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APPROXIMATING OF COMMON FIXED POINTS OF TWO MULTIVALUED NONEXPANSIVE MAPPINGS IN $CAT(0)$ SPACES

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Abstract. In this paper, we introduced one-step iterative process for approximating of common fixed points of two multivalued nonexpansive mappings in $CAT(0)$ spaces and established strong convergence theroems for proposed process under some conditions. Our results extend important results.

Keywords: CAT(0) space; nonexpansive multivalued mapping; strong convergence; one-step; iterative scheme.

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1. Introduction and Preliminaries

Let K be a nonempty subset of a Banach space X. We shall denote $CB(K)$, $C(K)$ and $P(K)$ by the families of nonempty closed and bounded subsets, nonempty compact subsets and nonempty proximinal bounded subsets of K, respectively. The set K is said to be proximinal if for each $x \in E$, there exists an element $y \in K$ such that $\|x-y\| = d(x,K)$, where $d(x,K) = \inf\{\|x-z\| : z \in K\}$. Let H be the Hausdorff metric induced by the metric d of X and given by

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$$H(A,B) = \max\left\{\sup_{x\in A} d(x,B),\ \sup_{y\in B} d(y,A)\right\}\quad A,B\in CB(X)$$

where $d(a,B) = \inf\{d(a,b) : b\in B\}$ is the distance from the point a to the set B.

A multivalued mapping $T : K \to P(K)$ is said to be nonexpansive if [6]

$$H(Tx,Ty) \leq \|x-y\| \quad \forall x,y\in K$$

A point $x\in K$ is called a fixed point of T if $x\in T(x)$.We shall denote by Fix(T) the set of all fixed points of T.

Two multivalued nonexpansive mappings $S,T : K \to CB(K)$ are said to satisfy condition (A$'$) if there exists a nondecreasing function $f : [0,\infty)\to[0,\infty)$ with $f(0) = 0$ and $f(r) > 0$ for all $r\in(0,\infty)$ such that either $d(x,Tx) \geq f(d(x,F))$ or $d(x,Sx) \geq f(d(x,F))$ for all $x\in K$.

A metric space X is a $CAT(0)$ space if it is geodesically connected, and if every geodesic triangle in X is at least as 'thin' as its comparison triangle in the Euclidean plane. The precise definition is given below. It is well known that any complete, simply connected Riemannian manifold having nonpositive sectional curvsture is a $CAT(0)$ space. other examples include pre-Hilbert spaces, $\mathbb{R}-$trees, Euclidean buildings, the complex Hilbert ball with a hyperbolic metric, and many others.

The study of metric spaces without linear structure has played a vital role in various branches of pure and applied sciences. In particular, fixed point theorems in $CAT(0)$ spaces for non-expansive single valued, as well as for multivalued mapping have been studied extensively by many authors.

For nonexpansive multivalued mappings, Sastry and Babu [19] defined the Mann and Ishikawa iteration process in a Hilbert space. Panyanak [20], and Song and Wang [16] extended the results of Sastry and Babu [19], to uniformly convex Banach spaces. Very recently, Shahzad and Zegeye [8] extended and improved results of [16]-[19]-[20]

Fixed point theory in CAT(0) spaces was first studied by Kirk [1]. He showed that every nonexpansive (single-valued) mapping defined on a bounded closed convex subset of a complete CAT(0) space always has a fixed point. Fixed point theory for single-valued and multivalued mappings in CAT(0) spaces has been rapidly developed and many of papers have appeared. It is worth mentioning that fixed point theorems in $CAT(0)$ spaces can be applied to graph theory, biology, and computer sciences.

In [5],Dhompongsa and Panyanak obtained $\Delta-$convergence thorems for the Mann and Ishikawa iterations for nonexpansive single valued mappings in $CAT(0)$. Inspired by Song and Wang [16], Laowang and Panyanak[14] extended results of [5] for multivalued nonexpansive mappings in $CAT(0)$ spaces. On the other hand, in [21], Garcia- Falset et al. introduced two new conditions on single valued mappings, called conditions (E) and (C_λ), which are weaker than nonexpansiveness and stronger than quasi-nonexpansiveness.

Akbar and Eslamian [12] proved that if K is a nonempty bounded closed convex subset of a complete $CAT(0)$ space $X, t : K \to K$ is a single-valued quasi-nonexpansive mapping and $T : K \to KC(K)$ is a mulivalued mapping satisfying conditions (E) and (C_λ) for some $\lambda \in (0,1)$ such that t and T are weakly commuting, then there exists a point $z \in K$ such that $z = t(z) \in T(z)$.

