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ON THE RATE OF CONVERGENCE OF NOOR, SP AND P-ITERATIONS FOR CONTINUOUS FUNCTIONS ON AN ARBITRARY INTERVAL

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Abstract. In this paper, we first give a necessary and sufficient condition for convergence of P-iteration to a fixed point of continuous functions on an arbitrary interval and prove equivalence of P-iteration, Noor and SP-iteration. We also compare the convergence speed of Noor, SP-iteration and P-iteration. It is proved that the P-iteration con verges faster than Noor and SP-iterations. Moreover, we also present numerical examples for the P-iteration to compare with the Noor and SP-iterations.

Keywords: rate of convergence; P-iteration; SP-iteration; non- decreasing function; fixed point; closed interval.

2010 AMS Subject Classification: 47H09, 47H10.

1. INTRODUCTION

Let E be a closed interval on the real line and $f : E \to E$ be a continuous function. A point $p \in E$ is a *fixed point* of f if $f(p) = p$.The set of all fixed points of f is denoted by $F(f)$. There are many fixed point iterations used for approximating a fixed point of a continuous mapping $f : E \to E$. The *Mann iteration* (see [1]) is defined by $v_1 \in E$ and

$$v_{n+1} = (1-\alpha_n)v_n + \alpha_n f(v_n) \tag{1.1}$$

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for all $n \geq 1$, where $\{\alpha_n\}_{n=1}^{\infty}$ is sequences in $[0,1]$, and will be denoted by $M(v_1,\alpha_n,f)$. The *Ishikawa iteration* (see [2]) is defined by $q_1 \in E$ and

$$\begin{cases} h_n = (1-\beta_n)\, g_n + \beta_n f(g_n) \\ g_{n+1} = (1-\alpha_n)\, g_n + \alpha_n f(h_n) \end{cases} \tag{1.2}$$

for all $n \geq 1$, where $\{\alpha_n\}_{n=1}^{\infty}$, $\{\beta_n\}_{n=1}^{\infty}$ are sequences in $[0,1]$, and will be denoted by $I(g_1,\alpha_n,\beta_n,f)$.The *Noor-iteration* (see [3]) is defined by $s_1 \in E$ and

$$\begin{cases} u_n = (1-\gamma_n)\, s_n + \gamma_n f(s_n) \\ t_n = (1-\beta_n)\, s_n + \beta_n f(u_n) \\ s_{n+1} = (1-\alpha_n)\, s_n + \alpha_n f(t_n) \end{cases} \tag{1.3}$$

for all $n \geq 1$, where $\{\alpha_n\}_{n=1}^{\infty}$, $\{\beta_n\}_{n=1}^{\infty}$ and $\{\gamma_n\}_{n=1}^{\infty}$ are sequences in $[0,1]$, and will be denoted by $N(x_1,\alpha_n,\beta_n,\gamma_n,f)$. The *SP-iteration* (see [4]) is defined by $w_1 \in E$ and

$$\begin{cases} r_n = (1-\gamma_n)\, w_n + \gamma_n f(w_n) \\ q_n = (1-\beta_n)\, r_n + \beta_n f(r_n) \\ w_{n+1} = (1-\alpha_n)\, q_n + \alpha_n f(q_n) \end{cases} \tag{1.4}$$

for all $n \geq 1$, where $\{\alpha_n\}_{n=1}^{\infty}$, $\{\beta_n\}_{n=1}^{\infty}$ and $\{\gamma_n\}_{n=1}^{\infty}$ are sequences in $[0,1]$, and will be denoted by $SP(x_1,\alpha_n,\beta_n,\gamma_n,f)$. The *P-iteration* (see [5])is defined by $x_1 \in E$ and

$$\begin{cases} z_n = (1-\gamma_n) x_n + \gamma_n f(x_n) \\ y_n = (1-\beta_n) z_n + \beta_n f(z_n) \\ x_{n+1} = (1-\alpha_n) f(z_n) + \alpha_n f(y_n) \end{cases} \tag{1.5}$$

for all $n \geq 1$, where $\{\alpha_n\}_{n=1}^{\infty}$, $\{\beta_n\}_{n=1}^{\infty}$ and $\{\gamma_n\}_{n=1}^{\infty}$ are sequences in $[0,1]$, and will be denoted by $P(x_1,\alpha_n,\beta_n,\gamma_n,f)$.

