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SOME FIXED POINT THEOREMS ON INTUITIONISTIC FUZZY METRIC SPACES USING NEW CONTRACTIONS

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Abstract. This paper concerns on some new contractions on intuitionistic fuzzy metric spaces. Here we have derived common fixed point for mappings under contractive conditions.

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1. Introduction and Preliminaries

Fuzzy set was defined by Zadeh [18] in 1965 which is a mathematical framework to vagueness or uncertainity in daily life. Kramosil and Michalek [11] introduced fuzzy metric spaces and this concept was modified by George and Veeramani in 1994[8].The concept of fuzzy mappings was first introduced by Heilpern [10]. Bose and Sahani [2], Chang and Huang [6], Chang [4], Som and Mukherjee [17] studied fixed point theorems for fuzzy mappings. Bose and Sahani [2] have extended the result of Heilperns for a pair of generalized fuzzy contraction mappings. A fixed point theorem was described by Lee and Cho [12] for contractive type fuzzy mappings which is generalization of Heilperns [10] result. Park [14] introduced the notion of intuitionistic fuzzy metric space in 2004. Several common fixed point theorems for contractive type mappings have

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been proved by many authors in the recent years. In this paper we have proved fixed point for mapping under contractive conditions using fuzzy metric triangular inequality [7].

Definition 1.1 [16]: A binary operation $*: [0,1] \times [0,1] \to [0,1]$ is continuous t-norm if $*$ is satisfying the following conditions:

(i) $*$ is commutative and associative;

(ii) $*$ is continuous;

(iii) $a * 1 = a$ for all $a \epsilon [0,1]$;

(iv) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$ for all $a, b, c, d \epsilon [0,1]$.

Definition 1.2 [16]: A binary operation $\Diamond: [0,1] \times [0,1] \to [0,1]$ is continuous t-norm if $\Diamond$ is satisfying the following conditions:

(i) $\Diamond$ is commutative and associative;

(ii) $\Diamond$ is continuous;

(iii) $a \Diamond 0 = a$ for all $a \epsilon [0,1]$;

(iv) $a \Diamond b \leq c \Diamond d$ whenever $a \leq c$ and $b \leq d$ for all $a, b, c, d \epsilon [0,1]$

Definition 1.3 [1]: A 5 – tuple $(X, M, N, *, \Diamond)$ is said to be an intuitionistic fuzzy metric space (shortly IFM- space) if X is an arbitrary set, $*$ is a continuous t- norm , $\Diamond$ is a continuous t-conorm and M, N are fuzzy sets on $X^2 \times (0, \infty)$ satisfying the following conditions for all $x, y, z \epsilon X$ and $t, s > 0$,

(i) $M(x, y, t) + N(x, y, t) \leq 1$; for all $x, y \epsilon X$ and $t > 0$

(ii) $M(x, y, 0) = 0$; for all $x, y \epsilon X$;

(iii) $M(x, y, t) = 1 \, for \, all \, t > 0$ if and only $x = y$;

(iv) $M(x, y, t) = M(y, x, t)$; for all $x, y \epsilon X$ and $t > 0$;

(v) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$ for all $x, y, z \epsilon X$ and $s, t > 0$;

(vi) $M(x, y, .) : [0, \infty) \to [0,1]$ is continuous;

(vii) $lim_{t \to \infty} M(x, y, t) = 1$ for all x, y in X and $t > 0$;

(viii) $N(x, y, 0) = 1$ for all $x, y \epsilon X$;

(ix) $N(x, y, t) = 0$ for all $x, y \epsilon X$ and $t > 0$ if and only if $x = y$;

(x) $N(x, y, t) = N(y, x, t)$ for all $x, y \epsilon X$ and $t > 0$;

(xi) $N(x, y, t) \Diamond N(y, z, s) \geq N(x, z, t + s)$ for all $x, y, z \epsilon X$ and $s, t > 0$;

(xii) $N(x, y, .) : [0, \infty) \to [0,1]$ is continuous for all $x, y \epsilon X$;

(xiii) $lim_{t \to \infty} N(x, y, t) = 0$ for all $x, y \epsilon X$ and $t > 0$;

Then (M, N) is called an intuitionistic fuzzy metric space. The functions $M(x, y, t)$ and $N(x, y, t)$ denote the degree of nearness and the degree of non-nearness between x and y with respect to t, respectively.

Definition 1.4 [14,16]: A sequence $\{x_n\}$ in an intuitionistic fuzzy metric space $(X, M, N,*,\lozenge)$ is said to be Cauchy sequence is convergent with respect to the topology $\tau_{(M,N)}$.

Let $(X, M, N,*,\lozenge)$ be an intuitionistic fuzzy metric space. According to [7], the fuzzy metric (M, N) is called triangular whenever

$$\frac{1}{M(x,y,t)} - 1 \leq \frac{1}{M(x,z,t)} - 1 + \frac{1}{M(z,y,t)} - 1$$

And

$$N(x, y, t) \leq N(x, z, t) + N(z, y, t)$$

For all $x, y, z \epsilon X$ and $t > 0$.

