$. Furthermore, $\pi_0(\underline{\mathcal{B}}) = \{\pi_0(\widetilde{B}) : \widetilde{B} \in \underline{\mathcal{B}}\} \subseteq \mathcal{P}(X)$, thus A attracts B for any $B \in \pi_0(\underline{\mathcal{B}})$. Therefore, $\pi_0(\widetilde{A})$ is an attractor of (X, f).

Definition 0.6. [9] *Let (X, f) be a dynamical system where (X, d) is a compact metric space and let A and B are two subsets of X. A absorbs B if there exists $m \in \mathbb{N}$ such that $\gamma_m(A, f) \subseteq \gamma(B, f)$.*

Theorem 0.5. *Let (X, f) be a dynamical system where (X, d) is a compact metric space and let $\widetilde{A}$ and $\widetilde{B}$ are two subsets of $\underleftarrow{\lim}(X, f)$. If $\widetilde{A}$ absorbs $\widetilde{B}$, then $\pi_0(\widetilde{A})$ absorbs $\pi_0(\widetilde{B})$.*

Proof. Since $\widetilde{A}$ absorbs $\widetilde{B}$, there exists $m \in \mathbb{N}$ such that $\gamma_m(\widetilde{B}, \sigma_f) \subseteq \gamma(\widetilde{A}, \sigma_f)$. By Lemma 0.3, we have $\pi_0(\gamma_m(\widetilde{B}, \sigma_f)) = \gamma(\pi_0(\widetilde{B}), f)$ and $\pi_0(\gamma(\widetilde{A}, \sigma_f)) = \gamma(\pi_0(\widetilde{A}), f)$, thus $\gamma(\pi_0(\widetilde{A}), f) \subseteq \gamma(\pi_0(\widetilde{A}), f)$. Therefore, $\pi_0(\widetilde{A})$ absorbs $\pi_0(\widetilde{B})$.

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