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A NEW COMMON FIXED POINT THEOREM FOR COMPATIBLE MAPPINGS OF TYPE (A) IN GENERALIZED METRIC SPACES

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Abstract: In this paper, we prove a new common fixed point theorem for six compatible mappings of type (A) in the framework of generalized metric spaces. An example is provided to support our new result. The results obtained in this paper differ from the recent relative results in literature.

Keywords: generalized metric space; compatible mappings of type (A); common fixed point.

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1. Introduction

In 2006, Mustafa and Sims [1] generalized the concept of a metric in which the real number is assigned to every triplet of an arbitrary set. Based on the notion of generalized metric spaces, Mustafa et al. [2-5], Obiedat and Mustafa [6], Aydi et al. [7,8], Gajié and Stojakovié [9], Zhou and Gu [10] obtained some fixed point results for mappings satisfying different contractive conditions. Shatanawi [11] obtained some fixed point results for Φ-maps in generalized metric spaces. Chugh et al. [12] obtained some fixed point results for maps satisfying property P in generalized metric spaces. Study of common fixed point problems in generalized metric spaces was

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initiated by Abbas and Rhoades [13]. Subsequently, many authors obtained many common fixed point theorems for the mappings satisfying different contractive conditions; see [14]-[32] for more details. Recently, Abbas et al. [33] and Mustafa et al. [34] obtained some common fixed point results for a pair of mappings satisfying $(E.A)$ property under certain generalized strict contractive conditions in G-metric spaces. Long et al. [35] obtained some common coincidence and common fixed points results of two pairs of mappings when only one pair satisfies $(E.A)$ property in the framework of a generalized metric space. Very recently, Gu and Yin [36] introduce the concept of the common $(E.A)$ property for two pairs of self-mappings in G-metric spaces, and study the existence and uniqueness of coincidence and common fixed points for three pairs of mappings satisfying Φ-contractive conditions.

In 2011, Vats et. al [14] introduced the notion of compatible mappings of type (A), and prove a common fixed point theorem for three and four compatible mappings of type (A) in the framework of generalized metric spaces.

The purpose of this paper is to use the concept of weakly compatible mappings of type (A) to discuss a new common fixed point problem for six self-mappings in G-metric spaces. The results presented in this paper extend and improve the corresponding results of Vats, Kumar and Sihag[14].

We now recall some definitions and properties in G-metric spaces.

Definition 1.1[1] Let X be a nonempty set, and let $G: X \times X \times X \to R^+$ be a function satisfying the following axioms:

(G_1) $G(x, y, z) = 0$ if $x = y = z$,

(G_2) $0 < G(x, x, y)$, for all $x, y \in X$ with $x \neq y$,

(G_3) $G(x, x, y) \leq G(x, y, z)$, for all $x, y, z \in X$

(G_4) $G(x, y, z) = G(x, z, y) = G(y, z, x) = \ldots$ (symmetry in all three variables),

(G_5) $G(x,y,z) \le G(x,a,a) + G(a,y,z)$ for all $x,y,z,a \in X$ (rectangle inequality).

Then the function G is called a generalized metric, or, more specifically, a G-metric on X, and the pair (X,G) is called a G-metric space.

Definition 1.2[1] Let (X,G) be a G-metric space, and let $\{x_n\}$ a sequence of points in X, a point x in X is said to be the limit of the sequence $\{x_n\}$, $\lim_{n\to\infty} G(x, x_n, x_m) = 0$, and one says that sequence $\{x_n\}$ is G-convergent to x.

Thus, that if $x_n \to x$ or $\lim_{n\to\infty} x_n = x$ in a G-metric space (X,G) then if for each $\varepsilon > 0$ □□ there exists a positive integer N such that $G(x, x_n, x_m) < \varepsilon$ for all $n,m \ge N$.

Proposition 1.1[1] Let (X,G) be a G-metric space. Then the followings are equivalent:

(1) $\{x_n\}$ is G convergent to x,

(2) $G(x_n, x_n, x) \to 0$ as $n \to \infty$,

(3) $G(x_n, x, x) \to 0$ as $n \to \infty$,

(4) $G(x_m, x_n, x) \to 0$ as $m,n \to \infty$.

Definition 1.3[1] Let (X,G) be a G-metric space. A sequence $\{x_n\}$ is called G-Cauchy if, for each $\varepsilon > 0$, there exists a positive integer N such that $G(x_n, x_m, x_l) < \varepsilon$ for all $n,m,l \ge N$; *i.e.* if $G(x_n, x_m, x_l) \to 0$ as $n,m,l \to \infty$.

Proposition 1.2[1] If (X,G) is a G-metric space then the following are equivalent:

(1) The sequence $\{x_n\}$ is G-Cauchy,

(2) for each $\varepsilon > 0$, there exist a positive integer N such that $G(x_n, x_m, x_l) < \varepsilon$ for

all $n, m, l \geq N$.

Proposition 1.3[1] Let (X, G) be a G-metric space. Then the function $G(x, y, z)$ is jointly continuous in all three of its variables.

Definition 1.4[1] A G-metric space (X, G) is called a symmetric G-metric space if $G(x, y, y) = G(y, x, x)$ for all x, y in X.

Definition 1.5[2] A G-metric space (X, G) is said to be G-complete if every G-Cauchy sequence in (X, G) is G-convergent in X.

Proposition 1.4[3] Let (X, G) be a G-metric space. Then, for any x, y, z, a in X it follows that:

(i) If $G(x, y, z) = 0$, then $x = y = z$,

(ii) $G(x, y, z) \leq G(x, x, y) + G(x, x, z)$,

(iii) $G(x, y, y) \leq 2G(y, x, x)$,

(iv) $G(x, y, z) \leq G(x, a, z) + G(a, y, z)$,

(v) $G(x, y, z) \leq \frac{2}{3}(G(x, y, a) + G(x, a, z) + G(a, y, z))$,

(vi) $G(x, y, z) \leq (G(x, a, a) + G(y, a, a) + G(z, a, a))$.

Definition 1.6[14] Self mappings S and T of a G-metric space (X, G) are said to be compatible if

$$\lim_{n\to\infty} G(TSx_n, STx_n, STx_n) = 0 \text{ and } \lim_{n\to\infty} G(STx_n, TSx_n, TSx_n) = 0,$$

whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = t$, for some $t \in X$.

