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SOME FIXED POINT THEOREMS IN MENGER PM-SPACES

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Abstract. In this paper, we establish a fixed point theorem by revisiting the notion of a new contractive mapping in Menger PM-spaces. A fixed point for generalized type contractive mappings in M-complete Menger PM-spaces under arbitrary t-norm.

Keywords: fixed point; Menger PM-space; contractive.

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1. Introduction

The paper is intended to prove a new contraction mapping principle in certain probabilistic metric spaces, namely Menger Probabilistic Metric spaces. As is well known that Banach Contraction Principle is one of the most important results of functional analysis, generalization of this principle in general metric spaces has been intensively investigated and currently is also an active branch of research. To cite a few examples, in [1] a new contraction principle was addressed by Khan et al., where they used a control function on the metric function; in [2] and [3] generalized Banach contraction conjecture has been established independently and in [4]

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Kirk investigated asymptotic contraction in metric spaces. Particularly, the work of Khan et al., in [1] initiated the study of a new category of fixed point theorems. Furthermore, contractive types of mappings occupy a very important position in the fixed point theory in metric spaces. Several types of contractions and their interrelations have been discussed in the review paper due to Rhoades [5] and comparison of various definitions of contraction mappings may be seen in [6]. Probabilistic metric spaces have been introduced as a probabilistic generalization of metric spaces. Schweizer and Sklar [7] have investigated several of these structures. And on contributing to these study, we try to find a new way of contraction in Menger PM-space.

2. Preliminaries

A lot of work has been done on the existence of fixed points of mappings in such spaces. In the following we review some notions connected with probabilistic metric spaces.

Definition 2.1.[7,8] A map $F:\mathbb{R}\to\mathbb{R}^+$ is called a distribution function if it is nondecreasing and left continuous with $\inf\limits_{t\in R}F(t)=0$ and $\sup\limits_{t\in R}F(t)=1$. We shall denote by D^+ the set of all distribution functions, while $H\in D^+$ will always denote the specific distribution function defined by

$$H(t)=\begin{cases}0 & if \quad t \le 0,\\ 1 & if \quad t > 0.\end{cases}$$

Definition 2.2.([7,8]) A binary operation $T:[0,1]\times[0,1]\to[0,1]$ is a continuous t-norm is the following condition hold

(a) T is commutative and associative ,

(b) T is continuous,

(c) $T(a,1)=a$ for all $a\in[0,1]$,

(d) $T(a,b)\le T(c,d)$ whenever $a\le c$ and $b\le d$ for all $a,b,c,d\in[0,1]$.

The following are three basic continuous t-norms from the literature:

(i) The minimum t-norm, say T_M , defined by $T_M(a,b)=min\{a,b\}$.

(ii) The product t-norm, say T_p, defined by $T_p(a,b)=a\cdot b$.

(iii) The Lukasiewicz t-norm, sayT_L , defined by $T_L(a,b)=\max\{a+b-1,0\}$.

These t-norms are related in the following way: $T_L \leq T_p \leq T_M$.

Definition 2.3.[7,8]A probabilistic metric space (PM-space) is an order pair (X,F) where S is a nonempty set, F is a function defined on $X \times X$ to the set of distribution functions which satisfies the following conditions:

(i) $F_{x,y}(0) = 0$,

(ii) $F_{x,y}(t) = 1$ for all $t > 0$ if and only if $x = y$,

(iii) $F_{x,y}(t) = F_{y,x}(t)$ for all $x,y \in X$ for all $t \in \mathbb{R}$,

(iv) If $F_{x,y}(t_1) = 1$ and $F_{y,z}(t_2) = 1$ then $F_{x,z}(t_1+t_2) = 1$, $t \in \mathbb{R}$.

Definition 2.4.[7,8]A Menger Space is an triplet (X,F,T) where X is a nonempty set, F is a function defined on $X \times X$ to the set of distribution functions such that the following are satisfied:

(i) $F_{x,y}(0) = 0$ for all $x,y \in X$,

(ii) $F_{x,y}(t) = 1$ for all $t > 0$ if and only if $x = y$,

(iii) $F_{x,y}(t) = F_{y,x}(t)$ for all $x,y \in X$ for all $t \in \mathbb{R}$,

(iv) $F_{x,y}(t+s) \geq T(F_{x,z}(t), F_{z,y}(s))$ for all $x,y,z \in X$ for all $s,t \in \mathbb{R}^+$ where T is a t-norm.

