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A COMMON FIXED POINT THEOREM FOR WEAKLY COMPATIBLE MAPPINGS IN MENGER PM-SPACES

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Abstract. In present paper we prove a unique common fixed point theorem for four weakly compatible self-mappings in Menger-PM spaces without using the notion of continuity.

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1.Introduction

There have been number of generalizations of metric spaces.One such is probabilistic metric spaces(in brief PM-spaces)introduced by K. Menger [3] in 1942.The study of these spaces expanded rapidly with pioneering work of Schweizer and Sklar [5,6].Further in 1972,Sehgal [7] initiated study of contraction mappings in PM-Spaces.Since then there have been great developments in fixed point theorems with different conditions on mappings or on spaces itself.The notion of weakly commuting maps was initiated by Sessa [8] in metric spaces.Jungck [6] gave the concept of compatible maps and showed that weakly commuting maps are compatible but converse is not true.Jungck [2] further weakened the notion of compatibility and showed that compatible maps are weakly compatible but the

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converse is not true.Mishra [4] introduced the concept of compatible self-maps in Menger spaces and obtained a common fixed point theorem for four self mappings using compatibility and continuity of two functions.Singh and Jain [9] obtained a common fixed point theorem in Menger spaces through weak compatibility and continuity of one function and thus generalized the results of Mishra [4].In present paper we prove a unique common fixed point theorem for four self-mappings using weak compatibility and without using continuity.Doing so we establish a unique common fixed point theorem with less number of conditions in comparision of Mishra [4].Later we extend oue result to sequence of mappings whereas B. Singh [9] extended the result of Mishra[4] upto six mappings.In paper let R^+ denotes set of all non-negative real numbers.

2. Preliminaries

Definition 2.1. A mapping $F : R \to R^+$ is called a distribution function if it is non-decreasing and left continuous with $inf_{t\in R}F(t) = 0$ and $sup_{t\in R}F(t) = 1$.Let D denotes the set of all distribution functions whereas H stands for specific distribution function(also known as Heaviside function) defined as

$$H(t) = \begin{cases} 0, & t \leq 0; \\ 1, & t > 0. \end{cases}$$

Definition 2.2. A PM-space is an ordered pair (X, F)consisting of non- empty set $X$and a mapping F from $X \times X$ into D.The value of F at $(x, y) \in X$ is represented by $F_{x,y}$.The functions $F_{x,y}$are assumed to satisfy the following conditions:

$(PM1)$ $F_{x,y}(t) = 1$ for all $t > 0$ if and only if $x = y$;

$(PM2)$ $F_{x,y}(0) = 0$;

$(PM3)$ $F_{x,y}(t) = F_{y,x}(t)$;

$(PM4)$ if $F_{x,y}(t) = 1$ and $F_{y,z}(s) = 1$,then $F_{x,z}(t + s) = 1$for all $x, y \in X$ and $t, s \geq 0$.

Every metric (X, d) space can always be realized as a PM-space by considering F from $X \times X$ into D as $F_{u,v}(s) = H(s - d(u, v))$ for all $u, v \in X$.

Definition 2.3. A mapping $\Delta : [0,1] \times [0,1] \to [0,1]$ is called a triangular norm (briefly t-norm) if the following conditions are satisfied:

(1)$\Delta(a,1) = a \ for \ all \ a \in [0,1]$;

(2) $\Delta(a,b) = \Delta(b,a)$;

(3) $\Delta(c,d) \geq \Delta(a,b)$ for $c \geq a, d \geq b$;

(4) $\Delta(\Delta(a,b),c) = \Delta(a,\Delta(b,c))$ for all $a,b,c,d \in [0,1]$.

Examples of t-norm are $\Delta(a,b) = min(a,b)$, $\Delta(a,b) = ab$ and $\Delta(a,b) = min(a+b-1,0)$ etc.

Definition 2.4. A Menger space is a triplet(X,F,Δ),where$(X,F)$is a PM-space,$\Delta$is t-norm and the following condition hold:

$(PM5) F_{x,z}(t+s) \geq \Delta(F_{x,y}(t), F_{y,z}(s))$ holds for all $x,y,z \in X$ and $t,s \geq 0$.

