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SOME FIXED POINT THEOREMS USING COMPATIBLE-TYPE MAPPINGS IN BANACH SPACES

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Abstract. In this paper, we establish some fixed point theorems for two pairs of compatible mappings of type (B) and for two pairs of weakly compatible mappings in Banach spaces.

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1. Introduction

The concept of compatible mappings of type (B) introduced by Pathak *et al.* (see [45]).

Definition 1.1 [45] Let S and T be mappings from a normed space E into itself. The mappings S and T are said to be compatible mappings of type (B) if

$$\lim_{n\to\infty} \|STx_n - TTx_n\| \leq \frac{1}{2}[\lim_{n\to\infty} \|STx_n - St\| + \lim_{n\to\infty} \|St - SSx_n\|]$$

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and

$$\lim_{n\to\infty}\|TSx_n - SSx_n\| \leq \frac{1}{2}[\lim_{n\to\infty}\|TSx_n - Tt\| + \lim_{n\to\infty}\|Tt - TTx_n\|]$$

whenever $\{x_n\}$ is a sequence in E such that $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = t$ for some $t \in E$.

Proposition 1.2 [45] Let S and T be compatible mappings of type (B) from a normed space E into itself. Suppose that

$$\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = t \text{ for some } t \in E \text{ then}$$

$$\lim_{n\to\infty} TTx_n = St \quad \text{if} \quad S \quad \text{is continuous at} \quad t,$$

$$\lim_{n\to\infty} SSx_n = Tt \quad \text{if} \quad T \quad \text{is continuous at} \quad t,$$

$$STt = TSt \quad \text{and} \quad St = Tt \quad \text{if } S \text{ and } T \text{ are continuous at} \quad t.$$

Let A and B be two mappings of a metric space (M,d) into itself. Pathak [44] defined A and B to be weakly compatible mappings with respect to B if and only if whenever

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = t \in M,$$

$$\lim_{n\to\infty} d(ABx_n, BAx_n) \leq d(At, Bt)$$

for all sequence $\{x_n\}$ in M and

$$d(At, Bt) \leq \lim_{n\to\infty} d(Bt, BAx_n)$$

for at least one sequence $\{x_n\}$ in M.

The following lemma is useful in the sequel.

Lemma 1.3 [44] *Let A, B:*$(M,d) \to (M,d)$ *be weak compatible with respect to B*

(a_1) *If* $At = Bt$, *then* $ABt = BAt$.

(a_2) *Suppose that* $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n$ *for some* $t \in X$.

(a_3) *If* A *is continuous at* t, *then* $\lim_{n\to\infty} d(BAx_n, At) \leq d(At, Bt)$.

(a_4) *If* A *and* B *are continuous at* t, *then* $At = Bt$ *and* $ABt = BAt$.

The paper is organized as follows: In Section 1, we explain some notations, concepts and the results as noted earlier which can be found in [2,13,29-37,41-48]. In Section 2, we prove a common fixed point theorem for two pairs of compatible mappings of type (B) in Banach spaces. Section 3 contains also a common fixed point theorem for two pairs of weakly compatible mappings in Banach spaces.

One of our main result to prove a common fixed point theorem for two pairs of compatible mappings of type (B) in Banach spaces will be given in the following section.

2. Fixed points by compatible maps of type (B)

Now we use the definition of compatible mappings of type (B) to obtain a common fixed point theorem in Banach spaces.

Theorem 2.1. *Let A, B, S, and T be mappings from a Banach space X into itself, and the pairs $\{A,S\}$ and $\{B,T\}$ are compatible of type (B), satisfying the following conditions:*

$$\|Ax-By\| \leq \Phi\big(\max\{\|Sx-Ty\|,\|Sx-Ax\|,\|Sx-Ax\|^{\frac{1}{2}}\|Ty-By\|^{\frac{1}{2}}, \|Ty-Ax\|^{\frac{1}{2}}\|Sx-By\|^{\frac{1}{2}}\}\big) \tag{2.1}$$

for all $x,y\in X$, and the function Φ satisfies the following conditions:

(b_1) $\Phi:[0,\infty)\to[0,\infty)$ is nondecreasing and right continuous.