Definition 2.5.[7,8]A Menger PM-space is a triple (X,F,T) where X is a nonempty set, T is a continuous t-norm and F is a mapping from $X \times X$ into D^+ such that, if $F_{x,y}$ denotes the value of F at the pair(x,y) , the following conditions hold:

(PM1) $F_{x,y}(t) = H(t)$ if and only if $x = y$ for all $t \in \mathbb{R}^+$,

(PM2) $F_{x,y}(t) = F_{y,x}(t)$ for all $x,y \in X$ for all $t \in \mathbb{R}^+$,

(PM3) $F_{x,y}(t+s) \geq T(F_{x,z}(t), F_{z,y}(s))$ for all $x,y,z \in X$ for all $s,t \in \mathbb{R}^+$.

Definition 2.6.[10]Let (X,F,T) be a Menger PM-space. Then

(i) A sequence $\{x_n\}$ in X is said to be convergent to $x \in X$ if, for every $\varepsilon > 0$ and $\lambda > 0$, there exists a positive integer N such that $F_{x_n,x}(\varepsilon) > 1-\lambda$ whenever $n \geq N$.

(ii) A sequence $\{x_n\}$ in X is called Cauchy sequence if, for every $\varepsilon > 0$ and $\lambda > 0$, there exists a positive integer N such that $F_{x_n,x_m}(\varepsilon) > 1-\lambda$ whenever $n,m \geq N$.

(iii) A Menger PM-space is said to be M-complete if every Cauchy sequence in X is convergent to a point in X.

(iv) A sequence $\{x_n\}$ in X is called G-Cauchy if $\lim_{n\to\infty} F_{x_n,x_n+m}(t) = 1$ for each $m \in \mathbb{N}$ and $t > 0$.

(v) The space (X,F,T) is said called G-Complete if every G-Cauchy sequence in X is convergent.

According to[7], the (ε,λ)-topology in a Menger PM-space (X,F,T) is introduced by the family of neighborhoods N_x of a point $x \in X$ given by

$$N_x = N_x(\varepsilon,\lambda) : \varepsilon > 0, \lambda \in (0,1),$$

where

$$N_x(\varepsilon,\lambda) = y \in X, : F_{x,y}(\varepsilon) > 1-\lambda.$$

The (ε,λ)-topology is a Hausdorff topology. In this topology a function f is continuous in $x \in X$ if and only if $f(x_n) \to f(x_0)$, for every sequence $x_n \to x$ as $n \to \infty$. The following class of functions was introduced in [10] and will be used in proving our results in the next section.

Definition 2.7.A function $\phi : \mathbb{R}^+ \to \mathbb{R}^+$ is said to be a ϕ-function if it satisfies the following conditions:

(i) $\phi(t) = 0$ if and only if $t = 0$,

(ii) $\phi(t)$ is strictly increasing and $\phi(t) \to \infty$ as $t \to \infty$,

(iii) ϕ is left continuous in $(0,\infty)$,

(iv) ϕ is continuous at 0.

Definition 2.8. [9] Let (X,F,T) be a Menger PM-space. The probabilistic metric F is triangular if it satisfies the condition

$$\frac{1}{F_{x,y}(t)} - 1 \le \left(\frac{1}{F_{x,z}(t)} - 1\right) + \left(\frac{1}{F_{z,y}(t)} - 1\right) \tag{2.1}$$

for every $x,y,z \in X$ and each $t > 0$.

In the sequel, the class of all ϕ-functions will be denoted by Φ.

