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UNIQUE COMMON FIXED POINTS FOR FOUR NON-CONTINUOUS MAPPINGS SATISFYING ψ-IMPLICIT CONTRACTIVE CONDITION ON NON-COMPLETE MULTIPLICATIVE METRIC SPACES

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Abstract. In this paper, we introduce a class Ψ of 5-dimensional real functions and consider a ψ-implicit contractive condition, and then discuss and obtain a unique common fixed point theorem for four non-continuous mappings defined on non-complete multiplicative metric spaces. Finally, we give several Ćirić type fixed point theorems on non-complete multiplicative metric spaces.

Keywords: multiplicative metric space; common fixed point; weakly compatible; ψ-implicit contractive condition.

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1. Introduction and preliminaries

The Banach contraction principle[1], i.e., Banach fixed point theorem is the basic and simple fixed point theorem, it is widely applied in mathematics and other fields. Hence the principle is very goodly generalized and improved in various metric spaces such as 2-metric space, G-metric space, cone metric space and complex valued metric space and so on. Especially, in

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2008, Bashirov et al[2] introduced the concept of multiplicative metric space and studied some basic properties. Afterwards, Florack and Assen[3] and Bashirove et al[4] also gave some other properties in this space. In 2012, Özavsar and Çevikel[5] introduced the concept of multiplicative contraction mappings on multiplicative metric spaces and obtained several existence theorems of fixed points; in 2013, He et al[6] proved the existence theorem of common fixed points for four mappings using the weakly commuting condition; in 2015, Abbas et al[7], Kang et al[8] and Gu et al[9] obtained common fixed point theorems using the locally contractive condition, compatible condition and weakly compatible condition respectively. Recently, Jiang and Gu[10] introduce the concept of ϕ-weakly commutative mappings and obtained common fixed point theorems for four mappings. Theses obtained results in [10] generalize and improve the corresponding conclusions in [6-9].

Now, we give some well-known definitions, examples, lemmas and theorems.

Definition 1.1.[2] Let X be a nonempty set, A multiplicative metric is a mapping $d : X \times X \to [0,\infty)$ satisfying:

(i) $d(x,y) \geq 1$ for all $x,y \in X$ and $d(x,y) = 1 \Longleftrightarrow x = y$;

(ii) $d(x,y) = d(y,x)$ for all $x,y \in X$;

(iii) $d(x,z) \leq d(x,y)\,d(y,z)$ for all $x,y,z \in X$.(multiplicative triangle inequality).

The pair (X,d) is called a multiplicative metric space.

Example 1.1.[5] Let $\mathbb{R}_+{}^n = \{(a_1,a_2,\cdots,a_n)|a_1,a_2,\cdots,a_n > 0\}$. Define $d : \mathbb{R}_+{}^n \times \mathbb{R}_+{}^n \to [0,\infty)$ as follows

$$d(x,y) = \mid \frac{x_1}{y_1} \mid \cdot \mid \frac{x_2}{y_2} \mid \cdots \mid \frac{x_2}{y_2} \mid,$$

where $x = (x_1,x_2,\cdots,x_n)$, $y = (y_1,y_2,\cdots,y_n) \in \mathbb{R}_+{}^n$, $|\cdot| : \mathbb{R}_+ \to \mathbb{R}_+$ is defined by :

$$\mid a \mid = \begin{cases} a, & \text{if}\, a \geq 1 \\ \frac{1}{a}, & \text{if}\, a < 1. \end{cases}$$

Then $(\mathbb{R}_+{}^n, d)$ is a multiplicative metric space.

Example 1.2.[9] Let $X = [0,\infty)$ and define $d(x,y) = e^{|x-y|}$, $\forall x,y \in X$. Then (X,d) is a also multiplicative metric space.

Definition 1.2.[2] Let (X,d) be a multiplicative metric space, $\{x_n\}$ be a sequence in X and $x \in X$. If for every multiplicative open ball $B_\varepsilon(x) = \{y \in X | d(x,y) < \varepsilon\}, \varepsilon > 1$, there exists a natural number N such that $x_n \in B_\varepsilon(x)$ for all $n > N$, then the sequence $\{x_n\}$ is said to be multiplicative converging to x, denoted by $x_n \to x(n \to \infty)$.

