Available online at http://scik.org
Advances in Fixed Point Theory, 2 (2012), No. 2, 176-196
ISSN: 1927-6303

COUPLED RANDOM FIXED POINT THEOREMS IN PARTIALLY ORDERED METRIC SPACES

SMRITI MEHTA[1,*] AND A.D.SINGH[2]

[1]Department of Mathematics, Truba Institute of Engineering and Information Technology, Bhopal, Madhya Pradesh, INDIA

[2]Department of Mathematics, Goverment M.V.M.College, Bhopal, Madhya Pradesh, INDIA

Abstract: In this paper, we prove some coupled random fixed point theorems for mapping having a mixed monotone property in partially ordered metric spaces for non linear contractions. Two coupled random coincidence and coupled random fixed point theorems are proved. Our result is a generalization of main result of Ciric and Lakshmikantham [Stochastic Analysis and Applications 27:6 (2009), 1246–1259].

Keywords: Coupled coincidence; Coupled fixed point; Measurable mapping; Mixed monotone mapping; Random operator.

2000 AMS Subject Classification: Primary 47H10; Secondary 54H25, 34B15.

1. Introduction:

A Fixed Point Theorem is a result that says that a function F will have at least one Fixed Point x for which (F(x) = x), under some conditions on F that can be stated in general terms. These results are the most generally useful in mathematics.

----

[*]Corresponding author

Received April 28, 2012

Random fixed point theorems for contraction mappings on separable complete metric spaces have been proved by several authors[8, 9, 10,12–14, 16–19, 26, 30, 31, 33–39]. The stochastic version of the well-known Schauder's fixed point theorem was proved by Sehgal and Singh[34].

Study of fixed point results in partially ordered metric spaces is at center of activity in field of research due to importance of this subject in differential equations. Fixed points of mappings in partially ordered spaces are of great importance and have been investigated by many researchers [1–7, 11, 15,20–25, 28, 29, 32]. Recently Bhaskar and Lakshmikantham [5], Nieto and Rodriguez-Lopez [28], Nieto, Pouso and Rodriguez-Lopez [29], Ranand Reurings [32], and Agarwal, El-Gebeily, and O'Regan [1] presented some new results for contractions in partially ordered metric spaces. V. Bhaskar and Lakshmikantham [5] introduced the concept of a coupled coincidence point of mapping F from X × X into X and a mapping g from X into X and studied fixed point theorems in partially ordered metric spaces.

Shatanawi [37] extended the results of Bhaskar and Lakshmikantham to partially ordered cone metric spaces. In [21] V. Lakshmikantham and L. Ciric studied some fixed point theorems for nonlinear contractions in partially ordered metric spaces. Recently, L. Ciric and Lakshmikantham [9] studied two coupled random coincidence and coupled random fixed point theorems for a pair of random mappings $F:\Omega\times(X\times X)\to X \text{ and } g:\Omega\times X\to X$ under some contractive conditions. Lakshmikantham and Ciric [21] introduced the concept of g-monotone mapping and proved some coupled coincidence and coupled common fixed point theorems in partially ordered complete metric spaces.

The purpose of this article is to prove coupled random coincidence and coupled random fixed point theorems for a pair of random mappings $F:\Omega\times\left(X\times X\right)\to X$ and $g:\Omega\times X\to X$. Thus we shall prove new results for random mixed monotone mappings, which are extensions of the corresponding results for deterministic mixed monotone mappings of Ciric and Lakshmikantham [9].

2. Preliminaries:

Definition 1.1 (Bhaskar and Lakshmikantham [5]). Let $(X,\leq)$ be a partially ordered set and $F: X\times X \to X$. The mapping F is said to has the mixed monotone property if F is monotone non-decreasing in its first argument and is monotone non-increasing in its second argument, that is, for any $x, y \in X$.

$$x_1, x_2 \in X;\ x_1 \leq x_2 \Rightarrow F(x_1, y) \leq F(x_2, y) \qquad .....(1)$$

and

$$y_1, y_2 \in X;\ y_1 \leq y_2 \Rightarrow F(x, y_1) \geq F(x, y_2) \qquad .....(2)$$

Definition 1.2 (Bhaskar and Lakshmikantham [5]). An element $(x, y) \in X\times X$ is called a coupled fixed point of the mapping $F: X\times X \to X$ if

$$F(x, y) = x, \quad F(y, x) = y.$$

The concept of the mixed monotone property is generalized in [20].

Definition 1.3 (Lakshmikantham and Ciric [21]). Let $(X,\leq)$ be a partially ordered set and $F: X\times X \to X$ and $g: X \to X$. The mapping F is said to has the mixed g-monotone property if F is monotone g-non-decreasing in its first argument and is monotone g-non-increasing in its second argument, that is for any $x, y \in X$.

$$x_1, x_2 \in X, \quad g(x_1) \leq g(x_2) \text{ implies } F(x_1, y) \leq F(x_2, y) \qquad .....(3)$$

and

$$y_1, y_2 \in X, \quad g(y_1) \leq g(y_2) \text{ implies } F(x, y_1) \geq F(x, y_2) \qquad .....(4)$$

Clearly, if g is the identity mapping, then Definition 1.3 reduces to Definition 1.1 .

Definition 1.4 An element $(x, y) \in X\times X$ is called a coupled coincidence point of a mapping $F: X\times X \to X$ and $g: X \to X$ if

$$F(x, y) = g(x), \qquad F(y, x) = g(y).$$

Definition 1.5 Let (X, d) be a separable metric space, (Ω, Σ) be a measurable space and $F : \Omega \times (X \times X) \to X$ and $g : \Omega \times X \to X$ be mapping. We say F and g are commutative if

$$F\Big(\omega, \big(g(\omega, x), g(\omega, y)\big)\Big) = g\Big(\omega, F\big(\omega, (x, y)\big)\Big)$$

for all $\omega \in \Omega$ and all $x, y \in X$.

Let (Ω, Σ) be a measurable space with Σ sigma algebra of subsets of Ω and let (X, d) be a metric space. A mapping $T : \Omega \to X$ is called Σ-measurable if for any open subset U of X, $T^{-1}(U) = \{\omega : T(\omega) \in U\} \in \Sigma$. In what follows, when we speak of measurability we will mean $\Sigma -$ measurability. A mapping $T : \Omega \times X \to X$ is called a random operator if for any $x \in X$, $T(., x)$ is measurable. A measurable mapping $\xi : \Omega \to X$ is called a random fixed point for a random function $T : \Omega \times X \to X$, if $\xi(\omega) = T(\omega, \xi(\omega))$ for every $\omega \in \Omega$. A measurable mapping $\xi : \Omega \to X$ is called a random coincidence of $T : \Omega \times X \to X$ and $g : \Omega \times X \to X$ if $g(\omega, \xi(\omega)) = T(\omega, \xi(\omega))$ for every $\omega \in \Omega$.

Let ϕ denote all functions $\varphi : [0, \infty) \to [0, \infty)$ which satisfy

(i) φ is continuous and non-decreasing,

(ii) $\varphi(t) = 0$ if and only if $t - 0$,

(iii) $\varphi(t + s) \le \varphi(t) + \varphi(s), \forall t, s \in [0, \infty)$

and ψ denote all functions $\psi : [0, \infty) \to [0, \infty)$ which satisfy $\lim_{t \to r} \psi(t) > 0$ for all $r > 0$ and $\lim_{t \to 0+} \psi(t) = 0$.

