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CONDITIONS OF NON-EMPTY INTERSECTIONS OF CLOSED SETS USING NON-SELF MAPS

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Abstract. In this paper, we find a set of conditions under which two closed subsets of a metric space have non-empty intersection. We show that the existence of a non-self map from one of the sets to the other satisfying a weak contraction inequality along with some other conditions is sufficient to ensure the nonempty intersection. Further it is shown that the intersection contains the unique fixed point of the mapping. The result has a corollary and is illustrated with two examples. One of the examples show that the main theorem properly contains its corollary.

Keywords: P-property, non-self map, fixed point, control function, weak contraction.

2000 AMS Subject Classification: 54H25, 47H09, 47H10.

1. Introduction and Mathematical Preliminaries

Non-self mappings appear in various contexts in the problems of functional analysis. In this paper we have considered two closed subsets of a metric space and a mapping from one of these sets to the other. The purpose is to find sufficient conditions for ensuring that the intersection of these two sets is non-empty. We assume P-property for the pair of closed sets. We have

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assumed two conditions on the non-self mapping. One of these is a weak contraction inequality assumed to be satisfied by the mapping on a specific subset of its domain. These are types of inequalities first introduced by Alber et al for weakening the contraction mapping principle in Hilbert spaces [1] and subsequently adapted to the metric space in general [21]. Following these works, a large number of papers have appeared using weak contractive inequalities of various types as, for instances, in [9, 10, 18, 25]. We have shown that the nonempty intersection of the two closed sets contains the unique fixed point of the non-self mapping. Fixed points of non-self mappings generally do not exist, but may exist under special circumstances. Some instances of works on fixed points of non-self mappings are in [2, 3, 6, 7, 8, 13, 15, 16, 19, 20, 24].

Our main theorem has a corollary and is supported with two examples. One of the examples show that the corollary in properly contained in the main theorem.

In the following we give some technical definitions which we use in our theorem.

Let A, B be two nonempty subsets of a metric space X.

Let $d(A,B) = \inf \{d(x,y) : x \in A \text{ and } y \in B\}$,

$A_0 = \{x \in A : d(x,y) = dist(A,B) \ for\ some\ y \in B\}$ and

$B_0 = \{y \in B : d(x,y) = dist(A,B) \ for\ some\ x \in A\}$.

There are some sufficient conditions which guarantee the nonemptiness of A_0 and B_0. For examples, some of these conditions are given in [11] and [17].

Definition 1.1. (Altering distance function [14]) A function $\psi : [0,\infty) \to [0,\infty)$ is called an altering distance function if the following properties are satisfied:

i) $\psi(0) = 0$,

ii) ψ is continuous and monotonically non-decreasing.

These functions are actually control functions which, acting on the metric, destroys the triangular inequality. This introduces an interesting complication in the problem concerned. This function and its generalizations have been used extensively in the problems of metric fixed point theory, some instances of these works are in [4, 5, 9, 12, 23].

Definition 1.2. (P-property) [22] Let (A,B) be a pair of nonempty subsets of a metric space X with $A_0 \neq \emptyset$. Then the pair (A,B) is said to have the P-property if and only if

$$\left.\begin{array}{l} d(x_1,y_1) = d(A,B) \\ d(x_2,y_2) = d(A,B) \end{array}\right\} \Rightarrow d(x_1,x_2) = d(y_1,y_2)$$

where $x_1,x_2 \in A_0$ and $y_1,y_2 \in B_0$.

This property of pairs of closed sets has been used in a proximity point problem in [22]. There it has been observed that any pair (A,B) of nonempty closed convex subsets of a real Hilbert space H satisfies the P-property.

Definition 1.3. Condition-I Let (A,B) be a pair of nonempty subsets of a metric space (X,d). The mapping $T : A \longrightarrow B$ is said to satisfy condition-I, if
$d(y,Tx) \leq d(x,y)$ whenever y is such that $d(y,Tx) = d(A,B)$.
The following example illustrates P-property and Condition-I.

