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SOME FIXED POINT THEOREMS USING EXPANSIVE TYPE MAPPINGS IN 2-BANACH SPACE

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Abstract. In present paper, we define expansive mappings in 2-Banach space and prove some common unique fixed point theorems.

Keywords: 2-normed space; 2-Banach space; Expansive mapping; Fixed point.

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1. Introduction

The research about fixed points of expansive mapping was initiated by Machuca [6]. Later Jungck discussed fixed points for other forms of expansive mapping; see [5]. In 1982, Wang *et al.* [12] presented some interesting work on expansive mappings in metric spaces which correspond to some contractive mapping in [10]. In order to generalize the results on fixed points of nonlinear operators, Zhang [14] studied fixed point problems for expansive mapping. As applications, he also investigated the existence of solutions of equations for locally condensing mapping and locally accretive mappings. On the other hand, Gahler ([2],[3]) investigated the

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idea of 2-metric and 2-Banach spaces and proved the same results. Subsequently, several authors including Iseki [4], Rhoades [8], White [13], Panja and Baisnab [7] and Saha *et al.* [11] studied various aspects of fixed points and proved fixed point theorems in 2-metric spaces and 2-Banach spaces. Recently, the study about fixed points for expansive mapping is deeply explored and has extended too many others directions. Motivated and inspired by the above work, we investigate fixed points for expansive mapping in 2-Banach spaces.

2. Preliminaries

Definition 2.1. Let X be a real linear space and $\| .,. \|$ be a non-negative real valued function defined on $X \times X$ satisfying the following conditions:

i) $\|x,y\| = 0$ if and only if x and y are linearly dependent in X,

ii) $\|x,y\| = \|y,x\|, \quad \forall x,y \in X$

iii) $\|x,ay\| = | a | \, \|x,y\|, \forall x,y \in X$

iv) $\|x,y+z\| = \|x,y\| + \|x,z\|, \forall x,y,z \in X$.

Then $\| .,. \|$ is called a 2-norm and the pair $(X, \| .,. \|)$ is called a linear 2-normed space.

Some of the basic properties of 2-norms are that they are non negative satisfying $\|x,y+ax\| = \|x,y\|$, for all $x,y \in X$ and all real numbers a.

Definition 2.2. A sequence $\{x_n\}$ in a linear 2-normed space $(X, \| .,. \|)$ is called Cauchy sequence if $\lim_{m,n\to\infty} \| x_m - x_n, y \| = 0 \ \forall y \in X$.

Definition 2.3. A sequence $\{x_n\}$ in a linear 2-normed space $(X, \| .,. \|)$ is said to be convergent if there is a point x in X such that $\lim_{n\to\infty} \| x_n - x, y \| = 0 \ \forall y \in X$. If $\{x_n\}$ converges to x, we write $\{x_n\} \to x$ as $n \to \infty$.

Definition 2.4. A linear 2-normed space X is said to be complete if every Cauchy sequence is convergent to an element of X. We then call X to be a 2-Banach space.

Definition 2.5. Let X be a 2-Banach space and T be a self-mapping of X. T is said to be continuous at x if for every sequence $\{x_n\}$ in X, $\{x_n\} \to x$ as $n \to \infty$ implies $\{T(x_n)\} \to T(x)$ as $n \to \infty$.

Example 2.6. *Let X is R^3 and consider the following 2-norm on X as*

$$\| \mathbf{x}, \mathbf{y} \| = \left| det \begin{pmatrix} i & j & k \\ x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{pmatrix} \right|,$$

where $x = (x_1, x_2, x_3)$ and $y = (y_1, y_2, y_3)$. Then ($X, \| .,. \|$) is a 2-Banach space.

Example 2.7. *Let P_n denotes the set of all real polynomials of degree $\leq n$, on the interval [0,1]. By considering usual addition and scalar multiplication, P_n is a linear vector space over the reals. Let $\{x_o, x_1, ..., x_{2n}\}$ be distinct fixed points in [0,1] and define the following 2-norm on P_n:$\| f, g \| = \sum_{k=0}^{2n} | f(x_k)\, g(x_k) |$, whenever f and g are linearly independent and $\| f, g \| = 0$, if f, g are linearly dependent. Then ($P_n, \| .,. \|$) is a 2-Banach space.*

Example 2.8. *Let X is Q^3, the field of rational number and consider the following 2-norm on X as:*

$$\| \mathbf{x}, \mathbf{y} \| = \left| det \begin{pmatrix} i & j & k \\ x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{pmatrix} \right|,$$

where $x = (x_1, x_2, x_3)$ and $y = (y_1, y_2, y_3)$. Then ($X, \| .,. \|$) is not a 2-Banach space but a 2-normed space.

