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COMMON FIXED POINT THEOREMS FOR FOUR WEAKLY COMPATIBLE MAPPINGS USING CLRg PROPERTY

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Abstract. The aim of this paper is to prove common fixed point theorems for weakly compatible maps in fuzzy metric spaces using the CLRg property, which generalizes the results of A.F. Roldan-Lopez-de-Hierro, W. Sintunavarat [17], and Manish Jain et al. [8].

Keywords: Fuzzy metric spaces; Common fixed point; Weakly compatible maps; Property (CLRg).

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1. Introduction

The first important result on fixed-point for contractive-type mappings was the well-known Banach fixed point theorem, published for the first time in 1922. In 1998, Jungck and Rhoades [10] introduced the notion of weakly compatible mappings and showed that compatible maps are weakly compatible but not conversely. The concept of fuzzy set was introduced by Zadeh [20] and after his work there has been a great endeavor to obtain fuzzy analogues of classical theories. In 1994, George and Veeramani [5] introduced the notion of fuzzy metric space and

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defined a Hausdorff topology on this fuzzy metric space. In 2006, Bhaskar and Lakshmikantham [3] introduces the concept of coupled fixed point for a given partially ordered set X. In 2002, Aamri and Moutawakil [2] introduced the notion of (E.A.)-property. In 2011, Sintunavarat and Kumam [16] coined the idea of common limit in the range property (called CLR) which relaxes the requirement of completeness.

The aim of this paper is to prove common fixed point theorems for weakly compatible maps in fuzzy metric spaces using the CLRg property, which generalizes the results of A.F. Roldan-Lopez-DeHierro, W. Sintunavarat [17], and M. Jain *et al.* [8].

2. Preliminaries

Definition 2.1. Let $f,g : X \to X$ be self-maps. A point $x \in X$ is called:

- a fixed point of f if $fx = x$.
- a coincidence point of f and g if $fx = gx$.
- a common fixed point of f and g if $fx = gx = x$.

Gnana-Bhaskar and Lakshmikantham (see [3]), given $F : X^2 \to X$ and $g : X \to X$, we will say that a point $(x,y) \in X^2$ is

- a coupled fixed point of F if $F(x,y) = x$ and $F(y,x) = y$;
- a coupled coincidence point of F and g if $F(x,y) = gx$ and $F(y,x) = gy$;
- a coupled common fixed point of F and g if $F(x,y) = gx = x$ and $F(y,x) = gy = y$.

Definition 2.2. [1] Let f and g be two self-maps defined on a set, then f and g are said to be weakly compatible if they commute at coincidence points. That is, if $fu = gu$ for some $u \in X$, then $fgu = gfu$.

The maps $F : X^2 \to X$ and $g : X \to X$ are weakly compatible (or the pair (F,g) is w-compatible) if

$$\left\{ \begin{array}{l} F(x,y) = gx \\ F(y,x) = gy \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} gF(x,y) = F(gx,gy) \\ gF(y,x) = F(gy,gx) \end{array} \right\}$$

Definition 2.3. (Schweizer and Sklar [18]) A map $* : I^2 \to I$ is called a triangular norm if it is associative, commutative, nondecreasing in both arguments and has 1 as identity.

A t-norm is continuous if it is continuous in I^2.

If $a_1, a_2, \ldots, a_m \in I^2$, then

$$\overset{m}{\underset{i=1}{*}} a_i = a_1 * a_2 * a_3 * \ldots * a_m \,.$$

Remark 2.4. If $m, n \in \mathbb{N}$, then $*^m(*^n a) = *^{mn} a$ for all $a \in I$.

Definition 2.5. (Schweizer and Sklar [18]) A t-norm $*$ is said to be positive if $a * b > 0$ whenever $a, b \in (0, 1]$.

Definition 2.6. (Hadzic and Pap [19]) A t-norm $*$ is said to be of H-type if the sequence $\{*^m a\}_{m=1}^{\infty}$ is equicontinuous at $a = 1$, i.e., for all $\varepsilon \in (0, 1)$, there exists $\eta \in (0, 1)$ such that if $a \in (1 - \eta, 1]$, then $*^m a > 1 - \varepsilon$ for all $m \in \mathbb{N}$.

The most important and well known continous t-norm of H-type is $* = \min$.