2. Main Results

Theorem 2.1: Let $(X, M, N,*,\lozenge)$ be a complete triangular intuitionistic fuzzy metric space, $k \epsilon (0,1)$ and let $G: X \to X$ be a continuous mapping satisfying the contractive condition

$$\frac{1}{M(Gx,Gy,t)} - 1 \leq k \, max\left(\frac{1}{M(x,Gx,t)} - 1, \frac{1}{M(y,Gy,t)} - 1, \frac{1}{M(x,Gy,t)} - 1, \frac{1}{M(x,y,t)} - 1\right) \quad \text{...(2.1.1)}$$

For all $x, y \epsilon X$. Then G has a fixed point.

Proof: Let $x_0 \epsilon X$. Put $x_1 = Gx_0$ and $x_{n+1} = G^{n+1}x_0$ for all $n \geq 1$.

If $Gx_n = x_{n+1}$ for some n, then we have nothing to prove.

Assume that $x_n \neq x_{n+1}$ for all n. Then

$$\frac{1}{M(x_{n+1},x_n,t)} - 1 = \frac{1}{M(Gx_n,Gx_{n-1},t)} - 1$$

$$\leq k \, max\left(\frac{1}{M(x_n,Gx_n,t)} - 1, \frac{1}{M(x_{n-1},Gx_{n-1},t)} - 1, \frac{1}{M(x_n,Gx_{n-1},t)} - 1, \frac{1}{M(x_n,x_{n-1},t)} - 1\right)$$

Now for each n, put

$$t_n = max\left(\frac{1}{M(x_n,Gx_n,t)} - 1, \frac{1}{M(x_{n-1},Gx_{n-1},t)} - 1, \frac{1}{M(x_n,Gx_{n-1},t)} - 1, \frac{1}{M(x_n,x_{n-1},t)} - 1\right)$$

Case I: If $t_n = \frac{1}{M(x_n,Gx_n,t)} - 1$, then

$$\frac{1}{M(x_{n+1},x_n,t)} - 1 \leq k\left(\frac{1}{M(x_n,Gx_n,t)} - 1\right) = k\left(\frac{1}{M(x_n,x_{n+1},t)} - 1\right)$$

which is a contradiction.

Case II: If $t_n = \frac{1}{M(x_{n-1},Gx_{n-1},t)} - 1$, then

$$\frac{1}{M(x_{n+1},x_n,t)} - 1 \leq k\left(\frac{1}{M(x_{n-1},Gx_{n-1},t)} - 1\right) = k\left(\frac{1}{M(x_{n-1},x_n,t)} - 1\right)$$

But $\frac{1}{M(x_n,x_{n-1},t)} - 1 = \frac{1}{M(Gx_{n-1},Gx_{n-2},t)} - 1$

$$\leq k\, max\left(\frac{1}{M(x_{n-1},Gx_{n-1},t)} - 1, \frac{1}{M(x_{n-2},Gx_{n-2},t)} - 1, \frac{1}{M(x_{n-1},Gx_{n-2},t)} - 1, \frac{1}{M(x_{n-1},x_{n-2},t)} - 1\right)$$

$$\leq k\, max\left(\frac{1}{M(x_{n-1},x_n,t)} - 1, \frac{1}{M(x_{n-2},x_{n-1},t)} - 1, \frac{1}{M(x_{n-1},x_{n-1},t)} - 1, \frac{1}{M(x_{n-1},x_{n-2},t)} - 1\right)$$

$\frac{1}{M(x_n,x_{n-1},t)} - 1 \leq k\left(\frac{1}{M(x_{n-1},x_{n-2},t)} - 1\right)$ for all n.

Thus, $\frac{1}{M(x_{n+1},x_n,t)} - 1 \leq k\left(\frac{1}{M(x_n,x_{n-1},t)} - 1\right) \leq \cdots k^n\left(\frac{1}{M(x_1,x_0,t)} - 1\right)$

Case III: If $t_n = \frac{1}{M(x_n,Gx_{n-1},t)} - 1$, then

$$\frac{1}{M(x_{n+1},x_n,t)} - 1 \leq k\left(\frac{1}{M(x_n,Gx_{n-1},t)} - 1\right) = k\left(\frac{1}{M(x_n,x_n,t)} - 1\right),$$

which is a contradiction .

Case IV: If $t_n = \frac{1}{M(x_n,x_{n-1},t)} - 1$, then

$$\frac{1}{M(x_{n+1},x_n,t)} - 1 \leq k\left(\frac{1}{M(x_n,x_{n-1},t)} - 1\right)$$

But $\frac{1}{M(x_n,x_{n-1},t)} - 1 = \frac{1}{M(Gx_{n-1},Gx_{n-2},t)} - 1$

$$\leq k\, max\left(\frac{1}{M(x_{n-1},Gx_{n-1},t)} - 1, \frac{1}{M(x_{n-2},Gx_{n-2},t)} - 1, \frac{1}{M(x_{n-1},Gx_{n-2},t)} - 1, \frac{1}{M(x_{n-1},x_{n-2},t)} - 1\right)$$

$$\leq kmax\left(\frac{1}{M(x_{n-1},x_n,t)} - 1, \frac{1}{M(x_{n-2},x_{n-1},t)} - 1, \frac{1}{M(x_{n-1},x_{n-1},t)} - 1, \frac{1}{M(x_{n-1},x_{n-2},t)} - 1\right)$$

$\frac{1}{M(x_n,x_{n-1},t)} - 1 \leq k\left(\frac{1}{M(x_{n-2},x_{n-1},t)} - 1\right)$ for all n.