Definition 2.9. Let $(X, \| .,. \|)$ be a 2-Banach space with 2-norm $\| .,. \|$. A mapping T of X into itself is said to be expansive if there exists a constant $h > 1$ such that $\| Tx - Ty, a \| \geq h \| x - y, a \|$, $\forall x, y \in X$.

Example 2.10. *Let $X = R^2$ and consider the following 2-norm on X as $\| x, y \| = | x_1 y_2 - x_2 y_1 |$, where $x = (x_1, x_2)$ and $y = (y_1, y_2)$. Then $(X, \| .,. \|)$ is a 2-Banach space. Define a self map T on X as follows $Tx = \beta x$, where $\beta > 1$ for all $x \in X$, clearly T is an expansive mapping.*

3. Main results

Theorem 3.1. *Let $(X, \| .,. \|)$ be a 2-Banach space and T be a surjective mapping of X into itself such that for every $x, y \in X$,*

$$\| Tx - Ty, a \| \geq \alpha_1 \frac{\| x - Tx, a \| \, \| y - Ty, a \|}{\| x - y, a \|} + \alpha_2 \frac{[1 + \| x - Tx, a \|] \, \| x - Ty, a \|}{\| x - y, a \|}$$
$$+ \alpha_3 \frac{\| y - Ty, a \| \, \| x - Ty, a \|}{\| x - y, a \|} + \beta_1 \| x - Tx, a \| + \beta_2 \| y - Ty, a \|$$
$$+ \beta_3 \| x - Ty, a \| + \beta_4 \| x - y, a \|, \tag{3.1.1}$$

where $\alpha_1, \alpha_2, \alpha_3, \beta_1, \beta_2, \beta_3, \beta_4 \geq 0$, $\alpha_1 + \beta_1 + \beta_2 + \beta_4 > 1$ and $\beta_2 + \beta_4 > 0$. Then T has a fixed point.

Proof. Let x_o be an arbitrary point in X. There is x_1 in X such that $T(x_1) = x_o$. In this way we define a sequence $\{x_n\}$ as follows.

$$x_n = Tx_{n+1} \; for \; n = 0, 1, 2, ... \tag{3.1.2}$$

If $x_n = x_{n+1}$ for some n then we see that x_n is a fixed point of T, therefore we suppose that no two consecutive terms of sequence $\{x_n\}$ are equal. Now, we consider

$$\| x_n - x_{n+1}, a \| = \| Tx_{n+1} - Tx_{n+2}, a \|$$
$$\geq \alpha_1 \frac{\| x_{n+1} - Tx_{n+1}, a \| \, \| x_{n+2} - Tx_{n+2}, a \|}{\| x_{n+1} - x_{n+2}, a \|}$$
$$+ \alpha_2 \frac{[1 + \| x_{n+1} - Tx_{n+1}, a \|] \, \| x_{n+1} - Tx_{n+2}, a \|}{\| x_{n+1} - x_{n+2}, a \|}$$
$$+ \alpha_3 \frac{\| x_{n+2} - Tx_{n+2}, a \| \, \| x_{n+1} - Tx_{n+2}, a \|}{\| x_{n+1} - x_{n+2}, a \|}$$
$$+ \beta_1 \| x_{n+1} - Tx_{n+1}, a \| + \beta_2 \| x_{n+2} - Tx_{n+2}, a \|$$
$$+ \beta_3 \| x_{n+1} - Tx_{n+2}, a \| + \beta_4 \| x_{n+1} - x_{n+2}, a \|$$
$$= \alpha_1 \frac{\| x_{n+1} - x_n, a \| \, \| x_{n+2} - x_{n+1}, a \|}{\| x_{n+1} - x_{n+2}, a \|} + \alpha_2 \frac{[1 + \| x_{n+1} - x_n, a \|] \, \| x_{n+1} - x_{n+1}, a \|}{\| x_{n+1} - x_{n+2}, a \|}$$
$$+ \alpha_3 \frac{\| x_{n+2} - x_{n+1}, a \| \, \| x_{n+1} - x_{n+1}, a \|}{\| x_{n+1} - x_{n+2}, a \|} + \beta_1 \| x_{n+1} - x_n, a \| + \beta_2 \| x_{n+2} - x_{n+1}, a \|$$
$$+ \beta_3 \| x_{n+1} - x_{n+1}, a \| + \beta_4 \| x_{n+1} - x_{n+2}, a \|$$

