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FIXED POINTS IN TOPOLOGICAL VECTOR SPACE (TVS)VALUED CONE METRIC SPACES

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Abstract. In this paper, we use the notion of topological vector space valued cone metric space and generalize a common fixed point theorem of a pair of multivalued mappings satisfying a generalized contractive type condition. Our results extend some well known recent results in the literature.

Keywords: Topological vector space; Cone metric space; Non-normal cones; Fixed point; Common fixed point.

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1. Introduction-preliminaries

Many authors [5, 6, 7, 8, 10, 22, 15, 27] studied fixed points results of mappings satisfying contractive type condition in Banach space valued cone metric spaces. In recent papers [9] the authors obtained common fixed points of a pair of mapping in a class of topological vector space -valued (tvs-valued) cone metric spaces.The class of tvs-cone metric spaces is bigger than the class of cone metric spaces studied in [8, 10, 22, 15, 27]. Recently Azam *et al.*[9] obtain

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common fixed points of mappings satisfying a generalized contractive type condition in tvs-cone metric spaces.In this paper we continue these investigations to generalize the results in [8, 15].

Let (E,τ) be always a topological vector space (tvs) and P a subset of E. Then, P is called a cone whenever

(i) P is closed, non-empty and $P \neq \{0\}$,

(ii) $ax+by \in P$ for all $x,y \in P$ and non-negative real numbers a,b,

(iii) $P \cap (-P) = \{0\}$.

For a given cone $P \subseteq E$, we can define a partial ordering $\leq$ with respect to P by $x \leq y$ if and only if $y-x \in P$. $x < y$ will stand for $x \leq y$ and $x \neq y$, while $x \ll y$ will stand for $y-x \in intP$, where $intP$ denotes the interior of P.

Definition 1.1. Let X be a non-empty set. Suppose the mapping $d : X \times X \to E$ satisfies

(d_1) $0 \leq d(x,y)$ for all $x,y \in X$ and $d(x,y) = 0$ if and only if $x = y$,

(d_2) $d(x,y) = d(y,x)$ for all $x,y \in X$,

(d_3) $d(x,y) \leq d(x,z) + d(z,y)$ for all $x,y,z \in X$.

Then d is called a topological vector space-valued cone metric on X and (X,d) is called a topological vector space-valued cone metric space.

If E is a real Banach space, then (X,d) is called (Banach space valued) cone metric space [8, 10, 22, 15, 27].

Definition 1.2. Let (X,d) be a tvs-cone metric space, $x \in X$ and $\{x_n\}_{n\geq 1}$ a sequence in X. Then

(i) $\{x_n\}_{n\geq 1}$ converges to x whenever for every $c \in E$ with $0 \ll c$ there is a natural number N such that $d(x_n,x) \ll c$ for all $n \geq N$. We denote this by $\lim_{n\to\infty} x_n = x$ or $x_n \to x$.

(ii) $\{x_n\}_{n\geq 1}$ is a Cauchy sequence whenever for every $c \in E$ with $0 \ll c$ there is a natural number N such that $d(x_n,x_m) \ll c$ for all $n,m \geq N$.

(iii) (X,d) is a complete cone metric space if every Cauchy sequence is convergent.

Lemma 1.3. [11] *Let (X,d) be a tvs-cone metric space, P be a cone. Let $\{x_n\}$ be a sequence in X and $\{a_n\}$ be a sequence in P converging to 0. If $d(x_n,x_m) \leq a_n$ for every $n \in \mathbb{N}$ with $m > n$, then $\{x_n\}$ is a Cauchy sequence.*

The fixed point theorems and other results, in the case of cone metric spaces with non-normal solid cones, cannot be proved by reducing to metric spaces. Further, the vector valued function cone metric is not continuous in the general case.

2. Main results

In the sequel, let $\mathbb{E}$ be a locally convex Hausdorff tvs with its zero vector θ, P be a proper, closed and convex pointed cone in $\mathbb{E}$ with $int\,P \neq \emptyset$ and $\preccurlyeq$ denotes the induced partial ordering with respect to P.