Let (X,d) be a metric space. A geodesic path joining $x \in X$ to $y \in X$ is a map c from a closed interval $[0,l] \subset R$ to X such that $c(0) = x$ and $c(l) = y$, and $d\left(c(t), c\left(t^{'}\right)\right) = \left|t - t^{'}\right|$. İn particular, c is an isometry and $d(x,y) = l$. The image α of c is called a geodesic (or metric) segment joining x and y. When itis unique this godesic is denoted by $[x,y]$. The space (X,d) is said to be a geodesic space if every two points of X are joined by a geodesic, and X is said to be uniquely geodesic if there is exactly one geodesic joining x and y for each $x,y \in X$. A subset $Y \subseteq X$ is said to be convex if y includes every geodesic segment joining any two of its points.

A geodesic triangle $\Delta(x_1,x_2,x_3)$ in a geodesic space X consists of three points x_1,x_2,x_3 of X and three geodesic segments joining each pair of vertices. A comparison triangle of a geodesic triangle $\Delta(x_1,x_2,x_3)$ is the triangle $\bar{\Delta}(x_1,x_2,x_3) := \Delta(\bar{x}_1,\bar{x}_2,\bar{x}_3)$ in the Euclidean space $\mathbb{R}^2$such that $d\left(x_i,x_j\right) = d_{\mathbb{R}^2}\left(\overline{x_i},\overline{x_j}\right)$ for all $i,j = 1,2,3$.

A geodesic space X is a $CAT(0)$ space if for each geodesic triangle $\Delta := \Delta(x_1,x_2,x_3)$ in X and its comparison triangle $\overline{\Delta} := \Delta(\bar{x}_1,\bar{x}_2,\bar{x}_3)$ in $\mathbb{R}^2$, the $CAT(0)$ inequality

$$d(x,y) \leq d_{\mathbb{R}^2}(\bar{x},\bar{y})$$

is satisfied by all $x,y \in \Delta$ and $\bar{x},\bar{y} \in \overline{\Delta}$. The meaning of the $CAT(0)$ inequality is that a geodesic triangle in X is at least thin as its comparison triangle in the Euclidean plane.

Lemma 1. *Let (X,d) be a $CAT(0)$ space. For $x,y \in X$ and $t \in [0,1]$, there exists a unique point $z \in [x,y]$ such that*

$$d(x,z) = td(x,y) \quad and \quad d(y,z) = (1-t)d(x,y) \tag{1.1}$$

We use the notation $(1-t)x \oplus ty$ for the unique point z satisfying (1.1). Dhompongsa and Panyanak [5] obtained the following lemma which will be used frequently in the proof our main theorems.

Lemma 2. *Let X be a $CAT(0)$ space. Then*

$$d((1-t)x \oplus ty, z) \leq (1-t)d(x,z) + d(y,z)$$

for all $x,y,z \in X$ and $t \in [0,1]$.

The existence of fixed points for multivalued nonexpansive mapping in a $CAT(0)$ space was proved by S. Dhompongsa et al., as follows.

Theorem 1. *let K be a closed convex subset of complete $CAT(0)$ space X, and let $T: K \to \kappa(K)$ be a nonexpansive nonself-mapping. Suppose*

$$\lim_{n\to\infty} dist(x_n, Tx_n) = 0$$

for some bounded sequence $\{x_n\}$ in K. Then T has a fixed point.

Now, we define the sequences of Mann and Ishikawa iterates in a $CAT(0)$ space which are analogs of the two defined in Banach spaces by Song and Wang [15]-[16]

Definition 1. *Let K be nonempty convex subset of a $CAT(0)$ space X and $T: K \to CB(K)$ be a multivalued mapping. The sequence of Mann iterates is defined as follows : let $\alpha_n \in [0,1]$ and $\gamma_n \in (0,\infty)$ such that* $\lim_{n\to\infty} \gamma_n = 0$.

$$x_{n+1} = (1-\alpha_n)x_n \oplus \alpha_n y_n$$

where $y_n \in Tx_n$ such that $d(y_{n+1}, y_n) \leq H(Tx_{n+1}, Tx_n) + \gamma_n$.

