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SOME COUPLED FIXED POINT THEOREMS IN TWO QUASI-PARTIAL b-METRIC SPACES WITH DIFFERENT COEFFICIENTS

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Abstract. In this paper, some coupled common fixed-point theorems are proved for mappings defined on a set equipped with two quasi-partial b-metric spaces with different coefficients.

Keywords: Common coupled fixed point; Coupled coincidence point; w-compatible mappings; Quasi-partial metric space; Quasi-partial b-metric space.

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1. Introduction

The notion of partial metric spaces was introduced by Matthews [14] in 1994 who then further extended the banach contraction principle from metric spaces to partial metric spaces. Since then several authors (for example, [2,3,4,9]) worked on this notion of partial metric spaces and obtained fixed point results for mappings satisfying different contractive conditions.

The concept of b-metric spaces was introduced by Bakhtin [5] which was further extended by Czerwick [8]. Later Shukla [16] generalized both the concept of b-metric and partial metric

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spaces by introducing the partial b-metric spaces. Motivated by this we introduced the notion of Quasi-partial b-metric space [10] and proved fixed point theorem on it. Then we extended this study to coupled fixed point theorems on Quasi-partial b-metric spaces [11]. Earlier in 2012, Karapinar et al. [12] had introduced the concept of quasi-partial metric space which is defined as follows:

Definition 1.1. [12] A *quasi-partial metric* on nonempty set X is a function $q : X \times X \to \mathbb{R}^+$ which satisfies:

(QPM$_1$) If $q(x,x) = q(x,y) = q(y,y)$, then $x = y$,

(QPM$_2$) $q(x,x) \leqslant q(x,y)$,

(QPM$_3$) $q(x,x) \leqslant q(y,x)$, and

(QPM$_4$) $q(x,y) + q(z,z) \leqslant q(x,z) + q(z,y)$

for all $x,y,z \in X$.

A *quasi-partial metric space* is a pair (X,q) such that X is a nonempty set and q is a quasi-partial metric on X.

Let q be a quasi-partial metric on the set X. Then

$$d_q(x,y) = q(x,y) + q(y,x) - q(x,x) - q(y,y) \text{ is a metric on } X.$$

Lemma 1.2. [12] *Let (X,q) be a quasi-partial metric space. Let (X,p_q) be the corresponding partial metric space, and let (X,d_{p_q}) be the corresponding metric space. Then the following statements are equivalent*

(i) *(X,q) is complete,*

(ii) *(X,p_q) is complete,*

(iii) *(X,d_{p_q}) is complete.*

Moreover,

$$\begin{aligned}
\lim_{n\to\infty} d_{p_q}(x,x_n) = 0 \;\Leftrightarrow\;& p_q(x,x) = \lim_{n\to\infty} p_q(x,x_n) = \lim_{n,m\to\infty} p_q(x_n,x_m) \\
\Leftrightarrow\;& q(x,x) = \lim_{n\to\infty} q(x,x_n) = \lim_{n,m\to\infty} q(x_n,x_m) \\
&= \lim_{n\to\infty} q(x_n,x) = \lim_{n,m\to\infty} q(x_m,x_n) .
\end{aligned}$$

Definition 1.3. [16] A *partial b-metric* on a nonempty set X is a mapping $p_b : X \times X \to \mathbb{R}^+$ such that for some real number $s \geqslant 1$ and for all $x, y, z \in X$

(P_{b_1}) $x = y$ if and only if $p_b(x,x) = p_b(x,y) = p_b(y,y)$,

(P_{b_2}) $p_b(x,x) \leqslant p_b(x,y)$,

(P_{b_3}) $p_b(x,y) = p_b(y,x)$,

(P_{b_4}) $p_b(x,y) \leqslant s[p_b(x,z) + p_b(z,y)] - p_b(z,z)$.

A *partial b-metric space* is a pair (X, p_b) such that X is a nonempty set and p_b is a partial b-metric on X. The number s is called the coefficient of (X, p_b).

Definition 1.4. [6] Let X be a nonempty set. An element $(x,y) \in X \times X$ is a *coupled fixed point* of the mapping

$$F : X \times X \to X \text{ if } F(x,y) = x \text{ and } F(y,x) = y.$$

Definition 1.5. [13] An element $(x,y) \in X \times X$ is called

(i) a *coupled coincidence point* of the mappings $F : X \times X \to X$ and $g : X \to X$ if $F(x,y) = gx$ and $F(y,x) = gy$; in this case (gx, gy) is called *coupled point of coincidence* of mappings F and g;

(ii) a *common coupled fixed point* of mappings $F : X \times X \to X$ and $g : X \to X$ if $F(x,y) = gx = x$ and $F(y,x) = gy = y$.

The concept of w-compatible mappings was introduced by Abbas et al. [1].

Definition 1.6. [1] Let X be a nonempty set. The mappings $F : X \times X \to X$ and $g : X \to X$ are *w-compatible* if $gF(x,y) = F(gx, gy)$ whenever $gx = F(x,y)$ and $gy = F(y,x)$.

Shatanawi and Pitea [15] obtained some common coupled fixed point results for a pair of mappings in quasi-partial metric space. Later Gu and Wang [9] proved coupled fixed-point theorems in two quasi-partial metric spaces.

Theorem 1.7 ([9], Theorem 2.1). *Let q_1 and q_2 be two quasi partial metrics on X such that $q_2(x,y) \leqslant q_1(x,y)$, for all $x, y \in X$, and let $F : X \times X \to X$, $g : X \to X$ be two mappings. Suppose that there exists k_1, k_2, k_3, k_4, and k_5 in $[0,1)$ with*

$$k_1 + k_2 + k_3 + 2k_4 + k_5 < 1 \tag{1}$$

such that the condition

$$
\begin{aligned}
q_1(F(x,y),F(u,v)) &+ q_1(F(y,x),F(v,u)) \\
&\leqslant k_1[q_2(gx,gu)+q_2(gy,gv)]+k_2[q_2(gx,F(x,y))+q_2(gy,F(y,x))] \\
&\quad + k_3[q_2(gu,F(u,v))+q_2(gv,F(v,u))]+k_4[q_2(gx,F(u,v))+q_2(gy,F(v,u))] \\
&\quad + k_5[q_2(gu,F(x,y))+q_2(gv,F(y,x))]
\end{aligned} \tag{2}
$$

holds for all $x,y,u,v \in X$. Also, suppose we have the following hypotheses:

(i) $F(X\times X)\subseteq g(X)$.