In 2005,Soltuz [6] showned that Mann and Ishikawa iterations are equivalent for the class of Zamfirescu operators. After that Babu and Prasad [7] showed that in the class of Zamfirescu operators, Mann iteration converges faster than Ishikawa iteration, but the claim is false,see Qing and Rhoades [8]. In 2011, Phuengrattana-Suantai [4] showed that the SP-iteration converges faster than the Mann, Ishikawa and Noor iterations on an arbitrary interval. In 2013, Kosol [9] showed that the S-iteration converges faster than the Ishikawa iteration. Recently, Sainuan [5] showed that the P-iteration converges faster than the Ishikawa and S-iterations.

In this paper, we give a necessary and sufficient condition for the convergence of the P-iteration of continuous non-decreasing functions on an arbitrary interval. We also prove that if the SP-iteration converges, then the

P-iteration converges and converges faster than Noor and the SP-iterations for the class of continuous and non-decreasing functions. Morover, we present the numerical examples for the P-iteration to compare with the Noor and SP-iterations.

2. Preliminaries

In this section we recall some lemmas, definitions, theorems and known results which will be used for our main results.

Lemma 2.1. *([4],Lemma 3.2) Let E be a closed interval on the real line and $f : E \to E$ be a continuous function. Let $\{\alpha_n\}_{n=1}^{\infty}$, $\{\beta_n\}_{n=1}^{\infty}$and $\{\gamma_n\}_{n=1}^{\infty}$ be sequences in $[0,1)$. Let $\{s_n\}_{n=1}^{\infty}$, $\{w_n\}_{n=1}^{\infty}$ be defined by Noor and SP-iterations, respectively. Then the following hold:*

(i) If $f(s_1) < s_1$, then $f(s_n) \leq s_n$ for all $n \geq 1$ and $\{s_n\}_{n=1}^{\infty}$ is non-increasing.
(ii) If $f(s_1) > s_1$, then $f(s_n) \geq s_n$ for all $n \geq 1$ and $\{s_n\}_{n=1}^{\infty}$ is non-decreasing.
(iii) If $f(w_1) < w_1$, then $f(w_n) \leq w_n$ for all $n \geq 1$ and $\{w_n\}_{n=1}^{\infty}$ is non-increasing.
(iv) If $f(w_1) > w_1$, then $f(w_n) \geq w_n$ for all $n \geq 1$ and $\{w_n\}_{n=1}^{\infty}$ is non-decreasing.

Lemma 2.2. *([5],Lemma 3.1) Let E be a closed interval on the real line and $f : E \to E$ be a continuous and non-decreasing function. Let $\{\alpha_n\}_{n=1}^{\infty}$, $\{\beta_n\}_{n=1}^{\infty}$and $\{\gamma_n\}_{n=1}^{\infty}$ be sequences in $[0,1]$. For $x_1 \in E$, let $\{x_n\}_{n=1}^{\infty}$ be defined by P-iteration. Then the following hold:*

(i) If $f(x_1) < x_1$, then $f(x_n) \leq x_n$ for all $n \geq 1$ and $\{x_n\}_{n=1}^{\infty}$ is non-increasing.
(ii) If $f(x_1) > x_1$, then $f(x_n) \geq x_n$ for all $n \geq 1$ and $\{x_n\}_{n=1}^{\infty}$ is non-decreasing.