For example, functions $\varphi_1(t)=kt$ where $k>0, \varphi_2(t)=\frac{t}{t+1}, \varphi_3(t)=\ln(t+1)$, and $\varphi_4(t)=\min\{t,1\}$ are in ϕ ; $\psi_1(t)=kt$ where $k>0, \psi_2(t)=\frac{\ln(2t+1)}{2}$, and

$$\psi_3(t)=\begin{cases}1, & t=0\\ \frac{t}{t+1}, & 0<t<1\\ 1, & t=1\\ \frac{1}{2}t, & t>1\end{cases}$$ are in ψ.

In [20] the following theorem is proved.

Theorem 1.1 (Lakshmikantham and Ciric [21]). Let $(X,\leq)$ be apartially ordered set and suppose there is a metric d on X such that (X,d) is a complete metric space. Assume there is a function

$\varphi:[0,+\infty)\to[0,+\infty)$ with $\varphi(t)<t$ and $\lim_{r\to t+}\varphi(r)<t$ for each $t>0$ and also suppose $F:X\times X\to X$ and $g:X\to X$ are such that F has the mixed g-monotone property and

$$d(F(x,y),F(u,v))\leq\varphi\left(\frac{d(g(x),g(u))+d(g(y),g(v))}{2}\right)$$

for all $x,y,u,v\in X$ for which $g(x)\leq g(u)$ and $g(y)\geq g(v)$. Suppose $F(X\times X)\subseteq g(X), g$ is continuous and commutes with F and also suppose either

(a) F is continuous or

(b) X has the following property:

i. If a non-decreasing sequence $\{x_n\}\to x$, then $x_n\leq x$ for all n,

ii. If a non-decreasing sequence $\{y_n\}\to y$, then $y\leq y_n$ for all n,

If there exists $x_0, y_0\in X$ such that

$$g(x_0)\leq F(x_0,y_0) \text{ and } g(y_0)\geq F(y_0,x_0),$$

then there exist $x,y\in X$ such that

$g(x)=F(x,y)$ and $g(y)=F(y,x)$

That is, F and g have a coupled coincidence.

Theorem 1.2 (Lakshmikantham and Ciric [9]): Let (X,d) be a complete separable metric space, (Ω,E) be a measurable space and $F:\Omega\times(X\to X)\to X$ and $g:\Omega\times X\to X$ be mappings such that

(i) $F(\omega,.),g(\omega,.)$ are continuous for all $\omega\in\Omega$,

(ii) $F(.,v),g(.,x)$ are measurable for all $v\in X\times X$ and $x\in X$, respectively,

(iii) $F:\Omega\times(X\times X)\to X$ and $g:\Omega\times X\to X$ are such that F has the mixed g-monotone property and

$$d\left(F(\omega,(x,y)),F(\omega,(u,v))\right)$$
$$\le\varphi\left(\frac{d\left(g(\omega,x),g(\omega,u)\right)+d\left(g(\omega,y),g(\omega,v)\right)}{2}\right)$$

for all $x,y,u,v\in X$ for which $g(\omega,x)\le g(\omega,u)$ and $g(\omega,y)\ge g(\omega,v)$ for all $\omega\in\Omega$. Suppose $g(\omega\times X)=X$ for each $\omega\in\Omega$, g is continuous and commutes with F and also suppose either

(a) F is continuous or

(b) X has the following property:

(i) if a non-decreasing sequence $\{x_n\}\to x$, then $x_n\le x$ for all n,

(ii) if a non-increasing sequence $\{y_n\}\to y$, then $y_n\ge y$ for all n.

If there exist measurable mappings $\xi_0,\eta_0:\Omega\to X$ such that

$g\left(\omega,\xi_0\left(\omega\right)\right)\le F\left(\omega,\left(\xi_0\left(\omega\right),\eta_0\left(\omega\right)\right)\right)$ and

$g\left(\omega,\eta_0\left(\omega\right)\right)\ge F\left(\omega,\left(\eta_0\left(\omega\right),\xi_0\left(\omega\right)\right)\right)$ for all $\omega\in\Omega$,

then there are measurable mappings $\zeta,\theta:\Omega\to X$ such that

$F\left(\omega,\left(\zeta\left(\omega\right),\theta\left(\omega\right)\right)\right)=g\left(\omega,\zeta\left(\omega\right)\right)$ and $F\left(\omega,\left(\theta\left(\omega\right),\zeta\left(\omega\right)\right)\right)=g\left(\omega,\theta\left(\omega\right)\right)$

for all $\omega\in\Omega$, that is , F and g have a coupled random coincidence.

3. Main Results:

The following theorem is our main result.

Theorem 2.1 Let $\left(X,d\right)$ be a complete separable metric space, $\left(\Omega,\Sigma\right)$ be a measurable space and $F:\Omega\times\left(X\times X\right)\to X$ and $g:\Omega\times X\to X$ be mapping s such that

(i) $F\left(\omega,\cdot\right)$, $g\left(\omega,\cdot\right)$ are continuous for all $\omega\in\Omega$,

(ii) $F\left(\cdot,v\right), g\left(\cdot,x\right)$ are measurable for all $v\in X\times X$ and $x\in X$ respectively,

(iii) $F:\Omega\times\left(X\times X\right)\to X$ and $g:\Omega\times X\to X$ are such that F has the mixed g-monotone property and

$$\begin{aligned}&\phi\left(d\left(F\left(\omega,\left(x,y\right)\right),F\left(\omega,\left(u,v\right)\right)\right)\right)\\&\le\frac{1}{2}\varphi\left(d\left(g\left(\omega,x\right),g\left(\omega,u\right)\right)+d\left(g\left(\omega,y\right),g\left(\omega,v\right)\right)\right)\\&-\psi\left(\frac{d\left(g\left(\omega,x\right),g\left(\omega,u\right)\right)+d\left(g\left(\omega,y\right),g\left(\omega,v\right)\right)}{2}\right)\end{aligned} \quad .....(5)$$

for all $x,y,u,v\in X$ for which $g\left(\omega,x\right)\le g\left(\omega,u\right)$ and $g\left(\omega,y\right)\ge g\left(\omega,v\right)$ for all $\omega\in\Omega$. Suppose $g\left(\omega\times X\right)=X$ for each $\omega\in\Omega$, g is continuous and commutes with F and also suppose either

(a) F is continuous or

(b) X has the following property:

(i) if a non-decreasing sequence $\{x_n\} \to x,$ then $x_n \le x$ for all n, …..(6)

(ii) if a non-increasing sequence $\{y_n\} \to y,$ then $y_n \ge y$ for all n, …..(7)

If there exist measurable mappings $\xi_0, \eta_0 : \Omega \to X$ such that

$$g(\omega, \xi_0(\omega)) \le F(\omega, (\xi_0(\omega), \eta_0(\omega))) \text{ and}$$

$$g(\omega, \eta_0(\omega)) \ge F(\omega, (\eta_0(\omega), \xi_0(\omega))) \text{ for all } \omega \in \Omega,$$

then there are measurable mappings $\xi, \theta : \Omega \to X$ such that

$$F(\omega, (\xi(\omega), \theta(\omega))) = g(\omega, \xi(\omega)) \text{and } F(\omega, (\theta(\omega), \xi(\omega))) = g(\omega, \theta(\omega))$$

for all $\omega \in \Omega$, that is, F and g have a coupled random coincidence.