3. Main results

In metric spaces we often use the contraction like $d(fx, fy) \le cd(x,y), c \in (0,1)$ to get a fixed point. And in PM spaces the conditions like $F_{fx,fy}(\phi(t)) \ge F_{x,y}(\phi(t/c)), t > 0, c \in (0,1)$

have been often used to study fixed point problems. In this section, we find a new contractive mapping in Menger PM-space. We start with a revised version of $F_{fx,fy}(\phi(t)) \geq F_{x,y}(\phi(t/c))$ to proof the existence problem of a fixed point in a G-complete Menger space.

Theorem 3.1. *Let (X,F,T) be a G-complete Menger space F is triangular and $f : X \to X$ be a mapping satisfying the following inequality:*

$$\frac{1}{F_{fx,fy}(\phi(t))} - 1 \leq r\left(\frac{1}{F_{fx,x}(\phi(t/c))} - 1\right) + r\left(\frac{1}{F_{fy,y}(\phi(t/c))} - 1\right) \tag{3.1}$$

where $x,y \in X, c \in (0,1), r \in [0,\frac{1}{2}), t > 0, \phi \in \Phi$ such that $F_{x,y}(\phi(t)) > 0$. Then f has a fixed point.

Proof. Let $x_0 \in X$. Define a sequence $\{x_n\}$ in X so that $x_{n+1} = f(x_n)$ for all $n \in \mathbb{N}\bigcup\{0\}$. We suppose $x_{n+1} \neq x_n$ for all $n \in \mathbb{N}$, otherwise f has trivially a fixed point.

We have known that $\sup\limits_{t\in\mathbb{R}} F_{x_0,x_1}(t) = 1$ and by (ii) of Definition 2.7, one can find $t_0 > 0$ such that $F_{x_0,x_1}(\phi(t_0)) > 0$. Since $F_{x_0,x_1}(\phi(t_0)) > 0$ implies that $F_{x_0,x_1}(\phi(t_0/c)) > 0$, therefore (3.1) gives that

$$\frac{1}{F_{x_2,x_1}(\phi(t_0))} - 1 \leq r\left(\frac{1}{F_{x_2,x_1}(\phi(t_0/c))} - 1\right) + r\left(\frac{1}{F_{x_1,x_0}(\phi(t_0/c))} - 1\right) \tag{3.2}$$

since ϕ is strictly increasing and F is non-decreasing, with $t_0 > 0, c \in (0,1)$we can get $\frac{1}{F_{x_2,x_1}(\phi(t_0/c))} \leq \frac{1}{F_{x_2,x_1}(\phi(t_0))}$ and that is

$$\frac{1}{F_{x_2,x_1}(\phi(t_0))} - 1 \leq r\left(\frac{1}{F_{x_2,x_1}(\phi(t_0))} - 1\right) + r\left(\frac{1}{F_{x_1,x_0}(\phi(t_0/c))} - 1\right). \tag{3.3}$$

From (3.3) we can get that

$$(1-r)\left(\frac{1}{F_{x_2,x_1}(\phi(t_0))} - 1\right) \leq r\left(\frac{1}{F_{x_1,x_0}(\phi(t_0/c))} - 1\right)$$
$$\left(\frac{1}{F_{x_2,x_1}(\phi(t_0))} - 1\right) \leq \frac{r}{1-r}\left(\frac{1}{F_{x_1,x_0}(\phi(t_0/c))} - 1\right).$$

Again we let $x = x_2, y = x_1$ from (3.1) we can also have the result that

$$\frac{1}{F_{x_3,x_2}(\phi(t_0))} - 1 \leq r\left(\frac{1}{F_{x_3,x_2}(\phi(t_0/c))} - 1\right) + r\left(\frac{1}{F_{x_2,x_1}(\phi(t_0/c))} - 1\right)$$
$$\frac{1}{F_{x_3,x_2}(\phi(t_0))} - 1 \leq r\left(\frac{1}{F_{x_3,x_2}(\phi(t_0))} - 1\right) + r\left(\frac{1}{F_{x_2,x_1}(\phi(t_0/c))} - 1\right)$$

$$\frac{1}{F_{x_3,x_2}(\phi(t_0))}-1 \le \frac{r}{1-r}\left(\frac{1}{F_{x_2,x_1}(\phi(t_0/c))}-1\right) \le (\frac{r}{1-r})^2\left(\frac{1}{F_{x_1,x_0}(\phi(t_0/c^2))}-1\right). \tag{3.4}$$

