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ABSORBING MAPS AND COMMON FIXED POINT THEOREM IN MENGER SPACE

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Abstract. In this paper, we prove a common fixed point theorem, using newly defined absorbing maps in Menger space. Our result generalizes the result of Razani et al [11].

Keywords: Menger space, absorbing map, reciprocal continuous, semi-compatible mapping.

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1. Introduction

There have been a number of generalizations of metric space. One such generalization is Menger space initiated by Menger [6]. The theory of probabilistic metric spaces is of fundamental importance in probabilistic function analysis. It is a probabilistic generalization in which we assign a distribution function $F_{x,y}$ to any two points x and y. Sehgal et al [14] initiated the study of fixed points in probabilistic metric spaces. Moreover, this theory is studied in Menger probabilistic metric space by many authors such as Schweizer and Sklar [16], Razani et-al [11] and etc. It is observed by many authors that contraction condition in metric space may be exactly translated into PM-space endowed with min

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norms. Mishra [7] introduced the concept of compatible self-maps in Menger space and proved the existence of a common fixed point of a pair of compatible maps using a contractive condition. Subsequently, Singh et al [17] introduced the concept of semi-compatible mapping in Menger space and proved a fixed point theorem using semi-compatibility. Ranadive et-al [12] introduced the concept of absorbing mapping in metric space and prove a common fixed point theorem in this space. Moreover they [12] observe that the new notion of absorbing map is neither a subclass of compatible maps nor a subclass of non-compatible maps. In [9], Ranadive et-al introduced absorbing maps in fuzzy metric space and prove a common fixed point theorem in this spaces. Recently we [13] introduce absorbing maps in Menger space and prove a fixed point theorem in this space.

In this paper we prove, a common fixed point theorem using reciprocally continuity and employing absorbing mapping with semi-compatibility.In order to do this, we recall some definitions,Lemmas, prepositions and known results from [7], [17], and [18].

2. Preliminaries

Definition 2.1. A mapping $F : R \to R^+$ is called a distribution if it is non decreasing left-continuous with $\inf\{F(t) : t \in R = 0\}$ and $\sup\{F(t) : t \in R = 1\}$.We shall denote by L the set of all distribution functions while H will always denote the specific distribution function denoted by

$$H(t) = \begin{cases} 0, & if\ t \leq 0, \\ 1, & if\ t > 0. \end{cases}$$

Definition 2.2. A probabilistic metric space (PM-space) is an ordered pair (X, F) where X is a nonempty set and $F : X \times X \to L$ is defined by $(p, q) \mapsto F_{p,q}$ where L is the set of all distribution function, i.e., $L = \{F_{p,q} : p, q \in X\}$, where the functions $F_{p,q}$ satisfy:

(1) $F_{p,q}(x) = 1$ for all $x > 0$, if and only if $p = q$,

(2) $F_{p,q}(0) = 0$,

(3) $F_{p,q} = F_{q,p}$,

(4) if $F_{p,q}(x) = 1$ and $F_{q,r}(y) = 1$ then $F_{p,r}(x + y) = 1$, for all $p, q, r \in X$ $x, y \geq 0$.

Definition 2.3. A mapping $t : [0,1] \times [0,1] \to [0,1]$ is called a t-norm if the following conditions are satisfied:

(1) $t(a,\ 1) = a$ for every $a \in [0,1]$,
(2) $t(a,\ b) = t(b,\ a)$ for every $a, b \in [0,1]$,
(3) $t(c,\ d) \geq t(a,\ b)$ for $c \geq a, d \geq b\ a, b, c, d \in [0,1]$;
(4) $t(t(a,\ b),\ c) = t(a,\ t(b,\ c))a, b, c \in [0,1]$.

Definition 2.4. A Menger probabilistic metric space is a triplet (X, F, t), where (X, F) is PM-space and t is a t-norm and the following inequality holds:

$$F_{p,r}(x+y) \geq t(F_{p,q}(x), F_{q,r}(y))$$

for all $p,\ q,\ r \in X$ and for all $x, y \geq 0$.

