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A COMMON FIXED POINT THEOREM OF PRESIC TYPE FOR FOUR MAPS IN G-METRIC SPACES

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Abstract. In this paper, we extended the idea of Presic type contraction for G-metric space to obtain a unique common fixed point result for four maps. The result generalizes several well known comparable results in the literature.

Keywords: Presic type fixed point theorem; G-metric space; G-Coincidence point; Weakly commuting, Coincidentally commuting.

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1. Introduction

In 1922, Banach proved a fixed point theorem for contraction mapping in metric space. This result has been extended and generalized for various settings (see, for instance [10], [12] and the references therein). The study of fixed points of mappings satisfying certain contractive condition has been at the centre of vigorous research activity. Mustafa and Sims [19] introduced the new concept of G-metric space. Since then many authors have been studying fixed point

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results in G-metric spaces and subsequently many fixed point results on such spaces appeared (see, for instance [1-6], [8], [18], [20-21], [27] and the references therein).

On the other hand, amongst the various generalization of Banach contraction principle, Presic [22] in 1965 gave a contractive condition on finite product of metric spaces and proved a fixed point theorem. Further Ciric- Presic [9], Gairola-Rawat [14], George - Khan [17] and Rao et. al. [23-24] extended and generalized these results. Also with a view to generalize the fixed point theorem for commuting maps, Sessa [25] introduced the concepts of weakly commuting maps. Later on, Singh and Gairola [26] extended the notion of weakly commuting maps to co-ordinatewise commuting and weakly commuting maps for two system of maps on finite product of metric spaces and proved some fixed point theorems. Gairola and Jangwan [15], Singh Gairola and [16] and Baillon and Singh [7] conceptualize co-ordinatewise R-weakly commuting mappings and compatible maps. George and Khan [17] used the concept of weakly commuting and coincidently commuting maps for k-tuples and generalized Presic type fixed point theorem for two maps and then later on Rao *et al.* [23] extended this work for three maps using the concept of 2k-weakly compatible pair.

The aim of this paper is to prove a Presic type common fixed point theorem for four mappings in complete G-metric space which extend and unify the results of Ciric-Presic [9], Dhasmana [11], Gairola-Dhasmana [13] and Rao *et al.* [23].

2. Definitions and propositions

We begin by briefly recalling some basic definitions and results will be needed in the sequel. Let (X, d) be a metric space, k a positive integer, $T : X^k \rightarrow X$ and $f : X \rightarrow X$ be mappings. An element $x \in X$ is said be a coincidence point of f and T if $fx = T(x, x, \dots, x)$, x is a common fixed point of f and T if $x = fx = T(x, x, \dots, x)$. The set of coincidence point of f and T is denoted by $C(f, T)$.

Definition 2.1. [17] (see also [26]) Mappings f and T are said to be commuting if $f(T(x, x, \dots, x)) = T(fx, fx, \dots, fx)$ for all $x \in X$.

Definition 2.2. [17] (see also [26]) Mappings f and T are said to be weakly commuting if $d(f(T(x, x, \dots, x)), T(fx, fx, \dots, fx)) \leq d(f(x), T(x, x, \dots, x))$ for all $x \in X$.

Definition 2.3. [17] Mappings f and T are said to be coincidentally commuting if they commute at their coincidence points.

Remark 2.4. [17] (see also [26]) For $k = 1$, above definitions reduce to the usual definition of commuting and weakly commuting mappings in a metric space.

Remark 2.5. It is notable that the above Definitions 2.1, 2.2 and 2.3 are special cases of definition 1 and 2 of Singh-Gairola [26]. See also the remarks of Gairola *et al.* [15-16].

Definition 2.6. [19] Let X be a nonempty set, and let $G : X \times X \times X \rightarrow \mathbb{R}^+$, be a function satisfying:

$$(G_1) G(x, y, z) = 0; \text{ if } x = y = z,$$

$$(G_2) 0 < G(x, x, y); \text{ for all } x, y \in X \text{ with } x \neq y$$

$$(G_3) G(x, x, y) \leq G(x, y, z); \text{ for all } x, y, z \in X \text{ with } z \neq y.$$

$$(G_4) G(x, y, z) = G(x, z, y) = G(y, z, x) = \dots ; \text{ (symmetry in all three variables) and}$$

$$(G_5) G(x, y, z) \leq G(x, a, a) + G(a, y, z) \text{ for all } x, y, z, a \in X \text{ (rectangle inequality).}$$

Then the function G is called a generalized metric or more specifically a G-metric on X , and the pair (X, G) is called a G-metric space.