(ii) $g(X)$ *is complete subspace of X with respect to the quasi-partial metric q_1.*

Then the mapping F and g have a coupled coincidence point (x,y) satisfying $gx = F(x,y) = F(y,x) = gy$. Moreover, if F and g are w-compatible, then F and g have a unique common coupled fixed point of the form (u,u).

The aim of this paper is to prove some coupled common fixed-point theorems on quasi-partial b-metrics spaces for mappings defined on a set equipped with two quasi-partial b-metrics with different coefficients s_1 and s_2 respectively such that $s_2 > s_1$.

2. Quasi-partial b-metric spaces

Definition 2.1. A quasi-partial b-metric on a nonempty set X is a mapping $qp_b : X\times X \to \mathbb{R}^+$ such that for some real number $s \geqslant 1$ and for all $x,y,z\in X$

($\mathrm{QP_{b_1}}$) $qp_b(x,x) = qp_b(x,y) = qp_b(y,y) \Rightarrow x = y$,

($\mathrm{QP_{b_2}}$) $qp_b(x,x) \leqslant qp_b(x,y)$,

($\mathrm{QP_{b_3}}$) $qp_b(x,x) \leqslant qp_b(y,x)$,

($\mathrm{QP_{b_4}}$) $qp_b(x,y) \leqslant s[qp_b(x,z)+qp_b(z,y)] - qp_b(z,z)$.

A *quasi-partial b-metric space* is a pair (X,qp_b) such that X is a nonempty set and qp_b is a quasi-partial b-metric on X. The number s is called the coefficient of (X,qp_b).

Let qp_b be a quasi-partial b-metric on the set X. Then

$$
d_{qp_b}(x,y) = qp_b(x,y)+qp_b(y,x)-qp_b(x,x)-qp_b(y,y)
$$

is a b-metric on X.

Lemma 2.2. *Every Partial b-metric space is a quasi-partial b-metric space. But the converse need not be true.*

Lemma 2.3. *Let (X,qp_b) be a quasi-partial b-metric space. Then the following hold*

(A) *If $qp_b(x,y)=0$ then $x=y$,*

(B) *If $x\neq y$, then $qp_b(x,y)>0$ and $qp_b(y,x)>0$.*

Proof is similar as for the case of quasi-partial metric space (Refer [12]).

Definition 2.4. *Let (X,qp_b) be a quasi-partial b-metric space. Then*

(i) *a sequence $\{x_n\}\subset X$* Converges *to $x\in X$ if and only if*

$$qp_b(x,x)=\lim_{n\to\infty}qp_b(x,x_n)=\lim_{n\to\infty}qp_b(x_n,x).$$

(ii) *a sequence $\{x_n\}\subset X$ is called a* Cauchy sequence *if and only if*

$$\lim_{n,m\to\infty}qp_b(x_n,x_m)\quad \textit{and}\lim_{n,m\to\infty}qp_b(x_m,x_n)\quad \textit{exist (and are finite).}$$

(iii) *the quasi partial b-metric space (X,qp_b) is said to be* Complete *if every cauchy sequence $\{x_n\}\subset X$ converges with respect to τ_{qp_b} to a point $x\in X$ such that*

$$qp_b(x,x)=\lim_{n,m\to\infty}qp_b(x_m,x_n)=\lim_{n,m\to\infty}qp_b(x_n,x_m).$$

(iv) *a mapping $f:X\to X$ is said to be* Continuous *at $x_0\in X$ if, for every $\varepsilon>0$, there exists $\delta>0$ such that $f(B(x_0,\delta))\subset B(f(x_0),\varepsilon)$.*

Lemma 2.5. *Let (X,qp_b) be a quasi-partial b-metric space and (X,d_{qp_b}) be the corresponding b-metric space. Then (X,d_{qp_b}) is complete if (X,qp_b) is complete.*

Proof. Since (X,qp_b) is complete, every cauchy sequence $\{x_n\}$ in X converges, with respect to τ_{qp_b} to a point $x\in X$ such that

$$qp_b(x,x)=\lim_{n,m\to\infty}qp_b(x_n,x_m)=\lim_{n,m\to\infty}qp_b(x_m,x_n). \tag{3}$$

Consider a Cauchy sequence $\{x_n\}$ in (X,d_{qp_b}). We will show that $\{x_n\}$ is Cauchy in (X,qp_b). Since $\{x_n\}$ is Cauchy in (X,d_{qp_b}), therefore $\lim_{n,m\to\infty}d_{qp_b}(x_n,x_m)$ exists and is finite.

Also,

$$d_{qp_b}(x_n,x_m) = qp_b(x_n,x_m) + qp_b(x_m,x_n) - qp_b(x_n,x_n) - qp_b(x_m,x_m).$$

Clearly, $\lim_{n,m\to\infty} qp_b(x_n,x_m)$ and $\lim_{n,m\to\infty} qp_b(x_m,x_n)$ exists and are finite. Therefore, $\{x_n\}$ is Cauchy sequence in (X,qp_b). Now, since (X,qp_b) is complete, the sequence $\{x_n\}$ converges with respect to τ_{qp_b} to a point $x \in X$ such that (3) holds. For $\{x_n\}$ to be convergent in (X,d_{qp_b}) we will show that

$$d_{qp_b}(x,x) = \lim_{n\to\infty} d_{qp_b}(x,x_n).$$

It follows from definition of d_{qp_b} that $d_{qp_b}(x,x) = 0$. Also,

$$\begin{aligned}\lim_{n\to\infty} d_{qp_b}(x,x_n) &= \lim_{n\to\infty} qp_b(x,x_n) + \lim_{n\to\infty} qp_b(x_n,x) - \lim_{n\to\infty} qp_b(x_n,x_n) - \lim_{n\to\infty} qp_b(x,x)\\ &= 0 \qquad \text{by (3) and definition of convergence in } (X,qp_b).\end{aligned}$$

Hence, $d_{qp_b}(x,x) = \lim_{n\to\infty} d_{qp_b}(x,x_n)$.

3. The main results

Now, we shall prove our main result.