Definition 2.5. Let (X, F, t) be a Menger space with t-norm
(1) A sequence $\{x_n\}$ in X is said to convergent to x in X (written as $x_n \to x$) if for every $\in > 0$ and $\lambda > 0$, there exists a positive integer $N = N(\in,\ \lambda)$ such that $F_{x_n,x}(\in) > 1 - \lambda$ whenever $n \geq N$.
(2) The sequence $\{x_n\}$ in X is called a Cauchy sequence if for any $\in > 0$ and $\lambda > 0$, there is a positive integer $N = N(\in,\ \lambda)$ such that $F_{x_n,x_m}(\in) \geq 1 - \lambda$, whenever $n, m \geq N$.
(3) A Menger space $(X,\ F,\ t)$ is said to be complete if every Cauchy sequence in X if each Cauchy sequence in X is convergent to some point in X.

Definition 2.6. Two self maps A and S of a Menger space (X, F, t) are said to be reciprocal continuous if and only if $\lim_{n\to\infty} ASx_n = Az$ and $\lim_{n\to\infty} SAx_n = Sz$, whenever there exists a subsequence in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z$, for some z in X. It is well known that if A and S are both continuous then obviously they are reciprocal continuous but converse is not true.

Definition 2.7. Self maps A and S of a Menger space (X, F, t) are said to be weakly compatible if they commute at their coincidence points, i.e. if $Ap = Sp$ for some $p \in X$ then $ASp = SAp$.

Definition 2.8. Self maps A and S of a Menger space (X, F, t) are said to be compatible if $F_{ASp_n,SAp_n}(x) \to 1$ for all $x > 0$, whenever $\{p_n\}$ is a sequence in X such that $Ap_n, Sp_n(x) \to z$, for some z in X, as $n \to \infty$.

Definition 2.9. Self maps A and S of a Menger space (X, F, t) are said to be semi-compatible if $F_{ASp_n,Sz}(x) \to 1$ for all $x > 0$, whenever $\{p_n\}$ is a sequence in X such that $Ap_n, Sp_n \to z$ for some z in X, as $n \to \infty$. It follows that if (A, S) is semi-compatible and $Ay = Sy$, then $ASy = SAy$. Thus if the pair (A, S) is semi-compatible, then it is weak-compatible. But the converse is not true.

Recently we [13] define a new notion of mappings called absorbing mapping in Menger space as follows.

Definition 2.10. Let A and S be two self maps on a Menger space (X, F, t), then A is called S-absorbing if there exist some real number $R > 0$ such that $F_{S_x,SA_x}(t) \geq F_{S_x,A_x}(\frac{t}{R})$ for all x in X. Similarly S is called A-absorbing if there exist some real number $R > 0$ such that $F_{A_x,AS_x}(t) \geq F_{A_x,S_x}(\frac{t}{R})$ for all x in X. Thus we see that, the notion of absorbing maps is different from other generalizations of commutativity.

Lemma 2.11. *Let $\{p_n\}$ be a sequence in a Menger space (X, F, t) with continuous t-norm and $t(x, x) \geq x$. Suppose for all $x \in [0, 1]$, $\exists k \in (0, 1)$ such that for all $x > 0$ and $n \in N$,*

$$Fp_n, p_{n+1}(kx) \geq Fp_{n-1}, p_n(x)$$

Then $\{p_n\}$ is a Cauchy sequence in X.

Lemma 2.12. *Let (X, F, t) be a Menger space if there exists $k \in (0, 1)$ such that for $p, q \in X$ and $x > 0$*

$$F_{p,q}(kx) \geq F_{p,q}(x).$$

Then $p = q$.

The following theorem is proved by Razani, and Shirdaryazdi [11].

