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SOME EXTENDED RESULTS USING NON COMPATIBLE MAPPINGS IN FUZZY METRIC SPACES

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Abstract. In this paper, we proved the common fixed point theorems for the class of non compatible mappings in the framework of fuzzy metric spaces. There are additional restrictions imposed on the t-norm.

Keywords: fuzzy metric space; fixed point; compatible mappings; continuous t-norm .

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1. Introduction and preliminaries

Zadeh [1] first introduced the concept of fuzzy sets in 1965 and the concept of fuzzy metric space was introduced by Kramosil and Michalek [2]. After that, they have been intensively studied by many authors, see, for example, Deng [3], Kaleva and Seikkala [4]. Recently, many authors investigated common fixed point theorems of nonlinear operators in fuzzy metric

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spaces; see, Pant [5], Singh and Chouhan [6]. In [7], O'Regan and Abbas obtained some necessary and sufficient conditions for the existence of common fixed points in the framework of fuzzy metric spaces. Cho *et al.* [8] established fixed point theorems for mappings which satisfy generalized contractive conditions in fuzzy metric spaces. In this paper, we proved the common fixed point theorems for the class of non compatible mappings in the framework of fuzzy metric spaces. There are additional restrictions imposed on the t-norm. Our results mainly extend the corresponding results in Cho *et al.* [8] Beg and Abbas [9], and Singh [10].

Definition 1.1. Let X be a set. A fuzzy set A in X is a function with domain X and values in $[0,1]$. A mapping $*:[0,1]\times[0,1]\to[0,1]$ is called a continuous t-norm if $([0,1],*)$ is an abelian topological monoid with unit 1 such that

$$a*b\le c*d,\ \texttt{for}\ a\le c,\ b\le d.$$

Examples of t-norms are $a*b=\min\{a,b\}$ (minimum t-norm), $a*b=ab$ (product t-norm), and $a*b=\max\{a+b-1,0\}$ (Lukasiewicz t-norm).

Definition 1.2. The 3-tuple $(X,M,*)$ is called fuzzy metric space if X is an arbitrary set $*$ is a continuous t-norm and M is a fuzzy set in $X^2\times[0,\infty)$ satisfying the following conditions:

(a) $M(x,y,t)>0$,
(b) $M(x,y,t)=1$ for all $t>0$ if and only if $x=y$,
(c) $M(x,y,t)=M(y,x,t)$,
(d) $M(x,y,t)*M(y,z,s)\le M(x,z,t+s)$,
(e) $M(x,y,\cdot):[0,\infty)\to[0,1]$ is a continuous function, for all $x,y,z\in X$ and $t,s>0$.

Note that, $M(x,y,t)$ can be considered as the definition of nearness between x and y with respect to t. It is known that $M(x,y,\cdot)$ is nondecreasing for all $x,y\in X$ [5].

Let $(X,M,*)$ be a fuzzy metric space. For $t>0$, the open ball $B(x,r,t)$ with center $x\in X$ and radius $0<r<1$ is defined by

$$B(x,r,t)=\{y\in X:M(x,y,t)>1-r\}.$$

The collection $\{B(x,r,t); x \in X,\ 0 < r < 1,\ t > 0\}$ is a neighborhood system for a topology T on X induced by the fuzzy metric M. This topology is Hausdorff and first countable.

A sequence $\{x_n\}$ in X converges to x [6] if and only if for each $\varepsilon > 0$ and each $t > 0$ there exists $n_0 \in N$

$$M(x_n, x, t) > 1 - \varepsilon$$

for all $n \geq n_0$.

Definition 1.3. Mappings f and g from a fuzzy metric space $(X, M, *)$ into itself are weakly compatible if they commute at their coincidence point, that is $fx = gx$ implies that $fgx = gfx$.

It is known that a pair $\{f, g\}$ of compatible maps is weakly compatible but converse is not true in general.

Definition 1.4. Mappings A, B, S and T on a fuzzy metric space $(X, M, *)$ are said to satisfy common (EA) property if there exists sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} By_n = \lim_{n\to\infty} Ty_n = x$$

for some $x \in X$.

For more on (EA) and common (EA) properties, we refer to [1] and [9]. Note that compatible, noncompatible, compatible of type (I) and compatible of type (II) satisfy (EA) property but converse is not true in general.

Definition 1.5. Let f and g be self maps on a fuzzy metric space $(X, M, *)$. They are compatible or asymptotically commuting if for all $t > 0$,

$$\lim_{n\to\infty} M(fgx_n, gfx_n, t) = 1$$

whenever $\{x_n\}$ is a sequnce in X such that

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = z, \ \texttt{for some} \ z \in X.$$

Mappings f and g are noncompatible maps, if there exists a sequence $\{x_n\}$ in X such that

$$\lim_{n\to\infty} fx_n = p = \lim_{n\to\infty} gx_n,$$

but either

$$\lim_{n\to\infty} M(fgx_n, gfx_n, t) \neq 1$$

or the limit does not exists for all $p \in X$.