Proof: Let $\Theta = \{\xi : \Omega \to X\}$ be a family of measurable mappings. Define a function $h : \Omega \times X \to R^+$ as follows:

$$h(\omega, x) = d(x, g(\omega, x)).$$

Since $x \to g(\omega, x)$ is continuous for all $\omega \in \Omega$, we conclude that $h(\omega, \cdot)$ is continuous for all $\omega \in \Omega$. Also, since $\omega \to g(\omega, x)$ is measurable for all $x \in X$, we conclude that $h(\cdot, x)$ is measurable for all $\omega \in \Omega$ (see Wagner [38], p. 868). Thus, $h(\omega, x)$ is the Caratheordory function. Therefore, if $\xi : \Omega \to X$ is a measurable mapping, then $\omega \to h(\omega, \xi(\omega))$ is also measurable (see [32]). Also, for each $\xi \in \Theta$ the function $\eta : \Omega \to X$ defined by $\eta(\omega) = g(\omega, \xi(\omega))$ is measurable, that is, $\eta \in \Theta$.

Now we shall construct two sequences of measurable mappings $\{\xi_n\}$ and $\{\eta_n\}$ in Θ, and two sequences $\{g(\omega, \xi_n(\omega))\}$ and $\{g(\omega, \eta_n(\omega))\}$ in X as follows. Let $\xi_0, \eta_0 \in \Theta$ be such that $g(\omega, \xi_0(\omega)) \le F(\omega, (\xi_0(\omega), \eta_0(\omega)))$ and $g(\omega, \eta_0(\omega)) \ge F(\omega, (\eta_0(\omega), \xi_0(\omega)))$ for all $\omega \in \Omega$. Since $F(\omega, (\xi_0(\omega), \eta_0(\omega))) \in X = g(\omega \times X)$, by a sort of Filippov measurable implicit function theorem [4, 15, 18, 26] there is $\xi_1 \in \Theta$ such that $g(\omega,\xi_1(\omega)) = F(\omega,(\xi_0(\omega),\eta_0(\omega)))$. Similarly, as $F(\omega,(\eta_0(\omega),\xi_0(\omega))) \in g(\omega \times X)$, there is $\eta_1(\omega) \in \Theta$ such that $g(\omega,\eta_1(\omega)) = F(\omega,(\eta_0(\omega),\xi_0(\omega)))$. Now $F(\omega,(\xi_1(\omega),\eta_1(\omega)))$ and $F(\omega,(\eta_1(\omega),\xi_1(\omega)))$ are well defined. Again from $F(\omega,(\xi_1(\omega),\eta_1(\omega)))$, $F(\omega,(\eta_1(\omega),\xi_1(\omega))) \in g(\omega \times X)$, there are $\xi_2,\eta_2 \in \Theta$ such that $g(\omega,\xi_2(\omega)) = F(\omega,(\xi_1(\omega),\eta_1(\omega)))$ and $g(\omega,\eta_2(\omega)) = F(\omega,(\eta_1(\omega),\xi_1(\omega)))$. Continuing this process we can construct sequences $\{\xi_n(\omega)\}$ and $\{\eta_n(\omega)\}$ in X such that

$$g(\omega,\xi_{n+1}(\omega)) = F(\omega,(\xi_n(\omega),\eta_n(\omega))) \quad \text{and}$$

$$g(\omega,\eta_{n+1}(\omega)) = F(\omega,(\eta_n(\omega),\xi_n(\omega))) \quad \text{for all } n \geq 0. \qquad \text{…..(8)}$$

We shall prove that

$$g(\omega,\xi_n(\omega)) \leq g(\omega,\xi_{n+1}(\omega)) \quad \text{fro all } n \geq 0 \qquad \text{…..(9)}$$

and

$$g(\omega,\eta_n(\omega)) \geq g(\omega,\eta_{n+1}(\omega)) \quad \text{fro all } n \geq 0 \qquad \text{…..(10)}$$

The proof will be given by the mathematical induction. Let $n = 0$. By assumption we have $g(\omega,\xi_0(\omega)) \leq F(\omega,(\xi_0(\omega),\eta_0(\omega)))$ and $g(\omega,\eta_0(\omega)) \geq F(\omega,(\eta_0(\omega),\xi_0(\omega)))$.

Since $g(\omega,\xi_1(\omega)) = F(\omega,(\xi_0(\omega),\eta_0(\omega)))$ and $g(\omega,\eta_1(\omega)) = F(\omega,(\eta_0(\omega),\xi_0(\omega)))$, we have

$$g(\omega,\xi_0(\omega)) \leq g(\omega,\xi_1(\omega)) \text{ and } g(\omega,\eta_0(\omega)) \geq g(\omega,\eta_1(\omega))$$

Therefore, (9) and (10) hold for $n = 0$.

Suppose now that (9) and (10) hold for some fixed $n \geq 0$. Then, since $g(\omega,\xi_n(\omega)) \leq g(\omega,\xi_{n+1}(\omega))$ and $g(\omega,\eta_n(\omega)) \leq g(\omega,\eta_{n+1}(\omega))$, and as F is monotone g-non-decreasing in its first argument, from (3) and (8),

$$F(\omega,(\xi_n(\omega),\eta_n(\omega))) \leq F(\omega,(\xi_{n+1}(\omega),\eta_n(w)));$$

$$F\left(\omega,\left(\eta_{n+1}(\omega),\xi_n(\omega)\right)\right)\leq F\left(\omega,\left(\eta_n(\omega),\xi_n(\omega)\right)\right). \qquad .....(11)$$

Similarly, from (4) and (8), as $g\left(\omega,\eta_{n+1}(\omega)\right)\leq g\left(\omega,\eta_n(\omega)\right)$ and $g\left(\omega,\xi_n(\omega)\right)\leq g\left(\omega,\xi_{n+1}(\omega)\right)$,

$$F\left(\omega,\left(\xi_{n+1}(\omega),\eta_{n+1}(\omega)\right)\right)\geq F\left(\omega,\left(\xi_{n+1}(\omega),\eta_n(\omega)\right)\right);$$

$$F\left(\omega,\left(\eta_{n+1}(\omega),\xi_n(\omega)\right)\right)\geq F\left(\omega,\left(\eta_{n+1}(\omega),\xi_{n+1}(\omega)\right)\right). \qquad .....(12)$$

Now from (11), (12), and (8) we get

$$g\left(\omega,\xi_{n+1}(\omega)\right)\leq g\left(\omega,\xi_{n+2}(\omega)\right) \qquad .....(13)$$

and

$$g\left(\omega,\eta_{n+1}(\omega)\right)\geq g\left(\omega,\eta_{n+2}(\omega)\right) \qquad .....(14)$$

Thus, by the mathematical induction we conclude that (9) and (10) hold for all $n\geq 0$.