Definition 1.7[14] Self mappings S and T of a G-metric space (X, G) are said to be compatible of type (A) if

$$\lim_{n\to\infty} G(TSx_n, SSx_n, SSx_n) = 0 \text{ and } \lim_{n\to\infty} G(STx_n, TTx_n, TTx_n) = 0,$$

Whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = t$, for some $t \in X$.

2. Main results

Theorem 2.1 Suppose $S, T, I, A, B,$ and C be six self mapping of a complete G-metric space (X, G) in to itself, satisfying the conditions:

(i) $S(X) \subset B(X), T(X) \subset C(X), I(X) \subset A(X)$,

(ii) $G(Sx,Ty,Iz) \le \alpha G(Ax,By,Cz) + \beta[G(Sx,Ty,By) + G(Sx,Ty,Cz)]$

$$+\gamma[G(Ax,Iz,Sx) + G(By,Iz,Sx)] \tag{2.1}$$

for all $x, y, z \in X$. Where $\alpha, \beta, \gamma \ge 0$, $\alpha + 3\beta + 3\gamma < 1$. Then one of the pairs $(S, A), (T, B)$ and (I, C) has a coincidence point in X. Moreover, if the following conditions is satisfied:

(iii) one of S, T, I, A, B, C is continuous,

(iv) the pairs (S, A), (T, B) and (I, C) are compatible of type (A).

Then the mappings S, T, I, A, B and C have unique common fixed point in X.

Proof. Let $x_0 \in X$ be any arbitrary. We choose a point x_1 in X so that $Sx_0 = Bx_1$, this can be done since $S(X) \subset B(X)$. Let x_2 be a point in X such that $Tx_1 = Cx_2$, this can be done since $T(X) \subset C(X)$. Let x_3 be a point in X such that $Ix_2 = Ax_3$, this can be done since $I(X) \subset A(X)$. In general we can choose $x_{3n}, x_{3n+1}, x_{3n+2}, ...$, and there is exists a sequence $\{y_n\}$ in X, such that

$$y_{3n} = Sx_{3n} = Bx_{3n+1}, y_{3n+1} = Tx_{3n+1} = Cx_{3n+2},\ y_{3n+2} = Ix_{3n+2} = Ax_{3n+3}. \tag{2.2}$$

If $y_n = y_{n+1}$ for some n, with $n = 3m$, then $p = x_{3m+1}$ is a coincidence point of the pair (T,B); If $y_{n+1}=y_{n+2}$ for some n, with $n=3m$, then $p=x_{3m+2}$ is a coincidence point of the pair (I,C); If $y_{n+2}=y_{n+3}$ for some n, with $n=3m$, then $p=x_{3m+3}$ is a coincidence point of the pair (S,A).

On the other hand, if there exist $n_0\in N$, such that $y_{n_0}=y_{n_0+1}=y_{n_0+2}$, then $y_n=y_{n_0}$ for any $n\ge n_0$, this implies that $\{y_n\}$ is a G-Cauchy sequence.

In fact, if there exist $p\in N$, such that $y_{3p}=y_{3p+1}=y_{3p+2}$, then applying the condition (ii), we have

$$\begin{aligned}
G(y_{3p},y_{3p},y_{3p+3}) &= G(y_{3p+1},y_{3p+2},y_{3p+3}) = G(Sx_{3p+3},Tx_{3p+1},Ix_{3p+2})\\
&\le \alpha G(Ax_{3p+3},Bx_{3p+1},Cx_{3p+2})+\beta[G(Sx_{3p+3},Tx_{3p+1},Bx_{3p+1})+G(Sx_{3p+3},Tx_{3p+1},Cx_{3p+2})]\\
&\quad +\gamma[G(Ax_{3p+3},Ix_{3p+2},Sx_{3p+3})+G(Bx_{3p+1},Ix_{3p+2},Sx_{3p+3})]\\
&= \alpha G(y_{3p+2},y_{3p},y_{3p+1})+\beta[G(y_{3p+3},y_{3p+1},y_{3p})+G(y_{3p+3},y_{3p+1},y_{3p+1})]\\
&\quad +\gamma[G(y_{3p+2},y_{3p+2},y_{3p+3})+G(y_{3p},y_{3p+2},y_{3p+3})]\\
&= \alpha G(y_{3p},y_{3p},y_{3p})+\beta[G(y_{3p+3},y_{3p},y_{3p})+G(y_{3p+3},y_{3p},y_{3p})]\\
&\quad +\gamma[G(y_{3p},y_{3p},y_{3p+3})+G(y_{3p},y_{3p},y_{3p+3})]\\
&= (2\beta+2\gamma)G(y_{3p},y_{3p},y_{3p+3}).
\end{aligned}$$

If $y_{3p+1}\ne y_{3p}$, then $2\beta+2\gamma\ge 1$, this is a contradiction, since $0\le 2\beta+2\gamma\le\alpha+3\beta+3\gamma<1$. Which implies that $y_{3p+3}=y_{3p}=y_{3p+1}=y_{3p+2}$. So we find $y_n=y_{3p}$ for any $n\ge 3p$. This implies that $\{y_n\}$ is a G-Cauchy sequence. The same conclusion holds if $y_{3p+1}=y_{3p+2}=y_{3p+3}$, or $y_{3p+2}=y_{3p+3}=y_{3p+4}$, for some $p\in N$. Without loss of generality, we can assume that $y_n\ne y_m$, for all $n,m\in N$, and $n\ne m$.