Thus, $\frac{1}{M(x_{n+1},x_n,t)} - 1 \leq k\left(\frac{1}{M(x_n,x_{n-1},t)} - 1\right) \leq \cdots k^n\left(\frac{1}{M(x_1,x_0,t)} - 1\right)$

Hence, for each $n > m$, we obtain

$$\frac{1}{M(x_n,x_m,t)} - 1 \leq \left(\frac{1}{M(x_n,x_{n-1},t)} - 1\right) + \left(\frac{1}{M(x_{n-1},x_{n-2},t)} - 1\right) + \cdots + \left(\frac{1}{M(x_{m+1},x_m,t)} - 1\right)$$

$$\leq (k^{n-1} + k^{n-2} + \cdots + k^m)\left(\frac{1}{M(x_1,x_0,t)} - 1\right)$$

$$\leq \frac{k^m}{1-k}\left(\frac{1}{M(x_1,x_0,t)} - 1\right)$$

Therefore, $\{x_n\}$ is a Cauchy sequence and so there exists $x^* \epsilon X$ such that $x_n \to x^*$. Since G is continuous, $x_{n+1} = Gx_n \to Gx^*$ and so $x^* = Gx^*$.

Theorem 2.2: Let $(X, M, N,*,\Diamond)$ be a complete triangular intuitionistic fuzzy metric space, $k \epsilon(0,1)$ and let $G: X \rightarrow X$ be a selfmap which satisfies the contractive condition

$$\frac{1}{M(Gx,Gy,t)} - 1 \leq k\left(\frac{1}{M(x,y,t)} - 1\right) \qquad ...(2.2.1)$$

For all $x, y \epsilon X$. Then G has a fixed point.

Proof: Let $x_0 \epsilon X$. Define the sequence $\{x_n\}$ by $x_{n+1} = Gx_n$ for all n. Then

$$\frac{1}{M(x_{n+1},x_n,t)} - 1 = \frac{1}{M(Gx_n,Gx_{n-1},t)} - 1$$

$$\leq k\left(\frac{1}{M(x_n,x_{n-1},t)} - 1\right)$$

$$\leq k^2\left(\frac{1}{M(x_{n-1},x_{n-2},t)} - 1\right)$$

....

....

$$\leq k^n\left(\frac{1}{M(x_1,x_0,t)} - 1\right) \text{ for all } n \text{ and } t > 0.$$

Therefore, $\left(\frac{1}{M(x_n,x_{n-1},t)} - 1\right)$ is a non- increasing sequence and so it is convergent to some $z \geq 0$.

Hence $\{x_n\}$ is a Cauchy sequence and so it converges to some $z \epsilon X$. We claim z is a fixed point of G.

Since

$$\frac{1}{M(x_{n+1},Gz,t)} - 1 = \frac{1}{M(Gx_n,Gz,t)} - 1$$

$$\leq k\left(\frac{1}{M(x_n,z,t)} - 1\right)$$

$$\frac{1}{M(z,Gz,t)} - 1 \leq 0$$

But $\frac{1}{M(z,Gz,t)} - 1 \geq 0$

Thus $\frac{1}{M(z,Gz,t)} - 1 = 0$

$\Rightarrow Gz = z$ therefore z is a fixed point of G.

Example 2.1 Let $X = (0, \sqrt{3} - 1)$ endowed with the usual distance $(x, y) = |x - y|$. Consider $M(x, y, t) = \frac{t}{t+d(x,y)}$ and $N(x, y, t) = \frac{d(x,y)}{t+d(x,y)}$ for all $x, y \epsilon X$ and $t \geq 0$. Define the selfmap G on X by

$$Gx = \begin{cases} 0 & x \epsilon (0, \sqrt{3} - 1) \\ \sqrt{3} - 1), & x = \sqrt{3} - 1 \end{cases}$$

It is easy to check that G satisfies the conditions of Theorem 2.2

Theorem 2.3: Let $(X, M, N, *, \lozenge)$ be a complete triangular intuitionistic fuzzy metric space, $\alpha, \beta \epsilon (0,1)$ with $\alpha + \beta < 1$ and let $G: X \to X$ be a continuous mapping which satisfies the contractive condition

$$\frac{1}{M(Gx,Gy,t)} - 1 \leq \alpha \left[\frac{\left(\frac{1}{M(x,Gx,t)} - 1\right)\left\{\frac{1}{M(y,Gx,t)} - 1 + \frac{1}{M(x,Gy,t)} - 1\right\}}{\frac{1}{M(x,Gx,t)} - 1 + \frac{1}{M(x,y,t)} - 1} \right] + \beta \left(\frac{1}{M(x,y,t)} - 1 \right) \qquad ...(2.3.1)$$

For all $x, y \epsilon X$. Then G has a unique fixed point in X.

Proof: Let $x_0 \epsilon X$. Put $x_1 = Gx_0$ and $x_{n+1} = G^{n+1}x_0$ for all $n \geq 1$.

If $Gx_n = x_{n+1}$ for some n, then we have nothing to prove.

