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COMMON FIXED POINT THEOREM IN FUZZY METRIC SPACE USING INTEGRAL TYPE IMPLICIT RELATION

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Abstract: The objective of this paper is to utilize the notion of integral type implicit relation in fuzzy metric space. In this paper, we prove the existence result for common fixed point theorem of integral type for six self-maps satisfying integral type implicit relations is obtained in fuzzy metric space. Our main result improves and extends several known results.

Keywords: common fixed point; fuzzy metric space; compatible and semi compatible mappings and implicit relation.

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1. Introduction

The concept of Fuzzy sets was initially investigated by Zadeh[11] as a new way to represent vagueness in everyday life. Subsequently, it was developed extensively by many authors and used in various field. To use this concept in Topology and Analysis several researcher have defined several fuzzy metric space in various ways.

Commutativity of two mappings was given by sessa[10] with weakly commuting mappings. Later on, Jungck[3] enlarged the class non-commuting mappings by compatible mappings which asserts that a pair of self-mappings S and T be a metric space (X, d) is compatible if

$$\lim_{n\to\infty} d(TSx_n, STx_n) = 0 \quad \text{whenever} \quad \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = t \quad \text{for some } t \in X.$$

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A concept of compatible mappings was further improved by Jungck and Rhoades[4] with the notion of coincidentally commuting mappings which merely commute at their coincidence points. Recently, implicit relations are used as a tool for finding common fixed point of contraction maps. (See [1],[5],[6],[7],[9],[8]). These implicit relations guarantee coincidence point of pair of maps that ultimately leads to the existence of common fixed points of quadruple of maps satisfying weak compatibility criterion. In 2008, Altun and Turkoglu[2] proved two common fixed points theorems on complete FM-Space with an implicit relation.

2. Preliminaries

Throughout this paper, we use following definitions.

Definition 2.1. Let X be any set. A set in X is a function with domain X and values in [0,1].

Definition 2.2. A binary operation $*$: [0, 1] $\rightarrow$ [0, 1] is continuous t-norm if $*$ is satisfying the following conditions:

a. $*$ is commutative and associative.

b. $*$ is continuous.

c. $a * 1 \leq a$ for all $a \in [0,1]$.

d. $a*b = c*d$ whenever $a \leq c$ and $b \leq d$ for all a, b, c, $d \in [0,1]$.

Example $a * b = \min a$ and $a * b = ab$

Definition 2.3. A triplet (X, M, $*$) is a fuzzy metric space whenever X is an arbitrary set, $*$ is continuous t – norm and M is fuzzy set on $X \times X \times [0, \infty] (X^2 \times [0, \infty])$satisfying for every x, y, $z \in X$ and S, T > 0, the following condition.

FM-1 $M(x, y, t) > 0$;

FM-2 $M(x, y, 0) = 0$;

FM-3 $M(x, y, t) = 1$ for all $t > 0$ if and only if $x=y$;

FM-4 $M(x, y, t) = M(y, x, t)$;

FM-5 $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$;

FM-6 $M(x, y, \cdot) : [0,\infty) \rightarrow [0, 1]$ is left continuous;

Definition 2.4. Let (X, M, $*$) be a fuzzy metric space. The sequence $\{x_m\}$ in X is said to be convergent to a point $u \in X$ if

$$\lim_{n\to\infty} M(x_m, u, t) = 1 \text{ for all } t > 0$$

A sequence $\{x_n\}$ in X is said to be Cauchy sequence in X if

$$\lim_{n\to\infty} M(x_n, x_{n+p}, t) = 1 \text{ for all } t > 0\ , p > 0$$

The space is said to be complete if every Cauchy sequence in it converges to a point of it.

In this paper (X, M, $*$) is considered to be the fuzzy metric space with condition

FM-7

$$\lim_{n\to\infty} M(x, y, t) = 1 \text{ for all } x, y \in X.$$

Lemma 2.1. Let $\{y_n\}$ be a sequence is a fuzzy metric space (X, M, $*$) with the condition (FM-7) is there exists a number $k \in (0, 1)$ such that

$$M(y_{n+2}, y_{n+1}, kt) \ \geq\ M(y_{n+1}, y_n, t) \text{ for all } t > 0,$$

then$\{y_n\}$ is a Cauchy sequence in X.