According to [9], let (X,d) be a tvs-valued cone metric space with a solid cone P and $CB(X)$ be a collection of nonempty closed and bounded subsets of X. Let $T : X \to CB(X)$ be a multi-valued mapping. For any $x \in X$, $A \in CB(X)$, define a set $W_x(A)$ as follows:

$$W_x(A) = \{d(x,a) : a \in A\}.$$

Thus, for any $x,y \in X$, we have

$$W_x(Ty) = \{d(x,u) : u \in Ty\}.$$

Definition 2.1. [14] *Let (X,d) be a cone metric space with the solid cone P. A multi-valued mapping $S : X \to 2^{\mathbb{E}}$ is said to be bounded from below if, for any $x \in X$, there exists $z(x) \in \mathbb{E}$ such that $Sx - z(x) \subset P$.*

Definition 2.2. [14] *Let (X,d) be a cone metric space with the solid cone P. A cone P is said to be complete if, for any bounded from above and nonempty subset A of $\mathbb{E}$, $\sup A$ exists in $\mathbb{E}$. Equivalently, a cone P is complete if, for any bounded from below and nonempty subset A of $\mathbb{E}$, $\inf A$ exists in $\mathbb{E}$.*

Definition 2.3. [9] *Let (X,d) be a tvs-valued cone metric space with the solid cone P. A multi-valued mapping $T : X \to CB(X)$ is said to have the lower bound property (l.b. property) on X if, for any $x \in X$, the multi-valued mapping $S_x : X \to 2^{\mathbb{E}}$ defined by $S_x(y) = W_x(Ty)$ is bounded from below, that is, for any $x,y \in X$, there exists an element $\ell_x(Ty) \in \mathbb{E}$ such that $W_x(Ty) - \ell_x(Ty) \subset P$. $\ell_x(Ty)$ is called the lower bound of T associated with (x,y).*

Definition 2.4. [9] *Let (X,d) be a tvs-valued cone metric space with the solid cone P. A multi-valued mapping $T : X \to CB(X)$ is said to have the greatest lower bound property (for short, g.l.b. property) on X if the greatest lower bound of $W_x(Ty)$ exists in $\mathbb{E}$ for all $x, y \in X$. We denote $d(x,Ty)$ by the greatest lower bound of $W_x(Ty)$, that is,*

$$d(x,Ty) = \inf\{d(x,u) : u \in Ty\}.$$

According to [26], *we denote $s(p) = \{q \in \mathbb{E} : p \preccurlyeq q\}$ for all $q \in \mathbb{E}$ and*

$$s(a,B) = \bigcup_{b\in B} s(d(a,b)) = \bigcup_{b\in B} \{x \in \mathbb{E} : \ d(a,b) \preccurlyeq x\}$$

for all $a \in X$ and $B \in CB(X)$. For any $A, B \in CB(X)$, we denote

$$s(A,B) = \Big(\bigcap_{a\in A} s(a,B)\Big) \cap \Big(\bigcap_{b\in B} s(b,A)\Big).$$

Remark 2.5. [26] *Let (X,d) be a tvs-valued cone metric space. If $\mathbb{E} = R$ and $P = [0,+\infty)$, then (X,d) is a metric space. Moreover, for any $A, B \in CB(X)$, $H(A,B) = \inf s(A,B)$ is the Hausdorff distance induced by d.*

Theorem 2.6. *Let (X,d) be a complete tvs-valued cone metric space with the solid (normal or non-normal) cone P and let $S,T : X \longrightarrow CB(X)$ be multivalued mappings with g.l.b property such that*

$$A\,d(x,y) + B\,d(x,Sx) + Cd(y,Ty) + Dd(x,Ty) + Ed(y,Sx)) \in s(Sx,Ty). \tag{2.1}$$

for all $x, y \in X$, where A,B,C,D and E are non negative real numbers with $A+B+C+D+E < 1$. Then S and T have a common fixed point.