Definition 2.5. A sequence $\{p_n\}$in a Menger space (X,F,Δ) is said to converge to a point p in X if for every $\epsilon > 0$ and $\lambda > 0$,there is an integer $N(\epsilon,\lambda)$ such that $F_{p_n,p}(\epsilon) > 1-\lambda$,for all $n \geq N(\epsilon,\lambda)$.The sequence is said to be Cauchy sequence if for every $\epsilon > 0$ and $\lambda > 0$,there is an integer $N(\epsilon,\lambda)$ such that$F_{p_n,p_m}(\epsilon) > 1-\lambda$,for all $n,m \geq N(\epsilon,\lambda)$.

Definition 2.6. Self mappings A and S of a Menger space (X,F,Δ) are said to be compatible if $F_{ASx_n,SAx_n}(\epsilon) \to 1$ for all $\epsilon > 0$ when $\{x_n\}$is a sequence in X such that $Ax_n, Sx_n \to u$ for some $u \in X$ as $n \to \infty$.

Definition 2.7. Self mappings A and S of a Menger space (X,F,Δ) are said to be weakly compatible if they commute at their coincidence point that is, $Ax = Sx$ for $x \in X$ implies $ASx = SAx$.

Lemma 2.1. [9]Let (X,F,Δ) be a Menger Space.If there exist $h \in (0,1)$ such that $F_{u,v}(ht) \geq F_{u,v}(t)$ for all $u,v \in X$ then $u = v$.

Let Φ be the class of all real-valued continuous functions $\phi : (R^+)^4 \to R$, non-decreasing in first argument and satisfying the following conditions:

for all $x,y \geq 0, \phi(x,y,x,y) \geq 0$ or $\phi(x,y,y,x) \geq 0$ implies $x \geq y$ (2.1)

$\phi(x,x,1,1) \geq 0$ for all $x \geq 1$ (2.2)

Example 2.1. Define $\phi : (R^+)^4 \to R$ as $\phi(x_1,x_2,x_3,x_4) = 2x_1 - max\{2x_2, x_3/2, x_4/2\}$.Let $x,y \geq 0$ such that $\phi(x,y,x,y) \geq 0$ implies $2x - max\{2y, x/2, y/2\} \geq 0$ implies $2x -$

$max\{2y, x/2\} \geq 0$.If $max\{2y, x/2\} = 2y$, then $2x-2y \geq 0$ implies $x \geq y$. If $max\{2y, x/2\} = x/2$ then $2x - x/2 \geq 0$ implies $2x \geq x/2 \geq 2y$ implies $x \geq y$.Similarly $x \geq y$ can be proved when $x, y \geq 0$ such that $\phi(x, y, y, x) \geq 0$.Let $x \geq 1$, then $\phi(x, x, 1, 1) = 2x - max\{2x, 1/2, 1/2\} = 2x - 2x = 0$. Hence $\phi \in \Phi$.

3. Main results

Theorem 3.1 Let A,B,S and T be self mappings on a complete Menger Space $(X, F, \Delta)$where $\Delta = min$ and satisfying

(3.1) $A(X) \subseteq T(X), B(X) \subseteq S(X)$.

(3.2) Pairs (A, S) and (B, T) are weakly compatible.

(3.3) $\phi(F_{Au,Bv}(ht), F_{Su,Tv}(t), F_{Au,Su}(t), F_{Bv,Tv}(ht)) \geq 0$.

for all $u, v \in X, t > 0, h \in (0, 1)$.Then A,B,S and T have a unique common fixed point in X.

Proof: Define sequences $< x_n >$ and $< y_n >$ in X such that $y_{2n+1} = Ax_{2n} = Tx_{2n+1}$ and $y_{2n+2} = Bx_{2n+1} = Sx_{2n+2}$ for $n = 0, 1, 2...$

Putting $u = x_{2n}, v = x_{2n+1}$in (3.3) we get

$\phi(F_{Ax_{2n},Bx_{2n+1}}(ht), F_{Sx_{2n},Tx_{2n+1}}(t), F_{Ax_{2n},Sx_{2n}}(t), F_{Bx_{2n+1},Tx_{2n+1}}(ht)) \geq 0$.