Now we prove that $\{y_n\}$ is a G-Cauchy sequence in X. Using condition (ii) and the (G_3) and (G_5), we have

$$
\begin{aligned}
G(y_{3n-1}, y_{3n}, y_{3n+1}) &= G(Sx_{3n}, Tx_{3n+1}, Ix_{3n-1}) \\
&\le \alpha G(Ax_{3n}, Bx_{3n+1}, Cx_{3n-1}) + \beta[G(Sx_{3n}, Tx_{3n+1}, Bx_{3n+1}) + G(Sx_{3n}, Tx_{3n+1}, Cx_{3n-1})] \\
&\quad + \gamma[G(Ax_{3n}, Ix_{3n-1}, Sx_{3n}) + G(Bx_{3n+1}, Ix_{3n-1}, Sx_{3n})] \\
&= \alpha G(y_{3n-2}, y_{3n-1}, y_{3n}) + \beta[G(y_{3n}, y_{3n}, y_{3n+1}) + G(y_{3n-2}, y_{3n}, y_{3n+1})] \\
&\quad + \gamma[G(y_{3n-1}, y_{3n-1}, y_{3n}) + G(y_{3n}, y_{3n}, y_{3n-1})] \\
&\le \alpha G(y_{3n-2}, y_{3n-1}, y_{3n}) + \beta[G(y_{3n-1}, y_{3n}, y_{3n+1}) + G(y_{3n-2}, y_{3n-1}, y_{3n-1}) + G(y_{3n-1}, y_{3n}, y_{3n+1})] \\
&\quad + \gamma[G(y_{3n-1}, y_{3n}, y_{3n+1}) + G(y_{3n-1}, y_{3n}, y_{3n+1})] \\
&\le (\alpha+\beta)G(y_{3n-2}, y_{3n-1}, y_{3n}) + (2\beta+2\gamma)G(y_{3n-1}, y_{3n}, y_{3n+1}).
\end{aligned}
$$

Which implies that

$$
G(y_{3n-1}, y_{3n}, y_{3n+1}) \le \left(\tfrac{\alpha+\beta}{1-2\beta-2\gamma}\right) G(y_{3n-2}, y_{3n-1}, y_{3n}) = h_1 G(y_{3n-2}, y_{3n-1}, y_{3n}), \tag{2.3}
$$

where $h_1 = \dfrac{\alpha+\beta}{1-2\beta-2\gamma} < 1.$

Again using condition (ii) and the (G_3) and (G_5), we obtain

$$
\begin{aligned}
G(y_{3n}, y_{3n+1}, y_{3n+2}) &= G(Sx_{3n}, Tx_{3n+1}, Ix_{3n+2}) \\
&\le \alpha G(Ax_{3n}, Bx_{3n+1}, Cx_{3n+2}) + \beta[G(Sx_{3n}, Tx_{3n+1}, Bx_{3n+1}) + G(Sx_{3n}, Tx_{3n+1}, Cx_{3n+2})] \\
&\quad + \gamma[G(Ax_{3n}, Ix_{3n+2}, Sx_{3n}) + G(Bx_{3n+1}, Ix_{3n+2}, Sx_{3n})] \\
&= \alpha G(y_{3n-1}, y_{3n}, y_{3n+1}) + \beta[G(y_{3n}, y_{3n+1}, y_{3n}) + G(y_{3n}, y_{3n+1}, y_{3n+1})] \\
&\quad + \gamma[G(y_{3n-1}, y_{3n+2}, y_{3n}) + G(y_{3n}, y_{3n+2}, y_{3n})] \\
&\le \alpha G(y_{3n-1}, y_{3n}, y_{3n+1}) + \beta[G(y_{3n}, y_{3n+1}, y_{3n+2}) + G(y_{3n}, y_{3n+1}, y_{3n+2})] \\
&\quad + \gamma[G(y_{3n-1}, y_{3n+1}, y_{3n+1}) + G(y_{3n+1}, y_{3n+2}, y_{3n}) + G(y_{3n}, y_{3n+1}, y_{3n+2})] \\
&\le (\alpha+\gamma)G(y_{3n-1}, y_{3n}, y_{3n+1}) + (2\beta+2\gamma)G(y_{3n}, y_{3n+1}, y_{3n+2}).
\end{aligned}
$$

This implies that

$$
G(y_{3n}, y_{3n+1}, y_{3n+2}) \le \frac{\alpha+\gamma}{1-2\beta-2\gamma} G(y_{3n-1}, y_{3n}, y_{3n+1}) = h_2 G(y_{3n-1}, y_{3n}, y_{3n+1}), \tag{2.4}
$$

where $h_2 = \dfrac{\alpha+\gamma}{1-2\beta-2\gamma} < 1.$

Again using condition (ii) and the (G_3) and (G_5), we get

$$
\begin{aligned}
G(y_{3n+1}, y_{3n+2}, y_{3n+3}) &= G(Sx_{3n+3}, Tx_{3n+1}, Ix_{3n+2}) \\
&\le \alpha G(Ax_{3n+3}, Bx_{3n+1}, Cx_{3n+2}) + \beta[G(Sx_{3n+3}, Tx_{3n+1}, Bx_{3n+1}) + G(Sx_{3n+3}, Tx_{3n+1}, Cx_{3n+2})] \\
&\quad + \gamma[G(Ax_{3n+3}, Ix_{3n+2}, Sx_{3n+3}) + G(Bx_{3n+1}, Ix_{3n+2}, Sx_{3n+3})] \\
&= \alpha G(y_{3n+2}, y_{3n}, y_{3n+1}) + \beta[G(y_{3n+3}, y_{3n+1}, y_{3n}) + G(y_{3n+3}, y_{3n+1}, y_{3n+1})] \\
&\quad + \gamma[G(y_{3n+2}, y_{3n+2}, y_{3n+3}) + G(y_{3n}, y_{3n+2}, y_{3n+3})] \\
&\le \alpha G(y_{3n}, y_{3n+1}, y_{3n+2}) + \beta[G(y_{3n+3}, y_{3n+2}, y_{3n+2}) + G(y_{3n+2}, y_{3n+1}, y_{3n}) + G(y_{3n+1}, y_{3n+2}, y_{3n+3})] \\
&\quad + \gamma[G(y_{3n+1}, y_{3n+2}, y_{3n+3}) + G(y_{3n}, y_{3n+1}, y_{3n+1}) + G(y_{3n+1}, y_{3n+2}, y_{3n+3})] \\
&\le (\alpha+\beta+\gamma) G(y_{3n}, y_{3n+1}, y_{3n+2}) + (2\beta + 2\gamma) G(y_{3n+1}, y_{3n+2}, y_{3n+3}).
\end{aligned}
$$

Which implies that

$$
G(y_{3n+1}, y_{3n+2}, y_{3n+3}) \le \frac{\alpha+\beta+\gamma}{1-2\beta-2\gamma} G(y_{3n}, y_{3n+1}, y_{3n+2}) = h_3 G(y_{3n}, y_{3n+1}, y_{3n+2}), \quad (2.5)
$$

where $h_3 = \dfrac{\alpha+\beta+\gamma}{1-2\beta-2\gamma} < 1$.