$$= (\alpha_1 + \beta_1) \| x_n - x_{n+1}, a \| + (\beta_2 + \beta_4) \| x_{n+1} - x_{n+2}, a \|$$
$$\Rightarrow \| x_{n+1} - x_{n+2}, a \| \leq \frac{[1 - (\alpha_1 + \beta_1)]}{\beta_2 + \beta_4} \| x_n - x_{n+1}, a \| .$$

So, in general, we have

$$\| x_n - x_{n+1}, a \| \leq k \ \| x_{n-1} - x_n, a \| \quad for \quad n = 1, 2, 3, ..,$$

where $k = \frac{1-(\alpha_1+\beta_1)}{\beta_2+\beta_4} < 1$ $[As\ \alpha_1 + \beta_1 + \beta_2 + \beta_4 > 1]$

$$\Rightarrow \| x_n - x_{n+1}, a \| \leq k^n \ \| x_0 - x_1, a \| \tag{3.1.3}$$

Now we shall prove that $\{x_n\}$ is a Cauchy sequence. For this,for every positive integer p, we have

$$\begin{aligned} \| x_n - x_{n+p}, a \| &\leq \| x_n - x_{n+1}, a \| + \| x_{n+1} - x_{n+2}, a \| + ... + \| x_{n+p-1} - x_{n+p}, a \| \\ &\leq (k^n + k^{n+1} + k^{n+2} + ... + k^{n+p-1}) \ \| x_0 - x_1, a \| \qquad [By\ (3.1.3)] \\ &= k^n (1 + k + k^2 + ... + k^{p-1}) \ \| x_0 - x_1, a \| \\ &< \frac{k^n}{(1-k)} \| x_0 - x_1, a \| . \end{aligned}$$

As $n \to \infty$, $\| x_n - x_{n+p}, a \| \to 0$, it follows that $\{x_n\}$ is a Cauchy sequence in X. As X is a complete Banach space, so there exist a point $x \in X$ such that $\{x_n\} \to x$.

Existence of fixed points Since T is a surjective self map and hence there exist point y in X such that

$$x = Ty. \tag{3.1.4}$$

Consider

$$\| x_n - x, a \| = \| Tx_{n+1} - Ty, a \|$$

$$\begin{aligned} &\geq \alpha_1 \frac{\| x_{n+1} - Tx_{n+1}, a \| \ \| y - Ty, a \|}{\| x_{n+1} - y \|} + \alpha_2 \frac{[1 + \| x_{n+1} - Tx_{n+1}, a \|] \ \| x_{n+1} - Ty, a \|}{\| x_{n+1} - y, a \|} \\ &+ \alpha_3 \frac{\| y - Ty, a \| \ \| x_{n+1} - Ty, a \|}{\| x_{n+1} - y, a \|} + \beta_1 \ \| x_{n+1} - Tx_{n+1}, a \| + \beta_2 \ \| y - Ty, a \| \\ &+ \beta_3 \ \| x_{n+1} - Ty, a \| \ + \beta_4 \ \| x_{n+1} - y, a \| \\ &= \alpha_1 \frac{\| x_{n+1} - x_n, a \| \ \| y - Ty, a \|}{\| x_{n+1} - y, a \|} + \alpha_2 \frac{[1 + \| x_{n+1} - x_n, a \|] \ \| x_{n+1} - Ty, a \|}{\| x_{n+1} - y, a \|} \\ &+ \alpha_3 \frac{\| y - Ty, a \| \ \| x_{n+1} - Ty, a \|}{\| x_{n+1} - y, a \|} + \beta_1 \ \| x_{n+1} - x_n, a \| + \beta_2 \ \| y - Ty, a \| \\ &+ \beta_3 \ \| x_{n+1} - Ty, a \| \ + \beta_4 \ \| x_{n+1} - y, a \| . \end{aligned}$$