Repeating the above procedure successively n times, we obtain

$$\frac{1}{F_{x_{n+1},x_n}(\phi(t_0))}-1 \le (\frac{r}{1-r})^n\left(\frac{1}{F_{x_1,x_0}(\phi(t_0/c^n))}-1\right) \tag{3.5}$$

If we change x_0 with x_k in the previous inequalities, then for all $n > k$ we get

$$\frac{1}{F_{x_n,x_{n+1}}(\phi(c^k t_0))}-1 \le (\frac{r}{1-r})^{n-k}(\frac{1}{F_{x_k,x_{k+1}}(\phi(\frac{c^k t_0}{c^{n-k}}))}-1) \tag{3.6}$$

Since $r \in [0,\frac{1}{2})$, then $0 \le \frac{r}{1-r} < 1$, we can get $(\frac{r}{1-r})^n \to 0$ as $n \to \infty$. Therefor the above inequality imply that

$$\lim_{n\to\infty} F_{x_{n+1},x_n}(\phi(c^k t_0)) \ge 1 \tag{3.7}$$

with $F(t) \le 1$ we can get

$$\lim_{n\to\infty} Fx_n, x_{n+1}(\phi(c^k t_0)) = 1 \tag{3.8}$$

Now let $\varepsilon > 0$ be given, then by using the properties (i) and (iv) of a function ϕ we can find $k \in \mathbb{N}$ such that $\phi(c^k t_0) < \varepsilon$. It follows from (3.8) that

$$\lim_{n\to\infty} F_{x_n,x_{n+1}}(\varepsilon) \ge \lim_{n\to\infty} F_{x_n,x_{n+1}}(\phi(c^k t_0)) = 1 \tag{3.9}$$

By using the (iv) of Definition(2.4), we obtain

$$F_{x_n,x_{n+p}}(\varepsilon) \ge T(F_{x_n,x_{n+1}}(\varepsilon/p), T(F_{x_{n+1},x_{n+2}}(\varepsilon/p)), ..., (F_{x_{n+p-1}x_{n+p}}(\varepsilon/p)...)). \tag{3.10}$$

Let $n \to \infty$ and making use of (3.9) and (c) of Definition (2.2), for any integer p, we get

$$\lim_{n\to\infty} F_{x_n,x_{n+p}}(\varepsilon) = 1 \quad \text{for every} \quad \varepsilon > 0. \tag{3.11}$$

Hence $\{x_n\}$ is a G-Cauchy sequence. Since (X,F,T) is G-complete, therefor $x_n \to u$, as $n \to \infty$, for some $u \in X$.

Now we show that u is a fixed point of f.

Since

$$F_{fu,u}(\varepsilon) \ge T(F_{fu,x_{n+1}}(\varepsilon/2), F_{x_{n+1},u}(\varepsilon/2)) \tag{3.12}$$

by using the properties (i) and (iv) of a function ϕ, we can find $s > 0$ such that $\phi(s) < \frac{\varepsilon}{2}$. Again, since $x_n \to u$ as $n \to \infty$, then there exists $n_0 \in \mathbb{N}$ such that, for all $n > n_0$, we have $F_{x_n,u}(\phi(s)) > 0$.

Therefor, for $n > n_0$, we obtain

$$F_{x_{n+1},fu}(\varepsilon/2) \geq F_{x_{n+1},fu}(\phi(s)) \tag{3.13}$$

$$\frac{1}{F_{x_{n+1},fu}(\phi(s))} - 1 \leq r\left(\frac{1}{F_{fx_n,x_n}(\phi(\frac{s}{c}))} - 1\right) + r\left(\frac{1}{F_{fu,u}(\phi(\frac{s}{c}))} - 1\right) \tag{3.14}$$