Proof. Let x_0 be an arbitrary point in X and $x_1 \in Sx_0$. From (2.1), we have

$$Ad(x_0,x_1) + B(x_0,Sx_0) + Cd(x_1,Tx_1) + Dd(x_0,Tx_1) + Ed(x_1,Sx_0) \in s(Sx_0,Tx_1).$$

This implies that

$$Ad(x_0,x_1) + B(x_0,Sx_0) + Cd(x_1,Tx_1) + Dd(x_0,Tx_1) + Ed(x_1,Sx_0) \in \left(\bigcap_{x\in Sx_0} s(x,Tx_1)\right)$$

and

$$Ad(x_0,x_1) + B(x_0,Sx_0) + Cd(x_1,Tx_1) + Dd(x_0,Tx_1) + Ed(x_1,Sx_0) \in \ s(x,Tx_1), \forall x \in Sx_0.$$

Since $x_1 \in Sx_0$, we have

$$Ad\,(x_0,x_1)+B(x_0,Sx_0)+Cd(x_1,Tx_1)+Dd(x_0,Tx_1)+Ed(x_1,Sx_0) \in\ s\,(x_1,Tx_1)$$

and

$$Ad\,(x_0,x_1)+B(x_0,Sx_0)+Cd(x_1,Tx_1)+Dd(x_0,Tx_1)+Ed(x_1,Sx_0) \in\ s\,(x_1,Tx_1)$$

$$= \bigcup_{x\in Tx_1} s\,(d\,(x_1,x))\,.$$

So there exists some $x_2 \in Tx_1$ such that

$$Ad\,(x_0,x_1)+B(x_0,Sx_0)+Cd(x_1,Tx_1)+Dd(x_0,Tx_1)+Ed(x_1,Sx_0) \in s\,(d(x_1,x_2)).$$

That is

$$d(x_1,x_2) \preceq Ad\,(x_0,x_1)+B(x_0,Sx_0)+Cd(x_1,Tx_1)+Dd(x_0,Tx_1)+Ed(x_1,Sx_0).$$

By using the greatest lower bound property (g.l.b property) of S and T, we get

$$d\,(x_1,x_2)\ \preceq Ad\,(x_0,x_1)+B(x_0,x_1)+Cd(x_1,x_2)+Dd(x_0,x_2)+Ed(x_1,x_1),$$

which implies that

$$d\,(x_1,x_2)\ \preceq (A+B+D)d\,(x_0,x_1)+(C+D)d(x_1,x_2).$$

This further implies that

$$d\,(x_1,x_2)\ \preceq \frac{A+B+D}{1-C-D}d\,(x_0,x_1)\,.$$

Similarly, from (2.1), we get

$$Ad\,(x_1,x_2)+B(x_2,Sx_2)+Cd(x_1,Tx_1)+Dd(x_2,Tx_1)+Ed(x_1,Sx_2) \in s\,(Tx_1,Sx_2)\,.$$

This implies that

$$Ad\,(x_1,x_2)+B(x_2,Sx_2)+Cd(x_1,Tx_1)+Dd(x_2,Tx_1)+Ed(x_1,Sx_2) \in \left(\bigcap_{x\in Tx_1} s\,(x,Sx_2)\right)$$

and

$$Ad\,(x_1,x_2)+B(x_2,Sx_2)+Cd(x_1,Tx_1)+Dd(x_2,Tx_1)+Ed(x_1,Sx_2) \in\ s\,(x,Sx_2)\,,\forall x\in Tx_1.$$

Since $x_2 \in Tx_1$, we have

$$Ad\,(x_1,x_2)+B(x_2,Sx_2)+Cd(x_1,Tx_1)+Dd(x_2,Tx_1)+Ed(x_1,Sx_2) \in\ s\,(x_2,Sx_2)$$

and

$$Ad\,(x_1,x_2)+B(x_2,Sx_2)+Cd(x_1,Tx_1)+Dd(x_2,Tx_1)+Ed(x_1,Sx_2)\in\ s\,(x_2,Sx_2)$$

$$=\bigcup_{x\in Sx_2} s\,(d\,(x_2,x))\,.$$

So there exists some $x_3\in Sx_2$ such that

$$Ad\,(x_1,x_2)+B(x_2,Sx_2)+Cd(x_1,Tx_1)+Dd(x_2,Tx_1)+Ed(x_1,Sx_2)\in s\,(d(x_2,x_3)).$$