Theorem 2.3. *([5],Theorem 3.2) Let E be a closed interval on the real line and $f : E \to E$ be a continuous and non-decreasing function. For $x_1 \in E$, let $\{x_n\}_{n=1}^{\infty}$ be defined by (1.5), where $\{\alpha_n\}_{n=1}^{\infty}$, $\{\beta_n\}_{n=1}^{\infty}$and $\{\gamma_n\}_{n=1}^{\infty}$ are sequences in $[0,1]$ and $\lim_{n\to\infty} \beta_n = \lim_{n\to\infty} \gamma_n = 0$. Then $\{x_n\}_{n=1}^{\infty}$ is bounded if and only if $\{x_n\}_{n=1}^{\infty}$ converges to a fixed point of f.*

Lemma 2.4. *([5],Lemma 3.3) Let E be a closed interval on the real line and $f : E \to E$ be a continuous and non-decreasing function. For $x_1 \in E$, let $\{x_n\}_{n=1}^{\infty}$ be the P-iteration defined by (1.5), where $\{\alpha_n\}_{n=1}^{\infty}$, $\{\beta_n\}_{n=1}^{\infty}$and $\{\gamma_n\}_{n=1}^{\infty}$ are sequences in $[0,1]$. Then we have the following :*

(i) If $p \in F(f)$ with $x_1 > p$, then $x_n \geq p$ for all $n \geq 1$.
(ii) If $p \in F(f)$ with $x_1 < p$, then $x_n \leq p$ for all $n \geq 1$.

Lemma 2.5. *(([4],Lemma 3.4) Let E be a closed interval on the real line and $f : E \to E$ be a continuous and non-decreasing function.Let $\{\alpha_n\}$, $\{\beta_n\}$ and $\{\gamma_n\}$ be sequences in $[0,1)$. For $v_1 = g_1 = s_1 = w_1 \in E$, let $\{v_n\}_{n=1}^{\infty}$, $\{g_n\}_{n=1}^{\infty}$, $\{s_n\}_{n=1}^{\infty}$, $\{w_n\}_{n=1}^{\infty}$ be the sequences defined by (1.1) - (1.4), respectively.Then the following are satisfied:*

(i) If $f(v_1) < v_1$, then $w_n \leq s_n \leq g_n \leq v_n$ for all $n \geq 1$.

(ii) If $f(v_1) > v_1$, then $w_n \geq s_n \geq g_n \geq v_n$ for all $n \geq 1$.

Proposition 2.6. *([4],Proposition3.5) Let E be a closed interval on the real line and $f : E \to E$ be a continuous and non-decreasing function such that $F(f)$ is nonempty and bounded with $x_1 > \sup\{p \in E : p = f(p)\}$. Let $\{\alpha_n\}$, $\{\beta_n\}$ and $\{\gamma_n\}$ be sequences in $[0,1)$. If $f(x_1) > x_1$, then the sequence $\{x_n\}$ defined by one of the following iteration methods: $M(x_1,\alpha_n,f)$, $I(x_1,\alpha_n,\beta_n,f)$, $N(x_1,\alpha_n,\beta_n,\gamma_n,f)$ and $SP(x_1,\alpha_n,\beta_n,\gamma_n,f)$ does not converge to a fixed point of f.*

Proposition 2.7. *([4],Proposition 3.6) Let E be a closed interval on the real line and $f : E \to E$ be a continuous and non-decreasing function such that $F(f)$ is nonempty and bounded with $x_1 < \inf\{p \in E : p = f(p)\}$. Let $\{\alpha_n\}$, $\{\beta_n\}$ and $\{\gamma_n\}$ be sequences in $[0,1]$. If $f(x_1) < x_1$, then the sequence $\{x_n\}$ defined by one of the following iteration methods: $M(x_1,\alpha_n,f)$, $I(x_1,\alpha_n,\beta_n,f)$, $N(x_1,\alpha_n,\beta_n,\gamma_n,f)$ and $SP(x_1,\alpha_n,\beta_n,\gamma_n,f)$ does not converge to a fixed point of f.*

Proposition 2.8. *([5],Proposition 3.5) Let E be a closed interval on the real line and $f : E \to E$ be a continuous and non-decreasing function such that $F(f)$ is nonempty and bounded with $x_1 < \inf\{p \in E : p = f(p)\}$. Let $\{\alpha_n\}$, $\{\beta_n\}$ and $\{\gamma_n\}$ be sequences in $[0,1]$. If $f(x_1) < x_1$, then the sequence $\{x_n\}$ defined by P-iteration does not converge to a fixed point of f.*

