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# COMMON FIXED POINT RESULTS IN INTUITIONISTIC FUZZY METRIC SPACES UNDER NONLINEAR TYPE CONTRACTIONS

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**Abstract:** In this paper, we establish some new common fixed point theorems for a class of nonlinear contractions in intuitionistic fuzzy metric spaces.

**Keywords:** intuitionistic fuzzy metric space; common fixed point theorems; nonlinear contraction; condensation point.

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## 1. Introduction

The concept of fuzzy set was introduced by Zadeh in 1965 [1]. Ten year later, in 1975, Kramosil and Michalek introduced the notion of fuzzy metric space [2] and George and Veeramani modified the concept in 1994 [6]. In 2004, Park introduced the notion of intuitionistic fuzzy metric space. In his elegant article [7], he showed that for each intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$, the topology generated by the intuitionistic fuzzy metric $(M, N)$ coincides with the topology generated by the fuzzy metric M. Actually, Park's notion is useful in modelling some phenomena where it is necessary to study the relationship between two probability functions. For more details on intuitionistic fuzzy metric space and related results we refer the reader to [8-11, 15-18]. In this paper, we establish some new common fixed point theorems for a class of nonlinear contractions in intuitionistic fuzzy metric spaces.

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## 2. Preliminaries

Throughout this paper $\mathbb{R}$ and $\mathbb{R}_+$ will represents the set of real numbers and nonnegative real numbers, respectively.

The following two definitions are required in the sequel which can be found in [7].

**Definition 2.1** A binary operation $* : [0,1]\times[0,1] \to [0,1]$ is continuous t-norm if $*$ satisfying the following conditions:

1. $*$ is commutative and associative;
2. $*$ is continuous;
3. $a * 1 = a, \forall\, a \in [0,1]$;
4. $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d, \forall\, a, b, c, d \in [0,1]$.

**Definition 2.2** A binary operation $\diamond : [0,1]\times[0,1] \to [0,1]$ is continuous t-conorm if $\diamond$ satisfying the following conditions:

1. $\diamond$ is commutative and associative;
2. $\diamond$ is continuous;
3. $a \diamond 0 = a, \forall\, a \in [0,1]$;
4. $a \diamond b \leq c \diamond d$ whenever $a \leq c$ and $b \leq d, \forall\, a, b, c, d \in [0,1]$.

**Definition 2.3** A 5-tuple $(X, M, N, *, \diamond)$ is said to be an intuitionistic fuzzy metric space if X is an arbitrary set, $*$ is a continuous t-norm, $\diamond$ is a continuous t-conorm, and $M, N$ are two fuzzy sets on $X^2\times(0,\infty)$satisfying the following conditions, for all $x, y, z \in X$ and $s, t > 0$:

(i). $M(x,y,t) + N(x,y,t) \leq 1$;

(ii). $M(x,y,t) > 0$;

(iii). $M(x,y,t) = 1$ for all $t > 0$ if and only if $x = y$;

(iv). $M(x,y,t) = M(y,x,t)$;

(v). $M(x,y,t) * M(y,z,s) \leq M(x,z,t+s)$;

(vi). $M(x,y,.) : (0,\infty) \to [0,1]$ is left continuous;

(vii). $\lim_{t\to\infty} M(x,y,t) = 1$;

(viii). $N(x,y,t) > 0$;

(ix). $N(x,y,t) = 0$ for all $t > 0$ if and only if $x = y$;

(x). $N(x,y,t) = N(y,x,t)$;

(v). $N(x,y,t) \diamond N(y,z,s) \geq N(x,z,t+s)$;

(xi). $N(x,y,.): (0,\infty) \to [0,1]$ is right continuous;

(xii). $\lim_{t\to\infty} N(x,y,t) = 0$;

Then $(M,N)$ is called an intuitionistic fuzzy metric space on X. The functions $M(x,y,t)$ and $N(x,y,t)$ denote the degree of nearness and the degree on non-nearness between x and y with respect to t, respectively.