Theorem RS. *Let $P_1,\ P_2, ...,\ P_{2n},\ Q_0$ and Q_1 are self maps on a complete Menger space (X, F, Δ) with continuous t-norm with $\Delta(x, x) \geq x$ for all $x \in [0, 1]$,satisfying conditions:*

(1) $Q_0(X) \subseteq P_1P_3...P_{2n-1}(X), Q_1(X) \subseteq P_2P_4...P_{2n}(X)$,

(2) $P_2(P_4...P_{2n}) = (P_4...P_{2n})P_2$,

$P_2P_4(P_6...P_{2n}) = (P_6...P_{2n})P_2P_4$,

$\vdots$

$P_2...P_{2n-2}(P_{2n}) = (P_{2n})P_2...P_{2n-2}$,

$Q_0(P_4...P_{2n}) = (P_4...P_{2n})Q_0$,

$Q_0(P_6...P_{2n}) = (P_6...P_{2n})Q_0$,

$\vdots$

$Q_0P_{2n} = P_{2n}Q_0$,

$P_1(P_3...P_{2n-1}) = (P_3...P_{2n-1})P_1$,

$P_1P_3(P_5...P_{2n-1}) = (P_5...P_{2n-1})P_1P_3$,

$\vdots$

$P_1...P_{2n-3}(P_{2n-1}) = (P_{2n-1})P_1...P_{2n-3}$,

$Q_1(P_3...P_{2n-1}) = (P_3...P_{2n-1})Q_1$,

$Q_1(P_5...P_{2n-1}) = (P_5...P_{2n-1})Q_1$,

$\vdots$

$Q_1P_{2n-1} = P_{2n-1}Q_1$,

(3) $P_2...P_{2n}$ *or* Q_0 *is continuous,*

(4) $(Q_0, P_2...P_{2n})$ *is compatible and* $(Q_1, P_1...P_{2n-1})$ *is weakly compatible,*

(5) *There exists* $\alpha \in (0,\ 1)$ *such that*

$$F_{Q_0u,Q_1v}(\alpha x) \geq Min\{F_{P_2P_4...P_{2n}u,Q_0v}(x), F_{P_1P_3...P_{2n-1}u,Q_1v}(x), F_{P_1P_3...P_{2n-1}v,Q_0u}(\beta x),$$
$$F_{P_2P_4...P_{2n}u,Q_1v}((2-\beta)x), F_{P_2P_4...P_{2n}u,P_1P_3...P_{2n-1}}v(x)\},$$

for all $u, v \in X, \beta \in (0, 2)$ *and* $x > 0$. *Then* $P_1, P_2, ...P_{2n}, Q_0$ *and* Q_1 *have a unique common fixed point in* X.

3. Main results

In this paper, we prove a fixed point theorem using reciprocally continuity and employing absorbing mappings with semi-compatibility.

Theorem 3.1. *Let $R_1,\ R_2, ...,\ R_{2n},\ S_0$ and S_1 are self maps on a complete Menger space (X, F, Δ) with continuous t-norm with $\Delta(x, x) \geq x$ for all $x \in [0, 1]$, satisfying conditions:*

(1) $S_0(X) \subseteq R_1R_3...R_{2n-1}(X), S_1(X) \subseteq R_2R_4...R_{2n}(X)$,

(2) $R_2(R_4...R_{2n}) = (R_4...R_{2n})R_2$,

$R_2R_4(R_6...R_{2n}) = (R_6...R_{2n})R_2R_4$,

$\vdots$

$R_2...R_{2n-2}(R_{2n}) = R_{2n}(R_2...R_{2n-2})$,

$S_0(R_4...R_{2n}) = (R_4...R_{2n})S_0$,

$S_0(R_6...R_{2n}) = (R_6...R_{2n})S_0$,

$\vdots$

$S_0R_{2n} = R_{2n}S_0$,

$R_1(R_3...R_{2n-1}) = (R_3...R_{2n-1})R_1$,

$R_1R_3(R_5...R_{2n-1}) = (R_5...R_{2n-1})R_1R_3$,

$\vdots$

$R_1...R_{2n-3}(R_{2n-1}) = R_{2n-1}(R_1...R_{2n-3})$,

$S_1(R_3...R_{2n-1}) = (R_3...R_{2n-1})S_1$,

$S_1(R_5...R_{2n-1}) = (R_5...R_{2n-1})S_1$,

$\vdots$

$S_1R_{2n-1} = R_{2n-1}S_1$;