Denote

$$\delta_n=d\left(g\left(\omega,\xi_n(\omega)\right),g\left(\omega,\xi_{n+1}(\omega)\right)\right)+d\left(g\left(\omega,\eta_n(\omega)\right),g\left(\omega,\eta_{n+1}(\omega)\right)\right). \qquad .....(15)$$

Since from (9) and (10) we have $g\left(\omega,\xi_{n-1}(\omega)\right)\leq g\left(\omega,\xi_n(\omega)\right)$ and $g\left(\omega,\eta_{n-1}(\omega)\right)\geq g\left(\omega,\eta_n(\omega)\right)$, then from (8) and (5) we get

$$\phi\Big(d\left(g\left(\omega,\xi_n(\omega)\right),g\left(\omega,\xi_{n+1}(\omega)\right)\right)\Big)$$

$$=\phi\Big(d\Big(F\left(\omega,\left(\xi_{n-1}(\omega),\eta_{n-1}(\omega)\right)\right),F\left(\omega,\left(\xi_n(\omega),\eta_n(\omega)\right)\right)\Big)\Big)$$

$$\leq\frac{1}{2}\varphi\Big(d\left(g\left(\omega,\xi_{n-1}(\omega)\right),g\left(\omega,\xi_n(\omega)\right)\right)+d\left(g\left(\omega,\eta_{n-1}(\omega)\right),g\left(\omega,\eta_n(\omega)\right)\right)\Big)$$

$$-\psi\left(\frac{\Big(d\left(g\left(\omega,\xi_{n-1}(\omega)\right),g\left(\omega,\xi_n(\omega)\right)\right)+d\left(g\left(\omega,\eta_{n-1}(\omega)\right),g\left(\omega,\eta_n(\omega)\right)\right)\Big)}{2}\right) \qquad .....(16)$$

Similarly, from (8) and (5), as $g\left(\omega,\eta_n(\omega)\right)\leq g\left(\omega,\eta_{n-1}(\omega)\right)$ and $g\left(\omega,\xi_n(\omega)\right)\geq g\left(\omega,\xi_{n-1}(\omega)\right)$,

$$\phi\Big(d\left(g\left(\omega,\eta_{n+1}(\omega)\right),g\left(\omega,\eta_n(\omega)\right)\right)\Big)$$

$$=\phi\Big(d\Big(F\left(\omega,\left(\eta_n(\omega),\xi_n(\omega)\right)\right),F\left(\omega,\left(\eta_{n-1}(\omega),\xi_{n-1}(\omega)\right)\right)\Big)\Big)$$

$$\leq\frac{1}{2}\varphi\Big(d\left(g\left(\omega,\eta_{n-1}(\omega)\right),g\left(\omega,\eta_n(\omega)\right)\right)+d\left(g\left(\omega,\xi_{n-1}(\omega)\right),g\left(\omega,\xi_n(\omega)\right)\right)\Big)$$

$$-\psi\left(\frac{d\left(g\left(\omega,\eta_{n-1}(\omega)\right),g\left(\omega,\eta_n(\omega)\right)\right)+d\left(g\left(\omega,\xi_{n-1}(\omega)\right),g\left(\omega,\xi_n(\omega)\right)\right)}{2}\right) \quad .....(17)$$

By adding (16) and (17),

$$\phi\left(d\left(g\left(\omega,\xi_n(\omega)\right),g\left(\omega,\xi_{n+1}(\omega)\right)\right)\right)+\phi\left(d\left(g\left(\omega,\eta_{n+1}(\omega)\right),g\left(\omega,\eta_n(\omega)\right)\right)\right)$$

$$\leq\phi\left(d\left(g\left(\omega,\xi_{n-1}(\omega)\right),g\left(\omega,\xi_n(\omega)\right)\right)+d\left(g\left(\omega,\eta_{n-1}(\omega)\right),g\left(\omega,\eta_n(\omega)\right)\right)\right)$$

$$-2\psi\frac{\left(d\left(g\left(\omega,\xi_{n-1}(\omega)\right),g\left(\omega,\xi_n(\omega)\right)\right)+d\left(g\left(\omega,\eta_{n-1}(\omega)\right),g\left(\omega,\eta_n(\omega)\right)\right)\right)}{2} \quad .....(18)$$

By property (iii) of ϕ, we have

$$\phi\left[d\left(g\left(\omega,\xi_n(\omega)\right),g\left(\omega,\xi_{n+1}(\omega)\right)\right)+d\left(g\left(\omega,\eta_{n+1}(\omega)\right),g\left(\omega,\eta_n(\omega)\right)\right)\right]$$

$$\leq\phi\left(d\left(g\left(\omega,\xi_n(\omega)\right),g\left(\omega,\xi_{n+1}(\omega)\right)\right)\right)+\phi\left(d\left(g\left(\omega,\eta_{n+1}(\omega)\right),g\left(\omega,\eta_n(\omega)\right)\right)\right) \quad .....(19)$$

From (18) and (19) we have

$$\phi\left[d\left(g\left(\omega,\xi_n(\omega)\right),g\left(\omega,\xi_{n+1}(\omega)\right)\right)+d\left(g\left(\omega,\eta_{n+1}(\omega)\right),g\left(\omega,\eta_n(\omega)\right)\right)\right]$$

$$\leq\phi\left[d\left(g\left(\omega,\xi_{n-1}(\omega)\right),g\left(\omega,\xi_n(\omega)\right)\right)+d\left(g\left(\omega,\eta_{n-1}(\omega)\right),g\left(\omega,\eta_n(\omega)\right)\right)\right]$$

$$\leq 2\psi\left(\frac{d\left(g\left(\omega,\xi_{n-1}(\omega)\right),g\left(\omega,\xi_n(\omega)\right)\right)+d\left(g\left(\omega,\eta_{n-1}(\omega)\right),g\left(\omega,\eta_n(\omega)\right)\right)}{2}\right) \quad .....(20)$$

which implies

$$\phi\left(d\left(g\left(\omega,\xi_n(\omega)\right),g\left(\omega,\xi_{n+1}(\omega)\right)\right)+d\left(g\left(\omega,\eta_{n+1}(\omega)\right),g\left(\omega,\eta_n(\omega)\right)\right)\right)$$

$$\leq\phi\left(d\left(g\left(\omega,\xi_{n-1}(\omega)\right),g\left(\omega,\xi_n(\omega)\right)\right)+d\left(g\left(\omega,\eta_{n-1}(\omega)\right),g\left(\omega,\eta_n(\omega)\right)\right)\right)$$

Using the fact that ϕ is non decreasing, we get

$$d\left(g\left(\omega,\xi_n(\omega)\right),g\left(\omega,\xi_{n+1}(\omega)\right)\right)+d\left(g\left(\omega,\eta_{n+1}(\omega)\right),g\left(\omega,\eta_n(\omega)\right)\right)$$

$$\leq d\left(g\left(\omega,\xi_{n-1}(\omega)\right),g\left(\omega,\xi_n(\omega)\right)\right)+d\left(g\left(\omega,\eta_{n-1}(\omega)\right),g\left(\omega,\eta_n(\omega)\right)\right)$$

Therefore $\delta_n\leq\delta_{n-1}$

It follows that $\{\delta_n\}$ is the monotone decreasing sequence of positive reals. Therefore, there is some $\delta \geq 0$ such that

$$\lim_{n\to\infty}\delta_n = \lim_{n\to\infty}\Big[d\big(g(\omega,\xi_n(\omega)),g(\omega,\xi_{n+1}(\omega))\big)+d\big(g(\omega,\eta_{n+1}(\omega)),g(\omega,\eta_n(\omega))\big)\Big]=\delta$$