Lemma 1.1.[5] Let (X,d) be a multiplicative metric space, $\{x_n\}$ be a sequence in X and $x \in X$. Then

$$x_n \to x\,(n \to \infty) \Longleftrightarrow d(x_n,x) \to 1\,(n \to \infty).$$

Definition 1.3.[5] Let (X,d) be a multiplicative metric space, $\{x_n\}$ be a sequence in X. The sequence $\{x_n\}$ is called multiplicative Cauchy sequence if, for each $\varepsilon > 1$, there exists a natural number N such that $d(x_n,x_m) < \varepsilon$ for all $n,m > N$.

Lemma 1.2.[5] Let (X,d) be a multiplicative metric space, $\{x_n\}$ be a sequence in X. Then $\{x_n\}$ is a multiplicative Cauchy sequence if and only if $d(x_m,x_n) \to 1(m,n \to \infty)$.

Definition 1.4.[5] A multiplicative metric space (X,d) is said to be multiplicative complete if, every multiplicative Cauchy sequence in (X,d) is multiplicative convergent in X.

Definition 1.5.[5] Let (X,d_X) and (Y,d_Y) be two multiplicative metric spaces, $f : X \to Y$ a mapping and $x \in X$. If for every $\varepsilon > 1$, there exists $\delta > 1$ such that $f(B_\delta(x)) \subset B_\varepsilon(f(x))$, then we call f multiplicative continuous at x. If f is multiplicative continuous at all $x \in X$, then we say that f is multiplicative continuous on X.

Lemma 1.3.[5] Let (X,d_X) and (Y,d_Y) be two multiplicative metric spaces, $f : X \to Y$ a mapping and $x \in X$. Then f is multiplicative continuous at x if and only if $fx_n \to fx$ for every sequence $\{x_n\} \subset X$ with $x_n \to x$.

Lemma 1.4.[5] Let (X,d) be a multiplicative metric space, $\{x_n\}$ and $\{y_n\}$ be two sequences and $x,y \in X$. Then

$$x_n \to x,\, y_n \to y\,(n \to \infty) \Longrightarrow d(x_n,y_n) \to d(x,y)\,(n \to \infty).$$

Definition 1.6.[5] Let (X,d) be a multiplicative metric space. A mapping $f : X \to X$ is called a multiplicative contraction if there exists a real constant $\lambda \in (0,1)$ such that $d(fx,fy) \leq [d(x,y)]^\lambda$ for all $x,y \in X$.

Theorem 1.1.[5] Let (X,d) be a complete multiplicative metric space. If $f : X \to X$ is a multiplicative contraction, then f has a unique fixed point.

He et al obtained the following existence theorems of common fixed point for four mappings with the weakly commutativity on multiplicative metric spaces:

Theorem 1.2.[6] If four self-mappings S,T,A,B on a multiplicative metric space X satisfy the following conditions:

(i) $SX \subset BX$, $TX \subset AX$;

(ii) A and S are weakly commutative mappings, B and T are also weakly commutative mappings;

(iii) one of $\{A,B,S,T\}$ is continuous;

(iv) there exists $\lambda \in (0,\frac{1}{2})$ such that for every $x,y \in X$,

$$d(Sx,Ty) \leq \{\max\{d(Ax,By),d(Ax,Sx),d(Ry,Ty),d(Ax,Ty),d(By,Sx)\}\}^{\lambda},$$

Then S,T,A,B have a unique common fixed point.

In [9], Gu and Cho introduced the concepts of the compatibility and weak compatibility of two self-mappings on multiplicative metric spaces and gave the following fact:

commutativity $\Longrightarrow$ weak commutativity $\Longrightarrow$ compatibility $\Longrightarrow$ weak compatibility.

But the converse of the above fact is not true.(see, Remark 2.1 in [9])

The next class of functions can be found in [9]:

The class of real functions Φ is defined as follows: $\phi \in \Phi$ if and only if $\phi : [1,\infty)^5 \to [0,\infty)$ satisfy

(ϕ_1) ϕ is non-decreasing and continuous in each coordinate variable;

(ϕ_2) for each $t \geq 1$,

$$\max\{\phi(t,t,t,1,t),\phi(t,t,t,t,1),\phi(t,1,1,t,t),\phi(1,t,1,t,1),\phi(1,1,t,1,t)\} \leq t.$$

Gu and Cho gave the next generalization of Theorem 1.2 using $\phi \in \Phi$ and the concepts of compatibility and weak compatibility,:

Theorem 1.3.[9] If four self-mappings S,T,A,B on a complete multiplicative metric space X satisfy the following conditions:

(i) $SX \subset BX$, $TX \subset AX$;

(ii) there exists $\lambda \in (0, \frac{1}{2})$ such that for each $x, y \in X$,

$$d(Sx, Ty) \leq \phi(d^{\lambda}(Ax, By), d^{\lambda}(Ax, Sx), d^{\lambda}(By, Ty), d^{\lambda}(Ax, Ty), d^{\lambda}(By, Sx));$$

(iii) one of the following conditions is satisfied:

(a) either A or S is continuous, the pair (A, S) is compatible and the pair (T, B) is weakly compatible;

(b) either B or T is continuous, the pair (T, B) is compatible and the pair (A, S) is weakly compatible.

Then S, T, A, B have a unique common fixed point.

In 2017, Jiang and Gu[10] introduced the concept of φ-weak commutativity of two mappings and obtained the following result:

Theorem 1.4.[10] If four self-mappings S, T, A, B on a complete multiplicative metric space X satisfy the following conditions:

(i) $SX \subset BX$, $TX \subset AX$;

(ii) there exists $\lambda \in (0, \frac{1}{2})$ such that for each $x, y \in X$,

$$d(Sx, Ty) \leq \phi(d^{\lambda}(Ax, By), d^{\lambda}(Ax, Sx), d^{\lambda}(By, Ty), d^{\lambda}(Ax, Ty), d^{\lambda}(By, Sx));$$

(iii) one of the following conditions is satisfied:

(a) either A or S is continuous, the pair (A, S) is compatible and the pair (T, B) is φ-weakly commutative;

(b) either B or T is continuous, the pair (T, B) is compatible and the pair (A, S) is φ-weakly commutative.

Then S, T, A, B have a unique common fixed point.

Note that at least one mapping must be continuous in Theorem 1.2–Theorem 1.4. Hence the main aim in this paper is to introduce a new class Ψ of 5-dimensional real functions and discuss the existence problems of common fixed points for four non-continuous mappings satisfying ψ-implicit contractive conditions on non-complete multiplicative metric spaces. Finally, we give several fixed point theorems.

Definition 1.7.[9] Let (X,d) be a multiplicative metric space and $f,g : X \to X$ be two mappings. If $fgx = gfx$ whenever $fx = gx\,(x \in X)$, i.e., $d(fx,gx) = 1\,(x \in X) \Longrightarrow fgx = gfx$, i.e., $d(fgx,gfx) = 1$, then the pair (f,g) is called weakly compatible.

Definition 1.8.[11] Let X be a nonempty set and $f,g : X \to X$ two mappings. If there exist $w,x \in X$ such that $w = fx = gx$, then x is said to be a coincidence point of the pair (f,g) and w is said to be a point of coincidence of the pair (f,g).

Lemma 1.5 Let (X,d) be a multiplicative metric space and $f,g : X \to X$ be the pair of weakly compatible mappings. If $w = fx = gx$ is the unique point of coincidence of the pair (f,g), then w is the unique common fixed point of the pair (f,g).

Proof. By the condition $w = fx = gx$ and the weak compatibility, we have $fw = ffx = fgx = gfx = gw$. This implies that $gw = fw$ is also a point of coincidence of the pair (f,g), hence $fw = gw = w$ by the uniqueness of the point of coincidence of the pair (f,g) , which shows that w is a common fixed point of f and g. Obviously, w is the unique common fixed point of f and g.

2. Common fixed point under ψ-implicit contractions

Now, we introduce a class Ψ of real functions as follows

$\psi \in \Psi$ if and only if $\psi : [1,\infty)^5 \to [0,\infty)$ satisfies the following three conditions:

(ψ_1) ψ is non-decreasing and continuous in each coordinate variable;

(ψ_2) there exist $h_1,h_2 \in (0,\infty)$ satisfying $h_1h_2 < 1$ such that for each $u,v \in [1,\infty)$,

$$u \le \psi(v,u,v,1,uv) \Longrightarrow u \le v^{h_1},\ \ u \le \psi(v,v,u,uv,1) \Longrightarrow u \le v^{h_2};$$

(ψ_3) for each $t > 1$, $\max\{\psi(t,1,1,t,t), \psi(1,t,1,1,t), \psi(1,1,t,t,1)\} < t$.