Theorem 3.1. *Let qp_{b_1} and qp_{b_2} be two quasi-partial b-metrics on X with different coefficients s_1 and s_2 respectively such that $s_2 > s_1$ and $qp_{b_2}(x,y) \leqslant qp_{b_1}(x,y)$, for all $x,y \in X$. Let $F : X\times X \to X$, $g : X \to X$ be two mappings. Suppose that there exist k_1,k_2,k_3,k_4, and k_5 in $[0,1)$ with*

$$k_1 + k_2 + k_3 + 2s_2k_4 + k_5 < \frac{1}{s_1} \tag{4}$$

such that the condition

$$\begin{aligned}&qp_{b_1}(F(x,y),F(u,v)) + qp_{b_1}(F(y,x),F(v,u))\\ &\quad\leqslant k_1[qp_{b_2}(gx,gu)+qp_{b_2}(gy,gv)]+k_2[qp_{b_2}(gx,F(x,y))+qp_{b_2}(gy,F(y,x))]\\ &\qquad + k_3[qp_{b_2}(gu,F(u,v)) + qp_{b_2}(gv,F(v,u))] + k_4[qp_{b_2}(gx,F(u,v)) + qp_{b_2}(gy,F(v,u))]\\ &\qquad + k_5[qp_{b_2}(gu,F(x,y)) + qp_{b_2}(gv,F(y,x))]\end{aligned} \tag{5}$$

holds for all $x,y,u,v \in X$. Also, suppose we have the following hypotheses:

(i) $F(X\times X) \subset g(X)$

(ii) *$g(X)$ is a complete subspace of X with respect to the quasi-partial b-metric qp_{b_1}.*

Then the mappings F and g have a coupled coincidence point (x,y) satisfying $gx = F(x,y) = F(y,x) = gy$. Moreover, if F and g are w-compatible, then F and g have a unique common coupled fixed point of the form (u,u).

Proof. Let $x_0, y_0 \in X$. Since $F(X \times X) \subset g(X)$, we can choose $x_1, y_1 \in X$ such that $gx_1 = F(x_0,y_0)$ and $gy_1 = F(y_0,x_0)$. Similarly, we can choose $x_2, y_2 \in X$ such that $gx_2 = F(x_1,y_1)$ and $gy_2 = F(y_1,x_1)$.

Continuing in this way we can construct two sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$gx_{n+1} = F(x_n,y_n) \quad \text{and} \quad gy_{n+1} = F(y_n,x_n), \quad \forall\, n \geqslant 0. \tag{6}$$

It follows from (5), ($\mathrm{QP_{b_4}}$) and ($\mathrm{QP_{b_2}}$) that,

$$\begin{aligned}
&qp_{b_1}(gx_n, gx_{n+1}) + qp_{b_1}(gy_n, gy_{n+1})\\
&\quad = qp_{b_1}(F(x_{n-1},y_{n-1}), F(x_n,y_n)) + qp_{b_1}(F(y_{n-1},x_{n-1}), F(y_n,x_n))\\
&\quad \leqslant k_1[qp_{b_2}(gx_{n-1}, gx_n) + qp_{b_2}(gy_{n-1}, gy_n)]\\
&\qquad + k_2[qp_{b_2}(gx_{n-1}, F(x_{n-1},y_{n-1})) + qp_{b_2}(gy_{n-1}, F(y_{n-1},x_{n-1}))]\\
&\qquad + k_3[qp_{b_2}(gx_n, F(x_n,y_n)) + qp_{b_2}(gy_n, F(y_n,x_n))]\\
&\qquad + k_4[qp_{b_2}(gx_{n-1}, F(x_n,y_n)) + qp_{b_2}(gy_{n-1}, F(y_n,x_n))]\\
&\qquad + k_5[qp_{b_2}(gx_n, F(x_{n-1},y_{n-1})) + qp_{b_2}(gy_n, F(y_{n-1},x_{n-1}))]\\
&\quad = (k_1+k_2)[qp_{b_2}(gx_{n-1}, gx_n) + qp_{b_2}(gy_{n-1}, gy_n)]\\
&\qquad + k_3[qp_{b_2}(gx_n, gx_{n+1}) + qp_{b_2}(gy_n, gy_{n+1})]\\
&\qquad + k_4[qp_{b_2}(gx_{n-1}, gx_{n+1}) + qp_{b_2}(gy_{n-1}, gy_{n+1})]\\
&\qquad + k_5[qp_{b_2}(gx_n, gx_n) + qp_{b_2}(gy_n, gy_n)]
\end{aligned}$$

$$\begin{aligned}
&\leqslant (k_1+k_2)[qp_{b_2}(gx_{n-1},gx_n)+qp_{b_2}(gy_{n-1},gy_n)]\\
&\quad + k_3[qp_{b_2}(gx_n,gx_{n+1})+qp_{b_2}(gy_n,gy_{n+1})]\\
&\quad + k_4[s_2\{qp_{b_2}(gx_{n-1},gx_n)+qp_{b_2}(gx_n,gx_{n+1})\}-qp_{b_2}(gx_n,gx_n)\\
&\quad + s_2\{qp_{b_2}(gy_{n-1},gy_n)+qp_{b_2}(gy_n,gy_{n+1})\}-qp_{b_2}(gy_n,gy_n)]\\
&\quad + k_5[qp_{b_2}(gx_n,gx_{n+1})+qp_{b_2}(gy_n,gy_{n+1})]\\
&\leqslant (k_1+k_2+s_2k_4)[qp_{b_2}(gx_{n-1},gx_n)+qp_{b_2}(gy_{n-1},gy_n)]\\
&\quad +(k_3+s_2k_4+k_5)[qp_{b_2}(gx_n,gx_{n+1})+qp_{b_2}(gy_n,gy_{n+1})]\\
&\leqslant (k_1+k_2+s_2k_4)[qp_{b_1}(gx_{n-1},gx_n)+qp_{b_1}(gy_{n-1},gy_n)]\\
&\quad +(k_3+s_2k_4+k_5)[qp_{b_1}(gx_n,gx_{n+1})+qp_{b_1}(gy_n,gy_{n+1})],
\end{aligned}$$

which implies that

$$\begin{aligned}
&qp_{b_1}(gx_n,gx_{n+1})+qp_{b_1}(gy_n,gy_{n+1})\\
&\quad\leqslant \frac{k_1+k_2+s_2k_4}{1-k_3-s_2k_4-k_5}[qp_{b_1}(gx_{n-1},gx_n)+qp_{b_1}(gy_{n-1},gy_n)].
\end{aligned} \tag{7}$$