That is,

$$d(x_2,x_3)\preceq Ad\,(x_1,x_2)+B(x_2,Sx_2)+Cd(x_1,Tx_1)+Dd(x_2,Tx_1)+Ed(x_1,Sx_2).$$

By using the greatest lower bound property (g.l.b property) of S and T, we get

$$d(x_2,x_3)\preceq Ad\,(x_1,x_2)+B(x_2,x_3)+Cd(x_1,x_2)+Dd(x_2,x_2)+Ed(x_1,x_3),$$

which implies that

$$d(x_2,x_3)\preceq(A+C+E)d\,(x_1,x_2)+(B+E)(x_2,x_3).$$

This further implies

$$d(x_2,x_3)\preceq\frac{A+C+E}{1-B-E}d\,(x_1,x_2)\,.$$

Let $\delta=\max\{\frac{A+B+D}{1-C-D},\frac{A+C+E}{1-B-E}\}$. Then $\delta<1$. Thus inductively, one can easily construct a sequence $\{x_n\}$ in X such that

$$x_{2n+1}\in Sx_{2n},\quad x_{2n+2}\in Tx_{2n+1}$$

and

$$d(x_{2n},x_{2n+1})\preccurlyeq\delta d(x_{2n-1},x_{2n}).$$

for each $n\geq 0$. We assume that $x_n\neq x_{n+1}$ for each $n\geq 0$. Otherwise, there exists n such that $x_{2n}=x_{2n+1}$. Then $x_{2n}\in Sx_{2n}$ and x_{2n} is a fixed point of S and hence a fixed point of T. Similarly, if $x_{2n+1}=x_{2n+2}$ for some n, then x_{2n+1} is a common fixed point of T and S. Similarly, one can show that

$$d(x_{2n+1},x_{2n+2})\preccurlyeq\delta d(x_{2n},x_{2n+1}).$$

Thus we have

$$d(x_n,x_{n+1}) \preccurlyeq \delta d(x_{n-1},x_n) \preccurlyeq \delta^2 d(x_{n-2},x_{n-1}) \preccurlyeq \cdots \preccurlyeq \delta^n d(x_0,x_1)$$

for each $n \geq 0$. Now, for any $m > n$, consider

$$\begin{aligned} d(x_m,\ x_n) &\preccurlyeq d(x_n,\ x_{n+1}) + d(x_{n+1},\ x_{n+2}) + \cdots + d(x_{m-1},\ x_m) \\ &\preccurlyeq \left[\delta^n + \delta^{n+1} + \cdots + \delta^{m-1}\right] d(x_0,x_1) \\ &\preccurlyeq \left[\frac{\delta^n}{1-\delta}\right] d(x_0,x_1). \end{aligned}$$

Let $\theta \ll c$ be given and choose a symmetric neighborhood V of θ such that $c+V \subseteq \text{int}P$. Also, choose a natural number k_1 such that $\left[\frac{\delta^n}{1-\delta}\right] d(x_0,x_1) \in V$ for all $n \geq k_1$. Then $\frac{\delta^n}{1-\delta} d(x_1,x_0) \ll c$ for all $n \geq k_1$. Thus we have

$$d(x_m,\ x_n) \preccurlyeq \left[\frac{\delta^n}{1-\delta}\right] d(x_0,x_1) \ll c$$

for all $m > n$. Therefore, $\{x_n\}$ is a Cauchy sequence. Since X is complete, there exists $\nu \in X$ such that $x_n \to \nu$. Choose a natural number k_2 such that

$$\frac{1+E}{1-C} d(\nu,x_{2n+1}) \ll \frac{c}{3}, \quad \frac{A}{1-C} d(x_{2n},v) \ll \frac{c}{3} \text{ and } \frac{B}{1-C} d(x_{2n},x_{2n}) \ll \frac{c}{3} \tag{2.2}$$

for all $n \geq k_2$. Then, for all $n \geq k_2$, we have

$$Ad(x_{2n},v) + Bd(x_{2n},Sx_{2n}) + Cd(v,Tv) + Dd(x_{2n},T\nu) + Ed(\nu,Sx_{2n}) \in s\left(Sx_{2n},T\nu\right).$$