Assume that $x_n \neq x_{n+1}$ for all n. Then

$$\begin{aligned}
\frac{1}{M(x_{n+1},x_n,t)} - 1 &= \frac{1}{M(Gx_n,Gx_{n-1},t)} - 1 \\
&\leq \alpha \left[\frac{\left(\frac{1}{M(x_n,Gx_n,t)} - 1\right)\left\{\frac{1}{M(x_{n-1},Gx_n,t)} - 1 + \frac{1}{M(x_n,Gx_{n-1},t)} - 1\right\}}{\frac{1}{M(x_n,Gx_n,t)} - 1 + \frac{1}{M(x_n,x_{n-1},t)} - 1} \right] + \beta \left(\frac{1}{M(x_n,x_{n-1},t)} - 1 \right) \\
&\leq \alpha \left[\frac{\left(\frac{1}{M(x_n,x_{n+1},t)} - 1\right)\left\{\frac{1}{M(x_{n-1},x_{n+1},t)} - 1 + \frac{1}{M(x_n,x_n,t)} - 1\right\}}{\frac{1}{M(x_n,x_{n+1},t)} - 1 + \frac{1}{M(x_n,x_{n-1},t)} - 1} \right] + \beta \left(\frac{1}{M(x_n,x_{n-1},t)} - 1 \right) \\
&\leq \alpha \left[\frac{\left(\frac{1}{M(x_n,x_{n+1},t)} - 1\right)\left\{\frac{1}{M(x_{n-1},x_n,t)} - 1 + \frac{1}{M(x_n,x_{n+1},t)} - 1\right\}}{\frac{1}{M(x_n,x_{n+1},t)} - 1 + \frac{1}{M(x_n,x_{n-1},t)} - 1} \right] + \beta \left(\frac{1}{M(x_n,x_{n-1},t)} - 1 \right) \\
&\leq \alpha \left(\frac{1}{M(x_n,x_{n+1},t)} - 1 \right) + \beta \left(\frac{1}{M(x_n,x_{n-1},t)} - 1 \right)
\end{aligned}$$

$$\begin{aligned}
\Rightarrow \frac{1}{M(x_{n+1},x_n,t)} - 1 &\leq \left(\frac{\beta}{1-\alpha}\right)\left(\frac{1}{M(x_n,x_{n-1},t)} - 1 \right) \\
&\leq \cdots \leq \left(\frac{\beta}{1-\alpha}\right)^n \left(\frac{1}{M(x_1,x_0,t)} - 1 \right) \text{ for all n.}
\end{aligned}$$

By using the triangular inequality, for each $m \geq n$, we obtain

$$\begin{aligned}
\frac{1}{M(x_n,x_m,t)} - 1 &\leq \frac{1}{M(x_n,x_{n+1},t)} - 1 + \frac{1}{M(x_{n+1},x_{n+2},t)} - 1 + \cdots + \frac{1}{M(x_{m-1},x_m,t)} - 1 \\
&\leq (h^n + h^{n+1} + \cdots + h^{m-1}) \left(\frac{1}{M(x_1,x_0,t)} - 1 \right) \\
&\leq \frac{h^n}{1-h} \left(\frac{1}{M(x_1,x_0,t)} - 1 \right),
\end{aligned}$$

where $h = \frac{\beta}{1-\alpha}$. Thus $\{x_n\}$ is a Cauchy sequence, therefore it converges to some $x^* \epsilon X$. Since G is continuous, it follows $Gx^* = x^*$, hence x^* is a fixed point of G.

Now, suppose that G has another fixed point $y^* \neq x^*$. Then we have

$$\frac{1}{M(x^*,y^*,t)} - 1 = \frac{1}{M(Gx^*,Gy^*,t)} - 1$$

$$\leq \alpha \left[\frac{\left(\frac{1}{M(x^*,Gx^*,t)}-1\right)\left\{\frac{1}{M(y^*,Gx^*,t)}-1+\frac{1}{M(x^*,Gy^*,t)}-1\right\}}{\frac{1}{M(x^*,Gx^*,t)}-1+\frac{1}{M(x^*,y^*,t)}-1} \right] + \beta \left(\frac{1}{M(x^*,y^*,t)} - 1\right)$$

$$\Rightarrow \left(\frac{1}{M(x^*,y^*,t)} - 1\right) \leq \beta \left(\frac{1}{M(x^*,y^*,t)} - 1\right)$$

which is a contradiction. Hence G has a unique fixed point.

Theorem 2.4: Let $(X, M, N,*,\lozenge)$ be a complete triangular intuitionistic fuzzy metric space, $\alpha, \beta \epsilon (0,1)$ with $3\alpha + 2\beta < 1$ and let $G: X \to X$ be a continuous mapping which satisfies the contractive condition For all $x, y \in X$

$$\frac{1}{M(Gx,Gy,t)} - 1 \leq \alpha \frac{\frac{1}{M(y,Gx,t)}-1+\frac{1}{M(x,Gx,t)}-1+\frac{1}{M(x,Gy,t)}-1}{1+\left(\frac{1}{M(y,Gx,t)}-1\right)\left(\frac{1}{M(x,Gx,t)}-1\right)\left(\frac{1}{M(x,Gy,t)}-1\right)}$$

$$+\beta \frac{\frac{1}{M(x,y,t)}-1+\frac{1}{M(y,Gy,t)}-1+\frac{1}{M(x,Gy,t)}-1}{1+\left(\frac{1}{M(x,y,t)}-1\right)\left(\frac{1}{M(y,Gy,t)}-1\right)\left(\frac{1}{M(x,Gy,t)}-1\right)} \qquad \text{...(2.4.1)}$$

Then G has a unique fixed point in X.