Let $h = \max\{h_1, h_2, h_3\}$, then from $0 \le \alpha + 3\beta + 3\gamma < 1$ we know that $0 \le h < 1$. Combining (2.3), (2.4) and (2.5), we have

$$
\begin{aligned}
G(y_n, y_{n+1}, y_{n+2}) &\le hG(y_{n-1}, y_n, y_{n+1}) \\
&\le h^2 G(y_{n-2}, y_{n-1}, y_n) \le \cdots \le h^n G(y_0, y_1, y_2).
\end{aligned} \quad (2.6)
$$

Moreover, for all $n, m \in N$; $n < m$, we have by rectangle inequality that

$$
\begin{aligned}
G(y_n, y_m, y_m) &\le G(y_n, y_{n+1}, y_{n+1}) + G(y_{n+1}, y_{n+2}, y_{n+2}) + \cdots + G(y_{m-1}, y_m, y_m) \\
&\le (h^n + h^{n+1} + \cdots + h^{m-1}) G(y_0, y_1, y_2) \\
&\le \frac{h^n}{1-h} G(y_0, y_1, y_2).
\end{aligned} \quad (2.7)
$$

And so $G(y_n, y_m, y_m) \to 0$, as $n, m \to \infty$. Thus $\{y_n\}$ is G-Cauchy sequence. Due to the completeness of (X, G), there exists $z \in X$ such that $\{y_n\}$ is G-converge to z.

Since the sequences $\{Sx_{3n}\} = \{Bx_{3n+1}\}, \{Tx_{3n+1}\} = \{Cx_{3n+2}\} and \{Ix_{3n+2}\} = \{Ax_{3n+3}\}$ are all subsequences of $\{y_n\}$, then they all converge to z, that is,

$$
y_{3n} = Sx_{3n} = Bx_{3n+1} \to z, y_{3n+1} = Tx_{3n+1} = Cx_{3n+2} \to z, y_{3n+2} = Ix_{3n+2} = Ax_{3n+3} \to z (n \to \infty).
$$

Now we prove that z is a common fixed point of S, T, I, A, B and C.

First we suppose that A is continuous, then $A^2 x_{3n} \to Az$ as $n \to \infty$. since the pair (S, A) is compatible of type (A) we get $SAx_{3n} \to Az$ as $n \to \infty$. Now by the condition (ii)

$$\begin{aligned} G(SAx_{3n}, Tx_{3n+1}, Ix_{3n+2}) \le \alpha G(A^2 x_{3n}, Bx_{3n+1}, Cx_{3n+2}) & \\ + \beta[G(SAx_{3n}, Tx_{3n+1}, Bx_{3n+1}) + G(SAx_{3n}, Tx_{3n+1}, Cx_{3n+2})] & \quad (2.8) \\ + \gamma[G(A^2 x_{3n}, Ix_{3n+2}, SAx_{3n}) + G(Bx_{3n+1}, Ix_{3n+2}, SAx_{3n})]. & \end{aligned}$$

Letting $n \to \infty$, we have

$$G(Az, z, z) \le \alpha G(Az, z, z) + \beta[G(Az, z, z) + G(Az, z, z)] + \gamma[G(Az, z, Az) + G(z, z, Az)].$$

By (iii) of Proposition 1.4, we get

$$G(Az, z, z) \le (\alpha + 2\beta + 3\gamma) G(Az, z, z),$$

this gives $G(Az, z, z) = 0$, since $0 \le \alpha + 2\beta + 3\gamma < 1$. Hence $Az = z$.

Again by the condition (ii), we have

$$\begin{aligned} G(Sz, Tx_{3n+1}, Ix_{3n+2}) \le \alpha G(Az, Bx_{3n+1}, Cx_{3n+2}) & \\ + \beta[G(Sz, Tx_{3n+1}, Bx_{3n+1}) + G(Sz, Tx_{3n+1}, Cx_{3n+2})] & \quad (2.9) \\ + \gamma[G(Az, Ix_{3n+2}, Sz) + G(Bx_{3n+1}, Ix_{3n+2}, Sz)]. & \end{aligned}$$

Letting $n \to \infty$ and using $Az = z$, we have

$$\begin{aligned} G(Sz, z, z) &\le \alpha G(z, z, z) + \beta[G(Sz, z, z) + G(Sz, z, z)] + \gamma[G(z, z, Sz) + G(z, z, Sz)] \\ &= 2(\beta + \gamma) G(Sz, z, z). \end{aligned}$$

Which implies that $G(Sz, z, z) = 0$, since $0 \le 2\beta + 2\gamma < 1$. Thus $Sz = z = Az$.

Since $S(X) \subset B(X)$ and $z = Sz \in S(x)$, there is a point $u \in X$ such that $z = Sz = Bu$. By the condition (ii), we have

$$\begin{aligned} G(Sz, Tu, Ix_{3n+2}) \le \alpha G(Az, Bu, Cx_{3n+2}) & \\ + \beta[G(Sz, Tu, Bv) + G(Sz, Tu, Cx_{3n+2})] & \quad (2.10) \\ + \gamma[G(Az, Ix_{3n+2}, Sz) + G(Bu, Ix_{3n+2}, Sz)]. & \end{aligned}$$

Letting $n \to \infty$ and using $z = Bu$, we have

$$\begin{aligned} G(z,Tu,z) &\le \alpha G(z,z,z)+\beta[G(z,Tu,z)+G(z,Tu,z)]+\gamma[G(z,z,z)+G(z,z,z)] \\ &= 2\beta G(z,Tu,z). \end{aligned}$$

This implies that $G(z,Tu,z)=0$, since $0\le 2\beta<1$. Hence $Tu=z=Bu$.