Definition 2. *Let K be nonempty convex subset of a $CAT(0)$ space X and $T : K \to CB(K)$ be a multivalued mapping. The sequence of Ishikawa iterates is defined as follows : let $\alpha_n, \beta_n \in [0,1]$ and $\gamma_n \in (0,\infty)$ such that* $\lim_{n\to\infty} \gamma_n = 0$.

$$\begin{aligned} y_n &= (1-\beta_n)x_n \oplus \beta_n z_n \\ x_{n+1} &= (1-\alpha_n)x_n \oplus \alpha_n z_n^{'} \end{aligned}$$

where $z_n \in Tx_n$ and $z_n^{'} \in Ty_n$ such that $d\left(z_n, z_n^{'}\right) \leq H(Tx_n, Ty_n) + \gamma_n$ and $d\left(z_{n+1}, z_n^{'}\right) \leq H(Tx_{n+1}, Ty_n) + \gamma$

Now we state some useful lemmas.

Lemma 3. [1]*Let $A,B \in CB(X)$ and $a \in A$. If $\eta > 0$, then there exists $b \in B$ such that $d(a,b) \leq H(A,B) + \eta$.*

Before proving our main results we state a lemma which is an analog of Lemma 2.2 of [14]. The proof is metric in nature and carries over to the present setting without change.

Lemma 4. *Let $\{x_n\}$ and $\{y_n\}$ be bounded sequences in a $CAT(0)$ space and let $\{\alpha_n\}$ be a sequence in $[0,1]$ with $0 < \liminf_n \alpha_n < \limsup_n \alpha_n < 1$. Suppose that $x_{n+1} = \alpha_n y_n \oplus (1-\alpha_n)x_n$ for all $n \in N$ and*

$$\lim_{n\to\infty} \sup \left(d\left(y_{n+1}, y_n\right) - d\left(x_{n+1}, x_n\right)\right) \leq 0$$

Then

$$\lim_{n\to\infty} d\left(x_n, y_n\right) = 0.$$

In this paper, we use a one-step iterative process to convergence common fixed points of two multivalued nonexpansive mappings in $CAT(0)$ space. Let $S,T : K \to CB(K)$ be two multivalued nonexpansive mappings. Our process reads as follows: For any $x_1 \in K$

$$x_{n+1} = \lambda x_n \oplus \mu y_n \oplus \upsilon z_n \quad n \in N \tag{1.2}$$

where $y_n \in Sx_n$ and $z_n \in Tx_n$ such that $d\left(y_{n+1}, y_n\right) \leq H\left(Sx_{n+1}, Sx_n\right) + \gamma_n$ and $d\left(z_{n+1}, z_n\right) \leq H\left(Tx_{n+1}, Tx_n\right) + \gamma_n$. Also, where $0 \leq \lambda, \mu, \upsilon < 1$ with $\lambda + \mu + \upsilon = 1$.

Lemma 5. *Let X be a complete $CAT(0)$ space, K a nonempty compact subset of X and S,T : $K\to CB(K)$ be two nonexpansive multivalued mappings with $\phi\neq F=F(S)\cap F(T)$ and for which $Tp=Sp=\{p\}$ for each $p\in F$. Let $\{x_n\}$ be the sequence as defined in (1.2) Also suppose that $\lambda+\mu+\upsilon=1$ and $\gamma_n\in(0,\infty)$ satisfying $\lim_{n\to\infty}\sup\gamma_n=0$.Then*

i) $\lim_{n\to\infty} d(x_n,p)$ exists for all $p\in F$.

ii) For any $p\in F$, $\lim_{n\to\infty} dist(x_n,Tx_n)=\lim_{n\to\infty} dist(x_n,Sx_n)=0$.

Proof. Let $p\in F$. It follows from (1.2) that

$$\begin{aligned}
d(x_{n+1},p) &= d(\lambda x_n\oplus\mu y_n\oplus\upsilon z_n,p)\\
&\leq \lambda d(x_n,p)+\mu d(y_n,p)+d(z_n,p)\\
&\leq \lambda d(x_n,p)+\mu d(y_n,Sp)+\upsilon d(z_n,Tp)\\
&\leq \lambda d(x_n,p)+\mu H(Sx_n,Sp)+\upsilon H(Tx_n,Tp)\\
&\leq \lambda d(x_n,p)+\mu d(x_n,p)+\upsilon d(x_n,p)\\
&= (\lambda+\mu+\upsilon)d(x_n,p)\\
&< d(x_n,p)
\end{aligned}$$