(b_2) For every $t>0$, $\Phi(t)<t$ and we suppose that

$$(1-k)A(X)+kS(X)\subset A(X), \quad \forall \ k\in(0,1),$$
$$(1-\acute{k})B(X)+\acute{k}T(X)\subset B(X), \quad \forall \ \acute{k}\in(0,1).$$

For some $x_0\in X$, the sequence $\{x_n\}$ is defined by

$$Ax_{2n+1}=(1-c_{2n})Ax_{2n}+c_{2n}Sx_{2n}, \tag{2.2}$$

$$Bx_{2n+2}=(1-c_{2n+1})Bx_{2n+1}+c_{2n+1}Tx_{2n+1}, \tag{2.3}$$

with (i) $0<c_n\leq 1$ and (ii) $\lim_{n\to\infty}c_n=h>0$ for $n=0,1,2,\dots$. Then $\{x_n\}$ converges to a point z in C and if A and B are continuous at z, then z is a common fixed point of A,B,S and T.

Proof. Let $z \in X$ such that $\lim_{n\to\infty} x_n = z$. Now since A is continuous at z, then we have $Ax_n \to Az$ as $n \to \infty$. From (2.2), we have

$$Sx_{2n} = \frac{Ax_{2n+1} - (1-c_{2n})Ax_{2n}}{c_{2n}} \to \frac{Az-(1-h)Az}{h} = Az \text{ as } n \to \infty.$$

Similarly, from (2.3) we have $Tx_{2n+1} \to Bz$ as $n \to \infty$

Assume $AAz \neq Bz$. Then using (2.1) with $x = Sx_{2n},\ y = x_{2n+1}$, we obtain

$$\|ASx_{2n} - Bx_{2n+1}\| \leq \Phi\big(\max\{\|SSx_{2n} - Tx_{2n+1}\|, \|SSx_{2n} - ASx_{2n}\|,$$
$$\|SSx_{2n} - ASx_{2n}\|^{\frac{1}{2}}\|Tx_{2n+1} - Bx_{2n+1}\|^{\frac{1}{2}}, \|Tx_{2n+1} - ASx_{2n}\|^{\frac{1}{2}}\|SSx_{2n} - Bx_{2n+1}\|^{\frac{1}{2}}\}\big).$$

Taking the limit as $n \to \infty$, we obtain

$$\begin{aligned}\|A^2z - Bz\| &\leq \Phi\big(\max\{\|SAz - Bz\|, \|SAz - A^2z\|, \|SAz - A^2z\|^{\frac{1}{2}}\|Bz - Bz\|^{\frac{1}{2}},\\ &\|Bz - A^2z\|^{\frac{1}{2}}\|SAz - Bz\|^{\frac{1}{2}}\}\big)\\ &\leq \Phi\big(\max\{\|A^2z - Bz\|^p, 0, 0, \|A^2z - Bz\|\}\big)\\ &\leq \Phi\big(\|A^2z - Bz\|\big)\\ &\leq \|A^2z - Bz\|.\end{aligned}$$

This is a contradiction if $\|A^2z - Bz\| > 0$, and hence $\|A^2z - Bz\| = 0$. Thus $AAz = Bz$.

Now suppose that $Tz \neq Az$. Then from (2.1) and Proposition 1.2, we obtain

$$\|ASx_{2n} - Bz\| \leq \Phi\big(\max\{\|SSx_{2n} - Tz\|, \|SSx_{2n} - ASx_{2n}\|,$$
$$\|SSx_{2n} - ASx_{2n}\|^{\frac{1}{2}}\|Tz - Bz\|^{\frac{1}{2}}, \|Tz - ASx_{2n}\|^{\frac{1}{2}}\|SSx_{2n} - Bz\|^{\frac{1}{2}}\}\big).$$

Letting $n \to \infty$, we get, as $Bz = AAz$ and $\|ASx_{2n} - Bx_{2n}\| \to 0$,

$$\|AAz - Tz\| \leq \Phi(\max\{\|AAz - Tz\|, 0, 0, \|AAz - Tz\|\}),$$

which implies, $AAz = Tz$. Similarly $Sz = BBz$ therefore, $Az = Bz = Sz = Tz$, and

$$SAz = S^2z = A^2z = STz = ATz = Tz.$$

So $Tz = u$ is common fixed point of A, B, S and T. Let v be a another common fixed point of $A,\ B,\ S$ and T. By (2.1), we have

$$\begin{aligned}\|u-v\| &= \|Au-Bv\| \\ &\leq \Phi\big(\max\{\|Su-Tv\|, \|Su-Au\|, \|Su-Au\|^{\frac{1}{2}} \\ &\times\|Tv-Bv\|^{\frac{1}{2}}, \|Tv-Au\|^{\frac{1}{2}}\ \|Su-Bv\|^{\frac{1}{2}}\}\big) \\ &\leq \Phi\big(\max\{\|u-v\|, 0, 0, \|u-v\|\}\big),\end{aligned}$$

which implies $u = v$. This completes the proof.