Taking $w_n=u$ for all $n\ge1$. Then $Tw_n\to Tu=z$ and $Bw_n\to Bu=z$ as $n\to\infty$. Since the pair (T,B) is compatible type (A), we obtain $\lim_{n\to\infty} G(BTw_n,TTw_n,TTw_n)=0$, it gives $G(Bz,Tz,Tz)=0$ since $Tw_n=z$ for all $n\ge1$, hence we have $Bz=Tz$.

Again by the condition (ii), we have

$$\begin{aligned} G(Sz,Tz,Ix_{3n+2}) &\le \alpha G(Az,Bz,Cx_{3n+2})+\beta[G(Sz,Tz,Bz)+G(Sz,Tz,Cx_{3n+2})] \\ &\quad +\gamma[G(Az,Ix_{3n+2},Sz)+G(Bz,Ix_{3n+2},Sz)]. \end{aligned} \tag{2.11}$$

Letting $n\to\infty$, using $Bz=Tz$ and the Proposition 1.4, we have

$$\begin{aligned} G(z,Tz,z) &\le \alpha G(z,Tz,z)+\beta[G(z,Tz,Tz)+G(z,Tz,z)]+\gamma[G(z,z,z)+G(Tz,z,z)] \\ &\le (\alpha+3\beta+\gamma)G(z,Tz,z). \end{aligned}$$

Which implies that $G(z,Tz,z)=0$, since $0\le\alpha+3\beta+\gamma\le\alpha+3\beta+3\gamma<1$. Thus $Tz=z=Bz$.

Since $T(X)\subset C(X)$ and $z=Tz\in T(x)$, there is a point $v\in X$ such that $z=Tz=Cv$.

By the condition(ii), using $Sz=Az=Tz=Bz=Cv=z$, we have

$$\begin{aligned} G(z,z,Iv)=G(z,z,Iv) &\le \alpha G(z,z,z)+\beta[G(z,z,z)+G(z,z,z)] \\ &\quad +\gamma[G(z,Iv,z)+G(z,Iv,z)] \\ &= 2\gamma G(z,Iv,z). \end{aligned} \tag{2.12}$$

Hence $G(z,z,Iv)=0$, since $0\le 2\gamma<1$, so $Iv=z=Cv$.

Taking $t_n=v$ for all $n\ge1$, then $It_n\to Iv=z$ and $Ct_n\to Cv=z$ as $n\to\infty$. Since the pair (I,C) is compatible type (A), we obtain $\lim_{n\to\infty} G(CIt_n,IIt_n,IIt_n)=0$, it gives $G(Cz,Iz,Iz)=0$, since $It_n=z$ for all $n\ge1$, hence $Cz=Iz$. By the condition (ii), we have

$$G(Sz,Tz,Iz)\le \alpha G(Az,Bz,Cz)+\beta[G(Sz,Tz,Bz)+G(Sz,Tz,Cz)] \\ +\gamma[G(Az,Iz,Sz)+G(Bz,Iz,Sz)]. \quad (2.13)$$

It gives

$$G(z,z,Iz)\le \alpha G(z,z,Iz)+\beta[G(z,z,z)+G(z,z,Iz)]+\gamma[G(z,Iz,z)+G(z,Iz,z)] \\ =(\alpha+\beta+2\gamma)G(z,z,Iz).$$

Hence $G(z,z,Iz)=0$, since $0\le \alpha+\beta+2\gamma<1$. Thus $Iz=z=Cz$

Therefore, z is the common fixed point of S,T,I,A,B and C when A is continuous and the pairs $(S,A),(T,B)$ and (I,C) are compatible of type (A).

The proof is similar when B or C is continuous and the pairs $(S,A),(T,B)$ and (I,C) are compatible of type (A).

Next we suppose S is continuous, then $S^2x_{3n}\to Sz$ as $n\to\infty$ since the pair (S,A) is compatible of type (A) we get $ASx_{3n}\to Sz$ as $n\to\infty$. Again by the condition (ii), we have

$$G(S^2x_{3n},Tx_{3n+1},Ix_{3n+2})\le \alpha G(ASx_{3n},Bx_{3n+1},Cx_{3n+2}) \\ +\beta[G(S^2x_{3n},Tx_{3n+1},Bx_{3n+1})+G(S^2x_{3n},Tx_{3n+1},Cx_{3n+2})] \\ +\gamma[G(ASx_{3n},Ix_{3n+2},S^2x_{3n})+G(Bx_{3n+1},Ix_{3n+2},S^2x_{3n})]. \quad (2.14)$$

Letting $n\to\infty$ and using the (iii) of Proposition 1.4, we have

$$G(Sz,z,z)\le \alpha G(Sz,z,z)+\beta[G(Sz,z,z)+G(Sz,z,z)]+\gamma[G(Sz,z,Sz)+G(z,z,Sz)] \\ \le(\alpha+2\beta+3\gamma)G(Sz,z,z).$$

It gives $G(Sz,z,z)=0$, since $0\le\alpha+2\beta+3\gamma<1$. Hence $Sz=z.$

Since $S(X)\subset B(X)$ and $z=Sz\in S(X)$, there is a point $p\in X$ such that $Sz=z=Bp$. By the condition (ii), we have

$$G(S^2x_{3n},Tp,Ix_{3n+2})\le \alpha G(ASx_{3n},Bp,Cx_{3n+2}) \\ +\beta[G(S^2x_{3n},Tp,Bp)+G(S^2x_{3n},Tp,Cx_{3n+2})] \\ +\gamma[G(ASx_{3n},Ix_{3n+2},S^2x_{3n})+G(Bp,Ix_{3n+2},S^2x_{3n})]. \quad (2.15)$$

Letting $n \to \infty$, we have

$$G(z,Tp,z) \le \alpha G(z,z,z) + \beta[G(z,Tp,z) + G(z,Tp,z)] + \gamma[G(z,z,z) + G(z,z,z)]$$
$$= 2\beta G(z,Tp,z).$$

Hence $G(z,Tp,z) = 0$, since $0 \le 2\beta < 1$. Thus $Tp = z = Bp$.

Taking $p_n = p$ for all $n \ge 1$, then $Tp_n \to Tp = z$ and $Tp_n \to Tp = z$ as $n \to \infty$. Since the pair (T,B) is compatible type (A), we obtain $\lim_{n\to\infty} G(BTp_n, TTp_n, TTp_n) = 0$, it gives $G(Bz,Tz,Tz) = 0$, since $Tp_n = z$ for all $n \ge 1$, hence $Bz = Tz$.