Since $\{x_{n+1}\}$ is a subsequence of $\{x_n\}$, so $\{x_n\} \to x$, $\{x_{n+1}\} \to x$ when $n \to \infty$

$$0 \geq \alpha_1 \frac{\| x-x,a \| \, \| y-x,a \|}{\| x-y,a \|} + \alpha_2 \frac{[1+ \| x-x,a \|] \, \| x-x,a \|}{\| x-y,a \|}$$
$$+ \alpha_3 \frac{\| y-x,a \| \, \| x-x,a \|}{\| x-y,a \|} + \beta_1 \| x-x,a \| + \beta_2 \| y-x,a \|$$
$$+ \beta_3 \| x-x,a \| + \beta_4 \| x-y,a \| .$$

$0 \geq (\beta_2+\beta_4) \| x-y,a \| \Rightarrow \| x-y,a \| = 0$. Hence

$$x = y. \tag{3.1.5}$$

The fact (3.1.5) along with (3.1.4) shows that x is a common fixed point of T. This completes the proof of the theorem 3.1.

Theorem 3.2. *Let $(X, \| .,. \|)$ be a 2-Banach space and T be a mapping of X into itself such that for every $x, y, a \in X$,*

$$\| Tx-Ty,a \| \geq q \max \Big\{ \| x-y,a \|, \frac{\| x-Tx,a \| \, \| y-Ty,a \|}{\| x-y,a \|},$$
$$\frac{[1+ \| x-Tx,a \|] \, \| x-Ty,a \|}{\| x-y,a \|}, \frac{\| y-Ty,a \| \, \| x-Ty,a \|}{\| x-y,a \|} \Big\} \tag{3.1.6}$$

and $q > 1$. Then T has a fixed point.

Proof. Construct a sequence $\{x_n\}$ as in proof of Theorem 3.1. We claim that the inequality (3.1.6) for $x = x_{n+1}$ and $y = x_{n+2}$ implies that

$$\| Tx_{n+1} - Tx_{n+2}, a \| \geq q \max \Big\{ \| x_{n+1} - x_{n+2}, a \|, \frac{\| x_{n+1} - Tx_{n+1}, a \| \, \| x_{n+2} - Tx_{n+2}, a \|}{\| x_{n+1} - x_{n+2}, a \|},$$
$$\frac{[1+ \| x_{n+1} - Tx_{n+1}, a \|] \, \| x_{n+1} - Tx_{n+2}, a \|}{\| x_{n+1} - x_{n+2}, a \|},$$
$$\frac{\| x_{n+2} - Tx_{n+2}, a \| \, \| x_{n+1} - Tx_{n+2}, a \|}{\| x_{n+1} - x_{n+2}, a \|} \Big\}$$

$$\Rightarrow \| x_n - x_{n+1}, a \| \geq q \max \Big\{ \| x_{n+1} - x_{n+2}, a \|, \frac{\| x_{n+1} - x_n, a \| \| x_{n+2} - x_{n+1}, a \|}{\| x_{n+1} - x_{n+2}, a \|},$$
$$\frac{[1+ \| x_{n+1} - x_n, a \|] \, \| x_{n+1} - x_{n+1}, a \|}{\| x_{n+1} - x_{n+2}, a \|}, \frac{\| x_{n+2} - x_{n+1}, a \| \, \| x_{n+1} - x_{n+1}, a \|}{\| x_{n+1} - x_{n+2}, a \|} \Big\}$$
$$= q \max \{ \| x_{n+1} - x_{n+2}, a \|, \| x_{n+1} - x_n, a \| \}.$$

Case I:

$\| x_n - x_{n+1}, a \| \geq q \, \| x_n - x_{n+1}, a \| \Rightarrow 1 \geq q$, which is a contradiction.

Case II:

$\| x_n - x_{n+1}, a \| \geq q \;\| x_{n+1} - x_{n+2}, a \|$

$\Rightarrow \| x_{n+1} - x_{n+2}, a \| \leq \frac{1}{q} \;\| x_n - x_{n+1}, a \|$

$\Rightarrow \| x_{n+1} - x_{n+2}, a \| \leq k \;\| x_n - x_{n+1}, a \| \left[\text{where } k = \frac{1}{q} < 1 \quad (As\, q > 1)\right]$. So, in general, we have

$$\| x_n - x_{n+1}, a \| \leq k \;\| x_{n-1} - x_n, a \| \quad for \quad n = 1, 2, 3, ...$$

$$\Rightarrow \| x_n - x_{n+1}, a \| \leq k^n \;\| x_0 - x_1, a \| . \tag{3.1.7}$$

We find that $\{x_n\}$ is a Cauchy sequence using (3.1.7) as proved in theorem 3.1. As X is a complete Banach space, so there exist a point $x \in X$ such that $\{x_n\} \to x$.