**Definition 2.4** Let $(X, M, N, *, \diamond)$ be an intuitionistic fuzzy metric space. Then

1) A sequence $\{x_n\}$ is said to be Cauchy sequence whenever $\lim_{m,n\to\infty} M(x_n,x_m,t) = 1$ and $\lim_{m,n\to\infty} N(x_n,x_m,t) = 0$ for all $t > 0$. That is, for each $\varepsilon > 0$ and $t > 0$, there exists a natural number $n_0$ such that $M(x_n,x_m,t) > 1-\varepsilon$ and $N(x_n,x_m,t) < \varepsilon$ for all $n,m \geq n_0$.
2) $(X, M, N, *, \diamond)$ is called complete whenever every Cauchy sequence is convergent with respect to the topology $\tau_{(M,N)}$.

**Remark 2.5** Note that, if $(M,N)$ is called an intuitionistic fuzzy metric space on X and $\{x_n\}$ is a sequence in X such that $\lim_{m,n\to\infty} M(x_n,x_m,t) = 1$ and $\lim_{m,n\to\infty} N(x_n,x_m,t) = 0$ for all $t > 0$ as from (i) of Definition 2.3, we know that $M(x,y,t) + N(x,y,t) \leq 1$ for all $x,y \in X$ and $t > 0$.

Let $(X, M, N, *, \diamond)$ be an intuitionistic fuzzy metric space. According to [8, 10], the fuzzy metric $(M,N)$ is called triangular whenever

$$\frac{1}{M(x,y,t)} - 1 \leq \frac{1}{M(x,z,t)} - 1 + \frac{1}{M(z,y,t)} - 1 \tag{2.1}$$

and

$$N(x,y,t) \leq N(x,z,t) + N(z,y,t) \tag{2.2}$$

for all $x,y,z \in X$ and $t > 0$.

**Example 2.6** Let $X = \{(0,0),(0,4),(4,0),(4,5),(5,4)\}$ endowed with the metric $d: X\times X \to [0,+\infty)$ given by

$$d\big((x_1,x_2),(y_1,y_2)\big) = |x_1-y_1| + |x_2-y_2| \tag{2.3}$$

for all $(x_1,x_2),(y_1,y_2) \in X$. Define intuitionistic fuzzy metric by

$$M\big((x_1,x_2),(y_1,y_2),t\big) = \frac{t}{t+d\big((x_1,x_2),(y_1,y_2)\big)} \tag{2.4}$$

and

$$N\big((x_1,x_2),(y_1,y_2),t\big) = \frac{d\big((x_1,x_2),(y_1,y_2)\big)}{t+d\big((x_1,x_2),(y_1,y_2)\big)} \tag{2.5}$$

for all $(x_1, x_2), (y_1, y_2) \in X$ and $t > 0$, where $a * b = \min\{a, b\}$ and $a \diamond b = \max\{a, b\}$. Then X is a complete triangular intuitionistic fuzzy metric space.

**Example 1.4** Let $X = \{0,1,3,4\}$ be a set with usual metric. Define intuitionistic fuzzy metric by

(2.6) $$M(x, y, t) = \frac{t}{t+|x-y|} \quad \text{and}$$

(2.7) $$N(x, y, t) = \frac{|x-y|}{t+|x-y|}$$

for all $x, y \in X$ and $t > 0$, where $a * b = \min\{a, b\}$ and $a \diamond b = \max\{a, b\}$. Then X is a complete triangular intuitionistic fuzzy metric space.

## 3. Main results

In this section, we establish that common fixed points for mapping satisfying nonlinear type contractions are proved in the frame of intuitionistic fuzzy metric spaces.

**Definition 3.1 (see [3])** There exists $\phi(t)$ that satisfy the condition $\phi'$, if one lets $\phi: [0, +\infty) \to [0, +\infty)$ be non-decreasing and non-negative, then $\lim \phi_n(t) = 0$, for a given $t > 0$.

**Lemma 3.2 (see [3]** If $\phi$ satisfy the condition $\phi'$, then $\phi(t) < t$, for a given $t > 0$.

**Lemma 2.3 (see [5])** Let $\mathcal{F}: \mathbb{R}_+^3 \to \mathbb{R}_+$, and satisfy the condition $\phi'$; for all $u, v \geq 0$, if $u \leq \mathcal{F}(v, v, u)$ or $u \leq \mathcal{F}(v, u, v)$ or $u \leq \mathcal{F}(u, v, v)$, then $u \leq \phi(v)$.

The following common fixed point theorem is our first main result.