(3) S_1 *is* $(R_1...R_{2n-1})$ *absorbing ;*

(4) *There exists* $k \in (0,\ 1)$ *such that*

$F_{S_0p,\ S_1q}(kx) \geq min\{F_{R_2R_4...R_{2n}p,\ S_1q}((2-\beta)x), F_{R_2R_4...R_{2n}p,\ S_0p}(x),$

$F_{R_1R_3...R_{2n-1}q,\ S_1q}(x), F_{R_2R_4...R_{2n}p,\ R_1R_3...R_{2n-1}q}(x)\}$

for all $p, q \in X, \beta \in (0, 2)$ *and* $x > 0$*.If* $(S_0,\ R_2...R_{2n})$ *is reciprocal continuous , semi-compatible maps.Then* $R_1, R_2, ...R_{2n}, S_0$ *and* S_1 *have a unique common fixed point in* X*.*

Proof Let $x_0 \in X$, from condition (1) there exists $x_1, x_2 \in X$ such that $S_0x_0 = R_1R_3...R_{2n-1}x_1 = y_0$ and $S_1x_1 = R_2R_4...R_{2n}x_2 = y_1$,in general we can construct $\{x_n\}$ and $\{y_n\}$ in X such that $S_0x_{2n} = R_1R_3...R_{2n-1}x_{2n+1} = y_{2n}$ or $S_0x_{2n} = R_1R_3...R_{2n-1}x_{2n+1} =$

$R'_2x_{2n+1} = y_{2n}$ and $S_1x_{2n+1} = R_2R_4...R_{2n}x_{2n+2} = R'_1x_{2n+2} = y_{2n+1}$ for $n \in N$.

Putting $p = x_{2n}, q = x_{2n+1},\ x > 0$ and $\beta = 1-\alpha$ with $\alpha \in (0,1)$ in contractive condition, we have

$$F_{S_0x_{2n},\ S_1x_{2n+1}}(kx) \geq min\{F_{R'_1x_{2n},\ S_1x_{2n+1}}((2-(1-\alpha)x), F_{R'_1x_{2n},\ S_0x_{2n}}(x),$$
$$F_{R'_2x_{2n+1},\ S_1x_{2n+1}}(x), F_{R'_1x_{2n},\ R'_2x_{2n+1}}(x)\},$$

$$F_{y_{2n},y_{2n+1}}(kx) \geq min\{F_{y_{2n-1},y_{2n+1}}((1+\alpha)x), F_{y_{2n-1},y_{2n}}(x), F_{y_{2n},y_{2n+1}}(x), F_{y_{2n-1},y_{2n}}(x)\},$$

$$\geq min\{F_{y_{2n-1},y_{2n}}(x), F_{y_{2n},y_{2n+1}}(\alpha x), F_{y_{2n-1},y_{2n}}(x), F_{y_{2n-1},y_{2n}}(x)\},$$