Consequently, the sequence $d(x_n,p)$ is decreasing and bounded below and thus $\lim_{n\to\infty} d(x_n,p)$ exists for any $p\in F$. Also $\{x_n\}$ is bounded and, so $\{y_n\}$ and $\{z_n\}$ is bounded.

ii)Applying Lemma 4, we have

$$\begin{aligned}
d(y_{n+1},y_n) &\leq H(Sx_{n+1},Sx_n)+\gamma_n\\
&\leq d(x_{n+1},x_n)+\gamma_n\\
d(y_{n+1},y_n)-d(x_{n+1},x_n) &\leq \gamma_n
\end{aligned}$$

Taking $\lim\sup$ on both sides, from hipotez $\lim_{n\to\infty}\sup\gamma_n=0$,

$$\lim_{n\to\infty}\sup\left(d(y_{n+1},y_n)-d(x_{n+1},x_n)\right)\leq\lim_{n\to\infty}\sup\gamma_n=0$$

This implies that

$$\lim_{n\to\infty} d(y_n,x_n)=0.$$

Also since $\lim_{n\to\infty} dist\,(Sx_n,x_n) \leq \lim_{n\to\infty} d\,(y_n,x_n)$, we have $\lim_{n\to\infty} dist\,(Sx_n,x_n)=0$. In a similar way, we can show that

$$\lim_{n\to\infty} d\,(z_n,x_n)=0$$

Also since $\lim_{n\to\infty} dist\,(Tx_n,x_n) \leq \lim_{n\to\infty} d\,(z_n,x_n)$, we obtain $\lim_{n\to\infty} dist\,(Tx_n,x_n)=0$. □

Now we will approximate common fixed points of the mapping T and S through strong convergence of the sequence $\{x_n\}$ defined in (1.2)

Theorem 2. *Let X be a complete $CAT(0)$ space, K a nonempty compact subset of X and S,T : $K\to CB(K)$ be two nonexpansive multivalued mappings with $\phi \neq F = F(S)\cap F(T)$ and for which $Tp = Sp = \{p\}$ for each $p\in F$. Let $\{x_n\}$ be the iterates defined by (1.2). Assume that $\lambda+\mu+\upsilon = 1$ and $\gamma_n \in (0,\infty)$ satisfying $\lim_{n\to\infty}\sup\gamma_n = 0$. Then $\{x_n\}$ converges strongly to a common fixed point of S andT if and only if $\lim_{n\to\infty}\inf d\,(x_n,F)=0$.*

Proof. Necessity is obvious.To prove the sufficiency, suppose that $\lim_{n\to\infty}\inf d\,(x_n,F)=0$. By Lemma (5) we have,

$$d\,(x_{n+1},p) \leq d\,(x_n,p)$$

for all $p\in F$. This implies that

$$d\,(x_{n+1},F) \leq d\,(x_n,F)$$

Hence $\lim_{n\to\infty} d\,(x_n,F)$ exists. By hypothesis $\lim_{n\to\infty}\inf d\,(x_n,F)=0$, so there must exists $\lim_{n\to\infty} d\,(x_n,F)=0$. Therefore, we show that $\{x_n\}$ is a Cauchy sequence in K. Let $\varepsilon>0$ be arbitrarly chosen. Since $\lim_{n\to\infty} d\,(x_n,F)=0$, there exists a positive integer n_0 such that

$$d\,(x_n,F) < \frac{\varepsilon}{4}, \quad \forall n \geq n_0$$

In particular, $\inf\{d\,(x_n,p) : p\in F\} < \frac{\varepsilon}{4}$; so there must exists a $p^*\in F$ such that $d\,(x_{n_0},p^*) < \frac{\varepsilon}{2}$. Now for $\forall m,n \geq n_0$, we have

$$\begin{aligned} d\,(x_{n+m},x_n) &\leq d\,(x_{n+m},p^*)+d\,(x_n,p^*) \\ &\leq 2d\,(x_{n_0},p^*) \\ &\leq 2.\left(\frac{\varepsilon}{2}\right)=\varepsilon \end{aligned}$$