3. Fixed points by weakly compatible maps

For the class of weakly compatible mappings, we have the following result.

Theoem 3.1. *Let C be a nonempty closed convex subset of a Banach space X and $A,\ B,\ S,$ and T be mappings from C into itself satisfying the following conditions:*

$$\|Sx-Ty\| \leq \Phi\big(\max\{\|Ax-By\|, \|Ax-Sx\|, \|By-Ty\|, \|Ax-Ty\|, \|By-Sx\|\}\big) \tag{3.1}$$

for all $x, y \in C$, and the function Φ satisfies the following conditions:

(c_1) $\Phi : [0,\infty) \to [0,\infty)$ *is nondecreasing and right continuous*

(c_2) For every $t > 0$, $\Phi(t) < t$.

Also, we suppose that

$$(1-k)A(C) + kS(C) \subset A(C), \quad \forall k \in (0,1),$$
$$(1-\acute{k})B(C) + \acute{k}T(C) \subset B(C), \quad \forall \acute{k} \in (0,1),$$

(3.2) $\{A, B\}$, $\{S, B\}$*and* $\{T, B\}$ *are weakly compatible pairs with respect to* B *of* X.

For some $x_0 \in X$, the sequence $\{x_n\}$ is defined by

$$Ax_{2n+1} = (1-c_{2n})Ax_{2n} + c_{2n}Sx_{2n}, \tag{3.3}$$

$$Bx_{2n+2} = (1-c_{2n+1})Bx_{2n+1} + c_{2n+1}Tx_{2n+1}\,, \tag{3.4}$$

with (i) $0 < c_n \leq 1$ *and (ii)* $\lim_{n\to\infty} c_n = h > 0$ *for* $n = 0,1,2,\dots$ *. Then* $\{x_n\}$ *converges to a point* z *in* C *and if* A *and* B *are continuous at* z*, then* z *is a coincidence point of* A, B, S *and* T*. Further, if* A *and* B *are continuous at* z*, then* S *and* T *are continuous at* z*.*

Proof. Let $z \in C$ such that $\lim\limits_{n\to\infty} x_n = z$. Now since A is continuous at z, then we have $Ax_n \to Az$ as $n \to \infty$, so from (3.3) we have

$$Sx_{2n} = \frac{Ax_{2n+1} - (1-c_{2n})Ax_{2n}}{c_{2n}} \to \frac{Az-(1-h)Az}{h} = Az \text{ as } n \to \infty.$$

Similarly, from (3.4) we have $Tx_{2n+1} \to Bz$ as $n \to \infty$. Assume $Az \neq Bz$. Then using (3.1) with $x = x_{2n},\ y = x_{2n+1}$, we obtain

$$\begin{aligned}\|Sx_{2n} - Tx_{2n+1}\| \quad &\leq \Phi\big(\max\{\|Ax_{2n} - Bx_{2n+1}\|, \|Ax_{2n} - Sx_{2n}\|, \|Bx_{2n+1} - Tx_{2n+1}\|,\\ &\|Ax_{2n} - Tx_{2n+1}\|, \|Bx_{2n+1} - Sx_{2n}\|\}\big).\end{aligned}$$