Proof: Let $x_0 \epsilon X$. Put $x_1 = Gx_0$ and $x_{n+1} = G^{n+1}x_0$ for all $n \geq 1$.

If $Gx_n = x_{n+1}$ for some n, then we have nothing to prove.

Assume that $x_n \neq x_{n+1}$ for all n. Then

$$\frac{1}{M(x_{n+1},x_n,t)} - 1 = \frac{1}{M(Gx_n,Gx_{n-1},t)} - 1$$

$$\leq \alpha \frac{\frac{1}{M(x_{n-1},x_{n+1},t)}-1+\frac{1}{M(x_n,x_{n+1},t)}-1+\frac{1}{M(x_n,x_n,t)}-1}{1+\left(\frac{1}{M(x_{n-1},x_{n+1},t)}-1\right)\left(\frac{1}{M(x_n,x_{n+1},t)}-1\right)\left(\frac{1}{M(x_n,x_n,t)}-1\right)}$$

$$+\beta \frac{\frac{1}{M(x_n,x_{n-1},t)}-1+\frac{1}{M(x_{n-1},x_n,t)}-1+\frac{1}{M(x_n,x_n,t)}-1}{1+\left(\frac{1}{M(x_n,x_{n-1},t)}-1\right)\left(\frac{1}{M(x_{n-1},x_n,t)}-1\right)\left(\frac{1}{M(x_n,x_n,t)}-1\right)}$$

$$\leq \alpha \frac{\frac{1}{M(x_{n-1},x_{n+1},t)}-1+\frac{1}{M(x_n,x_{n+1},t)}-1}{1} + \beta \frac{\frac{1}{M(x_n,x_{n-1},t)}-1+\frac{1}{M(x_{n-1},x_n,t)}-1}{1}$$

$$\leq \alpha \left(\frac{1}{M(x_{n-1},x_{n+1},t)} - 1\right) + \alpha \left(\frac{1}{M(x_n,x_{n+1},t)} - 1\right) + 2\beta \left(\frac{1}{M(x_n,x_{n-1},t)} - 1\right)$$

$$\Rightarrow \left(\frac{1}{M(x_n,x_{n+1},t)} - 1\right) \leq \frac{(\alpha+2\beta)}{(1-2\alpha)} \left(\frac{1}{M(x_{n-1},x_n,t)} - 1\right)$$

Therefore $\frac{1}{M(x_n,x_{n+1},t)} - 1 \leq k \left(\frac{1}{M(x_{n-1},x_n,t)} - 1\right)$ Let $k = \frac{(\alpha+2\beta)}{(1-2\alpha)}$,

Continuing this process n times, we get

$$\frac{1}{M(x_n,x_{n+1},t)} - 1 \leq k^n \left(\frac{1}{M(x_0,x_1,t)} - 1\right)$$

By using the triangular inequality, for each $m \geq n$, we obtain

$$\frac{1}{M(x_n,x_m,t)} - 1 \leq \frac{1}{M(x_n,x_{n+1},t)} - 1 + \frac{1}{M(x_{n+1},x_{n+2},t)} - 1 + \cdots + \frac{1}{M(x_{m-1},x_m,t)} - 1$$

$$\leq (k^n + k^{n+1} + \cdots + k^{m-1})\left(\frac{1}{M(x_1,x_0,t)} - 1\right)$$

$$\leq \frac{k^n}{1-k}\left(\frac{1}{M(x_1,x_0,t)} - 1\right)$$

Thus $\{x_n\}$ is a Cauchy sequence, therefore it converges to some $x^* \epsilon X$. Since G is continuous, it follows $Gx^* = x^*$, hence x^* is a fixed point of G.

Now, suppose that G has another fixed point $y^* \neq x^*$. Then we have

$$\frac{1}{M(x^*,y^*,t)} - 1 = \frac{1}{M(Gx^*,Gy^*,t)} - 1$$

$$\leq \alpha \frac{\frac{1}{M(y^*,Gx^*,t)} - 1 + \frac{1}{M(x^*Gx^*,t)} - 1 + \frac{1}{M(x^*,Gy^*,t)} - 1}{1 + \left(\frac{1}{M(y^*,Gx^*,t)} - 1\right)\left(\frac{1}{M(x^*,Gx^*,t)} - 1\right)\left(\frac{1}{M(x^*,Gy^*,t)} - 1\right)}$$

$$+\beta \frac{\frac{1}{M(x^*,y^*,t)} - 1 + \frac{1}{M(y^*,Gy^*,t)} - 1 + \frac{1}{M(x^*,Gy^*,t)} - 1}{1 + \left(\frac{1}{M(x^*,y^*,t)} - 1\right)\left(\frac{1}{M(y^*,Gy^*,t)} - 1\right)\left(\frac{1}{M(x^*,Gy^*,t)} - 1\right)}$$

$$\frac{1}{M(x^*,y^*,t)} - 1 \leq (2\alpha + 2\beta)\left(\frac{1}{M(x^*,y^*,t)} - 1\right)$$

Which is a contradiction. Hence G has a unique fixed point.