Lemma 2.2. Let A and B be two self-maps on a complete fuzzy metric space (X, M, $*$) such that for some $k \in (0, 1)$ for all $x, y \in X$ and $t > 0$.

$M(Ax, By, kt) \geq \min\{M(x, y, t), M(Ax, x, t)\}$then A and B have a unique common fixed point in X.

Definition 2.5. Let A and S be mappings from a fuzzy metric space (X, M, $*$) into itself then

a. The mappings are said to be weak compatible if they commute at their coincidence point i.e.Ax=Sx implies that$ASx = SAx$.

b. The mappings are said to be compatible if

$$\lim_{n\to\infty} M(ASx_n, SAx_n) = 1 \quad \forall t > 0$$

Where $\{x_n\}$ is a sequence in X such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = x \in X$$

c. The mappings are said to be semi compatible if

$$\lim_{n\to\infty} M(ASx_n, SAx_n, t) = 1, \quad t > 0$$

Where $\{x_n\}$ is a sequence in X such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = x \in X$$

Implicit Relation:

Let K_4 be the set of all real continuous functions $F: R_+^4 \to R$, non-decreasing in first argument and satisfying the following condition.

$$\int_0^{F(u,\ v,\ v,\ u)} \emptyset(t)\, dt \geq 0 \quad \text{or} \quad \int_0^{F(u,\ v,\ u,\ v)} \emptyset(t)\, dt \geq 0 \ \text{(i)}$$

Then implies $u \geq v$ for $u, v \geq 0$.

$\int_0^{F(u,\ u,\ 1,\ 1)} \emptyset(t)\, dt \ \geq 0$ implies $u \ \geq\ 1.$ (ii)

Example:

(i) Let $F(u_1,u_2,u_3,u_4) = u_1 - \min(u_2,u_3,u_4)$ and $\emptyset(t) = \frac{9\pi}{10(1+t)^2}\cos\left(\frac{9\pi t}{10(1+t)}\right)$ for all t in R_+ and $(u, v) \geq 0$ are non-decreasing function in first argument. Now suppose that

$\int_0^{F(u,\ v,\ v,\ u)} \emptyset(t)\, dt \ \geq 0$ implies

$$\int_0^{F(u,\ v,\ v,\ u)} \frac{9\pi}{10(1+t)^2}\cos\left(\frac{9\pi t}{10(1+t)}\right)\, dt \ \geq 0$$

$$\int_0^{(u-v)} \frac{9\pi}{10(1+t)^2}\cos\left(\frac{9\pi t}{10(1+t)}\right)\, dt \ \geq 0 \qquad \text{implies that}$$

$$\sin\left(\frac{9\pi(u-v)}{10\big(1+(u-v)\big)}\right) \geq 0$$

It shows that $u \geq v$ for all $u, v \geq 0$.

(ii) $\int_0^{F(u,\ u,\ 1,\ 1)} \emptyset(t)\, dt \ \geq 0$

$$\int_0^{F(u,\ u,\ 1,\ 1)} \frac{9\pi}{10(1+t)^2}\cos\left(\frac{9\pi t}{10(1+t)}\right)\, dt \ \geq 0$$

$$\int_0^{(u-1)} \frac{9\pi}{10(1+t)^2}\cos\left(\frac{9\pi t}{10(1+t)}\right)\, dt \ \geq 0 \qquad \text{implies that}$$

$$\sin\left(\frac{9\pi(u-1)}{10\big(1+(u-1)\big)}\right) \geq 0$$

It shows that $u \geq 1$.

3. Main Result

Theorem 3.1 Let (X, M, $*$) be a complete fuzzy metric space and let A, B, S, T, I and J be self-maps on a complete metric space (X, M, $*$) satisfying.

a. $AB(X) \subset J(X)$, $ST(X) \subset I(X)$

b. AB = BA, ST = TS, IB = BI and JT = TJ

c. Either AB or I is continuous

d. (AB, I) is semi compatible and (ST, J) is weak compatible.