Example 1.4. An example of a function satisfying condition -I is the following.
Let X=R^2 be a metric space with metric
$d((x_1,y_1),(x_2,y_2)) =\mid x_1 - x_2 \mid + \mid y_1 - y_2 \mid$.
Let $A = \{0\} \times R$ and $B = \{1\} \times R$ be two subsets of X. Then $d(A,B) = 1$.
Let $T : A \longrightarrow B$ be defined as
$Tx = T(0,\alpha) = (1,\alpha-2)$, for all $x = (0,\alpha) \in A$.
Then T satisfies condition-I. Also the pair (A,B) has P-property.

2. Main results

Theorem 2.1. *et (X,d) be a metric space. Let (A,B) be a pair of non empty closed subsets of X such that A_0 is non empty and that the pair (A,B) has the P- property. If there exists a continuous mapping $T : A \longrightarrow B$ satisfying condition -I such that*
i) $T(A_0) \subseteq B_0$,
ii) for all $x,y \in A_0$, $\psi(d(Tx,Ty)) \leq \psi(M(x,y)) - \phi(M(x,y))$, *(2.1)*
where $M(x,y) = max\ \{d(x,y), d(y,Tx)\}$,

ψ is an altering distance function and $\phi : [0,\infty) \to [0,\infty)$ is continuous function with $\phi(t) = 0$ if and only if $t = 0$, then $A \cap B$ is nonempty and contains the unique fixed point of T.

Proof. Let $x_0 \in A_0$. Since $Tx_0 \in T(A_0) \subseteq B_0$, there exists $x_1 \in A_0$ such that $d(x_1, Tx_0) = d(A,B)$. Again, since $Tx_1 \in T(A_0) \subseteq B_0$, there exists $x_2 \in A_0$ such that $d(x_2, Tx_1) = d(A,B)$. Continuing the process, we have a sequence $\{x_n\}$ in $A_0 \subset A$ such that

$$d(x_n, Tx_{n-1}) = d(A,B), \text{ for all } n \in N.$$

Then condition-I implies that, for all $n \in N$,

$$d(A,B) = d(x_n, Tx_{n-1}) \leq d(x_{n-1}, x_n). \tag{2.2}$$

Then, for all $n \in N$, $M(x_{n-1}, x_n) = \max\,\{d(x_{n-1}, x_n), d(x_n, Tx_{n-1})\}$

$$= d(x_{n-1}, x_n) \text{ (by (2.2))}. \tag{2.3}$$

Since (A,B) satisfies the P-property, by virtue of (2.2), we conclude that

$$d(x_n, x_{n+1}) = d(Tx_{n-1}, Tx_n), \text{ for all } n \in N.$$

Then, for any positive integer n,

$$\psi(d(x_n, x_{n+1})) = \psi(d(Tx_{n-1}, Tx_n))$$

$$\leq \psi(M(x_{n-1}, x_n)) - \phi(M(x_{n-1}, x_n)) \text{ (by (2.1))}. \tag{2.4}$$

Then, $\psi(d(x_n, x_{n+1})) \leq \psi(M(x_{n-1}, x_n))$ (by the properties of ϕ).

By the monotone property of ψ, for all $n \in N$, we have

$$d(x_n, x_{n+1}) \leq M(x_{n-1}, x_n) = d(x_{n-1}, x_n) \text{ (by (2.3))}. \tag{2.5}$$

Hence $\{d(x_n, x_{n+1})\}$ is a monotone decreasing sequence and, consequently, there exists $r \geq 0$ such that

$$d(x_n, x_{n+1}) \longrightarrow r \text{ as } n \longrightarrow \infty. \tag{2.6}$$

By (2.5), it follows that $M(x_{n-1}, x_n) \to r$ as $n \longrightarrow \infty$.