Proposition 2.9. *([5],Proposition 3.6) Let E be a closed interval on the real line and $f : E \to E$ be a continuous and non-decreasing function such that $F(f)$ is nonempty and bounded with $x_1 > \sup\{p \in E : p = f(p)\}$. Let $\{\alpha_n\}$, $\{\beta_n\}$ and $\{\gamma_n\}$ be sequences in $[0,1]$. If $f(x_1) > x_1$, then the sequence $\{x_n\}$ defined by P-iteration does not converge to a fixed point of f.*

For comparision the rate of convergence, we employ the concept given by Rhoades [10] as follows.

Definition 2.10. *([10]) Let E be a closed interval on the real line and $f : E \to E$ be a continuous function. Suppose that $\{x_n\}_{n=1}^{\infty}$ and $\{y_n\}_{n=1}^{\infty}$ are two iterations which converge to the fixed point p of f. Then $\{x_n\}_{n=1}^{\infty}$ is said to converge faster than $\{y_n\}_{n=1}^{\infty}$ if $|x_n - p| \leq |y_n - p|$ for all $n \geq 1$.*

In 2011, Phuengrattana and Suantai use above concept for comparing rate of convergence between SP and Noor iterations.

Theorem 2.11. *([4],Theorem 3.7) Let E be a closed interval on the real line and $f : E \to E$ be a continuous and non-decreasing function such that $F(f)$ is nonempty and bounded. For $s_1 = w_1 \in E$, let $\{s_n\}$ and $\{w_n\}$ be the sequences defined by (1.3) and (1.4), respectively. If the Noor-iteration $\{s_n\}$ converges to $p \in F(f)$, then the SP-iteration $\{w_n\}$ converges to p. Moreover, the SP-iteration converges faster than the Noor- iteration.*

3. Main results

We first give some useful facts for our main results.

Lemma 3.1. *Let E be a closed interval on the real line and $f : E \to E$ be a continuous and non-decreasing function. For $x_1 \in E$, let $\{\alpha_n\}_{n=1}^{\infty}$, $\{\beta_n\}_{n=1}^{\infty}$ and $\{\gamma_n\}_{n=1}^{\infty}$ be sequences in $[0,1]$. For $x_1 = w_1 \in E$, let $\{w_n\}_{n=1}^{\infty}$ and $\{x_n\}_{n=1}^{\infty}$ be sequences defined by (1.4) and (1.5) respectively. Then we have the following :*

(i) If $f(w_1) < w_1$, then $x_n \leq w_n$ for all $n \geq 1$.

(ii) If $f(w_1) > w_1$, then $x_n \geq w_n$ for all $n \geq 1$.

Proof. (i) Let $f(w_1) < w_1$. Since $x_1 = w_1$, we get $f(x_1) < x_1$. First, we show that $x_n \leq w_n$ for all $n \geq 1$.
From (1.5), we get $f(x_1) \leq z_1 \leq x_1$. Since f is non-decreasing, we have

$$f(z_1) \leq f(x_1) \leq z_1 \leq x_1.$$

By (1.5), we have $f(z_1) \leq y_1 \leq z_1$. Since f is non-decreasing, we obtain

$$f(y_1) \leq f(z_1) \leq y_1 \leq z_1 \leq x_1.$$

From (1.4) and (1.5), we get $z_1 - r_1 = (1-\gamma_1)(x_1 - w_1) + \gamma_1(f(x_1) - f(w_1)) = 0$, that is $z_1 = r_1$.
By (1.4) and (1.5), we get $y_1 - q_1 = (1-\beta_1)(z_1 - r_1) + \beta_1(f(z_1) - f(r_1)) = 0$. Thus $y_1 = q_1$.
Since $x_2 = (1-\alpha_1)f(z_1) + \alpha_1 f(y_1)$, it follows that