Theorem 2.5: Let $(X, M, N, *, \Diamond)$ be a complete triangular intuitionistic fuzzy metric space, $\alpha, \beta \epsilon (0,1)$ with $\alpha < \frac{1}{2}$ & $\beta < 1$ and let $G: X \to X$ be a continuous mapping which satisfies the contractive condition

$$\frac{1}{M(Gx,Gy,t)} - 1 \leq \alpha \frac{\max\left\{\left(\frac{1}{M(x,Gx,t)} - 1\right)\left(\frac{1}{M(y,Gx,t)} - 1\right), \left(\frac{1}{M(x,Gy,t)} - 1\right)\left(\frac{1}{M(y,Gy,t)} - 1\right)\right\}}{\frac{1}{M(x,Gx,t)} - 1 + \frac{1}{M(x,Gy,t)} - 1}$$

$$+\beta \frac{\left(\frac{1}{M(x,Gy,t)} - 1\right)\left(\frac{1}{M(y,Gx,t)} - 1\right)}{\frac{1}{M(x,Gx,t)} - 1 + \frac{1}{M(x,Gy,t)} - 1} \quad ..(2.5.1)$$

For all $x, y \epsilon X$. Then G has a unique fixed point in X.

Proof: Let $x_0 \epsilon X$. Put $x_1 = Gx_0$ and $x_{n+1} = G^{n+1}x_0$ for all $n \geq 1$.

If $Gx_n = x_{n+1}$ for some n, then we have nothing to prove.

Assume that $x_n \neq x_{n+1}$ for all n. Then

$$\frac{1}{M(x_{n+1},x_n,t)} - 1 = \frac{1}{M(Gx_n,Gx_{n-1},t)} - 1$$

$$\leq \alpha \frac{\max\left\{\left(\frac{1}{M(x_n,Gx_n,t)} - 1\right)\left(\frac{1}{M(x_{n-1},Gx_n,t)} - 1\right), \left(\frac{1}{M(x_n,Gx_{n-1},t)} - 1\right)\left(\frac{1}{M(x_{n-1},Gx_{n-1},t)} - 1\right)\right\}}{\frac{1}{M(x_n,Gx_n,t)} - 1 + \frac{1}{M(x_n,Gx_{n-1},t)} - 1}$$

$$+\beta \frac{\left(\frac{1}{M(x_n,Gx_{n-1},t)}-1\right)\left(\frac{1}{M(x_{n-1},Gx_n,t)}-1\right)}{\frac{1}{M(x_n,Gx_n,t)}-1+\frac{1}{M(x_n,Gx_{n-1},t)}-1}$$

$$\leq \alpha \frac{\max\left\{\left(\frac{1}{M(x_n,x_{n+1},t)}-1\right)\left(\frac{1}{M(x_{n-1},x_{n+1},t)}-1\right),\left(\frac{1}{M(x_n,x_n,t)}-1\right)\left(\frac{1}{M(x_{n-1},x_n,t)}-1\right)\right\}}{\frac{1}{M(x_n,x_{n+1},t)}-1+\frac{1}{M(x_n,x_n,t)}-1}$$

$$+\beta \frac{\left(\frac{1}{M(x_n,x_n,t)}-1\right)\left(\frac{1}{M(x_{n-1},x_{n+1},t)}-1\right)}{\frac{1}{M(x_n,x_{n+1},t)}-1+\frac{1}{M(x_n,x_n,t)}-1}$$

$$\leq \alpha \frac{\max\left\{\left(\frac{1}{M(x_n,x_{n+1},t)}-1\right)\left(\frac{1}{M(x_{n-1},x_{n+1},t)}-1\right),0\right\}}{\frac{1}{M(x_n,x_{n+1},t)}-1}$$

$$\Rightarrow \left(\frac{1}{M(x_{n+1},x_n,t)}-1\right) \leq \alpha\left(\frac{1}{M(x_{n-1},x_{n+1},t)}-1\right)$$

$$\Rightarrow \left(\frac{1}{M(x_{n+1},x_n,t)}-1\right) \leq \alpha\left\{\left(\frac{1}{M(x_{n-1},x_n,t)}-1\right)+\left(\frac{1}{M(x_n,x_{n+1},t)}-1\right)\right\}$$

$$\Rightarrow \left(\frac{1}{M(x_{n+1},x_n,t)}-1\right) \leq \frac{\alpha}{1-\alpha}\left(\frac{1}{M(x_{n-1},x_n,t)}-1\right)$$

$$\left(\frac{1}{M(x_n,x_{n+1},t)}-1\right) \leq k\left(\frac{1}{M(x_{n-1},x_n,t)}-1\right) \qquad \text{Let } k=\frac{\alpha}{1-\alpha}<1.$$

Continue this process n times we can easily

$$\left(\frac{1}{M(x_n,x_{n+1},t)}-1\right) \leq k^n\left(\frac{1}{M(x_0,x_1,t)}-1\right)$$

By using the triangular inequality, for each $m \geq n$, we obtain

$$\frac{1}{M(x_n,x_m,t)}-1 \leq \frac{1}{M(x_n,x_{n+1},t)}-1+\frac{1}{M(x_{n+1},x_{n+2},t)}-1+\cdots+\frac{1}{M(x_{m-1},x_m,t)}-1$$

$$\leq (k^n+k^{n+1}+\cdots+k^{m-1})\left(\frac{1}{M(x_1,x_0,t)}-1\right)$$

$$\leq \frac{k^n}{1-k}\left(\frac{1}{M(x_1,x_0,t)}-1\right)$$

Thus $\{x_n\}$ is a Cauchy sequence, therefore it converges to some $x^*\epsilon\, X$. Since G is continuous, it follows $Gx^*=x^*,$ hence x^* is a fixed point of G.