Definition 2.7. (Kramosil and Michalek [11], Grabiec [6]) A triple $(X, M, *)$ is called a fuzzy metric space (briefly, FMS) if X is an arbitrary non-empty set, $*$ is a continuous t-norm and $M : X \times X \times [0, \infty) \to I$ is a fuzzy set satisfying the following conditions, for each $x, y, z \in X$, and $t, s \geq 0$:

(KM-1) $M(x, y, 0) = 0$;

(KM-2) $M(x, y, t) = 1$ for all $t > 0$ if, and only if $x = y$;

(KM-3) $M(x, y, t) = M(y, x, t)$;

(KM-4) $M(x, y, \cdot) : [0, \infty) \to I$ is left-continuous;

(KM-5) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$.

In this case, we also say that (X, M) is a FMS under $*$. In the sequel, we will only consider FMS verifying:

(KM-6) $\lim_{t \to \infty} M(x, y, t) = 1$ for all $x, y \in X$.

Lemma 2.8. (Grabiec [6]) *If (X, M) is a FMS under some t-norm and $x, y \in X$, then $M(x, y, \cdot)$ is a non-decreasing function on $(0, \infty)$.*

Definition 2.9. (George and Veeramani [5]) A triple $(X, M, *)$ is called a fuzzy metric space (in the sense of George and Veeramani) if X is an arbitrary non-empty set, $*$ is a continuous t-norm and $M : X \times X \times (0,\infty) \to I$ is a fuzzy set satisfying, for each $x,y,z \in X$ and $t,s > 0$, conditions (KM-2), (KM-3) and (KM-5), and replacing (KM-1) and (KM-4) by the following properties:

(GV-1) $M(x,y,t) > 0$;

(GV-4) $M(x,y,\cdot) : (0,\infty) \to I$; is continuous.

Lemma 2.10. (Rodriguez-Lopez and Romaguera [12]) *If (X,M) is a GV-FMS under some t-norm, then M is a continuous mapping on $X^2 \times (0,\infty)$.*

Definition 2.11. (Jain *et al.* [8]) Let $(X,M,*)$ be a GV-FMS. Two maps $F : X \times X \to X$ and $g : X \to X$ satisfy CLRg property if there exist sequences $\{x_n\},\{y_n\} \subseteq X$ and points $p,q \in X$ such that $\{F(x_n,y_n)\}$ and $\{gx_n\}$ M-converges to gp and $\{F(y_n,x_n)\}$ and $\{gy_n\}$ M-converges to gq.

Let denote by Φ the family of all functions $\phi : (0,\infty) \to (0,\infty)$ such that the following properties are fulfilled.

(Φ_1) ϕ is non-decreasing

(Φ_2) ϕ is upper semi-continuous form the right;

(Φ_3) $\sum_{k=1}^{\infty} \phi^k(t) < \infty$ for all $t > 0$ (where $\phi^{k+1}(t) = \phi(\phi^k(t))$ for all $k \in \mathbb{N}$ and all $t > 0$).

In particular, by (Φ_3), it follows that $\phi(t) < t$ for all $t > 0$ and all $\phi \in \Phi$. Using this family, Jain et al. proved the following result.

Theorem 2.12. (Jain *et al.* [8, Theorem 3.2]) *Let $(X,M,*)$ be a GV-FMS, $*$ being continuous t-norm of H-type. Let $F : X \times X \to X$ and $g : X \times X$ be two mappings and there exists $\phi \in \Phi$ satisfying*

$$M(F(x,y),F(u,v),\phi(t)) \geq M(gx,gu,t) * M(gy,gv,t) \text{ for all } x,y,u,v \in X \text{ and all } t > 0,$$

with the following conditions:

- *the pair (F,g) is weakly compatible,*
- *the pair (F,g) satisfying CLRg property.*

Then F and g have a coupled coincidence point in X.

Moreover, there exists a unique point $x \in X$ such that $x = F(x,x) = gx$.

Definition 2.13. Let denote by Φ' the family of all functions $\phi : (0,\infty) \to (0,\infty)$ such that the following properties are fulfilled

(Φ'_1) $0 < \phi(t)$ for all $t > 0$.

(Φ'_2) $\lim_{k\to\infty} \phi^k(t) = 0$ for all $t > 0$.

By condition (Φ'_2) we also have that

$$\phi(t) < t \quad \text{for all } t > 0\,.$$

Obviously, $\Phi \subset \Phi'$. Using this family, A.F. Roldan-Lopez-DeHierro, W. Sintunavarat proved the following result.