We show that $\delta = 0$. Suppose, to the contrary, that $\delta > 0$. Then, taking the limit in (20) when $\delta_n \to \delta+$ and have in mind that we assume that $\lim_{t\to r}\psi(t) > 0$ for all $r > 0$ and ϕ in continuous, we have

$$\begin{aligned}\phi(\delta) = \lim_{n\to\infty}\phi(\delta_n) &\leq \lim_{n\to\infty}\left[\phi(\delta_{n-1}) - 2\psi\left(\frac{\delta_{n-1}}{2}\right)\right]\\ &= \phi(\delta) - 2\lim_{\delta_{n-1}\to\delta}\psi\left(\frac{\delta_{n-1}}{2}\right)\\ &< \phi(\delta)\end{aligned}$$

a contradiction. Thus, $\delta = 0$,
that is

$$d\big(g(\omega,\xi_n(\omega)),g(\omega,\xi_{n+1}(\omega))\big)+d\big(g(\omega,\eta_{n+1}(\omega)),g(\omega,\eta_n(\omega))\big)=0 \quad \text{.....(21)}$$

Now we prove that $\{g(\omega,\xi_n(\omega))\}$ and $\{g(\omega,\eta_n(\omega))\}$ are Cauchy sequences. Suppose, to the contrary, that at least one of $\{g(\omega,\xi_n(\omega))\}$ or $\{g(\omega,\eta_n(\omega))\}$ is not a Cauchy sequence. Then there exist an $\varepsilon > 0$ and two subsequences of positive integers $\{l(k)\},\{m(k)\}$, $m(k) > l(k) \geq k$ with

$$\begin{aligned}r_k &= d\big(g(\omega,\xi_{l(k)}(\omega)),g(\omega,\xi_{m(k)}(\omega))\big)\\ &\quad + d\big(g(\omega,\eta_{l(k)}(\omega)),g(\omega,\eta_{m(k)}(\omega))\big) \geq \varepsilon \quad \text{for } k\in\{1,2,\ldots\}.\end{aligned} \quad \text{.....(22)}$$

We may also assume

$$d\big(g(\omega,\xi_{l(k)}(\omega)),g(\omega,\xi_{m(k)-1}(\omega))\big)+d\big(g(\omega,\eta_{l(k)}(\omega)),g(\omega,\eta_{m(k)-1}(\omega))\big)<\varepsilon \quad \text{....(23)}$$

by choosing $m(k)$ to be the smallest number exceeding $l(k)$ for which (18) holds. Such $m(k)$ for which (23) holds exists, because $\delta_n \to 0$. From (22), (23) and by the triangle inequality,

$$\begin{aligned}\varepsilon \le r_k &\le d\left(g\left(\omega,\xi_{l(k)}(\omega)\right), g\left(\omega,\xi_{m(k)-1}(\omega)\right)\right)\\ &+d\left(g\left(\omega,\xi_{m(k)-1}(\omega)\right), g\left(\omega,\xi_{m(k)}(\omega)\right)\right)\\ &+d\left(g\left(\omega,\eta_{l(k)}(\omega)\right), g\left(\omega,\eta_{m(k)-1}(\omega)\right)\right)\\ &+d\left(g\left(\omega,\eta_{m(k)-1}(\omega)\right), g\left(\omega,\eta_{m(k)}(\omega)\right)\right)\\ &= d\left(g\left(\omega,\xi_{l(k)}(\omega)\right), g\left(\omega,\xi_{m(k)-1}(\omega)\right)\right)\\ &\quad +d\left(g\left(\omega,\eta_{l(k)}(\omega)\right), g\left(\omega,\eta_{m(k)-1}(\omega)\right)\right)+\delta_{m(k)-1}\\ &< \varepsilon+\delta_{m(k)-1} \quad .\end{aligned}$$

Taking the limit as $k\to\infty$ we get

$$\lim_{k\to\infty} r_k = \varepsilon + . \qquad .....(24)$$

By (22) and the triangle inequality,

$$\begin{aligned} r_k &= d\left(g\left(\omega,\xi_{l(k)}(\omega)\right), g\left(\omega,\xi_{m(k)}(\omega)\right)\right)+d\left(g\left(\omega,\eta_{l(k)}(\omega)\right), g\left(\omega,\eta_{m(k)}(\omega)\right)\right)\\ &\le d\left(g\left(\omega,\xi_{l(k)}(\omega)\right), g\left(\omega,\xi_{l(k)+1}(\omega)\right)\right)\\ &\quad +d\left(g\left(\omega,\xi_{l(k)+1}(\omega)\right), g\left(\omega,\xi_{m(k)+1}(\omega)\right)\right)\\ &\quad +d\left(g\left(\omega,\xi_{m(k)+1}(\omega)\right), g\left(\omega,\xi_{m(k)}(\omega)\right)\right)\\ &\quad +d\left(g\left(\omega,\eta_{l(k)}(\omega)\right), g\left(\omega,\eta_{l(k)+1}(\omega)\right)\right)\\ &\quad +d\left(g\left(\omega,\eta_{l(k)+1}(\omega)\right), g\left(\omega,\eta_{m(k)+1}(\omega)\right)\right)\\ &\quad +d\left(g\left(\omega,\eta_{m(k)+1}(\omega)\right), g\left(\omega,\eta_{m(k)}(\omega)\right)\right)\\ &= \left[d\left(g\left(\omega,\xi_{l(k)}(\omega)\right), g\left(\omega,\xi_{l(k)+1}(\omega)\right)\right)+d\left(g\left(\omega,\eta_{l(k)}(\omega)\right), g\left(\omega,\eta_{l(k)+1}(\omega)\right)\right)\right]\\ &\quad +\left[d\left(g\left(\omega,\xi_{m(k)}(\omega)\right), g\left(\omega,\xi_{m(k)+1}(\omega)\right)\right)+d\left(g\left(\omega,\eta_{m(k)}(\omega)\right), g\left(\omega,\eta_{m(k)+1}(\omega)\right)\right)\right]\end{aligned}$$

$$+d\left(g\left(\omega,\xi_{l(k)+1}(\omega)\right),g\left(\omega,\xi_{m(k)+1}(\omega)\right)\right)$$

$$+d\left(g\left(\omega,\eta_{l(k)+1}(\omega)\right),g\left(\omega,\eta_{m(k)+1}(\omega)\right)\right).$$

Hence,

$$r_k \le \delta_{l(k)}+\delta_{m(k)}+d\left(g\left(\omega,\xi_{l(k)+1}(\omega)\right),g\left(\omega,\xi_{m(k)+1}(\omega)\right)\right)+d\left(g\left(\omega,\eta_{l(k)+1}(\omega)\right),g\left(\omega,\eta_{m(k)+1}(\omega)\right)\right) \qquad .....(25)$$

Using the property of ϕ, we have

$$\phi(r_k)=\phi\left[\delta_{l(k)}+\delta_{m(k)}+d\left(g\left(\omega,\xi_{l(k)+1}(\omega)\right),g\left(\omega,\xi_{m(k)+1}(\omega)\right)\right)+d\left(g\left(\omega,\eta_{l(k)+1}(\omega)\right),g\left(\omega,\eta_{m(k)+1}(\omega)\right)\right)\right]$$

$$\le\phi\left(\delta_{n(k)}+\delta_{m(k)}\right)+\phi d\left(d\left(g\left(\omega,\xi_{l(k)+1}(\omega)\right),g\left(\omega,\xi_{m(k)+1}(\omega)\right)\right)\right)$$

$$+\phi\left(d\left(g\left(\omega,\eta_{l(k)+1}(\omega)\right),g\left(\omega,\eta_{m(k)+1}(\omega)\right)\right)\right) \qquad .....(26)$$