Now, suppose that G has another fixed point $y^* \neq x^*$. Then we have

$$\frac{1}{M(x^*,y^*,t)}-1=\frac{1}{M(Gx^*,Gy^*,t)}-1$$

$$\leq \alpha \frac{\max\left\{\left(\frac{1}{M(x^*,Gx^*,t)}-1\right)\left(\frac{1}{M(y^*,Gx^*,t)}-1\right),\left(\frac{1}{M(x^*,Gy^*,t)}-1\right)\left(\frac{1}{M(y^*,Gy^*,t)}-1\right)\right\}}{\frac{1}{M(x^*,Gx^*,t)}-1+\frac{1}{M(x^*,Gy^*,t)}-1}$$

$$+\beta \frac{\left(\frac{1}{M(x^*,Gy^*,t)}-1\right)\left(\frac{1}{M(y^*,Gx^*,t)}-1\right)}{\frac{1}{M(x^*,Gx^*,t)}-1+\frac{1}{M(x^*,Gy^*,t)}-1}$$

$$\leq \alpha \frac{\max\left\{\left(\frac{1}{M(x^*,x^*,t)}-1\right)\left(\frac{1}{M(y^*,x^*,t)}-1\right),\left(\frac{1}{M(x^*,y^*,t)}-1\right)\left(\frac{1}{M(y^*,y^*,t)}-1\right)\right\}}{\frac{1}{M(x^*,x^*,t)}-1+\frac{1}{M(x^*,y^*,t)}-1}$$

$$+\beta \frac{\left(\frac{1}{M(x^*y^*,t)}-1\right)\left(\frac{1}{M(y^*,x^*,t)}-1\right)}{\frac{1}{M(x^*,x^*,t)}-1+\frac{1}{M(x^*,y^*,t)}-1}$$

$$\leq \beta \frac{\left(\frac{1}{M(x^*y^*,t)}-1\right)\left(\frac{1}{M(y^*,x^*,t)}-1\right)}{\frac{1}{M(x^*,y^*,t)}-1}$$

$$\frac{1}{M(x^*,y^*,t)}-1 \leq \beta \frac{1}{M(x^*,y^*,t)}-1$$

This is a contradiction.

$\Rightarrow \frac{1}{M(x^*,y^*,t)}-1=0$ Since $\beta<1$.

$\Rightarrow x^*=y^*$ Hence the fixed point of G is unique.

Theorem 2.6: Let $(X,M,N,*,\lozenge)$ be a complete triangular intuitionistic fuzzy metric space, $\alpha,\beta,\gamma\epsilon(0,1)$ with $\alpha+\beta+\gamma<1$ and let $G\colon X\to X$ be a continuous mapping which satisfies the contractive condition

$$\frac{1}{M(Gx,Gy,t)}-1 \leq \alpha\left(\frac{1}{M(x,y,t)}-1\right)+\beta\left(\frac{1}{M(x,Gx,t)}-1\right)$$
$$+\gamma \frac{\left(\frac{1}{M(y,Gy,t)}-1\right)\left[1+\frac{1}{M(x,Gy,t)}-1+\frac{1}{M(y,Gx,t)}-1\right]}{1+\frac{1}{M(x,Gx,t)}-1+\frac{1}{M(y,Gy,t)}-1} \quad \text{...(2.6.1)}$$

For all $x,y\,\epsilon\, X$. Then G has a unique fixed point in X.

Proof: Let $x_0\,\epsilon X$. Put $x_1=Gx_0$ and $x_{n+1}=G^{n+1}x_0$ for all $n\geq 1$.

If $Gx_n=x_{n+1}$ for some n, then we have nothing to prove.

Assume that $x_n\neq x_{n+1}$ for all n. Then

$$\frac{1}{M(x_{n+1},x_n,t)}-1=\frac{1}{M(Gx_n,Gx_{n-1},t)}-1$$
$$\leq \alpha\left(\frac{1}{M(x_n,x_{n-1},t)}-1\right)+\beta\left(\frac{1}{M(x_n,x_{n+1},t)}-1\right)$$
$$+\gamma \frac{\left(\frac{1}{M(x_{n-1},x_n,t)}-1\right)\left[1+\frac{1}{M(x_n,x_n,t)}-1+\frac{1}{M(x_{n-1},x_{n+1},t)}-1\right]}{1+\frac{1}{M(x_n,x_{n+1},t)}-1+\frac{1}{M(x_{n-1},x_n,t)}-1}$$
$$\leq \alpha\left(\frac{1}{M(x_n,x_{n-1},t)}-1\right)+\beta\left(\frac{1}{M(x_n,x_{n+1},t)}-1\right)$$
$$+\gamma \frac{\left(\frac{1}{M(x_{n-1},x_n,t)}-1\right)\left[1+\frac{1}{M(x_{n-1},x_{n+1},t)}-1\right]}{1+\frac{1}{M(x_n,x_{n+1},t)}-1+\frac{1}{M(x_{n-1},x_n,t)}-1}$$
$$\leq \alpha\left(\frac{1}{M(x_n,x_{n-1},t)}-1\right)+\beta\left(\frac{1}{M(x_n,x_{n+1},t)}-1\right)$$