Theorem 2.14. (A.F. Roldan-Lopez-DeHierro, W. Sintunavarat [17, Theorem 21]) *Let $(X,M,*)$ be a FMS such that $*$ is a t-norm of H-type and let $f,g : X \to X$ be weakly compatible mappings having the CLRg property. Assume that there exists $\phi \in \Phi'$ and $N \in \mathbb{N}$ such that*

$$M(fx, fy, \phi(t)) \geq *^N M(gx, gy, t) \textit{ for all } x,y \in X \textit{ and all } t > 0,$$

Then f and g have a unique common fixed point.

In fact, if $z \in X$ is any coincidence point of f and g, then $w = f(z) = g(z)$ is their only common fixed point.

Example 2.15. Let $(X,M,*)$ be a fuzzy metric space, $*$ being a continuous norm, $X = \left[0, \frac{1}{2}\right)$, and $M(x,y,t) = \dfrac{t}{t+|x-y|}$ for all $x,y \in X$ and $t > 0$.

Define the maps $f,g : X \to X$ by $f(x) = x^2$, $g(x) = \dfrac{x}{2}$ respectively.

Consider the sequence $\{x_n\} = \dfrac{1}{n}$. Then

$$\lim_{n\to\infty} f(x_n) = 0 = \lim_{n\to\infty} g(x_n)$$

Further, there exists the point $0 \in X$ such that $g(0) = 0$, so that the pair (f,g) satisfies (CLRg) property.

Also $f(0) = g(0)$ so '0' is the coincidence point of (f,g).

$$fg(0) = f(0) = 0 \quad \text{and} \quad gf(0) = g(0) = 0\,.$$

So the pair (f,g) is weakly compatible.

Lemma 2.16. (A.F. Roldan-Lopez-de-Hierro, W. Sintunavarat [17]) *Let* $(X,M,*)$ *be a FMS and let* $N \in \mathbb{N}$. *Consider the product spaces* $X^N = X \times X \times X \times \overset{N}{\ldots} \times X$ *of* N *identical copies of* X. *Let define* $M^N : X^N \times X^N \times [0,\infty) \to I$ *given by*

$$M^N(A,B,t) = \overset{N}{\underset{i=1}{*}} M(a_i,b_i,t) \quad \text{for all } A = (a_1,a_2,\ldots,a_N)$$
$$B = (b_1,b_2,\ldots,b_N) \in X^N \ \text{ and all } t \geq 0\,.$$

Then the following properties hold:

(i) $(X^N,M^N,*)$ *is also a FMS.*

(ii) *Let* $A_n = \{(a_n^1,a_n^2,\ldots,a_n^N)\}$ *be a sequence on* X^N *and* $A = (a_1,a_2,\ldots,a_N) \in X^N$. *Then* $\{A_n\} \xrightarrow{M^N} A$ *if, and only if* $\{a_n^i\} \xrightarrow{M} a_i$ *for all* $i \in \{1,2,\ldots,N\}$.

(iii) *If* $\{A_n = (a_n^1,a_n^2,\ldots,a_n^N)\}$ *is a sequence on* X^N, *then* $\{A_n\}$ *is* M^N*-Cauchy if, and only if,* $\{a_n^i\}$ *is M-Cauchy for all* $i \in \{1,2,\ldots,N\}$.

(iv) $(X,M,*)$ *is complete if, and only if,* $(X^N,M^N,*)$ *is complete.*

Lemma 2.17. [17] *Let* $F,F' : X^2 \to X$ *and* $g,g' : X \to X$, *let denote by* $T_F^2, T_{F'}'^2, G^2, G'^2 : X^2 \to X^2$ *the mappings*

$$\begin{cases} T_F^2(x,y) = (F(x,y),F(y,x)), \\ T_{F'}'^2(x,y) = (F'(x,y),F'(y,x)), \end{cases}$$

and

$$\begin{cases} G^2(x,y) = (gx,gy) \\ G'^2(x,y) = (g'x,g'y) \end{cases}$$

Let $F : X^2 \to X$ *and* $g : X \to X$, *a point* $(x_1,x_2) \in X^2$ *is:*

(i) *a coupled fixed point of* F *if, and only if, it is a fixed point of* T_F^2;

(ii) *a coupled common fixed point of* F *and* g *if, and only if, it is a common fixed point of* T_F^2 *and* G^2.