**Theorem 3.4** Let $(X, M, N, *, \diamond)$ be a complete triangular intuitionistic fuzzy metric space and let S and T be self-mappings on X if

1) either S and T is continuous,
2) there exists $\mathcal{F}$ satisfying the condition $\phi'$ for all $x, y \in X$ and $t > 0$, such that

(3.1) $$\frac{1}{M(Sx,Ty,t)} - 1 \leq \mathcal{F}\left(\frac{1}{M(x,y,t)} - 1, \frac{1}{M(x,Sx,t)} - 1, \frac{1}{M(y,Ty,t)} - 1\right).$$

Then S and T have a unique common fixed point.

Proof: Let $x_0 \in X$ be arbitrary, T continuous in X, $\{x_n\}$ and $\{y_n\}$ the sequence of X, and

(3.2) $$x_n = (ST)^n(x_0) = ST(x_{n-1}), \; y_n = T(ST)^{n-1}(x_0), \;\; \forall\, n \in \mathbb{N}.$$

Obviously,

(3.3) $$y_n = T(x_{n-1}), \quad S(y_n) = x_n, \quad TS(y_n) = T(x_n) = y_{n+1}, \; \forall\, n \in \mathbb{N}.$$

From (3.1), we get

(3.4) $$\frac{1}{M(x_{n+1},y_{n+1},t)} - 1 = \frac{1}{M(ST(x_n),T(x_n),t)} - 1$$

$$\leq \mathcal{F}\left(\frac{1}{M(T(x_n),x_n,t)}-1,\frac{1}{M(T(x_n),ST(x_n),t)}-1,\frac{1}{M(x_n,T(x_n),t)}-1\right)$$
$$=\mathcal{F}\left(\frac{1}{M(y_{n+1},x_n,t)}-1,\frac{1}{M(y_{n+1},x_{n+1},t)}-1,\frac{1}{M(x_n,y_{n+1},t)}-1\right)$$

Hence by Lemma 3.3, we have

(3.5) $$\frac{1}{M(x_{n+1},y_{n+1},t)}-1\leq \phi\left(\frac{1}{M(x_n,y_{n+1},t)}-1\right)$$

Also,

(3.6) $$\frac{1}{M(x_n,y_{n+1},t)}-1=\frac{1}{M(x_n,T(x_n),t)}-1$$
$$=\frac{1}{M(S(y_n),T(x_n),t)}-1$$
$$\leq \mathcal{F}\left(\frac{1}{M(y_n,x_n,t)}-1,\frac{1}{M(y_n,S(y_n),t)}-1,\frac{1}{M(x_n,T(x_n),t)}-1\right)$$
$$=\mathcal{F}\left(\frac{1}{M(y_n,S(y_n),t)}-1,\frac{1}{M(y_n,S(y_n),t)}-1,\frac{1}{M(x_n,T(x_n),t)}-1\right)$$

Therefore,

$$\frac{1}{M(x_n,y_{n+1},t)}-1=\frac{1}{M(x_n,T(x_n),t)}-1\leq \phi\left(\frac{1}{M(y_n,S(y_n),t)}-1\right)$$

That is,

(3.7) $$\frac{1}{M(x_n,y_{n+1},t)}-1\leq \emptyset\left(\frac{1}{M(y_n,x_n,t)}-1\right)=\phi\left(\frac{1}{M(x_n,y_n,t)}-1\right)$$

From (3.5) and (3.7), we conclude that

(3.8) $$\frac{1}{M(x_{n+1},y_{n+1},t)}-1\leq \phi^2\left(\frac{1}{M(x_n,y_n,t)}-1\right)$$

Hence, by induction, for all $n\in\mathbb{N}$, we obtain that

(3.9) $$\frac{1}{M(x_{n+1},y_{n+1},t)}-1\leq \phi^{2n}\left(\frac{1}{M(x_1,y_1,t)}-1\right)=\phi^{2n}\left(\frac{1}{M(x_1,T(x_0),t)}-1\right)$$

In similar one obtains that

(3.10) $$\frac{1}{M(y_{n+1},x_n,t)}-1=\frac{1}{M(x_n,y_{n+1},t)}-1\leq \phi^{2n-1}\left(\frac{1}{M(x_1,y_1,t)}-1\right)$$
$$=\phi^{2n-1}\left(\frac{1}{M(x_1,T(x_0),t)}-1\right)$$