Since from (9) and (10) we conclude that

$g\left(\omega,\xi_{l(k)}(\omega)\right)\le g\left(\omega,\xi_{m(k)}(\omega)\right)$ and $g\left(\omega,\eta_{l(k)}(\omega)\right)\ge g\left(\omega,\eta_{m(k)}(\omega)\right)$,

from (5) and (8),

$$\phi\left(d\left(g\left(\omega,\xi_{l(k)+1}(\omega)\right),g\left(\omega,\xi_{m(k)+1}(\omega)\right)\right)\right)$$

$$=\phi\left(d\left(F\left(\omega,\left(\xi_{l(k)},\eta_{l(k)}(\omega)\right)\right),F\left(\omega,\left(\xi_{m(k)}(\omega),\eta_{m(k)}(\omega)\right)\right)\right)\right)$$

$$\le\frac{1}{2}\varphi\left(d\left(g\left(\omega,\xi_{l(k)}\right),g\left(\omega,\xi_{m(k)}(\omega)\right)\right)+d\left(g\left(\omega,\eta_{l(k)}(\omega)\right),g\left(\omega,\xi_{m(k)}(\omega)\right)\right)\right)$$

$$-\psi\left(\frac{d\left(g\left(\omega,\xi_{l(k)}\right),g\left(\omega,\xi_{m(k)}(\omega)\right)\right)+d\left(g\left(\omega,\eta_{l(k)}(\omega)\right),g\left(\omega,\xi_{m(k)}(\omega)\right)\right)}{2}\right)$$

$$=\frac{1}{2}\phi(r_k)-\psi\left(\frac{r_k}{2}\right) \qquad .....(27)$$

Also from (5) and (8), as $g\left(\omega,\eta_{m(k)}(\omega)\right)\le g\left(\omega,\eta_{l(k)}(\omega)\right)$ and $g\left(\omega,\xi_{m(k)}(\omega)\right)\ge g\left(\omega,\xi_{l(k)}(\omega)\right)$,

$$\phi\left(d\left(g\left(\omega,\eta_{m(k)+1}(\omega)\right),g\left(\omega,\eta_{l(k)+1}(\omega)\right)\right)\right)$$

$$=\phi\left(d\left(F\left(\omega,\left(\eta_{m(k)}(\omega),\xi_{m(k)}(\omega)\right)\right),F\left(\omega,\left(\eta_{l(k)}(\omega),\xi_{l(k)}\right)\right)\right)\right)$$

$$\leq \frac{1}{2}\varphi\Big(d\left(g\left(\omega,\eta_{m(k)}(\omega)\right),g\left(\omega,\eta_{l(k)}(\omega)\right)\right)+d\left(g\left(\omega,\xi_{m(k)}(\omega)\right),g\left(\omega,\xi_{l(k)}(\omega)\right)\right)\Big)$$

$$-\psi\left(\frac{d\left(g\left(\omega,\eta_{m(k)}(\omega)\right),g\left(\omega,\eta_{l(k)}(\omega)\right)\right)+d\left(g\left(\omega,\xi_{m(k)}(\omega)\right),g\left(\omega,\xi_{l(k)}(\omega)\right)\right)}{2}\right)$$

$$=\frac{1}{2}\phi(r_k)-\psi\left(\frac{r_k}{2}\right) \qquad .....(28)$$

Inserting (26) and (28) in (25) we obtain

$$\phi(r_k)\leq\phi\left(\delta_{n(k)}+\delta_{m(k)}\right)+\phi(r_k)-2\psi\left(\frac{r_k}{2}\right)$$

Letting $k\to\infty$ we get, by 17(a) and (20),

$$\phi(\varepsilon)\leq\phi(0)+\phi(\varepsilon)-2\lim_{k\to\infty}\psi\left(\frac{r_k}{2}\right)=\phi(\varepsilon)-2\lim_{r_k\to\varepsilon}\psi\left(\frac{r_k}{2}\right)<\phi(\varepsilon) \qquad .....(29)$$

a contradiction. Therefore, our supposition (22) was wrong. Thus, we proved that $\left\{g\left(\omega,\xi_n(\omega)\right)\right\}$ and $\left\{g\left(\omega,\eta_n(\omega)\right)\right\}$ are Cauchy sequences in X.

Since X is complete and $g(\omega\times X)=X$, there exist $\zeta_0,\theta_0\in\Theta$ such that $\lim_{n\to\infty} g\left(\omega,\xi_n(\omega)\right)=g\left(\omega,\zeta_0(\omega)\right)$ and $\lim_{n\to\infty} g\left(\omega,\eta_n(\omega)\right)=g\left(\omega,\theta_0(\omega)\right)$. Since $g\left(\omega,\zeta_0(\omega)\right)$ and $g\left(\omega,\theta_0(\omega)\right)$ are measurable, then the functions $\zeta(\omega)$ and $\theta(\omega)$, defined by $\zeta(\omega)=g\left(\omega,\zeta_0(\omega)\right)$ and $\theta(\omega)=g\left(\omega,\theta_0(\omega)\right)$, are measurable. Thus

$$\lim_{n\to\infty} g\left(\omega,\xi_n(\omega)\right)=\zeta(\omega) \quad \text{and} \quad \lim_{n\to\infty} g\left(\omega,\eta_n(\omega)\right)=\theta(\omega) \qquad .....(30)$$

From (30) and continuity of g,

$$\lim_{n\to\infty} g\left(\omega,g\left(\omega,\xi_n(\omega)\right)\right)=g\left(\omega,\zeta(\omega)\right) \quad \text{and} \quad \lim_{n\to\infty} g\left(\omega,g\left(\omega,\eta_n(\omega)\right)\right)=g\left(\omega,\theta(\omega)\right) \qquad .....(31)$$

Form (8) and commutativity of F and g,

$$F\left(\omega,\left(g\left(\omega,\xi_n(\omega)\right),g\left(\omega,\eta_n(\omega)\right)\right)\right)$$
$$=g\left(\omega,F\left(\omega,\left(\xi_n(\omega),\eta_n(\omega)\right)\right)\right)=g\left(\omega,g\left(\omega,\xi_{n+1}(\omega)\right)\right) \quad .....(32)$$

$$F\left(\omega,\left(g\left(\omega,\eta_n(\omega)\right),g\left(\omega,\xi_n(\omega)\right)\right)\right)$$
$$=g\left(\omega,F\left(\omega,\left(\eta_n(\omega),\xi_n(\omega)\right)\right)\right)=g\left(\omega,g\left(\omega,\eta_{n+1}(\omega)\right)\right) \quad .....(33)$$

We now show that if the assumption a) or b) holds, then

$$g\left(\omega,\zeta(\omega)\right)=F\left(\omega,\left(\zeta(\omega),\theta(\omega)\right)\right) \text{ and } g\left(\omega,\theta(\omega)\right)=F\left(\omega,\left(\theta(\omega),\zeta(\omega)\right)\right).$$