Again by the condition (ii), we have

$$\begin{aligned} G(Sx_{3n}, Tz, Ix_{3n+2}) \le{} & \alpha G(Ax_{3n}, Bz, Cx_{3n+2}) \\ & + \beta[G(Sx_{3n}, Tz, Bz) + G(Sx_{3n}, Tz, Cx_{3n+2})] \\ & + \gamma[G(Ax_{3n}, Ix_{3n+2}, Sx_{3n}) + G(Bz, Ix_{3n+2}, Sx_{3n})]. \end{aligned} \tag{2.16}$$

Letting $n \to \infty$ and using (iii) of Proposition 1.4, we have

$$G(z,Tz,z) \le \alpha G(z,Tz,z) + \beta[G(z,Tz,Tz) + G(z,Tz,z)] + \gamma[G(z,z,z) + G(Tz,z,z)]$$
$$\le (\alpha + 3\beta + \gamma) G(z,Tz,z)$$

Hence $G(z,Tz,z) = 0$, since $0 \le \alpha + 3\beta + \gamma < 1$. Thus $Tz = z = Bz.$

Since $T(X) \subset C(X)$ and $z = Tz \in T(X)$, there is a point $q \in X$ such that $Tz = z = Cq$. By the condition (ii), we have

$$\begin{aligned} G(Sx_{3n}, Tz, Iq) \le{} & \alpha G(Ax_{3n}, Bz, Cq) \\ & + \beta[G(Sx_{3n}, Tz, Bz) + G(Sx_{3n}, Tz, Cq)] \\ & + \gamma[G(Ax_{3n}, Iq, Sx_{3n}) + G(Bz, Iq, Sx_{3n})]. \end{aligned} \tag{2.17}$$

Letting $n \to \infty$, we have

$$G(z,z,Iq) \le \alpha G(z,z,z) + \beta[G(z,z,z) + G(z,z,z)] + \gamma[G(z,Iq,z) + G(z,Iq,z)]$$
$$= 2\gamma G(z,Iq,z).$$

Hence $G(z,Iq,z) = 0$, since $0 \le 2\gamma < 1$. Thus $Iq = z = Cq$.

Taking $q_n = q$ for all $n \ge 1$, then $Iq_n \to Iq = z$ and $Cq_n \to Cq = z$ as $n \to \infty$.

Since the pair (I,C) is compatible type (A), we obtain $\lim_{n\to\infty} G(CIq_n, IIq_n, IIq_n)=0$, it gives $G(Cz, Iz, Iz)=0$, since $Iq_n = z$ for all $n\geq 1$, hence $Cz = Iz$.

Again by the condition (ii), we have

$$\begin{aligned} G(Sx_{3n}, Tz, Iz) &\leq \alpha G(Ax_{3n}, Bz, Cz)+\beta[G(Sx_{3n}, Tz, Bz)+G(Sx_{3n}, Tz, Cz)] \\ &+\gamma[G(Ax_{3n}, Iz, Sx_{3n})+G(Bz, Iz, Sx_{3n})]. \end{aligned} \tag{2.18}$$

Letting $n\to\infty$, we have

$$\begin{aligned} G(z,z,Iz) &\leq \alpha G(z,z,Iz)+\beta[G(z,z,z)+G(z,z,Iz)]+\gamma[G(z,Iz,z)+G(z,Iz,z)] \\ &\leq (\alpha+\beta+2\gamma)G(z,z,Iz). \end{aligned}$$

Hence $G(z,z,Iz)=0$, since $0\leq\alpha+\beta+2\gamma<1$. Thus $Iz=z=Cz$.

Since $I(X)\subset A(X)$ and $z=Iz\in I(X)$, there is a point $w\in X$ such that $Iz=z=Aw$. By the condition (ii), we have

$$\begin{aligned} G(Sw, Tz, Iz) &\leq \alpha G(Aw, Bz, Cz)+\beta[G(Sw, Tz, Bz)+G(Sw, Tz, Cz)] \\ &+\gamma[G(Aw, Iz, Sw)+G(Bz, Iz, Sw)]. \end{aligned} \tag{2.19}$$

It gives

$$G(Sw,z,z)\leq(2\beta+2\gamma)G(Sw,z,z).$$

Hence $G(Sw,z,z)=0$, since $0\leq 2\beta+2\gamma<1$.Thus $Sw=z=Aw$.

Taking $u_n = w$ for all $n\geq 1$, then $Su_n\to Sw=z$ and $Au_n\to Aw=z$ as $n\to\infty$. Since the pair (S,A) is compatible type (A), we obtain $\lim_{n\to\infty} G(SAu_n, AAu_n, AAu_n)=0$, it gives $G(Sz, Az, Az)=0$, since $Au_n = z$ for all $n\geq 1$, hence $Sz=Az=z$.

Therefore, z is the common fixed point of S,T,I,A,B and C when S is continuous and the pairs $(S,A),(T,B),(I,C)$ are compatible of type (A).

The proof is similar when T or I is continuous and the pairs $(S,A),(T,B),(I,C)$ are compatible of type (A).

Finally, we prove uniqueness of common fixed point z .

Let z and t be two common fixed point of S, T, I, A, B and C, then using the condition (ii) and the (iii) of Proposition 1.4 we have

$$\begin{aligned} G(t,z,z) &= G(St,Tz,Iz) \\ &\leq \alpha G(At,Bz,Cz) + \beta[G(St,Tz,Bz) + G(St,Tz,Cz)] \\ &\quad + \gamma[G(At,Iz,St) + G(Bz,Iz,St)] \\ &\leq (\alpha + 2\beta + 3\gamma) G(t,z,z). \end{aligned} \tag{2.20}$$

Which implies that $G(t,z,z) = 0$, since $0 \leq \alpha + 2\beta + 3\gamma < 1$. Thus $t = z$. So common fixed point is unique.

Remark 2.1 Theorem 2.1 generalize and extend the corresponding results in Vats, Kumar and Sihag[14,Theorem 2.1 and 2.2].

