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FIXED POINT THEOREMS UNDER F-CONTRACTION IN ULTRAMETRIC SPACE

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Abstract. In this paper, we establish some results on coincidence and common fixed points for a single-valued map, a pair of single-valued maps and of single-valued map with a multi-valued map in an Ultrametric space which satisfy F-contraction. Our theorems generalize and extent the theorems of Mishra and Pant[Generalization of some fixed point theorems in ultrametric spaces, Adv. Fixed Point Theory, 4(1)(2014), 41- 47], thereby generalizes some known results in the existing literature.

Keywords: coincidence point; fixed point; F-contraction; spherically complete and coincidentally commuting.

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1. Introduction

Gajic[2] studied the fixed point theorem of contractive type maps on a spherically complete ultrametric space which is a generalization of the Banach fixed point theorem and later in 2002[3], he generalized [2] to a multi-valued map. Later in 2007, Rao et al.[9] proved some coincidence point theorems for three and four self maps in the Ultrametric space. In

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2008, Rao and Kishore[8] proved some common fixed point theorems for a pair of maps of Jungck type on a spherically complete ultrametric space. And one of the recent generalization of the Banach Contraction Principle for single-valued maps on a complete metric space called F-contraction was introduced by Wardowski[11]. Following Wardowski, Minak et al.[6], Cosentino and vetro[4], Piri and Kumam[7] generalized this F-contraction to Hardy-Roger type and Suzuki type F-contraction. In [1] Altun et al. extend the F-contraction from single-valued maps to multi-valued maps.

In this paper we use the F-contraction for a single-valued map, a pair of single-valued maps and of single-valued map with multi-valued maps in an ultrametric space and prove some fixed point theorems.

2. Preliminaries

We denote the class of all nonempty compact subsets of X by $C(X)$ and for $A,B \in C(X)$, the Hausdorff metric induced by d is defined by

$$\mathrm{H}(A,B) = max\{\sup_{x\in A} d(x,B), \sup_{y\in B} d(y,A)\}$$

where $d(x,A) = inf\{d(x,y) : y \in A\}$

Definition 1.1.[10] Let (X,d) be a metric space. If the metric d satisfies strong triangle inequality $d(x,y) \leq max\{d(x,z), d(z,y)\}$, for all x,y,z in X, then d is called an ultra metric on X and (X,d) is called an ultra metric space.

Definition 1.2.[10] An ultra metric space is said to be spherically complete if every shrinking collection of balls in X has a non empty intersection.

Definition 1.3. Let (X,d) be an ultra metric space, an element $x \in X$ is said to be a coincidence point of a multi-valued map $g : X \to C(X)$ and a single-valued map $T : X \to X$ if $Tx \in gx$.

Definition 1.4.[8] Let (X,d) be an ultra metric space. Let $g : X \to C(X)$ be a multi-valued map and $T : X \to X$ be a single-valued map then g and T are said to be coincidentally commuting at $z \in X$ if $Tz \in gz$ implies $Tgz \subseteq gTz$.

Let R be the set of all real numbers and $\mathscr{F}$ be the set of all functions $F:(0,\infty)\to$ R satisfying the following conditions:

(a) F is strictly increasing, that is, for all $\alpha,\beta\in(0,+\infty)$ if $\alpha<\beta$ then $F(\alpha)<F(\beta)$.

(b) For each sequence $\{\alpha_n\}$ of positive numbers, the following holds:
$lim_{n\to\infty}\alpha_n=0$ if and only if $lim_{n\to\infty}F(\alpha_n)=-\infty$.

(c) There exist $k\in(0,1)$ such that $lim_{\alpha\to 0^+}(\alpha^k F(\alpha))=0$.

Definition 1.5.[11] Let (X, d) be a metric space. A self map T on X is an F-contraction, if $F\in\mathscr{F}$ and there exist $\tau>0$ such that

$$d(Tx,Ty)>0\Rightarrow\tau+F(d(Tx,Ty))\leq F(d(x,y)) \tag{1}$$

for all $x,y\in X$.