This implies that

$$Ad(x_{2n},v) + Bd(x_{2n},Sx_{2n}) + Cd(v,Tv) + Dd(x_{2n},T\nu) + Ed(\nu,Sx_{2n}) \in \left(\bigcap_{x \in Sx_{2n}} s\left(x,Tv\right)\right)$$

and we have

$$Ad(x_{2n},v) + Bd(x_{2n},Sx_{2n}) + Cd(v,Tv) + Dd(x_{2n},T\nu) + Ed(\nu,Sx_{2n}) \in s\left(x,Tv\right) \ for\ all\ x \in Sx_{2n}.$$

Since $x_{2n+1} \in Sx_{2n}$, we have

$$Ad(x_{2n},v) + Bd(x_{2n},Sx_{2n}) + Cd(v,Tv) + Dd(x_{2n},T\nu) + Ed(\nu,Sx_{2n}) \in s\left(x_{2n+1},Tv\right).$$

It follows that

$$Ad(x_{2n},v)+Bd(x_{2n},Sx_{2n})+Cd(v,Tv)+Dd(x_{2n},T\nu)+Ed(\nu,Sx_{2n})\in s\left(x_{2n+1},Tv\right)$$
$$=\bigcup_{u'\in Tu}s\left(d\left(x_{2n+1},u'\right)\right).$$

There exists some $\nu_n\in Tv$ such that

$$\begin{aligned}&Ad(x_{2n},v)+Bd(x_{2n},Sx_{2n})+Cd(v,Tv)+Dd(x_{2n},T\nu)+Ed(\nu,Sx_{2n})\\&\in s\left(x_{2n+1},Tv\right)\\&\in s\left(d(x_{2n+1},\nu_n)\right),\end{aligned}$$

that is

$$d(x_{2n+1},\nu_n)\preceq Ad(x_{2n},v)+Bd(x_{2n},Sx_{2n})+Cd(v,Tv)+Dd(x_{2n},T\nu)+Ed(\nu,Sx_{2n}).$$

By using the greatest lower bound property (g.l.b property) of S and T, we have

$$d(x_{2n+1},\nu_n)\preceq Ad(x_{2n},v)+Bd(x_{2n},x_{2n})+Cd(v,\nu_n)+Dd(x_{2n},\nu_n)+Ed(\nu,x_{2n+1}).$$

Now by using the triangular inequality, we get

$$d\left(x_{2n+1},\nu_n\right)\preceq Ad(x_{2n},v)+Bd(x_{2n},x_{2n+1})+Cd(v,x_{2n+1})+Dd(x_{2n},\nu_n)+Ed(\nu,x_{2n+1})$$

and it follows that

$$d\left(x_{2n+1},\nu_n\right)\preceq\frac{A}{1-C}d(x_{2n},v)+\frac{B}{1-C}d(x_{2n},x_{2n}))+\frac{C+E}{1-C}d(\nu,x_{2n+1}).$$

By using again triangular inequality, we get

$$\begin{aligned}d(\nu,\nu_n)&\preceq d(\nu,x_{2n+1})+d(x_{2n+1},\nu_n)\\&\preceq d(\nu,x_{2n+1})+\frac{A}{1-C}d(x_{2n},v)+\frac{B}{1-C}d(x_{2n},x_{2n}))+\frac{C+E}{1-C}d(\nu,x_{2n+1})\\&\preceq\frac{1+E}{1-C}d(\nu,x_{2n+1})+\frac{A}{1-C}d(x_{2n},v)+\frac{B}{1-C}d(x_{2n},x_{2n})\\&\ll\frac{c}{3}+\frac{c}{3}+\frac{c}{3}=c.\end{aligned}$$

Thus, we get

$$d(v,v_n)\ll\frac{c}{m}$$

for all $m \geq 1$ and so $\frac{c}{m} - d(v, v_n) \in P$ for all $m \geq 1$. Since $\frac{c}{m} \to \theta$ as $m \to \infty$ and P is closed, it follows that $-d(v, v_n) \in P$. But $d(v, v_n) \in P$. Therefore, $d(v, v_n) = \theta$ and $v_n \to v \in Tv$, since Tv is closed. This implies that v is a common point of S and T. This completes the proof.