Put $k=\dfrac{k_1+k_2+s_2k_4}{1-k_3-s_2k_4-k_5}$. Obviously, $0\leqslant k<\dfrac{1}{s_1}<1$. By repetition of the above inequality (7) n times we get

$$qp_{b_1}(gx_n,gx_{n+1})+qp_{b_1}(gy_n,gy_{n+1})\leqslant k^n[qp_{b_1}(gx_0,gx_1)+qp_{b_1}(gy_0,gy_1)]. \tag{8}$$

Next, we shall prove that $\{gx_n\}$ and $\{gy_n\}$ are Cauchy sequences in $g(X)$. In fact, for each $n,m\in\mathbb{N}$, $m>n$, from ($\mathrm{QP_{b_4}}$) and (8), we have

$$\begin{aligned}
qp_{b_1}(gx_n,gx_m)+qp_{b_1}(gy_n,gy_m) &\leqslant \sum_{i=n}^{m-1} s_1^{m-i}[qp_{b_1}(gx_i,gx_{i+1})+qp_{b_1}(gy_i,gy_{i+1})]\\
&\leqslant \sum_{i=n}^{m-1} s_1^{m-i}\cdot k^i[qp_{b_1}(gx_0,gx_1)+qp_{b_1}(gy_0,gy_1)]\\
&= \sum_{i=n}^{m-1}\left(\frac{k}{s_1}\right)^i s_1^m[qp_{b_1}(gx_0,gx_1)+qp_{b_1}(gy_0,gy_1)]
\end{aligned}$$

$$\leqslant \sum_{i=n}^{\infty}\left(\frac{k}{s_1}\right)^i s_1^m[qp_{b_1}(gx_0,gx_1)+qp_{b_1}(gy_0,gy_1)]$$
$$=\frac{\left(\frac{k}{s_1}\right)^n}{\left(1-\frac{k}{s_1}\right)}\cdot s_1^m[qp_{b_1}(gx_0,gx_1)+qp_{b_1}(gy_0,gy_1)]. \tag{9}$$

On letting $n\to\infty$ in (9); holding m fixed, we get

$$\lim_{n\to\infty}[qp_{b_1}(gx_n,gx_m)+qp_{b_1}(gy_n,gy_m)]\leqslant 0\,.$$

But

$$\lim_{n\to\infty}[qp_{b_1}(gx_n,gx_m)+qp_{b_1}(gy_n,gy_m)]\geqslant 0.$$

This implies that

$$\lim_{n\to\infty}[qp_{b_1}(gx_n,gx_m)]=\lim_{n\to\infty}[qp_{b_1}(gy_n,gy_m)]=0\,.$$

Now letting $m\to+\infty$, one has

$$\lim_{n,m\to\infty}qp_{b_1}(gx_n,gx_m)=\lim_{n,m\to\infty}qp_{b_1}(gy_n,gy_m)=0\,. \tag{10}$$

By similar arguments as above, we can show that

$$\lim_{n,m\to\infty}qp_{b_1}(gx_m,gx_n)=0 \quad \text{and} \quad \lim_{n,m\to\infty}qp_{b_1}(gy_m,gy_n)=0\,. \tag{11}$$

So, $\{gx_n\}$ and $\{gy_n\}$ are Cauchy sequences in $(g(X),qp_{b_1})$. Since $(g(X),qp_{b_1})$ is complete, there exist $gx,gy\in g(X)$ such that $\{gx_n\}$ and $\{gy_n\}$ converges to gx and gy with respect to $\tau_{qp_{b_1}}$, that is,

$$\begin{aligned}qp_{b_1}(gx,gx)&=\lim_{n\to\infty}qp_{b_1}(gx,gx_n)=\lim_{n\to\infty}qp_{b_1}(gx_n,gx)\\&=\lim_{n,m\to\infty}qp_{b_1}(gx_m,gx_n)=\lim_{n,m\to\infty}qp_{b_1}(gx_n,gx_m)\end{aligned} \tag{12}$$

and

$$\begin{aligned}qp_{b_1}(gy,gy)&=\lim_{n\to\infty}qp_{b_1}(gy,gy_n)=\lim_{n\to\infty}qp_{b_1}(gy_n,gy)\\&=\lim_{n,m\to\infty}qp_{b_1}(gy_m,gy_n)=\lim_{n,m\to\infty}qp_{b_1}(gy_n,gy_m).\end{aligned} \tag{13}$$

Combining (10)-(13), we have

$$\begin{aligned}qp_{b_1}(gx,gx)&=\lim_{n\to\infty}qp_{b_1}(gx,gx_n)=\lim_{n\to\infty}qp_{b_1}(gx_n,gx)\\&=\lim_{n,m\to\infty}qp_{b_1}(gx_m,gx_n)=\lim_{n,m\to\infty}qp_{b_1}(gx_n,gx_m)=0\end{aligned} \tag{14}$$

and

$$
\begin{aligned}
qp_{b_1}(gy,gy) &= \lim_{n\to\infty} qp_{b_1}(gy,gy_n) = \lim_{n\to\infty} qp_{b_1}(gy_n,gy) \\
&= \lim_{n,m\to\infty} qp_{b_1}(gy_m,gy_n) = \lim_{n,m\to\infty} qp_{b_1}(gy_n,gy_m) = 0.
\end{aligned} \tag{15}
$$

By (QP_{b_4}), we have

$$
\begin{aligned}
qp_{b_1}(gx_{n+1},F(x,y)) &\leqslant s_1\{qp_{b_1}(gx_{n+1},gx)+qp_{b_1}(gx,F(x,y))\} - qp_{b_1}(gx,gx) \\
&\leqslant s_1\{qp_{b_1}(gx_{n+1},gx)+qp_{b_1}(gx,F(x,y))\} \\
&\leqslant s_1\ [qp_{b_1}(gx_{n+1},gx)+s_1\{qp_{b_1}(gx,gx_{n+1}) \\
&\qquad + qp_{b_1}(gx_{n+1},F(x,y))\} - qp_{b_1}(gx_{n+1},gx_{n+1})] \\
&\leqslant s_1[qp_{b_1}(gx_{n+1},gx)] + s_1^2[qp_{b_1}(gx,gx_{n+1})] \\
&\qquad + s_1^2[qp_{b_1}(gx_{n+1},F(x,y))].
\end{aligned}
$$