Definition 2.7. [19] Let (X, G) be a G-metric space and let $\{x_n\}$ be a sequence of points of X . We say that $\{x_n\}$ is G-convergent to x if $\lim_{n,m \rightarrow \infty} G(x, x_n, x_m) = 0$; that is, for any $\varepsilon > 0$, there exists $N \in \mathbb{N}$ such that $G(x, x_n, x_m) < \varepsilon$, for all $n, m \geq N$. We refer to x as the limit of the sequence $\{x_n\}$ and write $x_n \xrightarrow{G} x$.

Proposition 2.8. [19] Let (X, G) be a G-metric space. The following statements are equivalent.

(1) $\{x_n\}$ is G-convergent to x .

(2) $G(x_n, x_n, x) \rightarrow 0$, as $n \rightarrow \infty$.

(3) $G(x_n, x, x) \rightarrow 0$, as $n \rightarrow \infty$.

Definition 2.9. [19] Let (X, G) be a G-metric space. A sequence $\{x_n\}$ is called G-Cauchy if given $\varepsilon > 0$, there is $N \in \mathbb{N}$ such that $G(x_n, x_m, x_l) < \varepsilon$, for all $n, m, l \geq N$, that is, if $G(x_n, x_m, x_l) \rightarrow 0$, as $n, m, l \rightarrow \infty$.

Proposition 2.10. [19] *In a G-metric space (X, G) , the following two statements are equivalent.*

- (1) *The sequence $\{x_n\}$ is G-Cauchy.*
- (2) *For every $\varepsilon > 0$, there exists $N \in \mathbb{N}$ such that $G(x_n, x_m, x_m) < \varepsilon$ for all $n, m \geq N$.*

Definition 2.11. [19] *A G-metric space (X, G) is said to be G-complete (or a complete G-metric space) if every G-Cauchy sequence in (X, G) is G-convergent in (X, G) .*

Definition 2.12. [19] *A G-metric space (X, G) is called symmetric if $G(x, y, y) = G(y, x, x)$ for all $x, y \in X$.*

Proposition 2.13. [19] *Let (X, G) be a G-metric space. Then the function $G(x, y, z)$ is jointly continuous in all three of its variables.*

Proposition 2.14. [19] *Every G-metric space (X, G) defines a metric space (X, d_G) by $d_G(x, y) = G(x, y, y) + G(y, x, x)$ for all $x, y \in X$.*

Note that if (X, G) is a symmetric G-metric space, then

$$d_G(x, y) = 2G(x, y, y) \quad \forall x, y \in X.$$

3. Main results

Now we state our main result.

Theorem 3.1. *Let (X, G) be a G-metric space, k a positive integer and $S, T, R : X^k \rightarrow X, f : X \rightarrow X$ be mappings satisfying the following conditions*

$$(1) \quad S(X^k) \cup T(X^k) \cup R(X^k) \subseteq f(X)$$

$$(2) \quad G(S(x_1, x_2, \dots, x_{k-1}, x_k), T(x_2, x_3, \dots, x_k, x_{k+1}),$$

$$R(x_3, x_4, \dots, x_{k+1}, x_{k+2})) \leq \lambda \max\{G(fx_i, fx_{i+1}, fx_{i+2}), 1 \leq i \leq k\}$$

for all $x_1, x_2, \dots, x_k, x_{k+1}, x_{k+2}$ in X

$$(3) \quad G(T(y_1, y_2, \dots, y_{k-1}, y_k), R(y_2, y_3, \dots, y_k, y_{k+1}),$$

$$S(y_3, y_4, \dots, y_{k+1}, y_{k+2})) \leq \lambda \max\{G(fy_i, fy_{i+1}, fy_{i+2}), 1 \leq i \leq k\}$$

for all $y_1, y_2, \dots, y_k, y_{k+1}, y_{k+2}$ in X

$$(4) \quad G(R(z_1, z_2, \dots, z_{k-1}, z_k), S(z_2, z_3, \dots, z_k, z_{k+1}),$$

$$T(z_3, z_4, \dots, z_{k+1}, z_{k+2})) \leq \lambda \max\{G(fz_i, fz_{i+1}, fz_{i+2}), 1 \leq i \leq k\}$$

for all $z_1, z_2, \dots, z_k, z_{k+1}, z_{k+2}$ in X , where $0 \leq \lambda < 1$,

$$(5) \quad d \left(S(u, u, \dots, u), T(v, v, \dots, v), R(w, w, \dots, w) \right) < G(fu, fv, fw),$$

for all $u, v, w \in X$ with $u \neq v \neq w$. Suppose that $f(X)$ is complete and one of (f, S) , (f, T) or (f, R) is coincidently commuting pair. Then there exist a unique point $p \in X$ such that $fp = p = S(p, p, \dots, p) = T(p, p, \dots, p) = R(p, p, \dots, p)$.