Lemma 2.18. [17] *The maps* $F : X^2 \to X$ *and* $g : X \to X$ *are w-compatible if, and only if,* T_F^2 *and* G^2 *are w-compatible.*

Lemma 2.19. [17] *Let (X,M) be a FMS under some continuous t-norm and let (X^2,M^2) is a FMS. Let $F : X^2 \to X$ and $g : X \to X$ be mappings. Then F and g satisfying the CLRg property if, and only if, T_F^2 and G^2 satisfy the CLRg property.*

3. Main Result

Theorem 3.1. *Let $(X,M,*)$ be a FMS such that $*$ is a t-norm of H-type and let f, g, h and $k : X \to X$ be self maps satisfying*

(i) $f(X) \subseteq h(X)$, $g(x) \subseteq k(X)$.

(ii) *there exists $\phi \in \Phi'$ such that*

$$M(fx,gy,\phi(t)) \geq *^N M(kx,hy,t) \text{ for all } x,y \in X \text{ and all } t > 0 \tag{1}$$

with the following conditions.

- *one of the pair (f,k) or (g,h) satisfy (CLRf) or (CLRg) property.*
- *The pairs (f,k) and (g,h) are weakly compatible.*

Then f, g, h and k have a unique common fixed point.

Proof. Firstly, suppose that the pair (f,k) satisfy (CLRf) property, so there exists a sequence $\{x_n\}$ in X such that

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} kx_n = fx \text{ for some } x \in X. \tag{2}$$

Since $f(X) \subseteq h(X)$, so there exists a sequence $\{y_n\}$ in X such that

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} hy_n = fx. \tag{3}$$

Now, we claim that $\lim_{n\to\infty} gy_n = fx$.

Let $\lim_{n\to\infty} gy_n = z$. Taking $x = x_n$, $y = y_n$, in (1),

$$M(fx_n,gy_n,\phi(t)) \geq *^N M(kx_n,hy_n,t).$$

$M(x,y,\cdot)$ is a non-decreasing function, so

$$M(fx_n,gy_n,t) \geq M(fx_n,gy_n,\phi(t)) \geq *^N M(kx_n,hy_n,t).$$

Taking limit as $n \to \infty$, we have

$$
\begin{aligned}
M(fx,z,t) &\geq M(fx,z,\phi(t)) \\
&\geq *^N M(fx,fx,t) \\
&= *^N 1 \\
&= 1*1*\cdots*1 \\
&= 1 \\
\text{that is} \quad M(fx,z,t) &\geq 1,
\end{aligned}
$$

which implies by (KM-2), that $z = fx$.

Subsequently, we have fx_n, kx_n, gy_n, hy_n converges to z. Since $f(X) \subseteq h(X)$, so there exists a point $v \in X$, such that $z = fx = hv$. We claim that $gv = fx = z$. For this,put $x = x_n$ and $y = v$ in (1), we have

$$
M(fx_n, gv, \phi(t)) \geq *^N M(kx_n, hv, t)
$$

$$
M(fx_n, gv, t) \geq M(fx_n, gv, \phi(t)) \geq *^N M(kx_n, hv, t).
$$

Letting $n \to \infty$, we have

$$
M(z, gv, t) \geq *^N M(z, z, t) = 1*1*\ldots*1 = 1,
$$

which implies by (KM-2), that $z = gv$. Therefore

$$
gv = hv = z. \tag{4}
$$

Hence v is a coincidence point of (g,h). As the pair (g,h) is weakly compatible, therefore

$$
\left.
\begin{aligned}
ghv &= hgv, \\
gz &= hz.
\end{aligned}
\right\} \tag{5}
$$

Since $g(X) \subseteq k(X)$, so there exists a point $u \in X$, such that

$$
gv = ku. \tag{6}
$$

We claim that $fu = ku = gv = z$.

Put $x=u$, $y=v$ in (1), we have

$$M(fu,gv,\phi(t)) \geq *^N M(ku,hv,t)$$
$$M(fu,gv,t) \geq M(fu,gv,\phi(t)) \geq *^N M(ku,hv,t),$$

which implies by (4) and (6), that

$$M(fu,z,t) \geq *^N M(z,z,t) = 1*1*1*\cdots*1 = 1\,.$$

Therefore, by (KM-2), we have

$$fu = ku = z. \tag{7}$$

Hence u is a coincidence point of (f,k). As the pair, (f,k) is weakly compatible

$$\left.\begin{aligned} fku &= kfu \\ fz &= kz \end{aligned}\right\} \tag{8}$$

Next, we claim that $z = kz$. Fix $\varepsilon, t > 0$ arbitrary.