For some $F \in K_4$ there exists $k \in (0, 1)$ such that for all $x, y \in X$ and $t>0$

$$\int_0^{F(M(ABx,STy,kt),\ M(Ix,Jy,t),\ M(Ix,ABx,t),\ M(Jy,STy,kt))} \xi(\upsilon)\, d\upsilon \geq 0 \quad (1) \qquad \text{and}$$

$$\int_0^{F(M(ABx,STy,kt),\ M(Ix,Jy,t),\ M(Ix,ABx,kt),\ M(Jy,STy,t))} \xi(\upsilon)\, d\upsilon \geq 0 \qquad (2)$$

Where ξ: $[0, +\infty] \to [0, +\infty]$ is a lebesgue integral mapping which is summable.

$\int_0^{\varepsilon} \xi(\upsilon)\, d\upsilon > 0$.Then A, B, S, T, I and J have unique common fixed point in X.

Proof: Suppose x_0 be an arbitrary point in X. since $AB(X) \subset J(X)$ and since $ST(X) \subset I(X)$ there exist $x_1, x_2 \in X$ such that $ABx_0 \subset Jx_1$ and since $STx_1 \subset Ix_2$. In general, we can construct sequences $\{y_n\}$ and $\{x_n\}$ in X such that

$y_{2n+1} = ABx_{2n} = Jx_{2n+1}$ and $y_{2n+2} = STx_{2n+1} = Ix_{2n+2}$ for $n = 0, 1, 2, \ldots \ldots$

(I) put $x = x_{2n}$, $y = x_{2n+1}$ in (1) we get

$$\int_0^{F(M(ABx_{2n},STx_{2n+1},kt),\ M(Ix_{2n},Jx_{2n+1},t),\ M(Ix_{2n},ABx_{2n},t),\ M(Jx_{2n+1},STx_{2n+1},kt))} \xi(\upsilon) d\upsilon \geq 0$$

$$\int_0^{F(M(y_{2n+1},y_{2n+2},kt),\ M(y_{2n},y_{2n+1},t),\ M(y_{2n},y_{2n+1},t),\ M(y_{2n+1},y_{2n+2},kt))} \xi(\upsilon) d\upsilon \geq 0 \qquad (3)$$

using (i) we get

$$M(y_{2n+1}, y_{2n+2}, kt) \geq M(y_{2n+1}, y_{2n}, t) \qquad (4)$$

Similarly, by putting $x = x_{2n+2}$ and $y = x_{2n+1}$ in (2)

$$\int_0^{F(M(ABx_{2n+2},STx_{2n+1},kt),\ M(Ix_{2n+2},Jx_{2n+1},t),\ M(Ix_{2n+2},ABx_{2n+2},kt),\ M(Jx_{2n+1},STx_{2n+1},t))} \xi(\upsilon) d\upsilon \geq 0$$

$$\int_0^{F(M(y_{2n+3},y_{2n+2},kt),\ M(y_{2n+2},y_{2n+1},t),\ M(y_{2n+2},y_{2n+3},t),\ M(y_{2n+1},y_{2n+2},t))} \xi(\upsilon) d\upsilon \geq 0$$

Using (i) we get,

$$M(y_{2n+3}, y_{2n+2}, kt) \geq M(y_{2n+1}, y_{2n+2}, t) \qquad (5)$$

Thus from (4) and (5) for n and t, we have

$$M(y_n, y_{n+2}, kt) \geq M(y_{n+1}, y_n, t) \qquad (6)$$

Hence by lemma 2.2 $\{y_n\}$ is a Cauchy sequence in X, which complete therefore $\{y_n\}$ converges top$\in$X. The sequences $\{ABx_{2n}\}$, $\{STx_{2n+1}\}$, $\{Ix_{2n}\}$, $\{Jx_{2n+1}\}$ being subsequences of $\{y_n\}$ also converges to p that is

$ABx_{2n} \to p$, $STx_{2n+1} \to p$, $Ix_{2n} \to p$, $Jx_{2n+1} \to p$. (7)

Case 1: I is continuous, since I is continuous, we get

$I(ABx_{2n}) \to Ip, \quad I^2x_{2n} \to Ip$ (8)