$$x_2 - w_2 = (1-\alpha_1)(f(z_1) - q_1)] + \alpha_1[f(y_1) - f(q_1)] \leq 0.$$

Thus $x_2 \leq w_2$. Assume that $x_k \leq w_k$. Thus $f(x_k) \leq f(w_k)$. By *Lemma* 2.1, $f(w_k) \leq w_k$ and *Lemma* 2.2 $f(x_k) \leq x_k$. By (1.4),(1.5), we get $f(w_k) \leq r_k \leq w_k$ and $f(x_k) \leq z_k \leq x_k$. Since f is non-decreasing, we have $f(r_k) \leq f(w_k) \leq r_k$ and $f(z_k) \leq f(x_k) \leq z_k$, it follows that

$$z_k - r_k = (1-\gamma_k)(x_k - w_k) + \gamma_k(f(x_k) - f(w_k)) \leq 0.$$

Thus $z_k \leq r_k$. Since f is non-decreasing, we have $f(z_k) \leq f(r_k)$. By (1,4),(1.5), we get $f(r_k) \leq q_k \leq r_k$ and $f(z_k) \leq y_k \leq z_k$. Since f is non-decreasing, we obtain $f(q_k) \leq f(r_k) \leq q_k \leq r_k$ and $f(y_k) \leq f(z_k) \leq y_k \leq z_k$. It follows that

$$y_k - q_k = (1-\beta_k)(z_k - r_k) + \beta_k(f(z_k) - f(r_k)) \leq 0, \text{ that is } y_k \leq q_k.$$

Since f is non-decreasing, we get $f(y_k) \leq f(q_k)$.
By (1.5), again $x_{k+1} = (1-\alpha_k)f(z_k) + \alpha_k f(y_k) \leq (1-\alpha_k)y_k + \alpha_k f(y_k)$.
It follows that $x_{k+1} - w_{k+1} \leq (1-\alpha_k)(y_k - q_k) + \alpha_k(f(y_k) - f(q_k)) \leq 0$, that is $x_{k+1} \leq w_{k+1}$.

By Mathematical induction, we obtain $x_n \leq w_n$ for all $n \geq 1$.
(ii) By using the same argument as in (i), we obtain the desired result.

□

Lemma 3.2. *Let E be a closed interval on the real line and $f : E \to E$ be a continuous and non-decreasing function. For $x_1 \in E$, let $\{\alpha_n\}_{n=1}^{\infty}$, $\{\beta_n\}_{n=1}^{\infty}$ and $\{\gamma_n\}_{n=1}^{\infty}$ be sequences in $[0,1]$. For $x_1 = s_1 \in E$, let $\{s_n\}_{n=1}^{\infty}$ and $\{x_n\}_{n=1}^{\infty}$ be sequences defined by (1.3) and (1.5), respectively. Then we have the following :*

(i) If $f(s_1) < s_1$, then $x_n \leq s_n$ for all $n \geq 1$.

(ii) If $f(s_1) > s_1$, then $x_n \geq s_n$ for all $n \geq 1$.

Proof. (i) and (ii) follows directly By *Lemma* 2.5, and using the same proof as in *Lemma* 3.1, we obtain the desired result. □

Theorem 3.3. *Let E be a closed interval on the real line and $f : E \to E$ be a continuous and non-decreasing function such that $F(f)$ is nonempty and bounded. For $w_1 = x_1 \in E$, let $\{w_n\}$ and $\{x_n\}$ be the sequences defined by (1.4) and (1.5), respectively. If the SP-iteration $\{w_n\}$ converges to $p \in F(f)$, then the P-iteration $\{x_n\}$ converges to p. Moreover, the P-iteration converges faster than the SP- iteration.*