As $*$ is of H-type, there exists $\eta \in (0,1)$ such that if $a \in (1-\eta,1]$. Then

$$*^m a > 1-\varepsilon \quad \text{for all } m \in \mathbb{N}. \tag{9}$$

Using (KM-6) of Definition 7, we know that $\lim_{s\to\infty} M(kz,z,s) = 1$. So there exists $t_0 > 0$ such that $M(kz,z,t_0) > 1-\eta$. Applying (9), we have that

$$*^m M(kz,z,t_0) > 1-\varepsilon \quad \text{for all } \ m \in \mathbb{N}. \tag{10}$$

We notice that

$$\begin{aligned} M(kz,z,\phi(t_0)) &= M(kfu,gv,\phi(t_0)) \\ &= M(fku,gv,\phi(t_0)) \\ &\geq *^N M(kku,hv,t_0) \\ &= *^N M(kz,z,t_0). \end{aligned}$$

Furthermore, by Remark 4,

$$\begin{aligned} M(kz,z,\phi^2(t_0)) &= M(kfu,gv,\phi(\phi(t_0))) \\ &= M(fku,gv,\phi(\phi(t_0))) \\ &\geq *^N M(kku,hv,\phi(t_0)) \\ &\geq *^N(*^N(M(kz,z,t_0))) \\ &\geq *^{N^2} M(kz,z,t_0)\,. \end{aligned}$$

Repeating this argument, it can be possible to prove, by induction, that

$$M(kz,z,\phi^m(t_0)) \geq *^{N^m} M(kz,z,t_0) \quad \text{for all } m \in \mathbb{N}. \tag{11}$$

As $\phi \in \Phi'$, then $\phi^m(t_0) \to 0$. also, as $t > 0$, there is $m_0 \in \mathbb{N}$ such that $\phi^{m_0}(t_0) < t$. It follows from (10), (11) and Lemma 2.8 that

$$M(kz,z,t) \geq M(kz,z,\phi^{m_0}(t_0)) \geq *^{N^{m_0}} M(kz,z,t_0) > 1-\varepsilon.$$

Since $\varepsilon, t > 0$ are arbitrary, we deduce that $M(kz,z,t) = 1$ for all $t > 0$, that is, $kz = z$. Therefore $z = kz = fz$.

Similarly, we can prove that $z = gz = hz$. Then evidently, from (5) and (8), f, g, h and k have a common fixed point. Therefore, $z = fz = kz = gz = hz$, that is z is the common fixed point of the maps f, g, h and k.

Uniqueness of z follows immediately from (KM-6) of Definition 7 and (1) of main result.

Corollary 3.2. *Let $(X,M,*)$ be a FMS such that $*$ is a t-norm of H-type and let $f,g,h : X \to X$ be self-maps satisfying:*

(i) $f(X) \subseteq h(X)$, $g(X) \subseteq k(X)$

(ii) *there exists $\phi \in \Phi'$ such that*

$$M(fx,gy,\phi(t)) \geq *^N M(hx,hy,t) \;\; \text{for all } \; x,y \in X \text{ and all } \; t > 0 \tag{12}$$

with the following conditions:

- *one of the pair (f,h) or (g,h) satisfying (CLRf) or (CLRg)-property.*
- *The pairs (f,h) and (g,h) are weakly compatible.*

Then f, g and h have a unique common fixed point.

Proof. By taking $k = h$ in Theorem 3.1, we get the result.

Corollary 3.3. *Let $(X, M, *)$ be a FMS such that $*$ is a t-norm of H-type and let $g, h : X \to X$ be self-maps satisfying:*

(i) $g(X) \subseteq h(X)$;

(ii) *there exists $\phi \in \Phi'$ such that*

$$M(gx, gy, \phi(t)) \geq *^N M(hx, hy, t) \ \textit{for all} \ x, y \in X \textit{ and all } t > 0 \tag{13}$$

with the following conditions:

- *The pair (g, h) satisfy (CLRg)-property.*
- *The pair (g, h) is weakly compatible.*

Then g and h have a unique common fixed point.

Proof. By taking $f = g$ and $k = h$ in Theorem 3.1, we get the desired result.

Corollary 3.4. *Let $(X, M, *)$ be a FMS such that $*$ is a t-norm of H-type and let $F, F' : X^2 \to X$ and $g, g' : X \to X$ be mapping satisfying:*

(i) $F(X \times X) \subseteq g(X)$, $F'(X \times X) \subseteq g'(X)$

(ii) *there exists $\phi \in \Phi'$ such that*

$$M(F(x,y), F'(u,v), \phi(t)) \geq M(gx, g'u, t) * M(gy, g'v, t)$$
$$\textit{for all} \ x, y, u, v \in X \ \textit{ and all } \ t > 0 \tag{14}$$

with the following conditions:

- *one of the pair (F, g) or (F', g') satisfy (CLRg) or (CLRg$'$)-property.*
- *The pairs (F, g) and (F', g') are weakly compatible.*

Then F, F', g and g' have a unique coupled common fixed point w.