3. Main results

In this section, we prove the existence of coincidence point and fixed point for a single-valued map and a pair of single-valued maps.

Theorem 3.1. Let (X,d) be a spherically complete ultra metric space and let $T:X\to X$ be a single-valued map such that

$$d(Tx,Ty)>0\Rightarrow\tau+F(d(Tx,Ty))\leq F(max\{d(x,y),d(x,Tx),d(y,Ty)\}) \tag{2}$$

for all $x,y\in X$ where $F\in\mathscr{F}$, $\tau>0$. Then T has a unique fixed point.

Proof. For $\alpha\in X$, let $B_\alpha=B(\alpha,d(\alpha,T\alpha))$ denote the closed sphere with center at α and radius $d(\alpha,T\alpha)$. Let $\mathscr{A}$ be the collection of these spheres for all $\alpha\in X$. Then the relation $B_\alpha\leq B_\beta$ if and only if $B_\beta\subseteq B_\alpha$ is a partial order on $\mathscr{A}$.

Now, consider a totally ordered subfamily $\mathscr{A}_1$ of $\mathscr{A}$. Since (X,d) is spherically complete, we have $\bigcap_{B_\alpha\in\mathscr{A}_1} B_\alpha=B\neq\phi$.

Let $\beta \in B$ and $B_\alpha \in \mathscr{A}_1$. If $x \in B_\beta$, then

$$\begin{aligned} d(x,\beta) &\le d(\beta,T\beta) \\ &\le max\{d(\beta,\alpha),d(\alpha,T\beta)\}) \\ &\le max\{d(\beta,\alpha),d(\alpha,T\alpha),d(T\alpha,T\beta)\}) \\ &= max\{d(\alpha,T\alpha),d(T\alpha,T\beta)\}. \end{aligned}$$

Case:1 If $d(T\alpha,T\beta) \le d(\alpha,T\alpha)$ then $d(x,\beta) \le d(\alpha,T\alpha)$.

Case:2 If $d(a,T\alpha) \le d(T\alpha,T\beta)$ then

$$\begin{aligned} d(x,\beta) \le d(\beta,T\beta) &\le d(T\alpha,T\beta) \\ &< max\{d(\alpha,\beta),d(\alpha,T\alpha),d(\beta,T\beta)\}) \quad from(2) \\ &= max\{d(\alpha,T\alpha),d(\beta,T\beta)\} \end{aligned}$$

If $d(\beta,T\beta) \le d(\alpha,T\alpha)$ then $d(x,\beta) \le d(\alpha,T\alpha)$. And if $d(\alpha,T\alpha) \le d(\beta,T\beta)$ then $d(\beta,T\beta) < d(\beta,T\beta)$, a contradiction. Therefore $d(x,\beta) \le d(\alpha,T\alpha)$ for $x \in B_\beta$.

Now,

$$\begin{aligned} d(x,\alpha) &\le max\{d(x,\beta),d(\beta,\alpha)\} \\ &\le max\{d(x,\beta),d(\alpha,,T\alpha)\} = d(a,T\alpha). \end{aligned}$$

Hence $d(x,\alpha) \le d(\alpha,T\alpha)$. Thus, $x \in B_\alpha$. Hence $B_\beta \subseteq B_\alpha$ for any B_α in $\mathscr{A}_1$. Thus B_β is the upper bound for the family $\mathscr{A}_1$ in $\mathscr{A}$ and hence by Zorn's lemma $\mathscr{A}$ has a maximum element say B_z for some $z \in X$.

Now to prove that $z = Tz$. Suppose that $z \ne Tz$.

Using (2)

$$\begin{aligned} \tau + F(d(Tz,T^2z)) &\le F(max\{d(z,Tz),d(Tz,T^2z),d(z,T^2z)\}) \\ &\le F(max\{d(z,Tz),d(Tz,T^2z)\}) \end{aligned}$$

which implies $F(d(Tz,T^2z)) < F(d(z,Tz))$ and since F is a increasing function,

$$d(Tz,T^2z) < Fd(z,Tz). \tag{3}$$

Now if $y \in B_{Tz}$, then $d(y,Tz) \le d(Tz,T^2z) < d(z,Tz)$.