Suppose at first that the assumption (a) holds. Then from (31), (32) and (33), and continuity of F we get

$$\begin{aligned} g\left(\omega,\zeta(\omega)\right) &= \lim_{n\to\infty} g\left(\omega,g\left(\omega,\xi_{n+1}(\omega)\right)\right) \\ &= \lim_{n\to\infty} F\left(\omega,\left(g\left(\omega,\xi_n(\omega)\right),g\left(\omega,\eta_n(\omega)\right)\right)\right) \\ &= F\left(\omega,\left(\lim_{n\to\infty} g\left(\omega,\xi_n(\omega)\right),\lim_{n\to\infty} g\left(\omega,\eta_n(\omega)\right)\right)\right) \\ &= F\left(\omega,\left(\zeta(\omega),\theta(\omega)\right)\right) \end{aligned}$$

$$\begin{aligned} g\left(\omega,\theta(\omega)\right) &= \lim_{n\to\infty} g\left(\omega,g\left(\omega,\eta_{n+1}(\omega)\right)\right) \\ &= \lim_{n\to\infty} F\left(\omega,\left(g\left(\omega,\eta_n(\omega)\right),g\left(\omega,\xi_n(\omega)\right)\right)\right) \\ &= F\left(\omega,\left(\lim_{n\to\infty} g\left(\omega,\eta_n(\omega)\right),\lim_{n\to\infty} g\left(\omega,\xi_n(\omega)\right)\right)\right) \\ &= F\left(\omega,\left(\theta(\omega),\zeta(\omega)\right)\right). \end{aligned}$$

Thus, we proved that

$$F\left(\omega,\left(\zeta(\omega),\theta(\omega)\right)\right)=g\left(\omega,\zeta(\omega)\right) \text{ and } F\left(\omega,\left(\theta(\omega),\zeta(\omega)\right)\right)=g\left(\omega,\theta(\omega)\right),$$

that is, $\left(\zeta(\omega),\theta(\omega)\right)\in X\times X$ is a coupled random coincidence of F and g.

Suppose now that (b) holds. Since from (9), $\left\{g\left(\omega,\xi_n(\omega)\right)\right\}$ is non-decreasing, and as $g\left(\omega,\xi_n(\omega)\right)\to g\left(\omega,\zeta(\omega)\right)$, from (6) we have $g\left(\omega,\xi_n(\omega)\right)\le g\left(\omega,\theta(\omega)\right)$ for all n. Also, as from (10), $\{g(\omega,\eta_n(\omega))\}$ is non-increasing and $g(\omega,\eta_n(\omega))\to g(\omega,\theta(\omega))$, from (7) we have $g(\omega,\eta_n(\omega))\geq g(\omega,\theta(\omega))$ for all n. Thus by the triangle inequality, (32) and (5) we get

$$
\begin{aligned}
&d\Big(g(\omega,\zeta(\omega)),F\big(\omega,(\zeta(\omega),\theta(\omega))\big)\Big)\\
&\quad\leq d\Big(g(\omega,\zeta(\omega)),g\big(\omega,g(\omega,\xi_{n+1}(\omega))\big)\Big)+d\Big(g\big(\omega,g(\omega,\xi_{n+1}(\omega))\big),F\big(\omega,(\zeta(\omega),\theta(\omega))\big)\Big)\\
&\qquad= d\Big(g(\omega,\zeta(\omega)),g\big(\omega,g(\omega,\xi_{n+1}(\omega))\big)\Big)\\
&\qquad\quad+d\Big(F\big(\omega,(g(\omega,\xi_n(\omega)),g(\omega,\eta_n(\omega)))\big),F\big(\omega,(\zeta(\omega),\theta(\omega))\big)\Big)
\end{aligned}
$$

Therefore

$$
\begin{aligned}
&\phi\Big(d\big(g(\omega,\zeta(\omega)),F(\omega,\zeta(\omega),\theta(\omega))\big)\Big)\\
&\leq\phi\Big(d\big(g(\omega,\zeta(\omega)),g(\omega,\xi_{n+1}(\omega))\big)\Big)+\phi\Big(d\Big(F\big(\omega,g(\omega,\xi_n(\omega)),g(\omega,\eta_n(\omega))\big),F\big(\omega,(\zeta(\omega),\theta(\omega))\big)\Big)\Big)\\
&\leq\phi\Big(d\Big(g(\omega,\zeta(\omega)),g\big(\omega,g(\omega,\xi_{n+1}(\omega))\big)\Big)\Big)\\
&+\frac{1}{2}\phi\Big(d\Big(g\big(\omega,g(\omega,\xi_n(\omega))\big),g(\omega,\zeta(\omega))\Big)+d\Big(g\big(\omega,g(\omega,\eta_n(\omega))\big),g(\omega,\theta(\omega))\Big)\Big)\\
&-\psi\left[\frac{d\Big(g\big(\omega,g(\omega,\xi_n(\omega))\big),g(\omega,\zeta(\omega))\Big)+d\Big(g\big(\omega,g(\omega,\eta_n(\omega))\big),g(\omega,\theta(\omega))\Big)}{2}\right]
\end{aligned}
$$

Letting $n\to\infty$, from (30) and the property of ψ we get $\phi\Big(d\big(g(\omega,\zeta(\omega)),F(\omega,(\zeta(\omega),\theta(\omega)))\big)\Big)=0.$

Thus $d\big(g(\omega,\zeta(\omega)),F(\omega,(\zeta(\omega),\theta(\omega)))\big)=0$

Hence $F(\omega,(\zeta(\omega),\theta(\omega)))=g(\omega,\zeta(\omega))$.

Similarly one can show that $F(\omega,(\theta(\omega),\zeta(\omega)))=g(\omega,\theta(\omega))$. Thus we proved that $(\zeta(\omega),\theta(\omega))\in X\times X$ is a random coupled coincidence point of F and g.

Corollary 2.2: Let (X,d) be a complete separable metric space, (Ω,Σ) be a measurable space and $F:\Omega\times(X\times X)\to X$ has the mixed monotone property and such that

(i) $F(\omega,\cdot)$ is continuous for all $\omega\in\Omega$.

(ii) $F(\cdot,v)$ is measurable for all $v\in X\times X$,

(iii) There exists a $\psi\in\psi$ such that F satisfies the following condition:

$$d\left(F\left(\omega,(x,y)\right),F\left(\omega,(u,v)\right)\right)\le\frac{d\left(g(\omega,x),g(\omega,u)\right)+d\left(g(\omega,y),g(\omega,v)\right)}{2}$$

$$-\psi\,\frac{d\left(g(\omega,x),g(\omega,u)\right)+d\left(g(\omega,y),g(\omega,v)\right)}{2} \qquad .....(34)$$

for all $x,y,u,v\in X$ for which $g(\omega,x)\le g(\omega,u)$ and $g(\omega,v)\le g(\omega,y)$ for all $\omega\in\Omega$.

Assume that F and g satisfies the following conditions.

(i) $F(\omega,\cdot),g(\omega,\cdot)$ are continuous for all $\omega\in\Omega$.

(ii) $F(\cdot,v),g(\cdot,v)$ are measurable for all $v\in X\times X$ and $x\in X$, respectively,

(iii) $F(\omega\times X)\subseteq X$ for each $\omega\in\Omega$,

(iv) g is continuous and commutes with F and also suppose either.

(a) F is continuous or

(b) X has the following property:

(i) if a non-decreasing sequence $\{x_n\}\to x$, then $x_n\le x$ for all n, ...(35)

(ii) if a non-increasing sequence $\{y_n\}\to y$, then $y\le y_n$ for all n. .....(36)

If there exist measurable mappings $\xi_0,\eta_0:\Omega\to X$ such that

$$\xi_0(\omega)\le F\left(\omega,\left(\xi_0(\omega),\eta_0(\omega)\right)\right) \text{ and}$$
$$\eta_0(\omega)\ge F\left(\omega,\left(\eta_0(\omega),\xi_0(\omega)\right)\right) \text{ for all } \omega\in\Omega,$$

then there are measurable mappings $\zeta,\theta:\Omega\to X$ such that

$$F\left(\omega,\left(\zeta(\omega),\theta(\omega)\right)\right)=\zeta(\omega) \text{ and } F\left(\omega,\left(\theta(\omega),\zeta(\omega)\right)\right)=\theta(\omega)$$

for all $\omega \in \Omega,$ that is, F has a random coupled fixed point.