If $n\geq 2$, we have

(3.11) $$\frac{1}{M(x_{n+1},x_n,t)}-1\leq \frac{1}{M(x_{n+1},y_{n+1},t)}-1+\frac{1}{M(y_{n+1},x_n,t)}-1$$
$$=\phi^{2n}\left(\frac{1}{M(x_1,T(x_0),t)}-1\right)+\phi^{2n-1}\left(\frac{1}{M(x_1,T(x_0),t)}-1\right)$$
$$\leq 2\phi^{2n-1}\left(\frac{1}{M(x_1,T(x_0),t)}-1\right)$$

Note that the condition $\emptyset'$ we know, for $n, m \in \mathbb{N}$ such that $m > n$, we have

(3.12) $$\frac{1}{M(x_n, x_{n+m}, t)} - 1 \le \frac{1}{M(x_n, x_{n+1}, t)} - 1 + \frac{1}{M(x_{n+1}, x_{n+2}, t)} - 1 + \dots\dots\dots + \frac{1}{M(x_{n+m-1}, x_{n+m}, t)} - 1$$

$$\le 2\phi^{2n-1}\left(\frac{1}{M(x_1, T(x_0), t)} - 1\right) + 2\phi^{2(n+1)-1}\left(\frac{1}{M(x_1, T(x_0), t)} - 1\right) + \cdots\dots\dots + 2\phi^{2(n+m-1)-1}\left(\frac{1}{M(x_1, T(x_0), t)} - 1\right)$$

$$\le \sum_{i=2n-1}^{2(n+m-1)-1} \phi^i \left(\frac{1}{M(x_1, T(x_0), t)} - 1\right)$$

$$\le \sum_{i=2n-1}^{\infty} \phi^i \left(\frac{1}{M(x_1, T(x_0), t)} - 1\right) \to 0.$$

Hence, $\lim_{m,n\to\infty}\left(\frac{1}{M(x_n, x_{n+m}, t)} - 1\right) = 0$. Equivalently, $\lim_{m,n\to\infty} M(x_n, x_{n+m}, t) = 1$. This forces that $\{x_n\}$ is a Cauchy sequence in $X$. But $X$ is complete triangular intuitionistic fuzzy metric space, there must exist $x^* \in X$ such that

(3.13) $$\lim_{n\to\infty} x_n = x^*.$$

By continuity of T, we have

(3.14) $$\lim_{n\to\infty} y_n = \lim_{n\to\infty} T(x_{n-1}) = T\left(\lim_{n\to\infty} x_{n-1}\right) = Tx^*.$$

Now we want to show that $x^* = Sx^* = Tx^*$.First we show that $x^* = Tx^*$. Suppose on the contrary that $x^* \neq Tx^*$, then from (3.1) we get

(3.15) $$\frac{1}{M(x_{n+1}, y_{n+1}, t)} - 1 \le \mathcal{F}\left(\frac{1}{M(y_{n+1}, x_n, t)} - 1, \frac{1}{M(y_{n+1}, x_{n+1}, t)} - 1, \frac{1}{M(x_n, y_{n+1}, t)} - 1\right)$$

By taking limit $n \to +\infty$ in the above inequality and using (3.13) and (3.14), we obtain that

(3.16) $$\frac{1}{M(x^*, Tx^*, t)} - 1 \le \mathcal{F}\left(\frac{1}{M(Tx^*, x^*, t)} - 1, \frac{1}{M(Tx^*, x^*, t)} - 1, \frac{1}{M(x^*, Tx^*, t)} - 1\right)$$

Therefore,

(3.17) $$\frac{1}{M(x^*, Tx^*, t)} - 1 \le \phi\left(\frac{1}{M(x^*, Tx^*, t)} - 1\right) < \frac{1}{M(x^*, Tx^*, t)} - 1.$$

which is contradiction. Hence $x^* = Tx^*$and so $x^*$ is a fixed point of T. Again from (3.1), we have