Letting $n\to\infty$ in the above inequalities and using (14), we have

$$
\begin{aligned}
\frac{1}{s_1} qp_{b_1}(gx,F(x,y)) &\leqslant \lim_{n\to\infty} qp_{b_1}(gx_{n+1},F(x,y)) \\
&\leqslant s_1 qp_{b_1}(gx,F(x,y)).
\end{aligned} \tag{16}
$$

Similarly using (15), one has

$$
\begin{aligned}
\frac{1}{s_1} qp_{b_1}(gy,F(y,x)) &\leqslant \lim_{n\to\infty} qp_{b_1}(gy_{n+1},F(y,x)) \\
&\leqslant s_1 qp_{b_1}(gy,F(y,x)).
\end{aligned} \tag{17}
$$

Now, we prove that $F(x,y)=gx$ and $F(y,x)=gy$. Infact, it follows from (5) and (6) that

$$
\begin{aligned}
& qp_{b_1}(gx_{n+1},F(x,y)) + qp_{b_1}(gy_{n+1},F(y,x)) \\
&\quad = qp_{b_1}(F(x_n,y_n),F(x,y)) + qp_{b_1}(F(y_n,x_n),F(y,x)) \\
&\quad \leqslant k_1[qp_{b_2}(gx_n,gx)+qp_{b_2}(gy_n,gy)] + k_2[qp_{b_2}(gx_n,F(x_n,y_n))+qp_{b_2}(gy_n,F(y_n,x_n))] \\
&\qquad + k_3[qp_{b_2}(gx,F(x,y))+qp_{b_2}(gy,F(y,x))] + k_4[qp_{b_2}(gx_n,F(x,y))+qp_{b_2}(gy_n,F(y,x))] \\
&\qquad + k_5[qp_{b_2}(gx,F(x_n,y_n))+qp_{b_2}(gy,F(y_n,x_n))]
\end{aligned}
$$

$$
\begin{aligned}
&= k_1[qp_{b_2}(gx_n,gx)+qp_{b_2}(gy_n,gy)]+k_2[qp_{b_2}(gx_n,gx_{n+1})+qp_{b_2}(gy_n,gy_{n+1})]\\
&\quad + k_3[qp_{b_2}(gx,F(x,y))+qp_{b_2}(gy,F(y,x))]+k_4[qp_{b_2}(gx_n,F(x,y))+qp_{b_2}(gy_n,F(y,x))]\\
&\quad + k_5[qp_{b_2}(gx,gx_{n+1})+qp_{b_2}(gy,gy_{n+1})]\\
&\leqslant k_1[qp_{b_1}(gx_n,gx)+qp_{b_1}(gy_n,gy)]+k_2[qp_{b_1}(gx_n,gx_{n+1})+qp_{b_2}(gy_n,gy_{n+1})]\\
&\quad + k_3[qp_{b_1}(gx,F(x,y))+qp_{b_1}(gy,F(y,x))]+k_4[qp_{b_1}(gx_n,F(x,y))+qp_{b_1}(gy_n,F(y,x))]\\
&\quad + k_5[qp_{b_1}(gx,gx_{n+1})+qp_{b_1}(gy,gy_{n+1})].
\end{aligned}
$$

Letting $n\to\infty$ in the above inequality, using (14)-(17), we get

$$
\begin{aligned}
&\lim_{n\to\infty}[qp_{b_1}(gx_{n+1},F(x,y))+qp_{b_1}(gy_{n+1},F(y,x))]\\
&\quad\leqslant \lim_{n\to\infty}\{[k_1(qp_{b_1}(gx_n,gx)+qp_{b_1}(gy_n,gy)]+k_2[qp_{b_1}(gx_n,gx_{n+1})+qp_{b_1}(gy_n,gy_{n+1})]\\
&\qquad + k_3[qp_{b_1}(gx,F(x,y))+qp_{b_1}(gy,F(y,x))]+k_4[qp_{b_1}(gx_n,F(x,y))+qp_{b_1}(gy_n,F(y,x))]\\
&\qquad + k_5[qp_{b_1}(gx,gx_{n+1})+qp_{b_1}(gy,gy_{n+1})]\}.
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
&\lim_{n\to\infty}[qp_{b_1}(gx_{n+1},F(x,y))+qp_{b_1}(gy_{n+1},F(y,x))]\\
&\quad\leqslant k_1[qp_{b_1}(gx,gx)+qp_{b_1}(gy,gy)]+k_2[qp_{b_1}(gx,gx)+qp_{b_1}(gy,gy)]\\
&\qquad + k_3[qp_{b_1}(gx,F(x,y))+qp_{b_1}(gy,F(y,x))]+\lim_{n\to\infty}k_4[qp_{b_1}(gx_n,F(x,y))+qp_{b_1}(gy_n,F(y,x))]\\
&\qquad + k_5[qp_{b_1}(gx,gx)+qp_{b_1}(gy,gy)]\\
&\quad= k_3[qp_{b_1}(gx,F(x,y))+qp_{b_1}(gy,F(y,x))]+\lim_{n\to\infty}k_4[qp_{b_1}(gx_n,F(x,y))+qp_{b_1}(gy_n,F(y,x))].
\end{aligned}
$$

By using (14)-(17), we get

$$
\begin{aligned}
&\lim_{n\to\infty}[qp_{b_1}(gx_{n+1},F(x,y))+qp_{b_1}(gy_{n+1},F(y,x))]\\
&\quad\leqslant k_3[qp_{b_1}(gx,F(x,y))+qp_{b_1}(gy,F(y,x))]+k_4\cdot s_1[qp_{b_1}(gx,F(x,y))+qp_{b_1}(gy,F(y,x))]\\
&\quad= (k_3+s_1k_4)[qp_{b_1}(gx,F(x,y))+qp_{b_1}(gy,F(y,x))].
\end{aligned}
$$

And also

$$\begin{aligned} &\frac{1}{s_1}[qp_{b_1}(gx,F(x,y))+qp_{b_1}(gy,F(y,x))] \\ &\quad\leqslant (k_3+s_1k_4)[qp_{b_1}(gx,F(x,y))+qp_{b_1}(gy,F(y,x))] \qquad (18)\\ \Rightarrow &\quad \left[\frac{1}{s_1}-k_3-s_1k_4\right][qp_{b_1}(gx,F(x,y))+qp_{b_1}(gy,F(y,x))]\leqslant 0. \end{aligned}$$

Also $k_3+s_1k_4<k_3+s_2k_4$ since $s_2>s_1$. Further it follows from (4) that $k_3+s_2k_4<\frac{1}{s_1}$. Hence $k_3+s_1k_4<\frac{1}{s_1}$. Thus it follows from (18) that

$$qp_{b_1}(gx,F(x,y))=qp_{b_1}(gy,F(y,x))=0.$$

By Lemma 2.3, we get $F(x,y)=gx$ and $F(y,x)=gy$. Hence, (gx,gy) is a coupled point of coincidence of mappings F and g.