Existence of fixed point Since T is a surjective self map and hence there exist point y in X such that

$$x = Ty \tag{3.1.8}$$

Consider $\| x_n - x, a \| = \| Tx_{n+1} - Ty, a \|$

$$\geq q \max\Big\{ \| x_{n+1} - y, a \|, \frac{\| x_{n+1} - Tx_{n+1}, a \| \, \| y - Ty, a \|}{\| x_{n+1} - y, a \|}, \frac{[1 + \| x_{n+1} - Tx_{n+1}, a \|] \;\| x_{n+1} - Ty, a \|}{\| x_{n+1} - y, a \|}, \frac{\| y - Ty, a \| \, \| x_{n+1} - Ty, a \|}{\| x_{n+1} - y, a \|}\Big\}$$

$$\Rightarrow \| x_n - x, a \| \geq q \max \Big\{ \| x_{n+1} - y, a \| , \frac{\| x_{n+1} - x_n, a \| \| y - x, a \|}{\| x_{n+1} - y, a \|}, \frac{[1 + \| x_{n+1} - x_n, a \|] \, \| x_{n+1} - x, a \|}{\| x_{n+1} - y, a \|}, \frac{\| y - x, a \| \, \| x_{n+1} - x, a \|}{\| x_{n+1} - y, a \|}\Big\}.$$

Since $\{x_{n+1}\}$ is a subsequence of $\{x_n\}$, so $\{x_n\} \to x$, $\{x_{n+1}\} \to x$ when $n \to \infty$.

$$\Rightarrow \| x - x, a \| \geq q \max \Big\{ \| x - y, a \| , \frac{\| x - x, a \| \| y - x, a \|}{\| x - y, a \|},$$

$$\frac{[1 + \| x - x, a \|] \, \| x - x, a \|}{\| x - y, a \|}, \frac{\| y - x, a \| \, \| x - x, a \|}{\| x - y, a \|}\Big\}$$

$0 \geq q \parallel x-y,a \parallel \Rightarrow \parallel x-y,a \parallel = 0 \; x = y$ (3.1.9) The fact (3.1.9) along with (3.1.8) shows that x is a common fixed point of T. This completes the proof of Theorem 3.2.

Theorem 3.3. *Let $(X, \parallel .,. \parallel)$ be a 2-Banach space and T be a mapping of X into itself such that for every $x,y,a \in X$,*

$$\parallel Tx-Ty,a \parallel^2 \geq a \parallel x-Tx,a \parallel \parallel x-y,a \parallel + b \parallel y-Ty,a \parallel \parallel x-y,a \parallel$$

$$+ c \parallel x-Tx,a \parallel \parallel y-Ty,a \parallel \tag{3.1.10}$$

for all distinct $x,y \in X$ where $a,c \geq 0$, $b > 0$ and $a+b+c > 1$. Then T has a fixed point.

Proof. Construct a sequence $\{x_n\}$ as in proof of Theorem 3.1. We claim that the inequality (3.1.10) for $x = x_{n+1}$ and $y = x_{n+2}$ implies that

$$\parallel Tx_{n+1}-Tx_{n+2},a \parallel^2 \geq a \parallel x_{n+1}-Tx_{n+1},a \parallel \parallel x_{n+1}-x_{n+2},a \parallel$$

$$+ b \parallel x_{n+2}-Tx_{n+2},a \parallel \parallel x_{n+1}-x_{n+2},a \parallel$$

$$+ c \parallel x_{n+1}-Tx_{n+1},a \parallel \parallel x_{n+2}-Tx_{n+2},a \parallel$$

$$= a \parallel x_{n+1}-x_n,a \parallel \parallel x_{n+1}-x_{n+2},a \parallel$$

$$+ b \parallel x_{n+2}-x_{n+1},a \parallel \parallel x_{n+1}-x_{n+2},a \parallel$$

$$+ c \parallel x_{n+1}-x_n,a \parallel \parallel x_{n+2}-x_{n+1},a \parallel$$

$$\parallel x_n-x_{n+1},a \parallel^2 \geq \parallel x_{n+1},x_{n+2},a \parallel \; (a+b+c) \; \min\{\parallel x_n,x_{n+1},a \parallel, \parallel x_{n+1},x_{n+2},a \parallel\}$$