Remark 2.2 In Theorem 2.1, we taken: 1) $A = B = C$; 2) $S = T = I$; 3) $T = I$ and $B = C$; 4) $B = C = E$ (E is identity mapping), several new result can be obtained.

Now we introduce an example to support Theorem 2.1.

Example 2.1 Let $X = [0,1]$ be a G -metric space with

$$G(x,y,z) = |x-y| + |y-z| + |z-x|.$$

We define mappings S, T, I, A, B and C on X by

$$Sx = \frac{8}{9}, \quad x \in [0,1], \qquad Tx = \begin{cases} \frac{7}{8}, & x \in [0, \frac{1}{2}] \\ \frac{8}{9}, & x \in (\frac{1}{2}, 1] \end{cases}, \qquad Ix = \begin{cases} \frac{6}{7}, & x \in [0, \frac{1}{2}] \\ \frac{8}{9}, & x \in (\frac{1}{2}, 1] \end{cases},$$

$$Ax = \begin{cases} 0, & x \in [0, \frac{1}{2}] \\ \frac{8}{9}, & x \in (\frac{1}{2}, 1) \\ \frac{6}{7}, & x = 1 \end{cases}, \qquad Bx = \begin{cases} 1, & x \in [0, \frac{1}{2}] \\ \frac{8}{9}, & x \in (\frac{1}{2}, 1) \\ 0, & x = 1 \end{cases}, \qquad Cx = \begin{cases} 0, & x \in [0, \frac{1}{2}] \\ \frac{8}{9}, & x \in (\frac{1}{2}, 1) \\ \frac{7}{8}, & x = 1 \end{cases}$$

Clearly, S is continuous and T, I, A, B and C are discontinuous mappings, $S(X) \subset B(X)$, $T(X) \subset C(X)$, $I(X) \subset A(X)$ and the pairs (S, A), (T, B), and

(I, C) be compatible of type (A).

Now we prove that the mappings S, T, I, A, B and C are satisfying the condition (2.1) of Theorem 2.1 with $\alpha = \frac{1}{3}, \beta = \frac{1}{9}, \gamma = \frac{1}{10}$.

Case 1. If $x, y, z \in [0, \frac{1}{2}]$, then we have

$$G(Sx, Ty, Iz) = G\left(\frac{8}{9}, \frac{7}{8}, \frac{6}{7}\right) = \frac{4}{63}$$

and

$$G(Ax, By, Cz) = G(0, 1, 0) = 2.$$

Thus we get

$$\begin{aligned} G(Sx, Ty, Iz) = \frac{4}{63} < \frac{1}{3} \times 2 = \alpha G(Ax, By, Cz) \\ \le \alpha G(Ax, By, Cz) + \beta[G(Sx, Ty, By) + G(Sx, Ty, Cz)]. \\ + \gamma[G(Ax, Iz, Sx) + G(By, Iz, Sx)]. \end{aligned}$$

Case 2. If $x, y \in [0, \frac{1}{2}]$, $z \in (\frac{1}{2}, 1]$, then we have

$$G(Sx, Ty, Iz) = G\left(\frac{8}{9}, \frac{7}{8}, \frac{8}{9}\right) = \frac{1}{36}.$$

If $z = 1$

$$G(Ax, By, Cz) = G\left(0, 1, \frac{7}{8}\right) = 2.$$

If $z \in (\frac{1}{2}, 1)$

$$G(Ax, By, Cz) = G\left(0, 1, \frac{8}{9}\right) = 2.$$

So we know

$$\begin{aligned} G(Sx, Ty, Iz) = \frac{1}{36} < \frac{1}{3} \times 2 = \alpha G(Ax, By, Cz) \\ \le \alpha G(Ax, By, Cz) + \beta[G(Sx, Ty, By) + G(Sx, Ty, Cz)] \\ + \gamma[G(Ax, Iz, Sx) + G(By, Iz, Sx)]. \end{aligned}$$

Case 3. If $x, z \in [0, \frac{1}{2}]$, $y \in (\frac{1}{2}, 1]$, then we have

$$G(Sx, Ty, Iz) = G\left(\frac{8}{9}, \frac{8}{9}, \frac{6}{7}\right) = \frac{4}{63} \ .$$

If $y = 1$

$$G(Sx, Ty, By) + G(Sx, Ty, Cz) = G\left(\frac{8}{9}, \frac{8}{9}, 0\right) + G\left(\frac{8}{9}, \frac{8}{9}, 0\right) = \frac{32}{9}.$$

Thus we have

$$\begin{aligned} G(Sx, Ty, Iz) = \frac{4}{63} < \frac{1}{9} \times \frac{32}{9} &= \beta[G(Sx, Ty, By) + G(Sx, Ty, Cz)] \\ &\leq \alpha G(Ax, By, Cz) + \beta[G(Sx, Ty, By) + G(Sx, Ty, Cz)] \\ &\quad + \gamma[G(Ax, Iz, Sx) + G(By, Iz, Sx)]. \end{aligned}$$

If $y \in (\frac{1}{2}, 1)$

$$G(Ax, By, Cz) = G\left(0, \frac{8}{9}, 0\right) = \frac{16}{9}.$$

So we know

$$\begin{aligned} G(Sx, Ty, Iz) = \frac{4}{63} < \frac{1}{3} \times \frac{16}{9} &= \alpha G(Ax, By, Cz) \\ &\leq \alpha G(Ax, By, Cz) + \beta[G(Sx, Ty, By) + G(Sx, Ty, Cz)] \\ &\quad + \gamma[G(Ax, Iz, Sx) + G(By, Iz, Sx)]. \end{aligned}$$

Case 4. If $y, z \in [0, \frac{1}{2}]$, $x \in (\frac{1}{2}, 1]$, then we have

$$G(Sx, Ty, Iz) = G\left(\frac{8}{9}, \frac{7}{8}, \frac{6}{7}\right) = \frac{4}{63} \ .$$