$\phi(F_{y_{2n+1},y_{2n+2}}(ht), F_{y_{2n},y_{2n+1}}(t), F_{y_{2n+1},y_{2n}}(t), F_{y_{2n+2},y_{2n+1}}(ht)) \geq 0$.

Using (2.1) we get

$F_{y_{2n+1},y_{2n+2}}(ht) \geq F_{y_{2n},y_{2n+1}}(t)$

We can write $F_{y_n,y_{n+1}}(t) \geq F_{y_{n-1},y_n}(\frac{t}{h})$ for $n = 2, 3, ...$ (3.4)

Let ϵ, λ be positive reals.Then for $m > n$ by $(PM5)$ we have

$F_{y_n,y_m}(\epsilon) \geq \Delta(F_{y_n,y_{n+1}}(\epsilon - h\epsilon), F_{y_{n+1},y_m}(h\epsilon))$

$\geq \Delta(F_{y_1,y_2}(\frac{\epsilon-h\epsilon}{h^{n-1}}), F_{y_{n+1},y_m}(h\epsilon))$ by (3.4)

$\geq \Delta(F_{y_1,y_2}(\frac{\epsilon-h\epsilon}{h^{n-1}}), \Delta(F_{y_{n+1},y_{n+2}}(h\epsilon - h^2\epsilon), F_{y_{n+2},y_m}(h^2\epsilon)))$

$\geq \Delta(F_{y_1,y_2}(\frac{\epsilon-h\epsilon}{h^{n-1}}), \Delta(F_{y_1,y_2}(\frac{h\epsilon-h^2\epsilon}{h^n}), F_{y_{n+2},y_m}(h^2\epsilon)))$

$\geq \Delta(\Delta(F_{y_1,y_2}(\frac{\epsilon-h\epsilon}{h^{n-1}}), F_{y_1,y_2}(\frac{\epsilon-h\epsilon}{h^{n-1}})), F_{y_{n+2},y_m}(h^2\epsilon))$

$\geq \Delta(F_{y_1,y_2}(\frac{\epsilon-h\epsilon}{h^{n-1}}), F_{y_{n+2},y_m}(h^2\epsilon))$

Repeated use of these arguments gives

$\geq \Delta(F_{y_1,y_2}(\frac{\epsilon-h\epsilon}{h^{n-1}}), F_{y_{m-1},y_m}(h^{m-1-n}\epsilon))$

$\geq \Delta(F_{y_1,y_2}(\frac{\epsilon-h\epsilon}{h^{n-1}}), F_{y_1,y_2}(\frac{h^{m-1-n}\epsilon}{h^{m-2}}))$

$\geq \Delta(F_{y_1,y_2}(\frac{\epsilon-h\epsilon}{h^{n-1}}), F_{y_1,y_2}(\frac{\epsilon-h\epsilon}{h^{n-1}}))$

$\geq F_{y_1,y_2}(\frac{\epsilon-h\epsilon}{h^{n-1}})$

if N be chosen that $F_{y_1,y_2}(\frac{\epsilon-h\epsilon}{h^{N-1}}) > 1-\lambda$ it follows that $F_{y_n,y_m}(\epsilon) > 1-\lambda$ for all $n \geq N$.Hence $\{y_n\}$ is a Cauchy sequence in X which is complete so let $\{y_n\}$ converges to point z in X.Its subsequences $\{Ax_{2n}\}$,$\{Tx_{2n+1}\}$,$\{Bx_{2n+1}\}$,$\{Sx_{2n+2}\}$ also converges to z.Since $B(X) \subseteq S(X)$ there exist a point $p \in X$ such that $z = Sp$. Using (3.3) we have

$\phi(F_{Ap,Bx_{2n+1}}(ht), F_{Sp,Tx_{2n+1}}(t), F_{Ap,Sp}(t), F_{Bx_{2n+1},Tx_{2n+1}}(ht)) \geq 0.$

Taking $n \to \infty$,$\phi(F_{Ap,z}(ht), F_{Sp,z}(t), F_{Ap,Sp}(t), F_{z,z}(ht)) \geq 0.$