Proof. Suppose x_1, x_2, \dots, x_k are arbitrary points in X and for $n \in N$ and define

$$fx_{k+3n-2} = S(x_{3n-2}, x_{3n-1}, \dots, x_{3n+k-3}),$$

$$fx_{k+3n-1} = T(x_{3n-1}, x_{3n}, \dots, x_{3n+k-2}),$$

$$fx_{k+3n} = R(x_{3n}, x_{3n+1}, \dots, x_{3n+k-1}).$$

Let

$$(6) \quad \alpha_n = G(fx_n, fx_{n+1}, fx_{n+2}).$$

Let $\theta = \lambda^{\frac{1}{k}}$ and $K = \max\{\frac{\alpha_1}{\theta^1}, \frac{\alpha_2}{\theta^2}, \dots, \frac{\alpha_k}{\theta^k}\}$. Claim $\alpha_n \leq K\theta^n$ for all $n \in N$. By selection of K we have $\alpha_n \leq K\theta^n$ for $n = 1, 2, \dots, k$. Now,

$$\begin{aligned} \alpha_{k+1} &= G(fx_{k+1}, fx_{k+2}, fx_{k+3}) \\ &= G(S(x_1, x_2, \dots, x_k), T(x_2, x_3, \dots, x_{k+1}), R(x_3, x_4, \dots, x_{k+2})) \\ &\leq \lambda \max\{G(fx_i, fx_{i+1}, fx_{i+2}) : i = 1, 2, \dots, k\} \text{ by (2)} \\ &= \lambda \max\{\alpha_1, \alpha_2, \dots, \alpha_{k-1}, \alpha_k\} \\ &\leq \lambda \max\{K\theta^1, K\theta^2, \dots, K\theta^{k-1}, K\theta^k\} \\ &= \lambda K\theta = \theta^k K\theta \text{ as } \theta = \lambda^{\frac{1}{k}}. \end{aligned}$$

Thus $\alpha_{k+1} \leq K\theta^{k+1}$. Similarly, we have

$$\begin{aligned}
\alpha_{k+2} &= G(fx_{k+2}, fx_{k+3}, fx_{k+4}) \\
&= G(T(x_2, x_3, \dots, x_{k+1}), R(x_3, x_4, \dots, x_{k+2}), S(x_4, x_5, \dots, x_{k+3})) \\
&\leq \lambda \max\{G(fx_i, fx_{i+1}, fx_{i+2}) : i = 2, 3, \dots, k+1\} \text{ by (3)} \\
&= \lambda \max\{\alpha_2, \alpha_3, \dots, \alpha_k, \alpha_{k+1}\} \\
&\leq \lambda \max\{K\theta^2, K\theta^3, \dots, K\theta^k, K\theta^{k+1}\} \\
&= \lambda K\theta^2 = \theta^k K\theta^2 \text{ as } \theta = \lambda^{\frac{1}{k}} = K\theta^{k+2}.
\end{aligned}$$

Thus $\alpha_{k+2} \leq K\theta^{k+2}$. Also,

$$\begin{aligned}
\alpha_{k+3} &= G(fx_{k+3}, fx_{k+4}, fx_{k+5}) \\
&= G(R(x_3, x_4, \dots, x_{k+2}), S(x_4, x_5, \dots, x_{k+3}), T(x_5, x_6, \dots, x_{k+4})) \\
&\leq \lambda \max\{G(fx_i, fx_{i+1}, fx_{i+2}) : i = 3, 4, \dots, k+2\} \text{ by (4)} \\
&= \lambda \max\{\alpha_3, \alpha_4, \dots, \alpha_{k+1}, \alpha_{k+2}\} \\
&\leq \lambda \max\{K\theta^3, K\theta^4, \dots, K\theta^{k+1}, K\theta^{k+2}\} \\
&= \lambda K\theta^3 = \theta^k K\theta^3 \text{ as } \theta = \lambda^{\frac{1}{k}} = K\theta^{k+3}.
\end{aligned}$$

Thus $\alpha_{k+3} \leq K\theta^{k+3}$. Hence the claim is true.