Case I:

$$\parallel x_n-x_{n+1},a \parallel^2 \geq (a+b+c) \parallel x_{n+1}-x_{n+2},a \parallel \parallel x_n-x_{n+1},a \parallel$$

$$\Rightarrow \parallel x_{n+1}-x_{n+2},a \parallel \leq \frac{1}{a+b+c} \parallel x_n-x_{n+1},a \parallel$$

$$\Rightarrow \parallel x_{n+1}-x_{n+2},a \parallel \leq k_1 \parallel x_n-x_{n+1},a \parallel, \left[\text{where } k_1 = \frac{1}{a+b+c} < 1 \; (\text{As } a+b+c > 1)\right].$$

Case II:

$$\parallel x_n-x_{n+1},a \parallel^2 \geq (a+b+c) \parallel x_{n+1}-x_{n+2},a \parallel \parallel x_{n+1}-x_{n+2},a \parallel$$

$$\Rightarrow \parallel x_{n+1}-x_{n+2},a \parallel^2 \leq \frac{1}{a+b+c} \parallel x_n-x_{n+1},a \parallel^2$$

$$\Rightarrow \parallel x_{n+1}-x_{n+2},a \parallel \leq \left[\frac{1}{a+b+c}\right]^{\frac{1}{2}} \parallel x_n-x_{n+1},a \parallel$$

$$\Rightarrow \parallel x_{n+1}-x_{n+2},a \parallel \leq k_2 \parallel x_n-x_{n+1},a \parallel, \left[\text{where } k_2 = \left[\frac{1}{a+b+c}\right]^{\frac{1}{2}} < 1 \; (\text{As } a+b+c > 1)\right].$$

Let $k = \max\{k_1, k_2\}$. Hence $k < 1$. So, in general

$$\| x_n - x_{n+1}, a \| \leq k \ \| x_{n-1} - x_n, a \| \ for \quad n = 1, 2, 3...$$

$$\| x_n - x_{n+1}, a \| \leq k^n \ \| x_0 - x_1, a \| . \tag{3.1.11}$$

We can prove that $\{x_n\}$ is a Cauchy sequence using (3.1.3) as proved in theorem 3.1. As X is a complete Banach space, so there exist a point $x \in X$ such that $\{x_n\} \to x$.

Existence of fixed point Since T is a surjective self map and hence there exist point y in X such that

$$x = Ty \tag{3.1.12}$$

Consider $\| x_n - x, a \| = \| Tx_{n+1} - Ty, a \|$

$$\begin{aligned} \| Tx_{n+1} - Ty, a \|^2 \geq \ & a \ \| x_{n+1} - Tx_{n+1}, a \| \ \| x_{n+1} - y, a \| \\ & + b \ \| y - Ty, a \| \ \| x_{n+1} - y, a \| \\ & + c \ \| x_{n+1} - Tx_{n+1}, a \| \ \| y - Ty, a \| \end{aligned}$$

$$\begin{aligned} \| x_n - x, a \|^2 \geq \ & a \ \| x_{n+1} - x_n, a \| \ \| x_{n+1} - y, a \| \\ & + b \ \| y - x, a \| \ \| x_{n+1} - y, a \| \\ & + c \ \| x_{n+1} - x_n, a \| \ \| y - x, a \| . \end{aligned}$$

Since $\{x_{n+1}\}$ is a subsequence of $\{x_n\}$, so $\{x_n\} \to x$, $\{x_{n+1}\} \to x$ when $n \to \infty$

$$\begin{aligned} \| x - x, a \|^2 \geq \ & a \ \| x - x, a \| \ \| x - y, a \| \\ & + b \ \| y - x, a \| \ \| x - y, a \| \\ & + c \ \| x - x, a \| \ \| y - x, a \| \end{aligned}$$

$0 \geq b \| x - y, a \|^2 \Rightarrow \| x - y, a \| = 0$. This shows that x is a common fixed point of T. This completes the proof of Theorem 3.3.

Conflict of Interests

The authors declare that there is no conflict of interests.

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