As t-norm is continuous, letting $\alpha \to 1$, we have,

$$F_{y_{2n},y_{2n+1}}(kx) \geq min\{F_{y_{2n-1},y_{2n}}(x), F_{y_{2n},y_{2n+1}}(x)\}.$$

$$F_{y_{2n},y_{2n+1}}(kx) \geq F_{y_{2n-1},y_{2n}}(x).$$

Again $p = x_{2n+2}, q = x_{2n+1}$, in contractive condition, gives

$$F_{S_0x_{2n+2},\ S_1x_{2n+1}}(kx) \geq min\{F_{R'_1x_{2n+2},\ S_1x_{2n+1}}((1+\alpha)x), F_{R'_1x_{2n+2},\ S_0x_{2n+2}}(x),$$
$$F_{R'_2x_{2n+1},\ S_1x_{2n+1}}(x), F_{R'_1x_{2n+2},\ R'_2x_{2n+1}}(x)\},$$

$$F_{y_{2n+2},y_{2n+1}}(kx) \geq min\{F_{y_{2n+1},y_{2n+1}}((1+\alpha)x), F_{y_{2n+1},y_{2n+2}}(x),$$
$$F_{y_{2n},y_{2n+1}}(x), F_{y_{2n+1},y_{2n}}(x)\},$$

$$F_{y_{2n+2},y_{2n+1}}(kx) \geq min\{F_{y_{2n+1},y_{2n+2}}(x), F_{y_{2n},y_{2n+1}}(x)\}.$$

$$F_{y_{2n+2},y_{2n+1}}(kx) \geq F_{y_{2n},y_{2n+1}}(x).$$

Consequently; for all n we have

$$F_{y_n,y_{n+1}}(x) \geq min\{F_{y_{n-1},y_n}(k^{-1}x), F_{y_n,y_{n+1}}(k^{-m}x)\}.$$

So, $F_{y_n,y_{n+1}}(k^{-m}x) \to 1$ as $m \to \infty$ for any $t > 0$, it follows that

$$F_{y_n,y_{n+1}}(\alpha x) \geq F_{y_{n-1},y_n}(x),$$

for all $n \in N$ and $x > 0$. Therefore, by Lemma 2.11 $\{y_n\}$ is a Cauchy sequence in X. Since X is complete, therefore $\{y_n\} \to z$ in X and its subsequences $\{S_1x_{2n+1}\}, \{R_1R_3...R_{2n-1}x_{2n+1}\}, \{S_0x_{2n}\}$ and $\{R_2R_4...R_{2n}x_{2n}\} \to z$

Case (I): By the reciprocally continuity and Semi-compatibility of maps $(S_0, R_2R_4....R_{2n})$, we have $\lim_{n\to\infty} S_0(R_2R_4...R_{2n}x_{2n}) = S_0z$,

$$\lim_{n\to\infty} R_2R_4...R_{2n}(S_0x_{2n}) = R_2R_4...R_{2n}z = R_1'z,$$

and

$$\lim_{n\to\infty} S_0(R_2R_4...R_{2n}x_{2n}) = R_2R_4...R_{2n}z = R_1'z$$

which implies that $S_0z = R_1'z$.We claim $S_0z = R_1'z = z$.

Step(i): By using contractive condition, with $p = z$, $q = x_{2n+1}$ and $\beta = 1$, $R_1' = R_2R_4...R_{2n}, R_2' = R_1R_3...R_{2n-1}$ we have

$$F_{S_0z,\ S_1x_{2n+1}}(kx) \geq min\{F_{R_1'z,\ S_1x_{2n+1}}(x), F_{R_1'z,\ S_0z}(x), F_{R_2'x_{2n+1},\ S_1x_{2n+1}}(x), F_{R_1'z,\ R_2'x_{2n+1}}(x)\},$$

Letting $n \to \infty$, we get

$$F_{S_0z,z}(kx) \geq min\{F_{S_0z,z(x)}, F_{S_0z,S_0z}(x), F_{z,z}(x), F_{S_0z,z}(x)\}.$$