Proof: In Theorem 2.1 taking $\phi(t) = t$, we get Corollary 2.1

REFERENCES

[1] R. P. Agrawal, M. A. EI-Gebeily and D. O'Regan, Generalized contractions in partially ordered metric spaces, Appl. Anal. 87 (2008), 109-116.

[2] R. P. Agrawal, D. O'Regan, and M. Sambandham, Random and deterministic fixed point theory for generalized contractive maps, Appl. Anal. 83 (2004), 711-725.

[3] R. P. Agrawal, M. A. EI-Gebeily, and D. O'Regan, Generalized contractions in partially ordered metric spaces, Appl. Anal. 87 (2008), 1-8.

[4] I. Beg and N. Shahzad, Random fixed point theorems for nonexpansive and contractive-type random operators on Banach spaces, J. Appl. Math. Stochastic Anal. 7 (1994), 569-580.

[5] T. G. Bhaskar, and V. Lakshmikantham, Fixed point theorems in partially ordered metric spaces and applications, Nonlinear Anal. 65 (2006), 1379-1393.

[6] T. G. Bhaskar, V. Lakshmikantham, and J. Devi Vasundhara Devi, Monotone iterative technique for functional differential equations withretardation and anticipation, Nonlinear Anal. 66 (2007), 2237–2242.

[7] Lj. B. Ciric, N. Cakic ́, M. Rajovic ́and J. S. Ume, Monotone generalized nonlinear contractions in partially ordered metric spaces, Fixed Point Theory Appl. 2008 (2008), Article ID 131294.

[8] Lj. B. C ́iric ́, S. N. Ješic ́ and J. S. Ume, On random coincidence for a pair of measurable mappings, J. Inequal. Appl. 2006 (2006), Article ID 81045.

[9] Lj. B. ́Ciri ́c, and V. Lakshmikantham, Coupled random fixed point theorems for nonlinear contractions in partially ordered metric spaces, Stochastic Analysis Appl. 27 (2009), 1246–1259.

[10] Lj. B. C ́iric ́, M. M. Milovanovic ́-Arandjelovic ́and N. T. Nikolic ́, On random coincidence and fixed points for a pair of multi-valued and singlevalued mappings, Italian J. Pure Appl. Math. 23 (2008), 37–44.

[11] D. Guo, and V. Lakshmikantham, Nonlinear Problems in Abstract Cones, Academic Press, New York, (1988).

[12] O. Hadzi ́c, A random fixed point theorem for multi valued mappings of C ́ iric ́'s type. Mat. Vesnik, 3 (16) (31)4, (1979), 397–401.

[13] O. Han ̌s, Reduzierende Zufällige transformationen. Czech. Math. J. 7 (1957), 154–158.

[14] O. Han ̌s, Random operator equations. Proc. 4th Berkeley Symp.Mathematical Statistics and Probability. Vol. II, Part I, University of California Press, Berkeley, pp., (1961), 185–202.

[15] S. Heikkila, and V. Lakshmikantham, Monotone Iterative Techniques for Discontinuous Nonlinear Differential Equations. Marcel Dekker,New York, (1994).

[16] C. J. Himmelberg, Measurable relations, Fund. Math. 87 (1975), 53–72.

[17] N. J. Huang, A principle of randomization of coincidence points with applications, Applied Math. Lett. 12 (1999), 107–113.

[18] N. Hussain, Common fixed points in best approximation for Banach operator pairs with C ´ iric ´ Type I-contractions, J. Math. Anal. Appl. 338 (2008), 1351–1363.

[19] S. Itoh, A random fixed point theorem for a multi-valued contraction mapping, Pacific J. Math. 68 (1977), 85–90.

[20] G.S. Ladde, V. Lakshmikantham and A. S. Vatsala, Monotone Iterative Techniques for Nonlinear Differential Equations, Pitman, Cambridge, (1985).

[21] V. Lakshmikantham, and Lj. B. C ´iric ´, Coupled fixed point theorems for nonlinear contractions in partially ordered metric spaces, Nonlinear Anal. 70 (2009), 4341–4349.

[22] V. Lakshmikantham, T. G. Bhaskar and J. Devi Vasundhara, Theory of Set Differential Equations in Metric Spaces, Cambridge Scientific Publishers, Cambridge, UK, (2005).

[23] V. Lakshmikantham and R. N. Mohapatra, Theory of Fuzzy Differential Equations and Inclusions, Taylor & Francis, London, (2003).

[24] V. Lakshmikantham and S. Koksal, Monotone Flows and Rapid Convergence for Nonlinear Partial Differential Equations, Taylor & Francis, London, (2003).

[25] V. Lakshmikantham and A. S. Vatsala, General uniqueness and monotone iterative technique for fractional differential equations, Appl. Math. Lett. 21 (2008), 828–834.

[26] T. C. Lin, Random approximations and random fixed point theorems for non-self maps, Proc. Am. Math. Soc. 103 (1988), 1129–1135.

[27] E. J. McShane and Jr. R. B. Warfield, On Filippov's implicit functions lemma, Proc. Am. Math. Soc. 18 (1967), 41–47.

[28] J. J. Nieto and R. Rodriguez-Lopez, Contractive mapping theorems in partially ordered sets and applications to ordinary differential equations, Order 22 (2005), 223–239.

[29] J. J. Nieto, R. L. Pouso and R. Rodriguez-Lopez, Fixed point theorems in ordered abstract spaces., Proc. Amer. Math. Soc. 135 (2007), 2505–2517.

[30] N. S. Papageorgiou, Random fixed point theorems for multifunctions, Math. Japonica 29, (1984), 93–106.

[31] N. S. Papageorgiou, Random fixed point theorems for measurable multifunctions in Banach spaces, Proc. Amer. Math. Soc., 97 (1986), 507–514.

[32] A.C.M. Ran and M.C.B. Reurings, A fixed point theorem in partially ordered sets and some applications to matrix equations, Proc.Amer. Math. Soc. 132 (2004), 1435–1443.

[33] R. T. Rockafellar, Measurable dependence of convex sets and functions in parameters, J. Math. Anal. Appl. 28 (1969), 4–25.

[34] V. M. Sehgal and S. P. Singh, On random approximations and a random fixed point theorem for set valued mappings, Proc. Am. Math. Soc. 95 (1985), 91–94.

[35] N. Shahzad and N. Hussian, Deterministic and random coincidence point results for f-nonexpansive maps, J. Math. Anal. Appl. 323 (2006), 1038–1046.

[36] N. Shahzad and A. Latif, A random coincidence point theorem, J. Math. Anal. Appl. 245 (2000), 633–638.

[37] W. Shatanawi, Partially ordered cone metric spaces and coupled fixed point results, Comput. Math. Appl. 60 (2010), 2508–2515.

[38] A. Špaček, Zufällige Gleichungen. Czech. Math. J. 5 (1955), 462–466.

[39] K. K. Tan, X. Z. Yuan and N. J. Huang, Random fixed point theorems and approximations in cones, J. Math. Anal. Appl. 185 (1994), 378–390.

[40] D. H. Wagner, Survey of measurable selection theorems, SIAM J. Control Optim. 15 (1977) 859–903.