Next, we will show that the coupled point of coincidence is unique. Suppose that $(x^*,y^*)\in X\times X$ with $F(x^*,y^*)=gx^*$ and $F(y^*,x^*)=gy^*$. Using (5), (14), (15), and ($\mathrm{QP_{b_3}}$), we obtain

$$\begin{aligned} &qp_{b_1}(gx,gx^*)+qp_{b_1}(gy,gy^*)\\ &\quad=qp_{b_1}(F(x,y),F(x^*,y^*))+qp_{b_1}(F(y,x),F(y^*,x^*))\\ &\quad\leqslant k_1[qp_{b_2}(gx,gx^*)+qp_{b_2}(gy,gy^*)]+k_2[qp_{b_2}(gx,F(x,y))+qp_{b_2}(gy,F(y,x))]\\ &\qquad+k_3[qp_{b_2}(gx^*,F(x^*,y^*))+qp_{b_2}(gy^*,F(y^*,x^*))]+k_4[qp_{b_2}(gx,F(x^*,y^*))+qp_{b_2}(gy,F(y^*,x^*))]\\ &\qquad+k_5[qp_{b_2}(gx^*,F(x,y))+qp_{b_2}(gy^*,F(y,x))]\\ &\quad=k_1[qp_{b_2}(gx,gx^*)+qp_{b_2}(gy,gy^*)]+k_2[qp_{b_2}(gx,gx)+qp_{b_2}(gy,gy)]\\ &\qquad+k_3[qp_{b_2}(gx^*,gx^*)+qp_{b_2}(gy^*,gy^*)]+k_4[qp_{b_2}(gx,gx^*)+qp_{b_2}(gy,gy^*)]\\ &\qquad+k_5[qp_{b_2}(gx^*,gx)+qp_{b_2}(gy^*,gy)]\\ &\quad\leqslant (k_1+k_4)[qp_{b_1}(gx,gx^*)+qp_{b_1}(gy,gy^*)]+k_2[qp_{b_1}(gx,gx)+qp_{b_1}(gy,gy)]\\ &\qquad+k_3[qp_{b_1}(gx^*,gx^*)+qp_{b_1}(gy^*,gy^*)]+k_5[qp_{b_1}(gx^*,gx)+qp_{b_1}(gy^*,gy)]\\ &\quad\leqslant (k_1+k_3+k_4)[qp_{b_1}(gx,gx^*)+qp_{b_1}(gy,gy^*)]+k_5[qp_{b_1}(gx^*,gx)+qp_{b_1}(gy^*,gy)]. \end{aligned}$$

This implies that

$$qp_{b_1}(gx,gx^*)+qp_{b_1}(gy,gy^*)\leqslant\frac{k_5}{1-k_1-k_3-k_4}[qp_{b_1}(gx^*,gx)+qp_{b_1}(gy^*,gy)].\tag{19}$$

Similarly, we have

$$qp_{b_1}(gx^*,gx)+qp_{b_1}(gy^*,gy)\leqslant\frac{k_5}{1-k_1-k_3-k_4}[qp_{b_1}(gx,gx^*)+qp_{b_1}(gy,gy^*)].\tag{20}$$

Substituting (20) into (19), we obtain

$$qp_{b_1}(gx,gx^*)+qp_{b_1}(gy,gy^*)\leqslant\left(\frac{k_5}{1-k_1-k_3-k_4}\right)^2[qp_{b_1}(gx,gx^*)+qp_{b_1}(gy,gy^*)].\tag{21}$$

Since $\frac{k_5}{1-k_1-k_3-k_4}<1$, from (21), we must have

$$qp_{b_1}(gx,gx^*)=qp_{b_1}(gy,gy^*)=0.$$

By Lemma 2.3, we get $gx=gx^*$ and $gy=gy^*$, which implies the uniqueness of the coupled point of coincidence of F and g, that is, (gx,gy).

Next, we will show that $gx=gy$. Infact, from (5), (14) and (15), we have

$$\begin{aligned}
&qp_{b_1}(gx,gy)+qp_{b_1}(gy,gx)\\
&\quad=qp_{b_1}(F(x,y),F(y,x))+qp_{b_1}(F(y,x),F(x,y))\\
&\quad\leqslant k_1[qp_{b_2}(gx,gy)+qp_{b_2}(gy,gx)]+k_2[qp_{b_2}(gx,F(x,y))+qp_{b_2}(gy,F(y,x))]\\
&\qquad+k_3[qp_{b_2}(gy,F(y,x))+qp_{b_2}(gx,F(x,y))]+k_4[qp_{b_2}(gx,F(y,x))+qp_{b_2}(gy,F(x,y))]\\
&\qquad+k_5[qp_{b_2}(gy,F(x,y))+qp_{b_2}(gx,F(y,x))]\\
&\quad=k_1[qp_{b_2}(gx,gy)+qp_{b_2}(gy,gx)]+k_2[qp_{b_2}(gx,gx)+qp_{b_2}(gy,gy)]\\
&\qquad+k_3[qp_{b_2}(gy,gy)+qp_{b_2}(gx,gx)]+k_4[qp_{b_2}(gx,gy)+qp_{b_2}(gy,gx)]\\
&\qquad+k_5[qp_{b_2}(gy,gx)+qp_{b_2}(gx,gy)]\\
&\quad\leqslant k_1[qp_{b_1}(gx,gy)+qp_{b_1}(gy,gx)]+k_2[qp_{b_1}(gx,gx)+qp_{b_1}(gy,gy)]\\
&\qquad+k_3[qp_{b_1}(gy,gy)+qp_{b_1}(gx,gx)]+k_4[qp_{b_1}(gx,gy)+qp_{b_1}(gy,gx)]\\
&\qquad+k_5[qp_{b_1}(gy,gx)+qp_{b_1}(gx,gy)]\\
&\quad=(k_1+k_4+k_5)[qp_{b_1}(gx,gy)+qp_{b_1}(gy,gx)].
\end{aligned}\tag{22}$$

Since $k_1+k_4+k_5<1$ from (22) we have

$$qp_{b_1}(gx,gy)=qp_{b_1}(gy,gx)=0.$$

By Lemma 2.3, we get $gx=gy$.