(3.18) $$\frac{1}{M(Sx^*, x^*, t)} - 1 = \frac{1}{M(Sx^*, Tx^*, t)} - 1$$

$$\le \mathcal{F}\left(\frac{1}{M(x^*, x^*, t)} - 1, \frac{1}{M(x^*, Sx^*, t)} - 1, \frac{1}{M(x^*, Tx^*, t)} - 1\right)$$

$$= \mathcal{F}\left(0, \frac{1}{M(x^*, Sx^*, t)} - 1, 0\right)$$

Hence,

$$\frac{1}{M(Sx^*, x^*, t)} - 1 \leq \phi(0) = 0 \Rightarrow M(Sx^*, x^*, t) = 1 \Rightarrow Sx^* = x^*.$$

So $x^*$ is a common fixed point of S and T. To prove uniqueness, suppose that $x^* \neq x'$, such that $Sx^* = Tx^* = x^*$ and $Sx' = Tx' = x'$. From (3.1), we get

(3.19) $$\frac{1}{M(x^*, x', t)} - 1 = \frac{1}{M(Sx^*, Tx', t)} - 1$$
$$\leq \mathcal{F}\left(\frac{1}{M(x^*, x', t)} - 1, \frac{1}{M(x^*, Sx^*, t)} - 1, \frac{1}{M(x', Tx', t)} - 1\right)$$
$$= \mathcal{F}\left(\frac{1}{M(x^*, x', t)} - 1, \frac{1}{M(x^*, x^*, t)} - 1, \frac{1}{M(x', x', t)} - 1\right)$$
$$= \mathcal{F}\left(\frac{1}{M(x^*, x', t)} - 1, 0, 0\right)$$

Hence,

$$\frac{1}{M(x^*, x', t)} - 1 \leq \phi(0) = 0 \Rightarrow M(x^*, x', t) = 1 \Rightarrow x^* = x'.$$

So $x^*$ is a unique common fixed point of S and T.

Next, we prove the following theorem.

**Theorem 3.5** Let $(X, M, N, *, \diamond)$ be a complete triangular intuitionistic fuzzy metric space and let S and T be continuous mappings on X if

1) there exists $\mathcal{F}$ satisfying the condition $\phi'$ for all $x, y \in X$ with $x \neq y$ and $t > 0$, such that

(3.20) $$\frac{1}{M(Sx, Ty, t)} - 1 \leq \mathcal{F}\left(\frac{1}{M(x, y, t)} - 1, \frac{1}{M(x, Sx, t)} - 1, \frac{1}{M(y, Ty, t)} - 1\right).$$

2) there exists $x_0 \in X$ such that $\{(ST)^n(x_0)\}$ have a condensation point.

Then S and T have a unique common fixed point.

**Proof** Let $\{x_n\}$ , $\{y_n\}$ be the sequence of X, and for all n, $x_n \neq y_n$,

(3.21) $$x_n = (ST)^n(x_0) = ST(x_{n-1}),\ y_n = T(ST)^{n-1}(x_0),\ \ \forall\, n \in \mathbb{N}.$$

Obviously,

(3.22) $$y_n = T(x_{n-1}),\quad S(y_n) = x_n,\quad TS(y_n) = T(x_n) = y_{n+1},\ \forall\, n \in \mathbb{N}.$$

Suppose that $x^*$ is the condensation point of $\{x_n\}$; there exists the subsequence $\{x_{n_i}\}$ of $\{x_n\}$ such that $x_{n_i} \to x^*$. Since T is continuous, $\lim T(x_{n_i}) = T(x^*) = y^*$.

Consider

(3.23) $$\frac{1}{M(S(y^*),y^*,t)} - 1 = \frac{1}{M(S(y^*),T(x^*),t)} - 1$$
$$\leq \mathcal{F}\left(\frac{1}{M(y^*,x^*,t)} - 1, \frac{1}{M(y^*,S(y^*),t)} - 1, \frac{1}{M(x^*,T(x^*),t)} - 1\right)$$
$$= \mathcal{F}\left(\frac{1}{M(y^*,x^*,t)} - 1, \frac{1}{M(y^*,S(y^*),t)} - 1, \frac{1}{M(x^*,y^*,t)} - 1\right)$$

Hence by Lemma 3.3, we have

(3.24) $$\frac{1}{M(S(y^*),y^*,t)} - 1 \leq \phi\left(\frac{1}{M(x^*,y^*,t)} - 1\right)$$
$$\Rightarrow \quad \frac{1}{M(S(y^*),y^*,t)} - 1 \leq \frac{1}{M(x^*,y^*,t)} - 1.$$