Letting $n \longrightarrow \infty$ in (2.4), using the continuities of ψ and ϕ, and the above limit, we have

$$\psi(r) \leq \psi(r) - \phi(r), \text{ which is a contradiction unless } r = 0.$$

$$\text{Hence } d(x_n, x_{n+1}) \longrightarrow 0 \text{ as } n \longrightarrow \infty. \tag{2.7}$$

$$\text{Taking limit as } n \to \infty \text{ in (2.2), and using (2.7), we obtain } d(A,B) = 0, \tag{2.8}$$

which implies that $A \cap B \neq \emptyset$, since A and B are closed.

We next prove that $\{x_n\}$ is a Cauchy sequence in A_0. If possible, let $\{x_n\}$ be not a Cauchy sequence. Then there exists $\varepsilon > 0$ for which we can find subsequences $\{x_{m(k)}\}$ and $\{x_{n(k)}\}$ of

$\{x_n\}$ in A_0 with $m(k) > n(k) > k$ such that

$$d(x_{m(k)}, x_{n(k)}) \geq \varepsilon \text{ and}$$
$$d(x_{m(k)-1}, x_{n(k)}) < \varepsilon.$$

Then, for all $k \geq 0$,

$$\varepsilon \leq d(x_{m(k)}, x_{n(k)}) \leq d(x_{m(k)}, x_{m(k)-1}) + d(x_{m(k)-1}, x_{n(k)}) < \varepsilon + d(x_{m(k)}, x_{m(k)-1}). \tag{2.9}$$

Taking $k \longrightarrow \infty$ in the above inequality, and using (2.7), we have

$$\lim_{k\to\infty} d(x_{m(k)}, x_{n(k)}) = \varepsilon. \tag{2.10}$$

Again, for all $k \geq 0$,

$$d(x_{m(k)}, x_{n(k)}) \leq d(x_{m(k)}, x_{m(k)-1}) + d(x_{m(k)-1}, x_{n(k)-1}) + d(x_{n(k)-1}, x_{n(k)}), \tag{2.11}$$

and $d(x_{m(k)-1}, x_{n(k)-1}) \leq d(x_{m(k)-1}, x_{m(k)}) + d(x_{m(k)}, x_{n(k)}) + d(x_{n(k)}, x_{n(k)-1}).$ (2.12)

Taking $k \longrightarrow \infty$ in the above two inequalities, using (2.7) and (2.10), we have

$$\lim_{k\to\infty} d(x_{m(k)-1}, x_{n(k)-1}) = \varepsilon. \tag{2.13}$$

By (2.2), for all $k \geq 0$, $d(x_{m(k)}, Tx_{m(k)-1}) = d(A,B)$ and $d(x_{n(k)}, Tx_{n(k)-1}) = d(A,B)$.

Since (A,B) satisfies the P-property, for all $k \geq 0$, we have

$$d(x_{m(k)}, x_{n(k)}) = d(Tx_{m(k)-1}, Tx_{n(k)-1}).$$

Then, for all $k \geq 0$, we have

$$\begin{aligned}\psi(d(x_{m(k)}, x_{n(k)})) &= \psi(d(Tx_{m(k)-1}, Tx_{n(k)-1})) \\ &\leq \psi(M(x_{m(k)-1}, x_{n(k)-1})) - \phi(M(x_{m(k)-1}, x_{n(k)-1})).\end{aligned} \tag{2.14}$$

Now, for all $k \geq 0$,

$$M(x_{m(k)-1}, x_{n(k)-1}) = \max\,\{d(x_{m(k)-1}, x_{n(k)-1}), d(x_{n(k)-1}, Tx_{m(k)-1})\}. \tag{2.15}$$