Thus by Lemma 2.12, we have

$$S_0z = R_1'z = z.$$

Step (ii): Putting $p = R_4...R_{2n}z$, $q = x_{2n+1}$, $\alpha = 1$ with $R_1' = R_2R_4...R_{2n}$, $R_2' = R_1R_3...R_{2n-1}$, in contractive condition, we have

$$F_{S_0R_4...R_{2n}z,\ S_1x_{2n+1}}(kx) \geq min\{F_{R_1'R_4...R_{2n}z,\ S_1x_{2n+1}}(x), F_{R_1'R_4...R_{2n}z,\ S_0R_4...R_{2n}z}(x),$$
$$F_{R_2'x_{2n+1},\ S_1x_{2n+1}}(x), F_{R_1'R_4...R_{2n}z,\ R_2'x_{2n+1}}(x)\},$$

Letting $n \to \infty$, we see that

$$F_{R_4...R_{2n}z,z}(kx) \geq min\{F_{R_4...R_{2n}z,z}(x), F_{R_4...R_{2n}z,R_4...R_{2n}z(x)}, F_{z,z}(x), F_{R_4...R_{2n}z,z}(x)\},$$

$$F_{R_4...R_{2n}z,z}(kx) \geq F_{R_4...R_{2n}z,z}(x).$$

Because $R_2(R_4...R_{2n}) = (R_4...R_{2n})R_2$ and $S_0(R_4...R_{2n}) = (R_4...R_{2n})S_0$, in (2); So $R_4...R_{2n}z = z$. Therefore $R_2z = z$. Continuing this procedure, we can obtain the following result:

$$S_0z = z = R_2z = R_4z = ... = R_{2n}z = z.$$

Since $S_0(X) \subseteq R_1R_3...R_{2n-1}(X)$,there exists $u \in X$, such that

$$z = S_0z = R_1R_3...R_{2n-1}u \ or \ z = S_0z = R'_2u.$$

Step (iii): Putting $p = x_{2n}, q = u, R'_1 = R_2R_4...R_{2n}$ and $R'_2 = R_1R_3...R_{2n-1}$ with $\beta = 1$ in contractive condition, we have

$$F_{S_0x_{2n},\ S_1u}(kx) \geq min\{F_{R'_1x_{2n},\ S_1u}(x), F_{R'_1x_{2n},\ S_0x_{2n}}(x), F_{R'_2u,\ S_1u}(x), F_{R'_1x_{2n}z,\ R'_2u}(x)\},$$

Letting $n \to \infty$, we obtain that

$$F_{z, S_1u}(kx) \geq min\{F_{z, S_1u}(x), F_{z,z}(x), F_{z, S_1u}(x), F_{z,z}(x)\}.$$

Therefore $z = S_1u$. Hence $z = S_1u = R'_2u = R_1R_3...R_{2n-1}u$.

Since S_1 is $R_1R_3...R_{2n-1}$-absorbing, we have,

$$F_{R_1R_3...R_{2n-1}u, R_1R_3...R_{2n-1}S_1u}(x) \geq F_{R_1R_3...R_{2n-1}u, S_1u}(x/R) \geq 1.$$

$$\Rightarrow R_1R_3...R_{2n-1}u = R'_2u = R'_2S_1u.$$

Therefore

$$z = R'_2z$$

Step(iv): Putting $p = x_{2n}, q = z$ with $\beta = 1$ and $R'_1 = R_2R_4...R_{2n}$ and $R'_2 = R_1R_3...R_{2n-1}$ in contractive condition, we have

$$F_{S_0x_{2n},\ S_1z}(kx) \geq \min\{F_{R'_1x_{2n},\ S_1z}(x), F_{R'_1x_{2n},\ S_0x_{2n}}(x), F_{R'_2z,\ S_1z}(x), F_{R'_1x_{2n},\ R'_2z}(x)\}.$$

Letting $n \to \infty$, we get

$$F_{z,\ S_1z}(kx) \geq min\{F_{z,\ S_1z}(x), F_{z,\ z}(x), F_{z,\ S_1z}(x), F_{z,\ z}(x)\},$$