And $d(y,z) \le max\{d(y,Tz),d(Tz,z)\} = d(z,Tz)$ which implies $y \in B_z$. Hence $B_{Tz} \subseteq B_z$.

Since $d(z,Tz) > d(Tz,T^2z)$ implies $z \notin B_{Tz}$. Therefore $B_{Tz} \nsubseteq B_z$. This is a contradiction to the maximality of B_z. Hence $z = Tz$, z is the common fixed point of T.

Uniqueness: Let w be a different fixed point. By(2) for $w \neq z$ we have

$$\begin{aligned}\tau + F(d(z,w)) = \tau + F(d(Tz,Tw) &\leq F(max\{d(z,w),d(z,Tz),d(w,Tw)\}) \\ &= F(d(z,w))\end{aligned}$$

which implies $F(d(z,w)) < F(d(z,w))$, that is, $d(z,w) < d(z,w)$, a contradiction. Therefore $w = z$, hence z is the unique common fixed point of T.

Corollary 3.2. *Theorem 3.1 holds if the F-contraction (2) is replaced by*

$$\begin{aligned}d(Tx,Ty) > 0 \Rightarrow \tau + F(d(Tx,Ty)) \leq F(max\{&d(x,y),d(x,Tx),d(y,Ty),d(x,Ty), \\ &d(y,Tx)\})\,\forall x,y \in X.\end{aligned} \tag{4}$$

Proof. By strong triangle inequality, we have that $d(x,Ty) \leq max\{d(x,y),d(y,Ty)\}$ and $d(y,Tx) \leq max\{d(y,x),d(x,Tx)\}$. Hence we conclude that (4) implies (2).

Theorem 3.3. Let (X,d) be a spherically complete Ultra metric space. If T and $S$are single-valued maps on X satisfying

(i) $T(X) \subseteq S(X)$,
(ii) $d(Tx,Ty) > 0 \Rightarrow \tau + F(d(Tx,Ty)) \leq F(max\{d(Sx,Sy),d(Sx,Tx),d(Sy,Ty)\})$, for all $x,y \in X$, $x \neq y$ where $F \in \mathscr{F}$, $\tau > 0$.

then there exists $z \in X$ such that $Sz = Tz$. Further if T and S are coincidentally commuting at z then z is the unique common fixed point of T and S.

Proof. For $\alpha \in X$, let $B_\alpha = B(S\alpha, d(S\alpha, T\alpha))$ denote the closed sphere with center at $S\alpha$ and radius $d(S\alpha,T\alpha)$. Let $\mathscr{A}$ be the collection of these spheres for all $\alpha \in X$. Then the relation $B_\alpha \leq B_\beta$ if and only if $B_\beta \subseteq B_\alpha$ is a partial order on $\mathscr{A}$.
Now, consider a totally ordered subfamily $\mathscr{A}_1$ of $\mathscr{A}$. Since (X,d) is spherically complete, we have $\bigcap_{B_\alpha \in \mathscr{A}_1} B_\alpha = B \neq \phi$.
Let $S\beta \in B$ and $B_\alpha \in \mathscr{A}_1$. Then $S\beta \in B_\alpha$. Hence $d(S\beta,S\alpha) \leq d(S\alpha,T\alpha)$.