Finally, assume that g and F are w-compatible. Let $u=gx$, then we have $u=gx=F(x,y)=gy=F(y,x)$, so that

$$gu=ggx=g(F(x,y))=F(gx,gy)=F(u,u). \tag{23}$$

Consequently, (u,u) is a coupled coincidence point of F and g, and therefore (gu,gu) is a coupled point of coincidence of F and g, and by its uniqueness, we get $gu=gx$. Thus, we obtain $F(u,u)=gu=u$. Therefore, (u,u) is the unique common coupled fixed point of F and g. This completes the proof.

Corollary 3.2. *Let qp_{b_1} and qp_{b_2} be two quasi-partial b-metrics on X with different coefficients s_1 and s_2 respectively such that $s_2>s_1$ and $qp_{b_2}(x,y)\leqslant qp_{b_1}(x,y)$, for all $x,y\in X$. Let $F:X\times X\to X$, $g:X\to X$ be two mappings. Suppose that there exist $a_i\in[0,1)$ $(i=1,2,3,\ldots,10)$ with*

$$a_1+a_2+a_3+a_4+a_5+a_6+2s_2(a_7+a_8)+a_9+a_{10}<\frac{1}{s_1}$$

such that the condition

$$\begin{aligned} &qp_{b_1}(F(x,y),F(u,v))\\ &\quad\leqslant a_1qp_{b_2}(gx,gu)+a_2qp_{b_2}(gy,gv)+a_3qp_{b_2}(gx,F(x,y))\\ &\qquad+a_4qp_{b_2}(gy,F(y,x))+a_5qp_{b_2}(gu,F(u,v))+a_6qp_{b_2}(gv,F(v,u))\\ &\qquad+a_7qp_{b_2}(gx,F(u,v))+a_8qp_{b_2}(gy,F(v,u))+a_9qp_{b_2}(gu,F(x,y))\\ &\qquad+a_{10}qp_{b_2}(gv,F(y,x)) \end{aligned} \tag{24}$$

holds for all $x,y,u,v\in X$. Also suppose we have the following hypotheses:

(i) $F(X\times X)\subseteq g(X)$

(ii) *$g(X)$ is a complete subspace of X with respect to the quasi-partial b-metric qp_{b_1}.*

Then the mappings F and g have a coincidence point (x,y) satisfying $gx = F(x,y) = F(y,x) = gy$. Moreover, if F and g are w-compatible, then F and g have a unique common coupled fixed point of the form (u,u).

Proof. Given $x,y,u,v \in X$, it follows from (24) that

$$\begin{aligned} &qp_{b_1}(F(x,y),F(u,v)) \\ &\quad\leqslant a_1 qp_{b_2}(gx,gu) + a_2 qp_{b_2}(gy,gv) + a_3 qp_{b_2}(gx,F(x,y)) \\ &\qquad + a_4 qp_{b_2}(gy,F(y,x)) + a_5 qp_{b_2}(gu,F(u,v)) + a_6 qp_{b_2}(gv,F(v,u)) \\ &\qquad + a_7 qp_{b_2}(gx,F(u,v)) + a_8 qp_{b_2}(gy,F(v,u)) + a_9 qp_{b_2}(gu,F(x,y)) \\ &\qquad + a_{10} qp_{b_2}(gv,F(y,x)) \end{aligned} \tag{25}$$

and

$$\begin{aligned} &qp_{b_1}(F(y,x),F(v,u)) \\ &\quad\leqslant a_1 qp_{b_2}(gy,gv) + a_2 qp_{b_2}(gx,gu) + a_3 qp_{b_2}(gy,F(y,x)) \\ &\qquad + a_4 qp_{b_2}(gx,F(x,y)) + a_5 qp_{b_2}(gv,F(v,u)) + a_6 qp_{b_2}(gu,F(u,v)) \\ &\qquad + a_7 qp_{b_2}(gy,F(v,u)) + a_8 qp_{b_2}(gx,F(u,v)) + a_9 qp_{b_2}(gv,F(y,x)) \\ &\qquad + a_{10} qp_{b_2}(gu,F(x,y)). \end{aligned} \tag{26}$$

Adding inequality (25) to inequality (26), we get

$$\begin{aligned} &qp_{b_1}(F(x,y),F(u,v)) + qp_{b_1}(F(y,x),F(v,u)) \\ &\leqslant (a_1+a_2)[qp_{b_2}(gx,gu) + qp_{b_2}(gy,gv)] + (a_3+a_4)[qp_{b_2}(gx,F(x,y)) + qp_{b_2}(gy,F(y,x))] \\ &\quad + (a_5+a_6)[qp_{b_2}(gu,F(u,v)) + qp_{b_2}(gv,F(v,u) \\ &\quad + (a_7+a_8)[qp_{b_2}(gx,F(u,v)) + qp_{b_2}(gy,F(v,u))] \\ &\quad + (a_9+a_{10})[qp_{b_2}(gu,F(x,y)) + qp_{b_2}(gv,F(y,x))]. \end{aligned} \tag{27}$$

Therefore, letting $a_1+a_2 = k_1$, $a_3+a_4 = k_2$, $a_5+a_6 = k_3$, $a_7+a_8 = k_4$, $a_9+a_{10} = k_5$, the result follows from Theorem 3.1.