Taking the limit as $n \to \infty$, yields

$$\begin{aligned}\|Az - Bz\| \quad &\leq \Phi\big(\max\{\|Az-Bz\|, \|Az-Az\|, \|Bz-Bz\|,\\ &\|Az-Bz\|, \|Bz-Az\|\}\big)\\ &\leq \Phi\big(\max\{\|Az-Bz\|, 0, 0, \|Az-Bz\|, \|Bz-Az\|\}\big)\\ &\leq \Phi\big(\|Az-Bz\|\big)\end{aligned}$$

a contradiction, if $\|Az - Bz\| > 0$, and so $\|Az - Bz\| = 0$. Thus $Az = Bz$. Now suppose that $Tz \neq Az$. Then from (3.1), we have

$$\begin{aligned}\|Sx_{2n} - Tz\| \quad &\leq \Phi\big(\max\{\|Ax_{2n} - Bz\|, \|Ax_{2n} - Sx_{2n}\|, \|Bz - Tz\|,\\ &\|Ax_{2n} - Tz\|, \|Bz - Sx_{2n}\|\}\big).\end{aligned}$$

Letting $n \to \infty$, we get, as $Bz = Az$ and $\|Ax_{2n} - Sx_{2n}\| \to 0$,

$$\|Az - Tz\| \leq \Phi\big(\max\{0, \|Az-Sz\|, \|Bz-Tz\|, \|Az-Tz\|, 0\}\big),$$

which implies that, $Az = Tz$. Similarly $Sz = Bz$ therefore, $Az = Bz = Sz = Tz$.

From (3.2), since $\{A,B\}$ is weakly compatible with respect to B and $Az = Bz$, we obtain $ABz = BAz$ by Lemma 1.3. Similarly, $SBz = BSz$ since $Sz = Bz$ and $\{S,B\}$ is weakly compatible with respect to B. Similarly, $TBz = BTz$ since $Tz = Bz$ and $\{T,B\}$ is weakly compatible with respect to B. Hence, using (3.1), we have

$$\begin{aligned}\|S^2z - Tz\| &\leq \Phi\big(\max\{\|ASz - Bz\|, \|ASz - SAz\|, \|Bz - Tz\|,\\ &\quad \|ASz - Tz\|, \|Bz - SAz\|\}\big)\\ &\leq \Phi\big(\max\{\|S^2z - Tz\|, 0, 0, \|S^2z - Tz\|, \|Tz - S^2z\|\}\big),\end{aligned}$$

which implies that

$$S^2z = Tz = Az = Bz = Sz = SAz = SBz = STz.$$

So $Sz = u$ is common fixed point of A, B, S and T. Let v be a second common fixed point of A, B, S and T. By (c_2), we have

$$\begin{aligned}\|u - v\| &= \|Su - Tv\|\\ &\leq \Phi\big(\max\{\|Au - Bv\|, \|Au - Su\|, \|Bv - Tv\|, \|Au - Tv\|, \|Bv - Su\|\}\big)\\ &\leq \Phi\big(\max\{\|u - v\|, 0, 0, \|u - v\|, \|v - u\|\}\big),\end{aligned}$$

which implies $u = v$.

Now we prove that, If A and B are continuous at z, then S and T are continuous at z.

Let $\{y_n\}$ be an arbitrary sequence in C converging to z.

Form (3.1), we have

$$\begin{aligned}\|Sy_n - Sz\| &= \|Sy_n - Tz\| \leq \Phi\big(\max\{\|Ay_n - Bz\|, \|Ay_n - Sy_n\|, \|Bz - Tz\|,\\ &\quad \|Ay_n - Tz\|, \|Bz - Sy_n\|\}\big)\\ &\leq \Phi\big(\max\{\|Ay_n - Az\|, \|Ay_n - Az\|, 0, \|Ay_n - Az\|, \|Ay_n - Az\|\}\big)\\ &\leq \|Ay_n - Az\|.\end{aligned}$$

Letting $n \to \infty$ we obtain, as A is continuous, $\lim_{n\to\infty} Sy_n = Sz$. Thus, S is continuous at z. Similarly, we can prove that when B is continuous at z then T is continuous at z.

Remark 3.2. It is still an open problem to extend the results of this paper using the sense of doubly sequence iterations. For some studies on various doubly sequence iterations, we refer to [3, 14, 15, 16, 17, 18].

Remark 3.3. It is still an open problem to study the obtained results of this paper in cone metric spaces, for more information on cone metric spaces, we refer to [35, 37, 41, 43] and others.

Remark 3.4. How one can investigate fixed points for some spaces defined by integral norms? For details on such spaces, we refer to [1], [4-18], [19-28], [38, 39, 40] and others.

Conflict of Interests

The authors declare that there is no conflict of interests.

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