$$+\gamma \frac{\left(\frac{1}{M(x_{n-1},x_n,t)}-1\right)\left[1+\frac{1}{M(x_{n-1},x_n,t)}-1+\frac{1}{M(x_n,x_{n+1},t)}-1\right]}{1+\frac{1}{M(x_n,x_{n+1},t)}-1+\frac{1}{M(x_{n-1},x_n,t)}-1}$$

$$\leq \alpha\left(\frac{1}{M(x_n,x_{n-},t)}-1\right)+\beta\left(\frac{1}{M(x_n,x_{n+1},t)}-1\right)+\gamma\left(\frac{1}{M(x_{n-1},x_n,t)}-1\right)$$

$$\frac{1}{M(x_{n+1},x_n,t)}-1 \leq \frac{\alpha+\gamma}{1-\beta}\left(\frac{1}{M(x_{n-1},x_n,t)}-1\right)$$

$$\Rightarrow \left(\frac{1}{M(x_n,x_{n+1},t)}-1\right) \leq k\left(\frac{1}{M(x_{n-1},x_n,t)}-1\right) \qquad \text{Let } k=\frac{\alpha+\gamma}{1-\beta}<1.$$

Continue this process n times we can easily

$$\left(\frac{1}{M(x_n,x_{n+1},t)}-1\right) \leq k^n\left(\frac{1}{M(x_0,x_1,t)}-1\right)$$

By using the triangular inequality, for each $m \geq n$, we obtain

$$\frac{1}{M(x_n,x_m,t)}-1 \leq \frac{1}{M(x_n,x_{n+1},t)}-1+\frac{1}{M(x_{n+1},x_{n+2},t)}-1+\cdots+\frac{1}{M(x_{m-1},x_m,t)}-1$$

$$\leq (k^n+k^{n+1}+\cdots+k^{m-1})\left(\frac{1}{M(x_1,x_0,t)}-1\right)$$

$$\leq \frac{k^n}{1-k}\left(\frac{1}{M(x_1,x_0,t)}-1\right)$$

Thus $\{x_n\}$ is a Cauchy sequence, therefore it converges to some $x^* \epsilon X$. Since G is continuous, it follows $Gx^* = x^*$, hence x^* is a fixed point of G.

Now, suppose that G has another fixed point $y^* \neq x^*$. Then we have

$$\frac{1}{M(x^*,y^*,t)}-1 = \frac{1}{M(Gx^*,Gy^*,t)}-1$$

$$\leq \alpha\left(\frac{1}{M(x^*,y^*,t)}-1\right)+\beta\left(\frac{1}{M(x^*,Gx^*,t)}-1\right)$$

$$+\gamma \frac{\left(\frac{1}{M(y^*,Gy^*,t)}-1\right)\left[1+\frac{1}{M(x^*,Gy^*,t)}-1+\frac{1}{M(y^*,Gx^*,t)}-1\right]}{1+\frac{1}{M(x^*,Gx^*,t)}-1+\frac{1}{M(y^*,Gy^*,t)}-1}$$

$$\leq \alpha\left(\frac{1}{M(x^*,y^*,t)}-1\right)+\beta\left(\frac{1}{M(x^*,x^*,t)}-1\right)$$

$$+\gamma \frac{\left(\frac{1}{M(y^*,y^*,t)}-1\right)\left[1+\frac{1}{M(x^*,y^*,t)}-1+\frac{1}{M(y^*,x^*,t)}-1\right]}{1+\frac{1}{M(x^*,x^*,t)}-1+\frac{1}{M(y^*,y^*,t)}-1}$$

$$\leq \alpha\left(\frac{1}{M(x^*,y^*,t)}-1\right)+\beta\left(\frac{1}{M(x^*,x^*,t)}-1\right)$$

$$+\gamma \frac{\left(\frac{1}{M(y^*,y^*,t)}-1\right)\left[1+\frac{1}{M(x^*,y^*,t)}-1+\frac{1}{M(y^*,x^*,t)}-1\right]}{1+\frac{1}{M(x^*,x^*,t)}-1+\frac{1}{M(y^*,y^*,t)}-1}$$

$$\frac{1}{M(x^*,y^*,t)}-1 \leq \alpha\left(\frac{1}{M(x^*,y^*,t)}-1\right)$$

This is a contradiction

$\Rightarrow \frac{1}{M(x^*,y^*,t)} - 1 = 0$ Since $\alpha < 1$

$\Rightarrow x^* = y^*$ Hence the fixed point of G is unique.

Conflict of Interests

The authors declare that there is no conflict of interests.

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