Corollary 3.3. *Let qp_{b_1} and qp_{b_2} be two quasi-partial b-metrics on X with different coefficients s_1 and s_2 respectively such that $s_2 > s_1$ and $qp_{b_2}(x,y) \leqslant qp_{b_1}(x,y)$, for all $x,y \in X$. Let*

$F : X \times X \to X$, $g : X \to X$ be two mappings. Suppose that there exists $k \in \left[0, \frac{1}{s_1}\right)$ such that the condition

$$qp_{b_1}(F(x,y),F(u,v)) + qp_{b_1}(F(y,x),F(v,u)) \leqslant k[qp_{b_2}(gx,gu) + qp_{b_2}(gy,gv)]$$

holds for all $x,y,u,v \in X$. Also, suppose we have the following hypotheses:

(i) $F(X \times X) \subseteq g(X)$

(ii) *$g(X)$ is a complete subspace of X with respect to the quasi-partial b-metric qp_{b_1}.*

Then the mappings F and g have a coincidence point (x,y) satisfying $gx = F(x,y) = F(y,x) = gy$. Moreover, if F and g are w-compatible, then F and g have a unique common coupled fixed point of the form (u,u).

Proof. By putting $k_1 = k$ and $k_2 = k_3 = k_4 = k_5 = 0$ in Theorem 3.1 we get the result.

Corollary 3.4. *Let qp_{b_1} and qp_{b_2} be two quasi-partial b-metrics on X with different coefficients s_1 and s_2 respectively such that $s_2 > s_1$ and $qp_{b_2}(x,y) \leqslant qp_{b_1}(x,y)$, for all $x,y \in X$. Let $F : X \times X \to X$, $g : X \to X$ be two mappings.Suppose that there exists $k \in \left[0, \frac{1}{2s_1s_2}\right)$ such that the condition*

$$qp_{b_1}(F(x,y),F(u,v)) + qp_{b_1}(F(y,x),F(v,u)) \leqslant k[qp_{b_2}(gx,F(u,v)) + qp_{b_2}(gy,F(v,u))]$$

holds for all $x,y,u,v \in X$. Also, suppose we have the following hypotheses:

(i) $F(X \times X) \subseteq g(X)$

(ii) *$g(X)$ is a complete subspace of X with respect to the quasi-partial b-metric qp_{b_1}.*

Then the mappings F and g have a coincidence point (x,y) satisfying $gx = F(x,y) = F(y,x) = gy$. Moreover, if F and g are w-compatible, then F and g have a unique common coupled fixed point of the form (u,u).

Proof. By putting $k_4 = k$ and $k_1 = k_2 = k_3 = k_5 = 0$ in Theorem 3.1 we get the result.

Example 3.5. *Let $X = [0,1]$ and two quasi-partial b-metrics qp_{b_1} and qp_{b_2} on X be given as*

$$qp_{b_1}(x,y) = |x-y| + x \quad and \quad qp_{b_2}(x,y) = \frac{1}{2}(|x-y| + x)$$

for all $x, y \in X$ with different coefficients s_1 and s_2 respectively. Also, define $F : X \times X \to X$ and $g : X \to X$ as $F(x,y) = \frac{x+y}{32}$ and $g(x) = \frac{x}{4}$ for all $x, y \in X$. Then

(i) *(X, qp_{b_1}) is a complete quasi-partial b-metric space.*

(ii) *$F(X \times X) \subseteq g(X)$*

(iii) *F and g is w-compatible.*

(iv) *For any $x, y, u, v \in X$, we have*

$$qp_{b_1}(F(x,y), F(u,v)) + qp_{b_1}(F(y,x), F(v,u)) \leqslant \frac{1}{2}(qp_{b_2}(gx, gu) + qp_{b_2}(gy, gv))$$

Proof. Here qp_{b_1} and qp_{b_2} are quasi-partial b-metrics with coefficients $s_1 = 1$ and $s_2 = 2$, respectively. Also $qp_{b_2}(x,y) \leq qp_{b_1}(x,y)$ for all $x, y \in X$. To prove (i) we proceed by observing that $qp_{b_1}(x,y) = |x-y| + x$ is a quasi-partial b-metric with $s = 1$. Hence a quasi-partial metric.

By Lemma 1.2, $(g(X), qp_{b_1})$ is complete if and only if $(g(X), p_{qp_{b_1}})$ is complete if and only if $(g(X), d_{p_{qp_{b_1}}})$ is complete. Here

$$p_{qp_{b_1}}(x,y) = \frac{1}{2}[qp_{b_1}(x,y) + qp_{b_1}(y,x)] = |x-y| + \frac{x+y}{2}$$

and

$$\begin{aligned} d_{p_{qp_{b_1}}}(x,y) &= 2p_{qp_{b_1}}(x,y) - p_{qp_{b_1}}(x,x) - p_{qp_{b_1}}(y,y) \\ &= 2|x-y| + x + y - x - y \\ &= 2|x-y|. \end{aligned}$$

Clearly, $(g(X), d_{p_{qp_{b_1}}})$ is a complete metric space being a compact space.

The proof of (ii) and (iii) are clear.

Next, we prove (iv). In fact, for $x, y, u, v \in X$, we have

$$\begin{aligned} &qp_{b_1}(F(x,y), F(u,v)) + qp_{b_1}(F(y,x), F(v,u)) \\ &= qp_{b_1}\left(\frac{x+y}{16}, \frac{u+v}{16}\right) + qp_{b_1}\left(\frac{y+x}{16}, \frac{v+u}{16}\right) \\ &= \left|\frac{x+y}{16} - \frac{u+v}{16}\right| + \frac{x+y}{16} + \left|\frac{y+x}{16} - \frac{v+u}{16}\right| + \frac{y+x}{16} \\ &= \frac{1}{16}[2|(x+y) - (u+v)| + 2(x+y)] \end{aligned}$$

$$\begin{aligned}
&\leqslant \frac{1}{8}\left[|x-u|+|y-v|+(x+y)\right] \\
&\leqslant \frac{1}{2}\left[\frac{1}{2}|x-u|+\frac{1}{2}|y-v|+\frac{x}{4}+\frac{y}{4}\right] \\
&= \frac{1}{2}\left(qp_{b_2}\left(\frac{x}{2},\frac{u}{2}\right)+qp_{b_2}\left(\frac{y}{2},\frac{v}{2}\right)\right) \\
&= \frac{1}{2}(qp_{b_2}(gx,gu)+qp_{b_2}(gy,gv)).
\end{aligned}$$

Thus, F and g satisfy all the hypotheses of Corollary 3.4. So, F and g have a unique common coupled fixed point. Here $(0,0)$ is the unique common coupled fixed point of F and g.

Competing Interests

The authors declare that there is no competing interests.

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