Proof. Suppose the SP-iteration $\{w_n\}$ converges to $p \in F(f)$. Put $l = \inf\{x \in E : x = f(x)\}$ and $u = \sup\{x \in E : x = f(x)\}$. We devide our proof into the following three cases: Case 1: $w_1 = x_1 > u$. By Proposition 2.6 and Proposition 2.9, we get $f(w_1) < w_1$ and $f(x_1) < x_1$. By Lemma 3.1 (i), we have $x_n \leq w_n$ for all $n \geq 1$. By continuity of f, we have $f(u) = u$, so $u = f(u) \leq f(x_1) < x_1$. This implies by (1.5) that $f(x_1) \leq z_1 \leq x_1$, so $u \leq z_1 \leq x_1$. Since f is non-decreasing, we have $u = f(u) \leq f(z_1) \leq f(x_1) \leq z_1 \leq x_1$. It follows by (1.5), that $y_1 = (1-\beta_1)z_1 + \beta_1 f(z_1) \leq z_1$. Since f is non-decreasing, we have $u \leq f(y_1) \leq f(z_1) \leq f(x_1) \leq z_1 \leq x_1$ and $u \leq f(y_1) \leq x_2 \leq f(z_1)$. By mathematical induction, it can be shown that $u \leq x_n$ for all $n \geq 1$. Hence , we have $p \leq x_n \leq w_n$ for all $n \geq 1$, which implies $|x_n - p| \leq |w_n - p|$ for all $n \geq 1$. Thus $x_n \to p$ and the P-iteration converges to p faster than the SP- iteration.

Case 2: $w_1 = x_1 < l$. By Proposition 2.7 and Proposition 2.8 , we get $f(w_1) > w_1$ and $f(x_1) > x_1$. By Lemma 3.1 (ii), we have $x_n \geq w_n$ for all $n \geq 1$. We note that $x_1 < l$, by (1.5) and mathematical induction, we can show that $x_n < l$ for all $n \geq 1$. So $w_n \leq x_n \leq p$ for all $n \geq 1$. Hence $|x_n - p| \leq |w_n - p|$. It follows that $x_n \to p$ and the P-iteration converges to p faster than the SP-iteration.

Case 3: $l < w_1 = x_1 < u$. Suppose that $f(x_1) \neq x_1$. If $f(x_1) < x_1$, by Lemma 2.1(iv), we have that $\{w_n\}$ is non-increasing. It follows that $p \leq w_n$ for all $n \geq 1$. By Lemma 2.4 (i) and Lemma 3.1 (ii) , we get $p \leq x_n \leq q_n$ for all $n \geq 1$, This implies $|x_n - p| \leq |w_n - p|$. It follows that $x_n \to p$ and the P-iteration converges to p faster than the SP-iteration.

If $f(x_1) > x_1$, by Lemma 2.1 (iv), we have that $\{w_n\}$ is non-decreasing. This implies $w_n \leq p$ for all $n \geq 1$. By Lemma 2.4 (ii) and Lemma 3.1 (ii) , we get $w_n \leq x_n \leq p$ for all $n \geq 1$. It follows that $|x_n - p| \leq |w_n - p|$ for all $n \geq 1$. Hence $x_n \to p$ and the P-iteration converges to p faster than the SP-iteration.

□

Theorem 3.4. *Let E be a closed interval on the real line and $f : E \to E$ be a continuous and non-decreasing function such that $F(f)$ is nonempty and bounded. For $s_1 = x_1 \in E$, let $\{s_n\}$ and $\{x_n\}$ be the sequences defined by (1.3) and (1.5), respectively. If the Noor-iteration $\{s_n\}$ converges to $p \in F(f)$, then the P-iteration $\{x_n\}$ converges to p. Moreover, the P-iteration converges faster than the Noor- iteration.*

Proof. By Theorem 2.11 and Theorem 3.3, we obtain the desired result. □

Example 3.5. *Let $f : [0,2] \to [0,2]$ be a function defined by $f(x) = \frac{x^2+3}{4}$. Then f is a continuous and non-decreasing function. The comparisons of the convergence of the Noor iteration , SP-iteration and the P-iteration to the exact fixed point $p = 1$ are given in the following table with the initial point $x_1 = w_1 = s_1 = 2$ and $\alpha_n = \frac{n}{n+3}$, $\beta_n = \frac{1}{n}$, $\gamma_n = \frac{1}{n+3}$.*