Now, by claim, for l, n, p with $l > n > p$ and the rectangular inequality of G-metric space, we have

$$\begin{aligned}
G(fx_n, fx_p, fx_l) &\leq G(fx_n, fx_{n+1}, fx_{n+1}) + G(fx_{n+1}, fx_{n+2}, fx_{n+2}) + \dots + G(fx_{l-1}, fx_l, fx_l) \\
&\leq G(fx_n, fx_{n+1}, fx_{n+2}) + G(fx_{n+1}, fx_{n+2}, fx_{n+3}) + \dots + G(fx_{l-2}, fx_{l-1}, fx_l) \\
&= \alpha_n + \alpha_{n+1} + \dots + \alpha_{l-2} \\
&\leq K\theta^n + K\theta^{n+1} + \dots + K\theta^{l-2} \\
&\leq K[\theta^n + \theta^{n+1} + \dots + \theta^{l-2} + \dots] \\
&= K \frac{\theta^n}{1-\theta} \rightarrow 0 \text{ as } n \rightarrow \infty.
\end{aligned}$$

Hence $\{fx_n\}$ is a G-Cauchy sequence. Since $f(X)$ is a G-complete and there exists z in $f(X)$ such that $z = \lim fx_n$. There exist $p \in X$ such that $z = fp$. Then for any integer n , using (2), (3) and (4) we have

$$\begin{aligned}
& G(S(p, p, \dots, p), fx_{k+3n-2}, fx_{k+3n-2}) \\
&= G\left(S(p, p, \dots, p), S(x_{3n-2}, x_{3n-1}, \dots, x_{k+3n-3}), S(x_{3n-2}, x_{3n-1}, \dots, x_{k+3n-3})\right) \\
&\leq G\left(S(p, p, \dots, p), T(p, p, \dots, x_{3n-2}), T(p, p, \dots, x_{3n-2})\right) \\
&+ G\left(T(p, p, \dots, x_{3n-2}), R(p, p, \dots, x_{3n-1}), R(p, p, \dots, x_{3n-1})\right) \\
&+ G\left(R(p, p, \dots, x_{3n-1}), S(p, p, \dots, x_{3n}), S(p, p, \dots, x_{3n})\right) \\
&+ G\left(S(p, p, \dots, x_{3n}), T(p, p, \dots, x_{3n+1}), T(p, p, \dots, x_{3n+1})\right) \\
&+ \dots \\
&+ G\left(T(p, p, x_{3n-2}, \dots, x_{k+3n-5}), R(p, x_{3n-2}, \dots, x_{k+3n-4}), R(p, x_{3n-2}, \dots, x_{k+3n-4})\right) \\
&+ G\left(R(p, x_{3n-2}, \dots, x_{k+3n-4}), S(x_{3n-2}, \dots, x_{k+3n-3}), S(x_{3n-2}, \dots, x_{k+3n-3})\right) \\
&\leq G\left(S(p, p, \dots, p), T(p, p, \dots, x_{3n-2}), R(p, p, \dots, x_{3n-1})\right) \\
&+ G\left(T(p, p, \dots, x_{3n-2}), R(p, p, \dots, x_{3n-1}), S(p, p, \dots, x_{3n})\right) \\
&+ G\left(R(p, p, \dots, x_{3n-1}), S(p, p, \dots, x_{3n}), T(p, p, \dots, x_{3n+1})\right) \\
&+ G\left(S(p, p, \dots, x_{3n}), T(p, p, \dots, x_{3n+1}), R(p, p, \dots, x_{3n+2})\right) \\
&+ \dots \\
&+ G\left(S(p, p, \dots, x_{k+3n-6}), T(p, p, \dots, x_{k+3n-5}), R(p, x_{3n-2}, \dots, x_{k+3n-4})\right) \\
&+ G\left(T(p, p, x_{3n-2}, \dots, x_{k+3n-5}), R(p, x_{3n-2}, \dots, x_{k+3n-4}), S(x_{3n-2}, \dots, x_{k+3n-3})\right) \\
&\leq \lambda G(fp, fx_{3n-2}, fx_{3n-1}) \\
&+ \lambda \max\{G(fp, fx_{3n-2}, fx_{3n-1}), G(fx_{3n-2}, fx_{3n-1}, fx_{3n})\} \\
&+ \lambda \max\{G(fp, fx_{3n-2}, fx_{3n-1}), G(fx_{3n-2}, fx_{3n-1}, fx_{3n}), G(fx_{3n-1}, fx_{3n}, fx_{3n+1})\}
\end{aligned}$$