Therefore

$$z = S_1z = R'_2z$$

Step (v): Putting $p = x_{2n}, q = R_3...R_{2n-1}$ with $\beta = 1$ and $R'_1 = R_2R_4...R_{2n}$ and $R'_2 = R_1R_3...R_{2n-1}$ in contractive condition, we have

$$F_{S_0x_{2n},\ S_1R_3...R_{2n-1}z}(kx) \geq min\{F_{R'_1x_{2n},\ S_1R_3...R_{2n-1}z}(x), F_{R'_1x_{2n},\ S_0x_{2n}}(x),$$
$$F_{R'_2R_3...R_{2n-1}z,\ S_1R_3...R_{2n-1}z}(x), F_{R'_1x_{2n},\ R'_2R_3...R_{2n-1}z}(x)\},$$

Again letting $n \to \infty$, so that

$$F_{z,\ R_3...R_{2n-1}z}(kx) \geq min\{F_{z,\ R_3...R_{2n-1}z}(x), F_{z,\ z}(x), F_{R_3...R_{2n-1}z,\ R_3...R_{2n-1}z}(x), F_{z,\ R_3...R_{2n-1}z}(x)\},$$

$$F_{z,\ R_3...R_{2n-1}z}(kx) \geq min\{F_{z,\ R_3...R_{2n-1}z}(x)\}$$

Therefore

$$z = R_3...R_{2n-1}z.$$

Because $R_1\{R_3...R_{2n-1}\} = \{R_3...R_{2n-1}\}R_1$ and $S_1\{R_3...R_{2n-1}\} = \{R_3...R_{2n-1}\}S_1$, we obtain $R_3...R_{2n-1}z = z$. Therefore $R_1z = z$. Continuing this procedure, we obtain the following results; $S_1 = R_1z = R_3z = ... = R_{2n-1}z$. So,

$$S_0z = S_1z = R_1z = R_2z = ... = R_{2n-1} = R_{2n}z = z.$$

Uniqueness: Let w be another common fixed point of $S_0, S_1, R_1R_3...R_{2n-1}$ and $R_2R_4...R_{2n}$ putting $p = z$ and $q = w$ with $\beta = 1$, $R'_1 = R_2R_4...R_{2n}$, $R'_2 = R_1R_3...R_{2n-1}$ in contractive condition, we have

$$F_{S_0z,\ S_1w}(kx) \geq min\{F_{R'_1z,\ S_1w}(x), F_{R'_1z,\ S_0z}(x), F_{R'_2w,\ S_1w}(x), F_{R'_1z,\ R'_2w}(x)\},$$

$$F_{z,w}(kx) \geq min\{F_{z,w}(x), F_{z,z}(x), F_{w,w}(x), F_{z,w}(x)\},$$

i.e. $z = w$. Hence z is unique common fixed point of maps.

Example: Let (X, d) be a metric space with the usual metric d where $X=$ [0,1] and $(X, F, *)$ be the induced Menger space with $F_{x,y}(t) = H(t - d(x, y))$ for all $x, y \in X, t > 0$. Clearly $(X, F, *)$ is complete Menger space where t-norm * is defined by $a * b = min\{a, b\}$ for all $a, b \in [0, 1]$.

Let S_0, S_1, R'_1 and R'_2 be maps from X into it self defined as $S_0(X) = x/6$, $S_1(X) =$

0, $R_1'(X) = x/3$, $R_2'(X) = x/2$ $\forall x \in X$.Then $S_0(X)=$ [0,1/6] $\subseteq$ [0,1/2]$=$ $R_2'(X)$ and $S_1(X) = \{0\} \subseteq$ [0,1/3] $= R_1'(X)$. Clearly all conditions of main Theorem are satisfied if $lim_{n\to\infty}x_n = 0$, where $\{x_n\}$ is a sequence in X such that $lim_{n\to\infty}S_0x_n = lim_{n\to\infty}R_2'x_n = 0$ and $lim_{n\to\infty} = S_1x_n = lim_{n\to\infty}R_1'x_n = 0$ for some $0 \in X$. Thus all condition of the main Theorem are satisfied. This completes the proof.

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