If $x = 1$

$$G(Ax, By, Cz) = G\left(\frac{6}{7}, 1, 0\right) = 2.$$

If $x \in (\frac{1}{2}, 1)$

$$G(Ax, By, Cz) = G\left(\frac{8}{9}, 1, 0\right) = 2.$$

So we know

$$G(Sx,Ty,Iz)=\frac{4}{63}<\frac{1}{3}\times 2=\alpha G(Ax,By,Cz)$$
$$\leq \alpha G(Ax,By,Cz)+\beta[G(Sx,Ty,By)+G(Sx,Ty,Cz)]$$
$$+\gamma[G(Ax,Iz,Sx)+G(By,Iz,Sx)].$$

Case 5. If $x\in[0,\frac{1}{2}],\ y,z\in(\frac{1}{2},1]$, then we have

$$G(Sx,Ty,Iz)=G\left(\frac{8}{9},\frac{8}{9},\frac{8}{9}\right)=0\ .$$

So we know

$$G(Sx,Ty,Iz)\leq \alpha G(Ax,By,Cz)+\beta[G(Sx,Ty,By)+G(Sx,Ty,Cz)]$$
$$+\gamma[G(Ax,Iz,Sx)+G(By,Iz,Sx)].$$

Case 6. If $y\in[0,\frac{1}{2}],\ x,z\in(\frac{1}{2},1]$, then we have

$$G(Sx,Ty,Iz)=G\left(\frac{8}{9},\frac{7}{8},\frac{8}{9}\right)=\frac{1}{36}.$$

If $x=1,\ z=1$ or $z\in(\frac{1}{2},1)$, we have

$$G(Ax,By,Cz)=\begin{cases}G\left(\frac{6}{7},1,\frac{7}{8}\right), & z=1\\ G\left(\frac{6}{7},1,\frac{8}{9}\right), & z\in(\frac{1}{2},1)\end{cases}=\frac{2}{7}.$$

Thus we get

$$G(Sx,Ty,Iz)=\frac{1}{36}<\frac{1}{3}\times\frac{2}{7}=\alpha G(Ax,By,Cz)$$
$$\leq \alpha G(Ax,By,Cz)+\beta[G(Sx,Ty,By)+G(Sx,Ty,Cz)]$$
$$+\gamma[G(Ax,Iz,Sx)+G(By,Iz,Sx)].$$

If $x\in(\frac{1}{2},1),z=1$

$$G(Ax,By,Cz)=G\left(\frac{8}{9},1,\frac{7}{8}\right)=\frac{1}{4}.$$

Thus we obtain

$$G(Sx,Ty,Iz)=\frac{1}{36}<\frac{1}{3}\times\frac{1}{4}=\alpha G(Ax,By,Cz)$$
$$\leq \alpha G(Ax,By,Cz)+\beta[G(Sx,Ty,By)+G(Sx,Ty,Cz)]$$
$$+\gamma[G(Ax,Iz,Sx)+G(By,Iz,Sx)].$$

If $x, z \in (\frac{1}{2}, 1)$

$$G(Ax, By, Cz) = G\left(\frac{8}{9}, 1, \frac{8}{9}\right) = \frac{2}{9}.$$

So we know

$$\begin{aligned} G(Sx, Ty, Iz) &= \frac{1}{36} < \frac{1}{3} \times \frac{2}{9} = \alpha G(Ax, By, Cz) \\ &\le \alpha G(Ax, By, Cz) + \beta[G(Sx, Ty, By) + G(Sx, Ty, Cz)] \\ &\quad + \gamma[G(Ax, Iz, Sx) + G(By, Iz, Sx)]. \end{aligned}$$

Case 7. If $z \in [0, \frac{1}{2}]$, $x, y \in (\frac{1}{2}, 1]$, then we have

$$G(Sx, Ty, Iz) = G\left(\frac{8}{9}, \frac{8}{9}, \frac{6}{7}\right) = \frac{4}{63}.$$

If $x = 1, y = 1$

$$G(Ax, By, Cz) = G\left(\frac{6}{7}, 0, 0\right) = \frac{12}{7}.$$

Thus we have

$$\begin{aligned} G(Sx, Ty, Iz) &= \frac{4}{63} < \frac{1}{3} \times \frac{12}{7} = \alpha G(Ax, By, Cz) \\ &\le \alpha G(Ax, By, Cz) + \beta[G(Sx, Ty, By) + G(Sx, Ty, Cz)] \\ &\quad + \gamma[G(Ax, Iz, Sx) + G(By, Iz, Sx)]. \end{aligned}$$

If $x = 1, y \in (\frac{1}{2}, 1)$

$$G(Ax, By, Cz) = G\left(\frac{6}{7}, \frac{8}{9}, 0\right) = \frac{16}{9}.$$

If $x \in (\frac{1}{2}, 1), y = 1$

$$G(Ax, By, Cz) = G\left(\frac{8}{9}, 0, 0\right) = \frac{16}{9}.$$

If $x, y \in (\frac{1}{2}, 1)$

$$G(Ax, By, Cz) = G\left(\frac{8}{9}, \frac{8}{9}, 0\right) = \frac{16}{9}.$$

So we know

$$G(Sx,Ty,Iz)=\frac{4}{63}<\frac{1}{3}\times\frac{16}{9}=\alpha G(Ax,By,Cz)$$
$$\leq \alpha G(Ax,By,Cz)+\beta[G(Sx,Ty,By)+G(Sx,Ty,Cz)]$$
$$+\gamma[G(Ax,Iz,Sx)+G(By,Iz,Sx)].$$

Case 8. If $x,y,z\in(\frac{1}{2},1]$, then we have

$$G(Sx,Ty,Iz)=G\left(\frac{8}{9},\frac{8}{9},\frac{8}{9}\right)=0 .$$

So we know

$$G(Sx,Ty,Iz)\leq \alpha G(Ax,By,Cz)+\beta[G(Sx,Ty,By)+G(Sx,Ty,Cz)]$$
$$+\gamma[G(Ax,Iz,Sx)+G(By,Iz,Sx)].$$

Then in all the above cases, the mapping S,T,I,A,B and C are satisfying the condition (2.1) of Theorem 2.1 with $\alpha=\frac{1}{3},\beta=\frac{1}{9},\gamma=\frac{1}{10}$ .so that all the conditions of Theorem 2.1 are satisfied. Moreover, $\frac{8}{9}$ is the unique common fixed point of S,T,I,A,B and C.

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