Since F is triangular then we can get

$$\begin{aligned} r\left(\frac{1}{F_{fu,u}(\phi(\frac{s}{c}))} - 1\right) &\leq r\left(\frac{1}{F_{fu,x_{n+1}}(\phi(\frac{s}{c}))} - 1\right) + r\left(\frac{1}{F_{x_{n+1},u}(\phi(\frac{s}{c}))} - 1\right) \\ &\leq r\left(\frac{1}{F_{fu,x_{n+1}}(\phi(s))} - 1\right) + r\left(\frac{1}{F_{x_{n+1},u}(\phi(\frac{s}{c}))} - 1\right) \end{aligned} \tag{3.15}$$

So (3.14) can be treated as:

$$\begin{aligned} \frac{1}{F_{x_{n+1},fu}(\phi(s))} - 1 \leq r\left(\frac{1}{F_{fx_n,x_n}(\phi(\frac{s}{c}))} - 1\right) + r\left(\frac{1}{F_{fu,x_{n+1}}(\phi(s))} - 1\right) \\ + r\left(\frac{1}{F_{x_{n+1},u}(\phi(\frac{s}{c}))} - 1\right) \end{aligned} \tag{3.16}$$

that is

$$\begin{aligned} &(1-r)\left(\frac{1}{F_{fu,x_{n+1}}(\phi(s))} - 1\right) \\ &\leq r\left(\frac{1}{F_{x_{n+1},u}(\phi(\frac{s}{c}))} - 1\right) + r\left(\frac{1}{F_{fx_n,x_n}(\phi(\frac{s}{c}))} - 1\right) \\ &\leq r\left(\frac{1}{F_{x_{n+1},u}(\phi(\frac{s}{c}))} - 1\right) + r\left(\frac{1}{F_{x_{n+1},x_n}(\phi(\frac{s}{c}))} - 1\right) \end{aligned} \tag{3.17}$$

In (3.17) let $n \to \infty$, we get

$$\lim_{n\to\infty} F_{x_{n+1},fu}(\phi(s)) = 1 \tag{3.18}$$

With (3.13) we can obtain

$$\lim_{n\to\infty} F_{x_{n+1},fu}(\frac{\varepsilon}{2}) = 1 \tag{3.19}$$

From (3.12) and (3.19), we get $F_{fu,u}(\varepsilon) = 1$ for every $\varepsilon > 0$, which in turn yields that $fu = u$. This completes the proof.

Theorem 3.2. *$Fix(f) := \{x \in X : x = fx\}, F_{u,v}(0) = 0$ if $u, v \in Fix(f)$ then with the hypotheses of Theorem 3.1, we obtain uniqueness of the fixed point.*

Proof. We prove uniqueness of the fixed point. Let u and v be two fixed point of f, that is, $fu = u$ and $fv = v$. For all $s > 0$, by the condition of Definition of 2.7 we have $\phi(\frac{s}{c^n}) \to \infty$ as $n \to \infty$. Since $\sup\limits_{n\to\infty} F_{u,v}(\phi(\frac{s}{c^n})) = 1$ there exist $n_0 > N$ so that $F_{u,v}(\phi(\frac{s}{c^{n_0}})) > 0$. By using (3.1) we get

$$
\begin{aligned}
\frac{1}{F_{u,v}(\phi(\frac{s}{c^{n_0-1}}))} - 1 &= \frac{1}{F_{fu,fv}(\phi(\frac{s}{c^{n_0-1}}))} - 1 \\
&\leq r\left(\frac{1}{F_{fu,u}(\phi(\frac{s}{c^{n_0}}))} - 1\right) + r\left(\frac{1}{F_{fv,v}(\phi(\frac{s}{c^{n_0}}))} - 1\right) \\
&= r\left(\frac{1}{F_{u,u}(\phi(\frac{s}{c^{n_0}}))} - 1\right) + r\left(\frac{1}{F_{v,v}(\phi(\frac{s}{c^{n_0}}))} - 1\right) \\
&= 1
\end{aligned}
\tag{3.20}
$$

that implies $\frac{1}{F_{u,v}(\phi(\frac{s}{c^{n_0-1}}))} = 1$. By repeating (3.20) n times, We get $F_{u,v}(\phi(s)) = 1$. It follows that $F_{u,v}(t) = H(t)$ for all $t > 0$. In fact, if t is not range of ϕ, since ϕ is continuous at 0, then there exists $s > 0$ such that $\phi(s) < t$. This implies $F_{u,v}(t) \geq F_{u,v}(\phi(s)) = 1$, yielding thereby $u = v$.