Proof. By items (i) and (iv) of Lemma 2.16, $(X^2, M^2, *)$ is a complete FMS. By Lemma 2.17, (T_F^2, G^2) and $(T'^2_{F'}, G'^2)$ are w-compatiable. By Lemma 2.18, T_F^2 and G^2 satisfy the CLRg property.

Finally, the contractive condition (14) and the commutativity of $*$ yield:

$$\begin{aligned}
M^2(T_F^2(x,y),T_{F'}'^2(u,v),\phi(t)) &= M^2[(F(x,y),F(y,x)),(F'(u,v),F'(v,u),\phi(t))]\\
&= M(F(x,y),F'(u,v),\phi(t)*M(F(y,x),F'(v,u),\phi(t))\\
&\geq [M(gx,g'u,t)*M(gy,g'v,t)]*[M(gy,g'v,t)*M(gx,g'u,t)]\\
&= *^2[M(gx,g'u,t)*M(gy,g'v,t)]\\
&= *^2(M^2(gx,gy),(g'u,g'v),t)\\
&= *^2M^2(G^2(x,y),G'^2(u,v),t)\,.
\end{aligned}$$

Applying Theorem 3.1, T_F^2, $T_{F'}'^2$, G^2 and G'^2 have a unique common fixed, that is a point $w = (w_1,w_2) \in X^2$ such that $T_F^2 w = T_{F'}'^2 = G^2 w = G'^2 w = w$. By item (ii) of Lemma 2.16, w is the unique coupled common fixed point of F, F', g and g'.

Remark 3.5.

(i) If we put $f = g$ and $k = h$, then Theorem 3.1 reduces to Theorem 21 of [17].

(ii) If we put $F' = F$ and $g' = g$, then Corollary 3.4 reduces to Corollary 23 of [17].

Example 3.6. Let $X = (0,\infty)$ endowed with the standard fuzzy metric M associated to the Euclidean metric $d_e(x,y) = |x-y|$ on X, that is, M is given by

$$M(x,y,t) = \begin{cases} 0, & \text{if } t = 0,\\ \dfrac{t}{d_e(x,y)+t}, & \text{if } t > 0. \end{cases}$$

Then (X,M) is a FMS under $* = \min$.

Let $f,g,h,k : X \to X$ be given, for all $x \in X$, by

$$f(x) = \begin{cases} \dfrac{3}{2}, & \text{if } 0 < x \leq 1,\\ x, & \text{if } 1 < x < 2,\\ 2, & \text{if } x \geq 2, \end{cases}, \qquad g(x) = \begin{cases} \dfrac{5}{3}, & \text{if } 0 < x \leq 1,\\ x+1, & \text{if } 1 < x < 2,\\ 2, & \text{if } x \geq 2, \end{cases},$$

$$h(x)=\begin{cases}\dfrac{7}{2}, & \text{if } 0<x\leq 1,\\ 5-3x, & \text{if } 1<x<\dfrac{4}{3},\\ 2, & \text{if } x\geq \dfrac{4}{3},\end{cases}, \quad k(x)=\begin{cases}1, & \text{if } 0<x\leq \dfrac{1}{2},\\ \dfrac{2(2x-1)}{3}, & \text{if } \dfrac{1}{2}<x\leq \dfrac{7}{2},\\ 3.5, & \text{if } x>\dfrac{7}{2}.\end{cases}$$

Clearly, $f(X)=(1,2]\subseteq(1,2]\cup\left\{\dfrac{7}{2}\right\}=h(X)$, $g(X)=\left\{\dfrac{5}{3}\right\}\cup[2,3)\subseteq(0,4]=k(X)$, Let $x_n=\left\langle 2-\dfrac{1}{n}\right\rangle$ and $y_n=\left\langle 1+\dfrac{1}{n}\right\rangle$ be two sequences in X. The pairs (f,k) and (g,h) satisfy (E.A.) property. Also the pairs (f,k) or (g,h) satisfy (CLRf) or (CLRg) property. The pairs (f,k) and (g,h) are weakly compatible. All the conditions of Theorem 3.1 are satisfied. '2' is the unique common fixed point of f, g, h and k.

Conflict of Interests

The authors declare that there is no conflict of interests.

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