Corollary 2.7. [9] *Let (X,d) be a complete tvs-valued cone metric space with the solid (normal or non-normal) cone P and let $S,T : X \longrightarrow CB(X)$ be multivalued mappings with g.l.b property such that*

$$B\, d(x,Sx) + Cd(y,Ty) \in s\,(Sx,Ty)$$

for all $x,y \in X$, where B,C are non negative real numbers with$B + C < 1$. Then S and T have a common fixed point.

Corollary 2.8. [9] *Let (X,d) be a complete tvs-valued cone metric space with the solid (normal or non-normal) cone P and let $S,T : X \longrightarrow CB(X)$ be multivalued mappings with g.l.b property such that*

$$Dd(x,Ty) + Ed(y,Sx)) \in s\,(Sx,Ty)$$

for all $x,y \in X$, where D,E are non negative real numbers with $D + E < 1$. Then S and T have common fixed point.

Hence, we have the following theorem which improves/generalizes the results in [8, 11].

Theorem 2.9. *Let (X,d) be a complete topological vector space-valued cone metric space, P be a cone. If mappings $S,T : X \to X$ satisfies:*

$$d(Sx,Ty) \leq A\, d(x,y) + B\, d(x,Sx) + Cd(y,Ty) + D\, d(x,Ty) + E\, d(y,Sx)$$

for all $x,y \in X$, where A,B,C,D,E are non negative real numbers with $A+B+C+D+E < 1$, $B = C$ or $D = E$. Then S and T have a unique common fixed point.

By substituting $D = E = 0$ in Theorem 2.9 , we obtain the following result.

Corollary 2.10. *Let (X,d) be a complete topological vector space-valued cone metric space, P be a cone and m,n be positive integers. If mappings $S,T : X \to X$ satisfies:*

$$d(Sx,Ty) \leq A\, d(x,y) + B\, d(x,Sx) + Cd(y,Ty)$$

for all $x,y \in X$, where A,B,C are non negative real numbers with $A+B+C<1$. Then S and T have a unique common fixed point.

By substituting $B=C=0$ in Theorem 2.9 , we obtain the following result.

Corollary 2.11. *Let (X,d) be a complete topological vector space-valued cone metric space, P be a cone and m,n be positive integers. If mappings $S,T:X\to X$ satisfies:*

$$d(Sx,Ty) \leq A\ d(x,y)+D\ d(x,Ty)+E\ d(y,Sx)$$

for all $x,y \in X$, where A,D,E are non negative real numbers with $A+D+E<1$. Then S and T have a unique common fixed point.

Corollary 2.12. [8] *Let (X,d) be a complete Banach space-valued cone metric space, P be a cone. If a mapping $S,T:X\to X$ satisfies:*

$$d(Sx,Ty) \leq pd(x,y)+\ q\,[d(x,Sx)+d(y,Ty)]+\ r\,[d(x,Ty)+E\ d(y,Sx)] \tag{2.5}$$

for all $x,y \in X$, where p,q,r are non negative real numbers with $p+2q+2r<1$. Then S and T have a unique common fixed point.

Conflict of Interests

The author declares that there is no conflict of interests.

References

[1] M. Abbas and B.E. Rhoades, Fixed and periodic point results in cone metric spaces, Appl. Math. Lett. 22 (2009), 511–515.

[2] M. Abbas, Y.J. Cho and T. Nazir, Common fixed point theorems for four mappings in tvs-valued cone metric spaces, J. Math. Inequal. 5 (2011), 287–299.

[3] M. Abbas and G. Jungck, Common fixed point results for non-commuting mappings without continuity in cone metric spaces, J. Math. Anal. Appl. 341 (2008), 416–420.

[4] R.P. Agarwal, M. Meehan and D. O'Regan, Fixed Point Theory and Applications, Cambridge University Press, (2001).