$\phi(F_{Ap,z}(ht), F_{z,z}(t), F_{Ap,z}(t), F_{z,z}(ht)) \geq 0.$

$\phi(F_{Ap,z}(ht), 1, F_{Ap,z}(t), 1) \geq 0.$

ϕ is non-decreasing in first argument gives $\phi(F_{Ap,z}(t), 1, F_{Ap,z}(t), 1) \geq 0.$

By (2.1), $F_{Ap,z}(t) \geq 1$ which gives Ap=z.Therefore $Ap = Sp = z$. Since A and S weakly compatible mappings we have $SAp = ASp$ implies $Az = Sz$.From (3.3) we get

$\phi(F_{Az,Bx_{2n+1}}(ht), F_{Sz,Tx_{2n+1}}(t), F_{Az,Sz}(t), F_{Bx_{2n+1},Tx_{2n+1}}(ht)) \geq 0.$

Taking $n \to \infty$,$\phi(F_{Az,z}(ht), F_{Sz,z}(t), F_{Az,Sz}(t), F_{z,z}(ht)) \geq 0.$

$\phi(F_{Az,z}(ht), F_{Az,z}(t), 1, 1) \geq 0.$

ϕ is non-decreasing in first argument gives $\phi(F_{Az,z}(t), F_{Az,z}(t), 1, 1) \geq 0.$

By (2.2)$F_{Az,z}(t) \geq 1$ implies $Az = z$.Therefore $Az = Sz = z$. As $A(X) \subseteq T(X)$ there exist a point $q \in X$ such that $z = Tq$.By (3.3) we get

$\phi(F_{Ax_{2n},Bq}(ht), F_{Sx_{2n},Tq}(t), F_{Ax_{2n},Sx_{2n}}(t), F_{Bq,Tq}(ht)) \geq 0.$

Taking $n \to \infty$,$\phi(F_{z,Bq}(ht), F_{z,z}(t), F_{z,z}(t), F_{Bq,z}(ht)) \geq 0.$

$\phi(F_{z,Bq}(ht), 1, 1, F_{Bq,z}(ht)) \geq 0.$

By (2.1),$F_{z,Bq}(ht) \geq 1$ implies $z = Bq$.Therefore $z = Bq = Tq$.Similarly as B and T are weakly compatible mappings so $BTq = TBq$ implies $Bz = Tz$.Using (3.3)we get

$\phi(F_{Ax_{2n},Bz}(ht), F_{Sx_{2n},Tz}(t), F_{Ax_{2n},Sx_{2n}}(t), F_{Bz,Tz}(ht)) \geq 0.$

Taking $n \to \infty$,$\phi(F_{z,Bz}(ht), F_{z,Tz}(t), F_{z,z}(t), F_{Bz,Tz}(ht)) \geq 0.$

$\phi(F_{z,Bz}(ht), F_{z,Bz}(t), F_{z,z}(t), F_{Bz,Bz}(ht)) \geq 0.$

$\phi(F_{z,Bz}(ht), F_{z,Bz}(t), 1, 1) \geq 0.$

ϕ is non-decreasing in first argument gives $\phi(F_{z,Bz}(t), F_{z,Bz}(t), 1, 1) \geq 0.$

By (2.2),$z = Bz$.Therefore $z = Bz = Tz$.Hence $z = Bz = Tz = Az = Bz$.Therefore mappings A,B,S and T have a common fixed point in X.Let z_1 be another common fixed point of mappings A,B,S and T.Then $z_1 = Bz_1 = Tz_1 = Az_1 = Bz_1$.From (3.3) we get

$\phi(F_{Az,Bz_1}(ht), F_{Sz,Tz_1}(t), F_{Az,Sz}(t), F_{Bz_1,Tz_1}(ht)) \geq 0.$

$\phi(F_{z,z_1}(ht), F_{z,z_1}(t), F_{z,z}(t), F_{z_1,z_1}(ht)) \geq 0.$

$\phi(F_{z,z_1}(ht), F_{z,z_1}(t), 1, 1) \geq 0.$

ϕ is non-decreasing in first argument gives $\phi(F_{z,z_1}(t), F_{z,z_1}(t), 1, 1) \geq 0.$

By (2.2),$F_{z,z_1} \geq 1$ implies $z = z_1$.Hence z is a unique fixed point of mappings A,B,S and T.