Also, for all $k \geq 0$,

$$\begin{aligned}d(x_{n(k)-1}, Tx_{m(k)-1}) &\leq d(x_{n(k)-1}, x_{m(k)-1}) + d(x_{m(k)-1}, x_{m(k)}) + d(x_{m(k)}, Tx_{m(k)-1}) \\ &\leq d(x_{n(k)-1}, x_{m(k)-1}) + d(x_{m(k)-1}, x_{m(k)}) \text{ (by (2.2) and (2.8))}.\end{aligned}$$

Taking $k \longrightarrow \infty$ in the above inequality, and using (2.7), we have

$$\lim_{k\to\infty} d(x_{n(k)-1}, Tx_{m(k)-1}) \leq \lim_{k\to\infty} d(x_{n(k)-1}, x_{m(k)-1}). \tag{2.16}$$

Then taking $k \longrightarrow \infty$ in (2.15), using (2.13) and (2.16), we have

$$\lim_{k\to\infty} M(x_{m(k)-1}, x_{n(k)-1}) = \varepsilon. \tag{2.17}$$

Letting $k \longrightarrow \infty$ in (2.14), using (2.10) and (2.17), we obtain

$\psi(\varepsilon) \leq \psi(\varepsilon) - \phi(\varepsilon)$, which, by a property of ϕ, is a contradiction with our assumption that $\varepsilon > 0$.

Hence $\{x_n\}$ is a Cauchy sequence.

Since A is a closed subset of the metric space X, there exists $z \in A$

such that $x_n \longrightarrow z$ as $n \to \infty$. (2.18)

Since T is a continuous mapping, we have $Tx_n \longrightarrow Tz$ as $n \to \infty$. Then, $d(x_{n+1}, Tx_n) \longrightarrow d(z, Tz)$ as $n \to \infty$. But according to (2.2) and (2.8), the sequence $\{d(x_{n+1}, Tx_n)\}$ is a constant sequence with constant value $d(A,B) = 0$. Therefore, $d(z,Tz) = d(A,B) = 0$, that is, $d(z,Tz) = 0$ or $z = Tz$, that is, z is a fixed point of T. It then follows that $z = Tz \in A \cap B$.

For any fixed point of T, $d(u,Tu) = 0 = d(A,B)$. Hence $u \in A_0$. If $\bar{z}$ is any fixed point of T, then z, $\bar{z} \in A_0$.

So we have $\psi(d(z,\bar{z})) = \psi(d(Tz,T\bar{z})) \leq \psi(M(z,\bar{z})) - \phi(M(z,\bar{z}))$. (2.19)

Now $M(z,\bar{z}) = \max\{d(z,\bar{z}), d(\bar{z},Tz)\} = \max\{d(z,\bar{z}), d(\bar{z},z)\} = d(z,\bar{z})$.

Putting this value in (2.19), we have

$\psi(d(z,\bar{z})) \leq \psi(d(z,\bar{z})) - \phi(d(z,\bar{z}))$, which, by a property of ϕ, is a contradiction unless $z = \bar{z}$.

Thus z is the unique fixed point of T which is contained in $A \cap B$, that is, $A \cap B$ contains the unique fixed point of T.

Corollary 2.2. *Let (X,d) be a metric space. Let (A,B) be a pair of non empty closed subsets of X such that A_0 is non empty and that the pair (A,B) has the P- property. If there exists a continuous mapping $T : A \longrightarrow B$ satisfying condition -I such that*

i) $T(A_0) \subseteq B_0$,

ii) for all $x,y \in A$, $\psi(d(Tx,Ty)) \leq \psi(M(x,y)) - \phi(M(x,y))$,

where $M(x,y) = max\ \{d(x,y), d(y,Tx)\}$,

ψ is an altering distance function and $\phi : [0,\infty) \to [0,\infty)$ is continuous function with $\phi(t) = 0$ if and only if $t = 0$, then $A \cap B$ is nonempty and contains the unique fixed point of T.