Using (ii)

$$\tau + F(d(T\alpha, T\beta)) \leq F(max\{d(S\alpha, S\beta), d(S\alpha, T\alpha), d(S\beta, T\beta)\})$$

$$F(d(T\alpha, T\beta)) < F(max\{d(S\alpha, S\beta), d(S\alpha, T\alpha), d(S\beta, T\beta)\}).$$

Hence

(5) $$d(T\alpha, T\beta) < max\{d(S\alpha, S\beta), d(S\alpha, T\alpha), d(S\beta, T\beta)\}.$$

If $\alpha = \beta$ then $B_\alpha = B_\beta$. Let $\alpha \neq \beta$ and $x \in B_\beta$. Then

$$\begin{aligned} d(x, S\beta) &\leq d(S\beta, T\beta) \\ &\leq max\{d(S\beta, S\alpha), d(S\alpha, T\beta)\}) \\ &\leq max\{d(S\beta, S\alpha), d(S\alpha, T\alpha), d(T\alpha, T\beta)\}) \\ &= max\{d(S\alpha, T\alpha), d(T\alpha, T\beta)\} \\ &< max\{d(S\alpha, S\beta), d(S\alpha, T\alpha), d(S\beta, T\beta)\}) \quad from (5) \end{aligned}$$

Thus $d(x, S\beta) \leq d(S\alpha, T\alpha)$.

Now,

$$\begin{aligned} d(x, S\alpha) &\leq max\{d(x, S\beta), d(S\beta, S\alpha)\} \\ &\leq d(S\alpha, T\alpha). \end{aligned}$$

Thus, $x \in B_\alpha$. Hence $B_\beta \subseteq B_\alpha$ for any B_α in $\mathscr{A}_1$. Thus B_β is the upper bound for the family $\mathscr{A}_1$ in $\mathscr{A}$ and hence by Zorn's lemma $\mathscr{A}$ has a maximum element say B_z for some $z \in X$.

Now to prove that $Sz = Tz$. Suppose that $Sz \neq Tz$.

Since $Tz \in TX \subseteq SX$, there exists a $w \in X$ such that $Tz = Sw$. Clearly $z \neq w$.

Consider,

$$\begin{aligned} \tau + F(d(Sw, Tw)) &= \tau + F(d(Tz, Tw) \\ &< F(max\{d(Sz, Sw), d(Sz, Tz), d(Sw, Tw)\}) \\ &= F(d(Sz, Sw)) \end{aligned}$$

which implies $F(d(Sw, Tw)) < F(d(Sz, Sw))$. Thus $d(Sw, Tw) < d(Sz, Sw)$. Hence $Sz \notin B_w$. Therefore $B_z \nsubseteq B_w$. This is a contradiction to the maximality of B_z. Hence $Sz = Tz$.

Since S and T are coincidentally commuting at z, $S^2z = S(Sz) = S(Tz) = T(Sz) = T^2z$.

Now to show that $Sz = z$. Suppose $Sz \neq z$, then we have,

$$\tau + F(d(TSz,Tz)) < F(max\{d(S^2z,Sz),d(S^2z,TSz),d(Sz,Tz)\})$$
$$= F(d(STz,Tz)).$$

Hence, we have $F(d(TSz,Tz)) < F(d(STz,Tz))$, which gives $(d(TSz,Tz)) < (d(STz,Tz))$, a contradiction. Hence $Sz = z$. Thus $z = Sz = Tz$, therefore z is the common fixed point of S and T.

Uniqueness: Let w be a different fixed point. For $w \neq z$ we have,

$$\tau + F(d(z,w)) = \tau + F(d(Tz,Tw)$$
$$\leq F(max\{d(Sz,Sw),d(Sz,Tz),d(Sw,Tw)\})$$
$$F(d(z,w)) < F(max\{d(Sz,Sw),d(Sz,Tz),d(Sw,Tw)\})$$
$$= F(d(z,w))$$

which implies $d(z,w) < d(z,w)$ a contradiction. Therefore $w = z$. Hence z is the unique common fixed point of S and T.

Corollary 3.4. *Theorem 3.3 holds if the condition (ii) of theorem 3.3 is replaced by generalized condition*

$$d(Tx,Ty) > 0 \Rightarrow \tau + F(d(Tx,Ty)) \leq F(max\{d(Sx,Sy),d(Sx,Tx),d(Sy,Ty),d(Sx,Ty), d(Sy,Tx)\}) \forall x,y \in X. \tag{6}$$

Proof. By strong triangle inequality we have $d(Sx,Ty) \leq max\{d(Sx,Sy),d(Sy,Ty)\}$ and $d(Sy,Tx) \leq max\{d(Sy,Sx),d(Sx,Tx)\}$. Hence (6) implies condition(ii) of theorem 3.3.