Remark 3.1. In Theorem 3.1 we have used less number of conditions in comparison of Mishra [4] in the sense that continuity of functions has not been used.Also one more notable point is that we have used weak compatibility in comparison of compatibility in Mishra [4].

Corollary 3.1. Let A,S and T be self mappings on a complete Menger Space $(X, F, \Delta)$where $\Delta = min$ and satisfying

(3.5) $A(X) \subseteq T(X) \cap S(X)$.

(3.6) Pairs (A, S) and (A, T) are weakly compatible.

(3.7) $\phi(F_{Au,Av}(ht), F_{Su,Tv}(t), F_{Au,Su}(t), F_{Av,Tv}(ht)) \geq 0.$

for all $u, v \in X, t > 0, h \in (0, 1)$.Then A,S and T have a unique common fixed point in X.

Corollary 3.2. Let A and S be self mappings on a complete Menger Space $(X, F, \Delta)$where $\Delta = min$ and satisfying

(3.8 $A(X) \subseteq S(X)$.

(3.9) Pairs (A, S) is weakly compatible.

(3.10) $\phi(F_{Au,Av}(ht), F_{Su,Sv}(t), F_{Au,Su}(t), F_{Av,Sv}(ht)) \geq 0.$

for all $u, v \in X, t > 0, h \in (0, 1)$.Then A and S have a unique common fixed point in X.

Corollary 3.3. If in hypotheses of Theorem 3.1, condition (3.3) is replaced by the following condition

$F_{Au,Bv}(ht) \geq min\{F_{Su,Tv}(t), F_{Au,Su}(t), F_{Bv,Tv}(t)\}$.Then mappings A,B,S and T have a unique common fixed point in X.

Proof:By following the proof of Theorem 3.1 and using Lemma 2.1.

Example 3.1. Let $X = R$ with the metric $d(u, v) = |u - v|$ and define $F_{u,v}(s) = H(s - d(u, v))$ for all $u, v \in X$.clearly (X, F, min) is a Menger space.Let A, B, S and T be self- mappings from X into itself defined as $T(x) = 2x + 1$ for all $x \in X$,$S(x) = x$ for all $x \in X$,A(x) = B(x)= -1 for all $x \in X$.

Then we see that

(1) $A(X) \subseteq T(X)$ and $B(X) \subseteq S(X)$.

(2) pairs (A, S) and (B, T) are weakly compatible.

(3) Let $\phi : (R^+)^4 \to R$ be defined as $\phi(x_1, x_2, x_3, x_4) = x_1 - x_2$.Then $\phi \in \Phi$ and condition (3.3) of Theorem 3.1 is satisfied for $h \in (0, 1)$ and $t > 0$.Thus all conditions of Theorem 3.1 is satisfied and -1 is a unique common fixed point of mappings A, B, S and T.

4. An application

Theorem 4.1 Let (X, F, min) be complete Menger space.Let A,B,S and T be mappings from $X \times X$ into X such that

(3.11) $A(X \times \{v\}) \subseteq T(X \times \{v\}), B(X \times \{v\}) \subseteq S(X \times \{v\}$ for all $v \in X$.

(3.12) $A(S(u, v), v) = S(A(u, v), v)$ for all $(u, v) \in C[A, S]$ where $C[A, S]$ denotes collection of coincidence points of A and S.

$B(T(u_1, v_1), v_1) = T(B(u_1, v_1), v_1)$ for all $(u_1, v_1) \in C[B, T]$ where $C[B, T]$ denotes collection of coincidence points of B and T.

(3.13) $\phi(F_{A(u,v),B(u_1,v_1)}(ht), F_{S(u,v),T(u_1,v_1)}(t), F_{A(u,v),S(u,v)}(t), F_{B(u_1,v_1),T(u_1,v_1)}(ht)) \geq 0.$

for all $u, v, u_1, v_1 \in X, t > 0, h \in (0,1)$.Then there exist exactly one point p in X such that $A(p,v) = B(p,v) = S(p,v) = T(p,v) = p$ for all $v \in X$.