Proof. All the conditions of the corollary are the same as that of the theorem 2.1 expect that the inequality (2.1) is now required to be satisfied for any pairs of points $x,y \in A$. Then (2.1) is automatically satisfied by all pairs $x,y \in A_0$.The corollary then follows by an application of theorem 2.1.

In the following section, with the help of an example, we show that the corollary is properly contained in the theorem.

3. Example

Example 3.1. Let X=R is a metric space with the usual metric $d(x,y)=\mid x-y\mid$.
We consider the two closed subsets of X, $A=[-1,0]$ and $B=[0,1]$.
Then (A,B) satisfies the P- property. Here $A_0=B_0=\{0\}$.
Let $T:A\longrightarrow B$ be defined as $Tx=-x-\frac{1}{2}x^2$, for all $x\in[-1,0]$.
Then T is continuous on A, satisfies Condition- I with respect to A and B and $T(A_0)\subseteq B_0$.
Let $\psi:[0,\infty)\longrightarrow[0,\infty)$ be defined as $\psi(t)=t$, for all $t\in[0,\infty)$,
and let $\phi:[0,\infty)\longrightarrow[0,\infty)$ be defined as $\phi(t)=\frac{t^2}{2}$, for all $t\in[0,\infty)$.
Since $x,y\in[-1,0]$, let $x=-a$ and $y=-b$ where $a,b\geq 0$.
Without loss of generality we assume that $a\geq b$.
Then $Tx=-x-\frac{1}{2}x^2=a-\frac{1}{2}a^2$ and $Ty=-y-\frac{1}{2}y^2=b-\frac{1}{2}b^2$.

$$\begin{aligned}\text{Now } \psi(d(Tx,Ty)) &= d(a-\tfrac{1}{2}a^2,b-\tfrac{1}{2}b^2) \text{ (since } \psi(t)=t)\\ &=\mid a-b-\tfrac{1}{2}a^2+\tfrac{1}{2}b^2\mid\\ &=\mid a-b\mid\mid 1-\tfrac{1}{2}(a+b)\mid\\ &\leq\mid a-b\mid\mid 1-\tfrac{1}{2}(a-b)\mid\\ &=\mid a-b\mid-\tfrac{1}{2}\mid a-b\mid^2.\end{aligned}$$

$$\begin{aligned}\text{Also, for any } x,y\in A,\ M(x,y) &= \max\{d(x,y),d(y,Tx)\}\\ &\geq d(x,y)=\mid a-b\mid.\end{aligned}$$

Since the function $f(t)=t-\frac{1}{2}t^2$ is monotone increasing in [0,1], we have
$\mid a-b\mid-\frac{1}{2}\mid a-b\mid^2\leq M(x,y)-\frac{1}{2}(M(x,y))^2=\psi(M(x,y))-\phi(M(x,y))$.
Thus, all the conditions of corollary 2.2 are satisfied. By an application of the corollary it follows that $A\cap B$ contains the unique fixed point of T. Here 0 is the unique fixed point of T included in $A\cap B=\{0\}$.

Our next example shows that theorem 2.1 properly contains its Corollary 2.2.

Example 3.2. Let X=R^2 is a metric space with

$d((x_1,y_1),(x_2,y_2)) = \mid x_1 - x_2 \mid + \mid y_1 - y_2 \mid$.

We consider the two closed subsets

$A = \{(p,q) : p \in (-\infty,0), q = 0\} \cup \{(p,q) : p = 0, q \in [0,1]\}$

and $B = \{(p,q) : p \in (0,\infty), q = 0\} \cup \{(p,q) : p = 0, q \in [0,1]\}$.

Then (A,B) satisfies the P- property. Here, $A_0 = B_0 = \{(p,q) : p = 0, q \in [0,1]\}$.

Let $T : A \longrightarrow B$ be defined as

$T(\alpha,0) = (-\alpha,0)$ for $(\alpha,0) \in \{(p,q)) : p \in (-\infty,0), q = 0\}$

and $T(0,\beta) = (0,\beta - \frac{1}{2}\beta^2)$ for $(0,\beta) \in \{(p,q) : p = 0, q \in [0,1]\}$.