[5] Muhammad Arshad, Fixed points of a pair of dominating mappings on a closed ball in ordered partial metric spaces, J. Adv. Math. Stud. 7 (2014), 123-134.

[6] M. Arshad, Some fixed point results for contractive multi-valued mapping in partial metric spaces, J. Adv. Res. Pure Math. 6 (2014), 47-57.

[7] M. Arshad , Fahimuddin, A. Shoaib and A. Hussain, Fixed point results for α-ψ-locally graphic contraction in dislocated qusai metric spaces, Math Sci. doi;10.1007/s40096-014-0132.

[8] M. Arshad, A. Azam and P. Vetro, Some common fixed point results in cone metric spaces, Fixed Point Theory Appl. 2009 (2009), Article ID 493965.

[9] A. Azam, N. Mehmood, Multivalued Fixed Point Theorems in tvs-Cone Metric Spaces, Fixed Point Theory Appl. 2013 (2013), Article ID 184.

[10] A. Azam, M. Arshad and I. Beg, Common fixed points of two maps in cone metric spaces, Rend. Circ. Mat. Palermo 57 (2008), 433–441.

[11] A. Azam, I. Beg and M. Arshad, Fixed point in topological vector space-valued cone metric spaces, Fixed Point Theory Appl. 2010 (2010), Article ID 604084.

[12] I. Beg, A. Azam and M. Arshad, Common fixed points for maps on topological vector space valued cone metric spaces, Interant. J. Math. Math. Sci. 2009 (2009), Article ID 604084.

[13] S.H. Cho and J.S. Bae, Fixed point theorems for multivalued maps in cone metric spaces, Fixed Point Theory Appl. (2011).

[14] S.H. Cho and J.S. Bae, Fixed points and variational principle with applications to equiliberium problems, J. Korean Math. Soc. 50 (2013), 95–109.

[15] L. Huang and X. Zhang, Cone metric spaces and fixed point theorems of contractive mappings, J. Math. Anal. Appl. 332 (2007) 1468–1476.

[16] D. Ilić and V. Pavlović, Common fixed points for maps on cone metric space, J. Math. Anal. Appl. 341 (2008), 876–882.

[17] Z. Kadelburg and S. Janković and S. Radenović, A note on the equivalence of some metric and cone metric fixed point results, Appl. Math. Lett. 24 (2011), 370–374.

[18] S. Janković, Z. Kadelburg and S. Radenović, On cone metric spaces, A survey, Nonlinear Anal. 74 (2011), 2591–260.

[19] M. Khani and M. Pourmahdian, On the metrizability of cone metric spaces, Topology Appl. 158 (2011), 190–193.

[20] A. Latif and F.Y. Shaddad, Fixed point results for multivalued maps in cone metric spaces, Fixed Point Theory Appl. 2010 (2010), Article ID 941371.

[21] S. Radenovic and B.E. Rhoades, Fixed point theorem for two non-self mappings in cone metric spaces, Comp. Math. Appl. 57 (2009), 1701–1707.

[22] S. Rezapour and R. Hamlbarani, Some notes on paper ”Cone metric spaces and fixed point theorems of contractive mappings”, J. Math. Anal. Appl. 345(2008), 719–724.

[23] S. Rezapour and R.H. Haghi, Fixed points of multifunctions on cone metric spaces, Numer. Funct. Anal. Optim. 30 (2009), 1–8.

[24] S. Rezapour, H. Khandani and S.M. Vaezpour, Efficacy of cones on topological vector spaces and application to common fixed points of multifunctions, Rend. Circ. Mat. Palermo 59 (2010), 185–197.

[25] H. H. Schaefer, Toplogical vector spaces, Springer-Verlag New York Heidelberg Berlin, 1970.

[26] W. Shatanawi, V. Ćojbašić, S. Radenović and A. Al-Rawashdeh, Mizoguchi-Takahashi-type theorems in tvs-cone metric spaces, Fixed Point Theory Appl. 2012 (2012), Article ID 106.

[27] P. Vetro, Common fixed points in cone metric spaces, Rend. Circ. Mat. Palermo 56 (2007), 464–468.