Proof:For a fixed $v \in X$ and $v = v_1$,(3.11),(3.12),(3.13)corresponds to (3.1),(3.2),(3.3) of Theorem 3.1 so by Theorem 3.1 for each $v \in X$ there exist unique point $u(v)$ in X such that

$A(u(v),v) = S(u(v),v) = B(u(v),v) = T(u(v),v) = u(v)$

Now for every v, v_1 in X from (3.13) we get

$\phi(F_{A(u(v),v),B(u(v_1),v_1)}(ht), F_{S(u(v),v),T(u(v_1),v_1)}(t),$

$F_{A(u(v),v),S(u(v),v)}(t), F_{B(u(v_1),v_1),T(u(v_1),v_1)}(ht)) \geq 0.$

$\phi(F_{u(v),u(v_1)}(ht), F_{u(v),u(v_1)}(t), F_{u(v),u(v)}(t), F_{u(v_1),u(v_1)}(ht)) \geq 0.$

$\phi(F_{u(v),u(v_1)}(ht), F_{u(v),u(v_1)}(t), 1, 1) \geq 0.$

ϕ is non-decreasing in first argument gives

$\phi(F_{u(v),u(v_1)}(t), F_{u(v),u(v_1)}(t), 1, 1) \geq 0.$

By (2.2) $F_{u(v),u(v_1)}(t) \geq 1$ implies $u(v) = u(v_1)$.Hence $u(.)$is some point $p \in X$ and so $A(p,v) = B(p,v) = S(p,v) = T(p,v) = p$ for all $v \in X$.

Theorem 4.2 Let$S$,$T$and$\{A_i\}_{i \in N}$be self mappings on a complete Menger Space $(X, F, \Delta)$where $\Delta = min$ and satisfying

(3.14) $A_i(X) \subseteq T(X), A_{i+1}(X) \subseteq S(X)$.

(3.15) Pairs (A_i, S) and (A_{i+1}, T) are weakly compatible.

(3.16) $\phi(F_{A_iu,A_{i+1}v}(ht), F_{Su,Tv}(t), F_{A_iu,Su}(t), F_{A_{i+1}v,Tv}(ht)) \geq 0.$

for all $u, v \in X, t > 0, h \in (0,1)$.Then S,T and $\{A_i\}_{i \in N}$ have a unique common fixed point in X.

Proof:Let $i = 1$, we get hypothesis of Theorem 3.1 for maps A_1,A_2,T and S.By using Theorem 3.1 we get z is a unique common fixed point of maps A_1,A_2,T and S.Now z is a unique common fixed point of T,S,A_1 and T,S,A_2.Otherwise, if $z_1$is a second fixed point of T,S and A_1 then by (3.3) we have

$\phi(F_{A_1z_1,A_2z}(ht), F_{Sz_1,Tz}(t), F_{A_1z_1,Sz_1}(t), F_{A_2z,Tz}(ht)) \geq 0.$

$\phi(F_{z_1,z}(ht), F_{z_1,z}(t), F_{z_1,z_1}(t), F_{z,z}(ht)) \geq 0.$

$\phi(F_{z_1,z}(ht, F_{z_1,z}(t), 1, 1) \geq 0.$

By (2.2) we get $F_{z_1,z} \geq 1$ implies $z_1 = z$.

Similarly we can show z is a unique common fixed point of mappings T,S,A_2.

Now by putting $i = 2$,we get hypothesis of same theorem for maps T,S,A_2 and A_3.consequently there exist a unique common fixed point for maps T,S,A_2 andA_3.Let this point be z_2.Similarly z_2 is a unique common fixed point of T,S,A_2 and T,S,A_3.Thus $z = z_2$.Hence we get z is a unique common fixed point for maps T,S,A_1,A_2 and A_3.Continuing in this way we see that z is a unique common fixed point for S,T and $\{A_i\}_{i \in N}$.

Remark 4.1. B. Singh [9] generalized the result of Mishra [4] to six mappings by using weak compatibility and continuity of one function and we have extended our result to sequence of mappings without using continuity of any function.

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