Also consider

(3.25) $$\frac{1}{M(x_{n+1},y_{n+1},t)} - 1 = \frac{1}{M(ST(x_n),T(x_n),t)} - 1$$
$$\leq \mathcal{F}\left(\frac{1}{M(T(x_n),x_n,t)} - 1, \frac{1}{M(T(x_n),ST(x_n),t)} - 1, \frac{1}{M(x_n,T(x_n),t)} - 1\right)$$
$$= \mathcal{F}\left(\frac{1}{M(y_{n+1},x_n,t)} - 1, \frac{1}{M(y_{n+1},x_{n+1},t)} - 1, \frac{1}{M(x_n,y_{n+1},t)} - 1\right)$$

Hence by Lemma 3.3, we have

(3.26) $$\frac{1}{M(x_{n+1},y_{n+1},t)} - 1 \leq \phi\left(\frac{1}{M(x_n,y_{n+1},t)} - 1\right)$$

Also,

(3.27) $$\frac{1}{M(x_n,y_{n+1},t)} - 1 = \frac{1}{M(x_n,T(x_n),t)} - 1$$
$$= \frac{1}{M(S(y_n),T(x_n),t)} - 1$$
$$\leq \mathcal{F}\left(\frac{1}{M(y_n,x_n,t)} - 1, \frac{1}{M(y_n,S(y_n),t)} - 1, \frac{1}{M(x_n,T(x_n),t)} - 1\right)$$
$$= \mathcal{F}\left(\frac{1}{M(y_n,S(y_n),t)} - 1, \frac{1}{M(y_n,S(y_n),t)} - 1, \frac{1}{M(x_n,T(x_n),t)} - 1\right)$$

Therefore,

(3.28) $$\frac{1}{M(x_n,y_{n+1},t)} - 1 = \frac{1}{M(x_n,T(x_n),t)} - 1 \leq \phi\left(\frac{1}{M(y_n,S(y_n),t)} - 1\right)$$

That is,

(3.29) $$\frac{1}{M(x_n,y_{n+1},t)} - 1 \leq \phi\left(\frac{1}{M(y_n,x_n,t)} - 1\right) = \phi\left(\frac{1}{M(x_n,y_n,t)} - 1\right)$$

From (3.26) and (3.27), we conclude that

(3.30) $$\frac{1}{M(x_{n+1},y_{n+1},t)} - 1 \leq \phi^2\left(\frac{1}{M(x_n,y_n,t)} - 1\right) < \frac{1}{M(x_n,y_n,t)} - 1$$

Hence, $\left\{\left(\frac{1}{M(x_{n+1},y_{n+1},t)} - 1\right)\right\}$ is decreasing. Let $\epsilon = \lim\left(\frac{1}{M(x_{n+1},y_{n+1},t)} - 1\right)$.

Now,

$$(3.31)\qquad \lim\left(\frac{1}{M\left(y_{n_i+1},x_{n_i+1},t\right)}-1\right)=\lim\left(\frac{1}{M\left(T\left(x_{n_i}\right),x_{n_i+1},t\right)}-1\right)$$

$$=\frac{1}{M(y^*,x^*,t)}-1$$

$$\leq \lim\left(\frac{1}{M\left(y_{n_i},x_{n_i},t\right)}-1\right)=\epsilon.$$

Since $\{x_{n_i}\}$ is the subsequence of $\{x_n\}$, we have

$$(3.32)\qquad \frac{1}{M(S(y^*),y^*,t)}-1=\lim\left(\frac{1}{M\left(S\left(y_{n_i+1}\right),y_{n_i+1},t\right)}-1\right)$$

$$=\lim\left(\frac{1}{M\left(x_{n_i},y_{n_i},t\right)}-1\right)=\epsilon.$$

Hence, from (3.30) and (3.31) we conclude that

$$(3.33)\qquad \frac{1}{M(y^*,x^*,t)}-1\leq\frac{1}{M(S(y^*),y^*,t)}-1.$$

So $x^*=y^*$, $y^*$ is the fixed point of T. Similarly, $y^*$ is a fixed point of $S$. To prove uniqueness, suppose that $y^*\neq y'$, such that $Sy^*=Ty^*=y^*$ and $Sy'=Ty'=y'$. From (3.1), we get