2. Main results

Theorem 2.1. *Let $(X, M, *)$ be a fuzzy metric space. Let A, B, S, T be maps from X into itself with $A^a(X) \subseteq T^t(X)$ and $B^b(X) \subseteq S^s(X)$ and there exists a constant $k \in (0,1)$ such that*

$$\begin{aligned} M(A^a x, B^b y, kt) \geq \psi\Big\{ & M(S^s x, T^t y, t), M(A^a x, S^s x, t), M(B^b y, T^t y, t), \\ & M(B^b y, S^s x, t), M(A^a x, T^t y, t), M(T^t x, B^b y, t)\Big\} \end{aligned} \tag{1}$$

for all $x, y \in X$, $t > 0$, $\forall \in \phi$ and $a, b, s, t \in N$. Then A, B, S, T have a unique common fixed point in X provided the pair $\{A, S\}$ or $\{B, T\}$ satisfies (EA) property. One of $A^a(X), T^t(X), B^b(X), S^s(X)$ is a closed subset of X and the pairs $\{B, T\}$ and $\{A, S\}$ are weakly compatible.

Proof. Since pair $\{B, T\}$ satisfies property (EA), we see there exists a sequence $\{x_n\}$ in X such that

$$\lim_{n\to\infty} B^b x_n = z = \lim_{n\to\infty} T^t x_n = \lim_{n\to\infty} T^t y_n.$$

Now $B^b(X) \subseteq S^s(X)$ implies that there exists a sequence $\{y_n\}$ in X such that

$$B^b x_n = S^s y_n = T^t y_n$$

for $x = y_n$ and $y = x_n$. Hence, (1) becomes

$$\begin{aligned} M(A^a y_n, B^b x_n, kt) \geq \psi\Big\{ & M(S^s y_n, T^t x_n, t), M(A^a y_n, S^s y_n, t), M(B^b x_n, T^t x_n, t), \\ & M(B^b x_n, S^s y_n, t), M(A^a y_n, T^t x_n, t), M(T^t y_n, B^b x_n, t)\Big\}. \end{aligned}$$

Taking $\lim_{n\to\infty}$ on the above inequality, one has

$$M\lim_{n\to\infty}(A^a y_n, B^b x_n, kt) \geq \psi \lim_{n\to\infty} M(S^s y_n, T^t x_n, kt)$$

$$\lim_{n\to\infty} M(A^a y_n, S^s y_n, t), \lim_{n\to\infty} M(B^b x_n, T^t x_n, t)$$

$$\lim_{n\to\infty} M(B^b x_n, S^s y_n, t), \lim_{n\to\infty} M(A^a y_n, T^t x_n, t)$$

$$\lim_{n\to\infty} M(T^t y_n, B^b x_n, t).$$

Therefore, one has

$$M(A^a y_n, z, kt) \geq \psi\Big(M(z,z,t), M(A^a y_n, z, t), M(z,z,t), M(z,z,t),$$
$$M(A^a y_n, z, t), M(z,z,t).$$

Since ψ is increasing in each of its coordinate and $\psi(t,t,t,t,t) > t$ for all $t \in [0,1]$, one has

$$M(\lim_{n\to\infty} A^a y_n, z, kt) > M(\lim_{n\to\infty} A^a y_n, z, t).$$

From Mishra, Sharma and Singh [11], we have

$$\lim_{n\to\infty} A^a y_n = z.$$

Suppose $S^s(X)$ in a closed subspace of X. Then $z = S^s u = T^t u$ and $Bx_{2n+1} = Tx_{2n+1}$, Replacing x by u and y by x_{2n+1} in (1), we find

$$M(A^a u, Bx_{2n+1}, kt) \geq \phi\Big\{M(S^s u, T^t x_{2n+1}, t), M(A^a u, S^s u, t),$$
$$M(B^b x_{2n+1}, T^t x_{2n+1}, t), M(B^b x_{2n+1}, S^s u, t),$$
$$M(A^a u, T^t x_{2n+1}, t), M(T^t u, B^b x_{2n+1}, t).$$

Letting $n \to \infty$, we obtain

$$M(A^a u, z, kt) \geq \psi\Big\{M(z,z,t), M(A^a u, z, t), M(z,z,t), M(z,z,t),$$
$$M(A^a u, z, t), M(z,z,t)$$
$$> M(A^a u, z, t),$$

which implies $A^a u = z$. Hence

$$A^a u = z = S^s u.$$

Since

$$A^a(X) \subseteq T^t(X),$$

there exists $v \in X$ such that $z = T^t v$. Following the argument similar to those given above, we obtain

$$z = B^b v = T^t v.$$

Since u is the coincidence point of the pair (A,S), therefore

$$S^s A^a u = A^a S^s u \texttt{ and } A^a z = S^s z.$$

Note that

$$\begin{aligned} M(A^a z, B^b v, kt) \geq \psi\Big\{ & M(S^s z, T^t v, t), M(A^a z, S^s z, t), M(B^b v, T^t v, t), \\ & M(B^b v, S^s z, t), M(A^a z, T^t v, t), M(T^t z, B^b v, t). \end{aligned}$$

Putting

$$Sz = Az = Bv = Tv = Tz,$$

we have

$$\begin{aligned} M(A^a z, B^b v, kt) \geq\geq \psi\Big\{ & M(A^a z, z, t), M(A^a z, A^a z, t), M(z, z, t), \\ & M(z, Az, t), M(Az, z, t), M(z, z, t) \\ \geq & (Az, z, t). \end{aligned}$$

This is a contradiction. Hence $z = A^a z = S^s z$. Similarly, we can prove $z = B^b z = T^t z$. This completes the proof.

Conflict of Interests

The authors declare that there is no conflict of interests.

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