The semi compatibility of the pair (AB, I) gives

$$\lim_{n\to\infty} AB(Ix_{2n}) = Ip \qquad (9)$$

1.1. Putting $x=Ix_n$, $y=x_{2n+1}$ in (1) we obtain

$$\int_0^{F(M((AB)Ix_{2n},(ST)x_{2n+1},kt),\ M(I.Ix_{2n},Jx_{2n+1},t),\ M(I.Ix_{2n},(AB)Ix_{2n},t),\ M(Jx_{2n+1},(ST)x_{2n+1},kt))} \xi(\upsilon)d\upsilon \geq 0 \qquad (10)$$

Letting $n\to\infty$ and using (8), (9), (10) and the continuity of the t-norm $*$ we have

$$\int_0^{F(M(Ip,p,kt),\ M(Ip,p,t),\ M(Ip,Ip,t),\ M(p,p,kt))} \xi(\upsilon)d\upsilon \geq 0 \qquad (11)$$

that is

$$\int_0^{F(M(Ip,p,kt),\ M(Ip,p,t),\ 1,\ 1)} \xi(\upsilon)d\upsilon \geq 0 \qquad (12)$$

Using (ii) we obtain

$$M(Ip,p,t) \geq 1 \text{ for all } t > 0 \textit{ which gives} M(Ip,p,t) = 1 \ \text{ i.e. } Ip = p \qquad (13)$$

1.2. Putting $x=p$, $y=x_{2n+1}$ in (1) we obtain

$$\int_0^{F(M(ABp,STx_{2n+1},kt),\ M(Ip,Jx_{2n+1},t),\ M(Ip,ABp,t),\ M(Jx_{2n+1},STx_{2n+1},kt))} \xi(\upsilon)d\upsilon \geq 0 \qquad (14)$$

letting $n\to\infty$ and using (8) and (14) we obtain

$$\int_0^{F(M(ABp,p,kt),\ M(p,p,t),\ M(p,ABp,t),\ M(p,p,kt))} \xi(\upsilon)d\upsilon \geq 0$$

$$\int_0^{F(M(ABp,p,Kt),\ 1,\ M(ABp,p,t),\ 1)} \xi(\upsilon)d\upsilon \geq 0 \qquad (15)$$

As F is non-decreasing in first argument, we have

$$\int_0^{F(M(ABp,p,t),\ 1,\ M(ABp,p,t),\ 1)} \xi(\upsilon)d\upsilon \geq 0$$

Using (i) we obtain $ABp = p$ (16)

from equation (13) and (16) $ABp = p = Ip$ (17)

Now put $x=Bp$ and $y=x_{2n+1}$ in (1) we get

$$\int_0^{F(M((AB)Bp,(ST)x_{2n+1},kt),\ M(I(Bp),Jx_{2n+1},t),\ M(I(Bp),(AB)Bp,t),\ M(Jx_{2n+1},STx_{2n+1},kt))} \xi(\upsilon)d\upsilon \geq 0$$

As $IB = BI, \quad AB = BA$ (18)

We have

$$I(Bp) = B(Ip) = Bp \text{ and } AB(Bp) = B(AB)p = Bp.$$

Letting $n \to \infty$ and using (7) and the continuity of t–norms $*$ we get

$$I(Bp) = B(Ip) = Bp \text{ and } AB(Bp) = B(AB)p = Bp.$$

$$\int_0^{F(M(Bp,p,kt),\ M(Bp,p,t),\ M(p,p,t),\ M(p,p,kt))} \xi(\upsilon)d\upsilon \geq 0$$

That is

$$\int_0^{F(M(Bp,p,kt),\ M(Bp,p,t),\ 1,\ 1)} \xi(\upsilon)d\upsilon \geq 0 \tag{19}$$

Using (ii) we get

$$M(Bp,p,t) \geq 1, \quad \forall t \geq 0 \text{ which gives } M(Bp,p,t) = 1$$

$Bp = p$ and so $p = ABp = Ap$

$$\text{therefore} \quad Ip = Ap = Bp = p \tag{20}$$

1.3. Since $AB(X) \subset J(X)$ there exist $u \in x$ such that

$ABP = Ip = p = Ju$.