Then T is continuous, satisfies Condition- I with respect to A and B and $T(A_0) \subseteq B_0$.

Let $\psi : [0,\infty) \longrightarrow [0,\infty)$ be defined as $\psi(t) = t$, for all $t \in [0,\infty)$,

and let $\phi : [0,\infty) \longrightarrow [0,\infty)$ be defined as $\phi(t) = \frac{t^2}{2}$, for all $t \in [0,\infty)$.

Let $x = (0,a) \in A_0 = \{(p,q) : p = 0, q \in [0,1]\}$ and $y = (0,b) \in B_0 = \{(p,q) : p = 0, q \in [0,1]\}$ where $a,b \geq 0$.

$$
\begin{aligned}
\text{Now, } \psi(d(Tx,Ty)) &= \psi(d(T(0,a),T(0,b))) \\
&= d((0,a-\tfrac{1}{2}a^2),(0,b-\tfrac{1}{2}b^2)) \text{ (since } \psi(t) = t) \\
&= \mid a - b - \tfrac{1}{2}a^2 + \tfrac{1}{2}b^2 \mid \\
&= \mid a - b \mid\mid 1 - \tfrac{1}{2}(a+b) \mid \\
&\leq \mid a - b \mid\mid 1 - \tfrac{1}{2}(a-b) \mid \\
&= \mid a - b \mid - \tfrac{1}{2} \mid a - b \mid^2.
\end{aligned}
$$

$$
\begin{aligned}
\text{Also, } M(x,y) &= \max \{d(x,y), d(y,Tx)\} \\
&= \max \{\mid a - b \mid, d(y,Tx)\} \geq \mid a - b \mid .
\end{aligned}
$$

Since the function $f(t) = t - \frac{1}{2}t^2$ is monotone increasing in [0,1], we have

$\mid a - b \mid - \frac{1}{2} \mid a - b \mid^2 \leq M(x,y) - \frac{1}{2}(M(x,y))^2 = \psi(M(x,y)) - \phi(M(x,y))$.

Then the inequality (2.1) is satisfied for all $x,y \in A_0$.

Thus, all the conditions of theorem 2.1 are satisfied. By an application of the theorem it follows that $A \cap B$ contains the unique fixed point of T. Here $(0,0)$ is the unique fixed point of T included in $A \cap B = \{(p,q) : p = 0, q \in [0,1]\}$.

Remark 3.3. We note that in example 3.2 the inequality (2.1) does not hold for $x = (-1,0)$, $y = (0,1)$. It shows that we cannot apply the corollary to this example. Thus we conclude that the corollary 2.2 is properly contained in the theorem 2.1.

Remark 3.4. In the example 1.4 we have that the function T is continuous and satisfies condition-I but has no fixed point. Also in this case $A \cap B = \emptyset$. The function T does not satisfy the inequality (2.1). This is seen by choosing $x = (0,0)$ and $y = (0,2)$, in which case $d(x,y) = 2$, $d(Tx,Ty) = 2$ which implies that (2.1) is not satisfied. It follows that some additional conditions are to be added to the condition-I to obtain the conclusions of theorem 2.1. The conditions i) and ii) in the theorem 2.1 are one set of these conditions. It is to be noted that the inequality (2.1) is required to be satisfied for any pair of points x and y in A_0. If (2.1) is satisfied for any pairs of points in A, then the conclusion of the theorem follow immediately. This is in fact the result noted in corollary 2.2 as mentioned in the remark 3.3, theorem 2.1. properly contains its corollary 2.2.

Remark 3.5. The continuity of T is a condition in our theorem. It remains to be investigated whether this can be replaced by some other condition.

Conflict of Interests

The authors declare that there is no conflict of interests.

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