	Noor	SP-iteration	P-iteration	
n	s_n	w_n	x_n	$\lvert f(x_n) - x_n \rvert$
7	1.36586986	1.034490736	1.007754827	0.008446877
8	1.086867017	1.019853146	1.003546709	0.003862379
⋮	⋮	⋮	⋮	⋮
27	1.000001719	1.000000193	1.000000003	0.000000003
28	1.000000929	1.000000102	1.000000001	0.000000001
29	1.000000500	1.000000054	1.000000001	0.000000001
30	1.000000269	1.000000029	1.000000000	0.000000000

Table 1:

Comparison of rate of convergence of Noor iteration, SP-iteration and P-iteration for the given function in Example 3.5 are shown in Table 1. We see that the P-iteration converges to $p = 1$ faster than the Noor and SP-iterations.

Example 3.6. *Let $f : [0,5] \to [0,5]$ be a function defined by $f(x) = \sqrt[3]{x^2+4}$. Then f is a continuous and non-decreasing function. The comparisons of the convergence of the Noor iteration, SP-iteration and the P-iteration to the exact fixed point $p = 2$ are given in the following table with the initial point $x_1 = w_1 = s_1 = 4$ and $\alpha_n = \frac{1}{n+2}$, $\beta_n = \frac{1}{n^2}$, $\gamma_n = \frac{1}{n^2}$.*

	Noor	SP-iteration	P-iteration	
n	s_n	w_n	x_n	$\lvert f(x_n) - x_n \rvert$
1	3.361458263	2.193800565	2.193800565	1.285582383
2	3.119954578	2.112579385	2.052307422	0.128242874
⋮	⋮	⋮	⋮	⋮
16	2.421482182	2.026681996	2.000000008	0.000000017
17	2.406891324	2.025628291	2.000000003	0.000000006
18	2.393506001	2.024673170	2.000000001	0.000000002
19	2.381173994	2.023802621	2.000000000	0.000000001

Table 2:

Table 2 shows comparison of rate of convergence of Noor iteration, SP-iteration and P-iteration for the given function in Example 3.6. We see that the P-iteration converges to $p = 2$ faster than the Noor and SP-iterations.

Conflict of Interests

The authors declare that there is no conflict of interests.

REFERENCES

[1] W.R. Mann, Mean value methods in iteration, Proc. Amer. Math. Soc. 4 (1953) 506-510.

[2] S. Ishikawa, Fixed points by a new iteration method, Proc. Amer. Math. Soc. 44 (1974) 147-150.

[3] M.A. Noor, New approximation schemes for general variational inequalities, J. Math. Anal. Appl. 251 (2000) 217-229.

[4] W.Phuengrattana, S-Suantai, On the rate of convergence of Mann, Ishikawa, Noor and SP-iterations continuous functions on an arbitrary interval, J. Comput. Appl. Math. 235(2011) 3006-3014.

[5] P. Sainuan, Rate of convergence of P-iteration and S-iteration for continuous functions on closed intervals, Thai J. Math. 13(2015) 449 - 457.

[6] S.M. Soltuz, The equivalence of Picard, Mann and Ishikawa iterations dealing with quasi-contractive operators, Math. Commun. 10 (2005), 81-88.

[7] G.V. Babu, K.N. Prasad, Mann iteration converges faster than Ishikawa iteration for the class of Zamfirescu operators, Fixed Point Theory Appl. 2006 (2006), Article ID 49615.

[8] Y. Qing, B.E. Rhoades, Comments on the Rate of Convergence between Mann and Ishikawa Iterations Applied to Zamfirescu Operators, Fixed Point Theory Appl. 2008 (2008) Article ID 387504.

[9] S. Kosol, Rate of convergence of S-iteration and Ishikawa iteration for continuous functions on closed intervals, Thai J. Math. 11 (2013), 703-709.

[10] B.E.Rhoades, Comments on two fixed point iteration methods, J. Math. Anal. Appl. 56 (1976), 741-750.