Remark 3.5. Taking $S = I$ Identity map in Theorem 3.3, we obtain Theorem 3.1.

Now we prove the existence of coincidence point and fixed point for a single-valued map and a multi-valued map, an extension of the Theorem 3.3.

Theorem 3.6. Let (X,d) be a spherically complete Ultra metric space. If $T : X \to X$ is a single-valued map and $g : X \to C(X)$ is a multi-valued map satisfying $gx \subseteq TX$ for all $x \in X$

and

$$H(gx,gy) > 0 \Rightarrow \tau + F(H(gx,gy)) \leq F(max\{d(Tx,Ty),d(Tx,gx),d(Ty,gy)\}) \tag{7}$$

for all $x,y \in X$ where $F \in \mathscr{F}$, $\tau > 0$, then there exists $z \in X$ such that $Tz \in gz$. Further if $d(Tx,Tu) \leq H(gTy,gu)$ for all $x,y,u \in X$ with $Tx \in gy$ and T and g are coincidentally commuting at z then Tz is the unique common fixed point of T and g.

Proof. For $\alpha \in X$, let $B_\alpha = B(T\alpha, d(T\alpha, g\alpha))$ denote the closed sphere with center at $T\alpha$ and radius $d(T\alpha, g\alpha)$. Let $\mathscr{A}$ be the collection of these spheres for all $\alpha \in X$. Then the relation $B_\alpha \leq B_\beta$ if and only if $B_\beta \subseteq B_\alpha$ is a partial order on $\mathscr{A}$.

Now, consider a totally ordered subfamily $\mathscr{A}_1$ of $\mathscr{A}$. Since (X,d) is spherically complete, we have $\bigcap_{B_\alpha \in \mathscr{A}_1} B_\alpha = B \neq \phi$.

Let $T\beta \in B$ and $B_\alpha \in \mathscr{A}_1$. Then $T\beta \in B_\alpha$. Hence $d(T\beta, T\alpha) \leq d(T\alpha, g\alpha)$.

If $\alpha = \beta$ then $B_\alpha = B_\beta$. Let $\alpha \neq \beta$ and $x \in B_\beta$. Then $d(x, T\beta) \leq d(T\beta, g\beta)$. Since $g\alpha$ is compact, there exists $v \in g\alpha$ such that $d(T\alpha, v) = d(T\alpha, g\alpha)$.

Using (7),

$$\begin{aligned}
\tau + F(H(g\alpha, g\beta)) &\leq F(max\{d(T\alpha, T\beta), d(T\alpha, g\alpha), d(T\beta, g\beta)\}) \\
&\leq F(max\{d(T\alpha, g\alpha), d(T\beta, g\beta)\}) \\
F(H(g\alpha, g\beta)) &< F(max\{d(T\alpha, g\alpha), d(T\beta, g\beta)\})
\end{aligned}$$

which implies $H(g\alpha, g\beta) < max\{d(T\alpha, g\alpha), d(T\beta, g\beta)\}$

Consider,

$$\begin{aligned}
d(T\beta, g\beta) &= \inf_{c \in g\beta} d(T\beta, c) \\
&\leq max\{d(T\beta, T\alpha), d(T\alpha, v), \inf_{c \in g\beta} d(v, c)\}) \\
&\leq max\{d(T\alpha, g\alpha), d(g\alpha, g\beta)\}) \\
&< max\{d(T\alpha, g\alpha), d(T\beta, g\beta)\}.
\end{aligned}$$

Thus $d(T\beta, g\beta) < d(T\alpha, g\alpha)$.