Our next step is to furnish a fixed point theorem in an M-complete Menger PM-space.

Theorem 3.3. *Let (X,F,T) be an M-complete Menger PM-space and : $f : X \to X$ be a contractive mapping, with the condition in Theorem 3.1. Then f has a fixed point.*

Proof In view of the assumptions in Theorem 3.1. Then, following similar arguments to those given in Theorem 3.1, we obtain $F_{x_n,x_{n+1}}(\varepsilon) \to 1$ as $n \to \infty$. Now we shall show that $\{x_n\}$ is a M-Cauchy sequence.By the properties of ϕ, given $\varepsilon > 0$, we can find $s > 0$ such that $\varepsilon > \phi(s) > 0$ Therefor,

$$
\frac{1}{F_{x_n,x_{n+p}}(\varepsilon)} - 1 \leq \frac{1}{F_{x_n,x_{n+p}}(\phi(s))} - 1 \tag{3.21}
$$

Now since F is triangular, we get

$$\begin{aligned}\frac{1}{F_{x_n,x_{n+p}}(\varepsilon)}-1 &\leq \frac{1}{F_{x_n,x_{n+1}}(\varepsilon)}-1+\frac{1}{F_{x_{n+1},x_{n+2}}(\varepsilon)}-1+\ldots\\ &+\frac{1}{F_{x_{n+p-1},x_{n+p}}(\varepsilon)}-1\\ &\leq \frac{1}{F_{x_n,x_{n+1}}(\phi(s))}-1+\frac{1}{F_{x_{n+1},x_{n+2}}(\phi(s))}-1+\ldots\\ &+\frac{1}{F_{x_{n+p-1},x_{n+p}}(\phi(s))}-1\end{aligned} \tag{3.22}$$

again we use the inequality (3.5) we get

$$\begin{aligned}\frac{1}{F_{x_n,x_{n+p}}(\varepsilon)}-1 &\leq (\frac{r}{1-r})^n\left(\frac{1}{F_{x_0,x_1}(\phi(s/c^n))}-1\right)\\ &+(\frac{r}{1-r})^{n+1}\left(\frac{1}{F_{x_0,x_1}(\phi(s/c^{n+1}))}-1\right)+\ldots\\ &+(\frac{r}{1-r})^{n+p-1}\left(\frac{1}{F_{x_0,x_1}(\phi(s/c^{n+p-1}))}-1\right)\\ &\leq (\frac{r}{1-r})^n\left(\frac{1}{F_{x_0,x_1}(\phi(s/c^n))}-1\right)\\ &+(\frac{r}{1-r})^{n+1}\left(\frac{1}{F_{x_0,x_1}(\phi(s/c^n))}-1\right)+\ldots\\ &+(\frac{r}{1-r})^{n+p-1}\left(\frac{1}{F_{x_0,x_1}(\phi(s/c^n))}-1\right)\\ &=(\frac{r}{1-r})^n\left(\frac{1}{F_{x_0,x_1}(\phi(s/c^n))}-1\right)\left(\frac{1-(\frac{r}{1-r})^p}{1-\frac{r}{1-r}}\right)\end{aligned} \tag{3.23}$$

Since $r\in[0,\frac{1}{2})$, $\frac{r}{1-r}\in(0,1)$ then $(\frac{r}{1-r})^n\to 0$ as $n\to\infty$, we obtain $F_{x_n,x_{n+p}}(\varepsilon)\to 1$ as $n\to\infty$. Thus $\{x_n\}$ is an M-Cauchy sequence in X. The rest of this theorem can be completed on the lines of Theorem 3.1. This concludes the proof.

Theorem 3.4. *The uniqueness of the fixed point can be proofed in the same way we have seen in Theorem 3.2.*

Conflict of Interests

The authors declare that there is no conflict of interests.

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