Putting $x=x_{2n}$, $y=u$ in (1) we have

$$\int_0^{F(M(ABx_{2n},STu,kt),\ M(Ix_{2n},Ju,t),\ M(Ix_{2n},ABx_{2n},t),\ M(Ju,STu,kt))} \xi(\upsilon)d\upsilon \geq 0 \tag{21}$$

Letting $n \to \infty$ and using (7) we get

$$\int_0^{F(M(p,STu,kt),\ M(p,Ju,t),\ M(p,p,t),\ M(Ju,STu,kt))} \xi(\upsilon)d\upsilon \geq 0$$

$$\int_0^{F(M(p,STu,kt),\ M(p,p,t),\ M(p,p,t),\ M(p,STu,kt))} \xi(\upsilon)d\upsilon \geq 0$$

$$\int_0^{F(M(p,STu,kt),\ 1,\ 1,\ M(p,STu,kt))} \xi(\upsilon)d\upsilon \geq 0$$

Using (i) we have $M(p,STu,kt) = 1$

Thus $p = STu$

Therefore $STu = Ju = p$. since (ST,J) is weak compatible. we have $J(ST)u = (ST)Ju$,

That is

$$STp = Jp \tag{22}$$

1.4. Put $x=x_{2n}$, $y=p$ in (1) we get

$$\int_0^{F(M(ABx_{2n},STp,Kt),\ M(Ix_{2n},Jp,t),\ M(Ix_{2n},ABx_{2n},t),\ M(Jp,STp,Kt))} \xi(\upsilon)d\upsilon \geq 0$$

Letting $n\to\infty$ and using (7) and (22) we get

$$\int_0^{F(M(p,STp,kt),\ M(p,STp,t),\ M(p,p,t),\ M(STp,STp,kt))} \xi(\upsilon)d\upsilon \geq 0$$

$$\int_0^{F(M(p,STp,kt),\ M(p,STp,t),\ 1,\ 1)} \xi(\upsilon)d\upsilon \geq 0$$

As F is non-decreasing in first argument, we have

$$\int_0^{F(M(p,STp,t),\ M(p,STp,t),\ 1,\ 1)} \xi(\upsilon)d\upsilon \geq 0 \tag{23}$$

Using (ii) we get $M(p,STp,t) \geq 1$ for all $t > 0$.

Thus we have $STp = p$

Put $x = x_{2n}$ and $y = Tp$ in (1) we get

$$\int_0^{F(M(ABx_{2n},(ST)Tp,kt),\ M(Ix_{2n},JTp,t),\ M(Ix_{2n},ABx_{2n},t),\ M(JTp,(ST)Tp,kt))} \xi(\upsilon)d\upsilon \geq 0$$

As JT = TJ and ST = TS we have

$JTp = TJp = Tp$ and $ST(Tp) = T(STp) = Tp$.

Letting $n\to\infty$, we get

$$\int_0^{F(M(p,Tp,kt),\ M(p,p,t),\ M(p,p,t),\ M(Tp,Tp,kt))} \xi(\upsilon)d\upsilon \geq 0$$

$$\int_0^{F(M(p,Tp,Kt),\ 1,\ 1,\ 1)} \xi(\upsilon)d\upsilon \geq 0$$

As F is non-decreasing in first argument, we have

$$\int_0^{F(M(p,Tp,Kt),\ 1,\ 1,\ 1)} \xi(\upsilon)d\upsilon \geq 0$$

Thus $M(p,Tp,t) \geq 1$

Then $Tp = p$

Now $STp = Tp = p$ implies $Sp = p$

Hence $Sp = Tp = Jp = p$ (24)

Combining (20) and (24)

Ap = Bp= Ip = Jp = Sp = Tp

i.e. p is common fixed point of A, B, I, J

Case 2: AB is continuous, since AB is continuous and (AB, I) is semi compatible, we get

$(AB)\, Ix_{2n} \to ABp,\ (AB)^2x_{2n} \to Bp,\ IABx_{2n} \to ABp$ (25)