$$\begin{aligned}
 &+ \lambda \max\{G(fp, fx_{3n-2}, fx_{3n-1}), G(fx_{3n-2}, fx_{3n-1}, fx_{3n}), \dots, G(fx_{3n}, fx_{3n+1}, fx_{3n+2})\} \\
 &+ \dots \\
 &+ \lambda \max\{G(fp, fx_{3n-2}, fx_{3n-1}), \dots, G(fx_{k+3n-6}, fx_{k+3n-5}, fx_{k+3n-4})\} \\
 &+ \lambda \max\{G(fp, fx_{3n-2}, fx_{3n-1}), \dots, G(fx_{k+3n-5}, fx_{k+3n-4}, fx_{k+3n-3})\}.
 \end{aligned}$$

Taking limit as $n \rightarrow \infty$, we get $G(S(p, p, \dots, p), fp, fp) \leq 0$ so that

$$(7) \quad S(p, p, \dots, p) = fp.$$

Consider,

$$G(fp, T(p, p, \dots, p), T(p, p, \dots, p)) = G(S(p, p, \dots, p), T(p, p, \dots, p), T(p, p, \dots, p)) \leq \lambda(0) = 0.$$

Thus

$$(8) \quad T(p, p, \dots, p) = fp.$$

Also

$$G(fp, R(p, p, \dots, p), R(p, p, \dots, p)) = G(T(p, p, \dots, p), R(p, p, \dots, p), R(p, p, \dots, p)) \leq \lambda(0) = 0.$$

Thus

$$(9) \quad R(p, p, \dots, p) = fp.$$

Now suppose that (f, S) is a coincidentally commuting pair. Then we have

$$\begin{aligned}
 f(S(p, p, \dots, p)) &= S(fp, fp, \dots, fp), \\
 f^2p &= f(f(p)) = f(S(p, p, \dots, p)) = S(fp, fp, \dots, fp).
 \end{aligned}$$

Suppose $fp \neq p$

$$G(f^2p, fp, fp) = G(S(fp, fp, \dots, fp), T(p, p, \dots, p), R(p, p, \dots, p)) < d(f^2p, fp, fp).$$

It is a contradiction. Therefore $fp = p$. Now from (7), (8) and (9) we have

$$fp = p = S(p, p, \dots, p) = T(p, p, \dots, p) = R(p, p, \dots, p).$$

Uniqueness of p : Suppose there exists a point $q \neq p$ in X such that

$$fq = q = S(q, q, \dots, q) = T(q, q, \dots, q) = R(q, q, \dots, q).$$

Consider from (5)

$$G(fp, fq, fq) = G(S(p, p, \dots, p), T(q, q, \dots, q), R(q, q, \dots, q)) < G(fp, fq, fq).$$

It is a contradiction. Therefore $q = p$.

Now we can get the following result of Gairola-Dhasmana [13] as a corollary.

Corollary 3.2. *Let (X, G) be a G -metric space, k a positive integer and $T : X^k \rightarrow X, f : X \rightarrow X$ be mappings satisfying the following conditions*

$$(10) \quad T(X^k) \subseteq f(X)$$

$$(11) \quad G(T(x_1, x_2, \dots, x_{k-1}, x_k), T(x_2, x_3, \dots, x_k, x_{k+1}),$$

$$T(x_3, x_4, \dots, x_{k+1}, x_{k+2})) \leq \lambda \max\{G(fx_i, fx_{i+1}, fx_{i+2}), 1 \leq i \leq k\}$$

for all $x_1, x_2, \dots, x_k, x_{k+1}, x_{k+2}$ in X

$$(12) \quad G\left(T(u, u, \dots, u), T(v, v, \dots, v), T(w, w, \dots, w)\right) < G(fu, fv, fw),$$

for all $u, v, w \in X$ with $u \neq v \neq w$. Suppose that $f(X)$ is G -complete and (f, T) is coincidentally commuting pair. Then there exist a unique point $p \in X$ such that $fp = p = T(p, p, \dots, p)$.

Proof. Putting $S = R = I$ (Identity map) in Theorem 3.1 we can get the required proof.

Remark 3.3. If $f = I$ (Identity map) in Corollary 3.2, we get the main Theorem of [11].

Conflict of Interests

The authors declare that there is no conflict of interests.

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