Hence $\{x_n\}$ is Cauchy sequence in a closed subset K , and therefore it must convergence in K. Let $\lim x_n = q$. Now

$$\begin{aligned} d(q,Tq) &\leq d(q,x_n) + d(x_n,Tx_n) + H(Tx_n,Tq) \\ &\leq d(x_n,q) + d(x_n,z_n) + d(x_n,q) \to 0 \end{aligned}$$

gives that $d(q,Tq) = 0$, which implies that $q \in Tq$. Similarly,

$$\begin{aligned} d(q,Sq) &\leq d(q,x_n) + d(x_n,Sx_n) + H(Sx_n,Sq) \\ &\leq d(x_n,q) + d(x_n,z_n) + d(x_n,q) \to 0 \end{aligned}$$

implies that $q \in Sq$. Consequently $q \in F = F(T) \cap F(S)$. □

Theorem 3. *Let X be a complete $CAT(0)$ space, K a nonempty compact subset of X and S,T : $K \to CB(K)$ be two nonexpansive multivalued mappings satisfying condition (A') with $\phi \neq F = F(S) \cap F(T)$ and for which $Tp = Sp = \{p\}$ for each $p \in F$. Let $\{x_n\}$ be the iterates defined by (1.2). Assume that $\lambda + \mu + \upsilon = 1$ and $\gamma_n \in (0,\infty)$ satisfying $\lim\limits_{n\to\infty} \sup \gamma_n = 0$ Then $\{x_n\}$ converges strongly to a common fixed point of S and T.*

Proof. Let $p \in F$. By Lemma 5, $\lim_{n\to\infty} d(x_n,p)$ exists for all $p \in F$ and,we have $d(x_n,Tx_n) \leq d(x_n,z_n) \to 0$ and $d(x_n,Sx_n) \leq d(x_n,y_n) \to 0$ for $n \to \infty$. Also, in the proof of Theorem 2 $\lim_{n\to\infty} d(x_n,F)$ exists and using the condition (A') either

$$\lim_{n\to\infty} f(d(x_n,F)) \leq \lim_{n\to\infty} d(x_n,Tx_n)$$

or

$$\lim_{n\to\infty} f(d(x_n,F)) \leq \lim_{n\to\infty} d(x_n,Sx_n)$$

In both cases, we have $\lim\limits_{n\to\infty} f(d(x_n,F)) = 0$. Since f is a nondecreasing function and $f(0) = 0$, so it follows that $\lim\limits_{n\to\infty} d(x_n,F) = 0$. Thus there is a subsequence $\{x_{n_k}\}$ of $\{x_n\}$and a sequence $\{p_k\} \subset F(T)$ such that $d(x_{n_k},p_k) < \frac{1}{2^k}$for all k. From (5), we obtain

$$d\left(x_{n_{k+1}},p_k\right) \leq d(x_{n_k},p_k) < \frac{1}{2^k}$$

Now we show $\{p_k\}$ is a Cauchy sequence in K. Noted that

$$\begin{aligned} d\left(p_{k+1},p_k\right) &\leq d\left(p_{k+1},x_{n_{k+1}}\right)+d\left(x_{n_{k+1}},p_k\right) \\ &< 2^{\frac{1}{k+1}}+2^{\frac{1}{k}} \\ &< 2^{\frac{1}{k-1}} \end{aligned}$$

This implies that $\{p_k\}$ is a Cauchy sequence in K and thus $q \in K$. Now we show that $q \in F$. Therefore

$$\begin{aligned} d\left(p_k,T\left(q\right)\right) &\leq H\left(T\left(q\right),T\left(p_k\right)\right) \\ &\leq d\left(q,p_k\right) \end{aligned}$$

$p_k \to q$ as $k \to \infty$, it follows that $d\left(q,Tq\right)=0$. Thus $q \in F(T)$. and $\{x_{n_k}\}$ converges strongly to q. Since $\lim\limits_{n\to\infty} d\left(x_n,q\right)$ exists, it follows that $\{x_n\}$ converges strongly to q. Similarly

$$\begin{aligned} d\left(p_k,S(q)\right) &\leq H\left(S(q),S(p_k)\right) \\ &\leq d\left(q,p_k\right) \end{aligned}$$

$p_k \to q$ as $k \to \infty$, it follows that $d\left(q,Sq\right)=0$. Thus $q \in F(S)$. and $\{x_{n_k}\}$ converges strongly to q. Since $\lim\limits_{n\to\infty} d\left(x_n,q\right)$ exists, it follows that $\{x_n\}$ converges strongly to q. So $q \in F = F(T) \cap F(S)$, this completes the proof. □

Conflict of Interests

The authors declare that there is no conflict of interests.

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