Thus

$$\lim_{n\to\infty} ABIx_{2n} = \lim_{n\to\infty} IABx_{2n} = ABp.$$

Now we prove $ABp = p$

2.1. Put x = ABx_{2n}, y = x_{2n+1} in (1) and assuming $ABp \neq p$.

$$\int_0^{F\left(M((AB)^2x_{2n},STx_{2n+1},kt),\ M(IABx_{2n},Jx_{2n+1},t),\ M(IABx_{2n},(AB)^2x_{2n},t),\ M(Jx_{2n+1},STx_{2n},kt)\right)} \xi(\upsilon)d\upsilon \geq 0$$

Letting n→∞ and using (25)

$$\int_0^{F\left(M(ABp,p,kt),\ M(ABp,p,t),\ M(ABp,ABp,t),\ M(p,p,kt)\right)} \xi(\upsilon)d\upsilon \geq 0$$

$$\int_0^{F(M(ABp,p,kt),\ M(ABp,p,t),\ 1,\ 1)} \xi(\upsilon)d\upsilon \geq 0$$

As F is non-decreasing in first argument, we have

$$\int_0^{F(M(ABp,p,kt),\ M(ABp,p,t),\ 1,\ 1)} \xi(\upsilon)d\upsilon \geq 0$$

Using (ii) we have $M(ABp, p, t) \geq 1$ for all $t > 0$

Thus$ABp = p$ (26)

2.2. Put x = p, y = x_{2n+1} in (1) we get

$$\int_0^{F\left(M(ABp,STx_{2n+1},kt),\ M(Ip,Jx_{2n+1},t),\ M(Ip,ABp,t),\ M(Jx_{2n+1},STx_{2n+1},kt)\right)} \xi(\upsilon)d\upsilon \geq 0$$

Letting n→∞ and using (8) and (26), we get

$$\int_0^{F\left(M(p,p,kt),\ M(Ip,p,t),\ M(Ip,p,t),\ M(p,p,kt)\right)} \xi(\upsilon)d\upsilon \geq 0$$

$$\int_0^{F(1,\ M(Ip,p,t),\ M(Ip,p,t),\ 1)} \xi(\upsilon)d\upsilon \geq 0$$

Using (ii) we obtain

$$M(Ip, p, t) \geq 1, \quad \forall t > 0$$

Thus $Ip = p$

Hence $Ip = p = ABp$

Further using (1.2) we get $Bp = p$

Thus $ABp = p$ gives $Ap = p$ and so $Ap = Bp = Ip = p$

also it follows from step (1.3) that $Sp = Tp = Jp = p$

Hence we get

$$\mathbf{Ap = Bp = Ip = Jp = Sp = Tp = p}$$

The uniqueness of common fixed point is an easy consequence of inequality (1) and (2) in view of ϕ_1 and ϕ_2.

Corollary 3.1

Let (X, M, ∗) be a complete fuzzy metric space and let A, B, S, T, I and J be self-maps on a complete metric space (X, M, ∗) satisfying.

a. $A^aB^b(X) \subset J^j(X)$, $S^sT^t(X) \subset I^i(X)$
b. AB = BA, ST = TS, IB = BI and JT = TJ
c. Either AB or I is continuous
d. (AB, I) is semi compatible and (ST, J) is weak compatible.

For some $F \in K_4$ there exists $k \in (0, 1)$ such that for all $x, y \in X$ and $t>0$

$$\int_0^{F\left(M(A^aB^bx, S^sT^ty, kt),\ M(I^ix, J^jy, t),\ M(I^ix, A^aB^bx, t),\ M(J^jy, S^sT^ty, kt)\right)} \xi(\upsilon)\, d\upsilon \geq 0 \qquad (1)$$

and

$$\int_0^{F\left(M(A^aB^bx, S^sT^ty, kt),\ M(I^ix, J^jy, t),\ M(I^ix, A^aB^bx, kt),\ M(J^jy, S^sT^ty, t)\right)} \xi(\upsilon)\, d\upsilon \geq 0 \qquad (2)$$

for all $x, y \in X$ and $a, b, s, t, i, j \in N$.

Where $\xi: [0, +\infty] \to [0, +\infty]$ is a lebesgue integral mapping which is summable. $\int_0^{\varepsilon} \xi(\upsilon)\, d\upsilon > 0$.Then A, B, S, T, I and J have unique common fixed point in X.

Conflict of Interests

The authors declare that there is no conflict of interests.

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