$$(3.34)\qquad \frac{1}{M(y^*,y\prime,t)}-1=\frac{1}{M(Sy^*,Ty\prime,t)}-1$$

$$\leq\mathcal{F}\left(\frac{1}{M(y^*y\prime,t)}-1,\frac{1}{M(y^*,Sy^*,t)}-1,\frac{1}{M(y\prime,Ty\prime,t)}-1\right)$$

$$=\mathcal{F}\left(\frac{1}{M(y^*,y\prime,t)}-1,\frac{1}{M(y^*,y^*,t)}-1,\frac{1}{M(y\prime,y\prime,t)}-1\right)$$

$$=\mathcal{F}\left(\frac{1}{M(y^*,y',t)}-1,0,0\right)$$

Hence,

$$\frac{1}{M(y^*,y',t)}-1\leq\phi(0)=0\Rightarrow M(y^*,y',t)=1\Rightarrow y^*=y'.$$

So $y^*$ is a unique common fixed point of S and T.

## Conflict of Interests

The authors declare that there is no conflict of interests.

## Author's Contributions

Both authors contributed equally and significantly to writing this paper. Both authors read and approved the final manuscript.

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REFERENCES

[1] LA Zadeh: Fuzzy sets. Inf. Control, l8 (1965), 338-353.

[2] O. Kramosil, J. Michalek: Fuzzy metric and statistical metric spaces, Kybernetika, 11 (1975), 326-334.

[3] R. Chen, Fixed point Theory and Applications, National Defense Industry press, 2012.

[4] Y. Yu and D. Sheng, On the strong convergence of an algorithm about firmly pseudo-demi-contractive mappings for the split common fixed point problem, Journal of Applied Mathematics, 2012 (2012), Article ID 256930.

[5] D.F. Xia, M. F. Fu, and B. Jiang, Common fixed points for two self-mappings on symmetric sets and complete metric spaces, Advances in Mathematics, 36 (2007), no.4, 415-420.

[6] A. George, P. Veeramani : On some results in fuzzy metric spaces, Fuzzy Sets Syst., 64 (1994), 395-399.

[7] J. H. Park, Intuitionistic fuzzy metric spaces, Chaos, Solitons and Fractals, 22 (2004), no.5, 1039–1046.

[8] C. Alaca, D. Turkoglu, and C. Yildiz, Fixed points in intuitionistic fuzzy metric spaces, Chaos, Solitons & Fractals, 29 (2006), no. 5, 1073–1078.

[9] K. Atanassov: Intuitionistic fuzzy sets. Fuzzy Sets Syst., 20 (1986), 87-96.

[10] C. Ionescu, S. Rezapour and M .E. Samei, Fixed points of some new contractions on intuitionistic fuzzy metric spaces, Fixed Point Theory and Applications, 2013 (2013), Article ID 168.

[11] D. Coker, An introduction to intuitionistic fuzzy topological spaces, Fuzzy Sets and Systems, 88 (1997), no.1, 81–89.

[12] C. Di Bari, C. Vetro: A fixed point theorem for a family of mappings in a fuzzy metric space, Rend. Circ. Mat. Palermo, 52 (2003), 315-321.

[13] H. Karayilan, M. Telci: Common fixed point theorem for contractive type mappings in fuzzy metric spaces, Rend. Circ. Mat. Palermo 60 (2011), 145-152.

[14] D. Mihet: A Banach contraction theorem in fuzzy metric spaces, Fuzzy Sets Syst. 144 (2004), 431-439.

[15] J. S. Park, Y. C. Kwun, and J. H. Park, A fixed point theorem in the intuitionistic fuzzy metric spaces, Far East Journal of Mathematical Sciences, 16 (2005), no.2, 137–149.

[16] N. Hussain, S. Khaleghizadeh, P.Salimi, andAfrahA.N.Abdou, A New Approach to Fixed Point Results in Triangular Intuitionistic Fuzzy Metric Spaces, Abstract and Applied Analysis, 2014 (2014), Article ID 690139.

[17] M. Rafi and M. S. M. Noorani, Fixed point theorem on intuitionistic fuzzy metric spaces, Iranian Journal of Fuzzy Systems, 3 (2006), no.1, 23–29.

[18] B. Schweizer and A. Sklar, Statistical metric spaces, Pacific Journal of Mathematics, 10 (1960), 313–334.