Now,

$$\begin{aligned} d(x,T\alpha) &\leq max\{d(x,T\beta),d(T\beta,T\alpha)\} \\ &\leq max\{d(T\beta,g\beta),d(T\alpha,g\alpha)\} \\ &= d(T\alpha,g\alpha) \end{aligned}$$

Thus, $x \in B_\alpha$. Hence $B_\beta \subseteq B_\alpha$ for any B_α in $\mathscr{A}_1$. Thus B_β is the upper bound for the family $\mathscr{A}_1$ in $\mathscr{A}$ and hence by Zorn's lemma $\mathscr{A}$ has a maximum element say B_z for some $z \in X$.

Now to prove that $Tz \in gz$. Suppose that $Tz \notin gz$.

Since gz is compact, there exists $k \in gz$ such that $d(Tz,gz) = d(Tz,k)$. Since $gx \subseteq TX$, there exists a $w \in X$ such that $k = Tw$. Therefore $d(Tz,gz) = d(Tz,Tw)$. Clearly $z \neq w$.

Using (7)

$$\begin{aligned} \tau + F(H(gz,gw)) &\leq F(max\{d(Tz,Tw),d(Tz,gz),d(Tw,gw)\}) \\ F(H(gz,gw)) &< F(max\{d(Tz,Tw),d(Tz,gz),d(Tw,gw)\}) \end{aligned}$$

which implies $H(gz,gw) < max\{d(Tz,Tw),d(Tz,gz),d(Tw,gw)\}$.

Consider,

$$\begin{aligned} d(Tw,gw) &\leq H(gz,gw) \\ &< max\{d(Tz,Tw),d(Tz,gz),d(Tw,gw)\} \\ &= d(Tz,Tw). \end{aligned}$$

Thus $d(Tw,gw) < d(Tz,Tw)$ which implies $Tz \notin B_w$. Therefore $B_z \nsubseteq B_w$. This is a contradiction to the maximality of B_z. Hence $Tz \in gz$.

Consider, $d(Tz,T^2z) = d(Tz,TTz) \leq H(gTz,gTz) = 0$ which implies $TTz = Tz$. Thus $Tz = TTz \in Tgz \subseteq gTz$. Hence Tz is the common fixed point of T and g.

Uniqueness: Let Tw be a another fixed point such that $Tz \neq Tw$.

Using (7)

$$\begin{aligned} \tau + F(H(gTz,gw)) &\leq F(max\{d(TTz,Tw),d(TTz,gTz),d(Tw,gw)\}) \\ F(H(gTz,gw)) &< F(max\{d(TTz,Tw),d(TTz,gTz),d(Tw,gw)\}) \end{aligned}$$

Which implies $H(gTz,gw) < max\{d(TTz,Tw),d(TTz,gTz),d(Tw,gw)\}$.

Now consider,

$$\begin{aligned} d(Tz,Tw)) &\leq H(gTz,gw) \\ &< max\{d(TTz,Tw),d(TTz,gTz),d(Tw,gw)\} \\ &< d(Tz,Tw) \end{aligned}$$

which implies $d(Tz,Tw) < d(Tz,Tw)$, a contradiction. Therefore $Tz = Tw$. Hence Tz is the unique common fixed point of T and g.

Corollary 3.7. *If the condition (ii) in the Theorem 3.6 is replaced by*

$$\begin{aligned} H(gx,gy) > 0 \Rightarrow \tau + F(H(gx,gy)) \leq F(max\{&d(Tx,Ty),d(Tx,gx),d(Ty,gy),d(Tx,gy), \\ &d(Ty,gx)\})\forall x,y \in X \end{aligned} \tag{8}$$

then the Theorem 3.6 holds.

Proof.

By strong triangle inequality, we have $d(Tx,gy) \leq max\{d(Tx,Ty),d(Ty,gy)\}$ and $d(Ty,gx) \leq max\{d(Ty,Tx),d(Tx,gx)\}$, which gives that (8) implies (7).

Conflict of Interests

The authors declare that there is no conflict of interests.

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