Theorem 2.1. If (X,d) is a multiplicative metric space and $\{S,T,A,B\}$ are four self-mappings on X. Suppose that

(i) $SX \subset BX$ and $TX \subset AX$;

(ii) for each $x,y \in X$,

$$d(Sx,Ty) \le \psi(d(Ax,By), d(Ax,Sx), d(By,Ty), d(Ax,Ty), d(By,Sx)); \tag{2.1}$$

(iii) the pair (A,S) and the pair (T,B) are all weakly compatible;

(iv) one of $\{SX,AX,BX,TX\}$ is complete.

Then S,T,A,B have a unique common fixed point.

Proof. Take any element $x_0 \in X$. In view of (i), we construct two sequences $\{x_n\}$ and $\{y_n\}$ in X satisfying

$$y_{2n} = Sx_{2n} = Bx_{2n+1},\ \ y_{2n+1} = Tx_{2n+1} = Ax_{2n+2}, n = 0,1,2,\cdots. \tag{2.2}$$

For each $n = 01,2,\cdots$, by (2.1) and (ψ_1),

$$\begin{aligned}
&d(y_{2n},y_{2n+1}) = d(Sx_{2n},Tx_{2n+1})\\
&\leq \psi(d(Ax_{2n},Bx_{2n+1}),d(Ax_{2n},Sx_{2n}),d(Bx_{2n+1},Tx_{2n+1}),d(Ax_{2n},Tx_{2n+1}),d(Bx_{2n+1},Sx_{2n}))\\
&= \psi(d(y_{2n-1},y_{2n}),d(y_{2n-1},y_{2n}),d(y_{2n},y_{2n+1}),d(y_{2n-1},y_{2n+1}),d(y_{2n},y_{2n}))\\
&\leq \psi(d(y_{2n-1},y_{2n}),d(y_{2n-1},y_{2n}),d(y_{2n},y_{2n+1}),d(y_{2n-1},y_{2n})\cdot d(y_{2n},y_{2n+1}),1),
\end{aligned}$$

hence by (ψ_2), we obtain

$$d(y_{2n},y_{2n+1}) \leq [d(y_{2n-1},y_{2n})]^{h_2}, \forall n = 0,1,2,\cdots. \tag{2.3}$$

Similarly,

$$\begin{aligned}
&d(y_{2n+1},y_{2n+2}) = d(Sx_{2n+2},Tx_{2n+1})\\
&\leq \psi(d(Ax_{2n+2},Bx_{2n+1}),d(Ax_{2n+2},Sx_{2n+2}),d(Bx_{2n+1},Tx_{2n+1}),d(Ax_{2n+2},Tx_{2n+1}),\\
&\qquad\qquad d(Bx_{2n+1},Sx_{2n+2}))\\
&= \psi(d(y_{2n+1},y_{2n}),d(y_{2n+1},y_{2n+2}),d(y_{2n},y_{2n+1}),d(y_{2n+1},y_{2n+1}),d(y_{2n},y_{2n+2}))\\
&\leq \psi(d(y_{2n+1},y_{2n}),d(y_{2n+1},y_{2n+2}),d(y_{2n},y_{2n+1}),1,d(y_{2n},y_{2n+1})\cdot d(y_{2n+1},y_{2n+2})),
\end{aligned}$$

hence by (ψ_2) again, we have

$$d(y_{2n+1},y_{2n+2}) \leq [d(y_{2n},y_{2n+1})]^{h_1}, \forall n = 0,1,2,\cdots. \tag{2.4}$$

Using (2.3) and (2.4), we obtain

$$\begin{aligned}
&d(y_{2n},y_{2n+1}) \leq [d(y_{2n-1},y_{2n})]^{h_2} \leq [d(y_{2n-2},y_{2n-1})]^{h_1h_2} \leq \cdots \leq [d(y_0,y_1)]^{(h_1h_2)^n};\\
&d(y_{2n+1},y_{2n+2}) \leq [d(y_{2n},y_{2n+1})]^{h_1} \leq [d(y_0,y_1)]^{(h_1h_2)^n h_1}.
\end{aligned} \tag{2.5}$$

For each two natural numbers p,q with $q>p$, by (2.5),

$$\begin{aligned}
& d(y_{2p+1},y_{2q+1}) \\
& \leq d(y_{2p+1},y_{2p+2})\cdot d(y_{2p+2},y_{2p+3})\cdots d(y_{2q},y_{2q+1}) \\
& \leq [d(y_0,y_1)]^{(h_1h_2)^p h_1}\cdot [d(y_0,y_1)]^{(h_1h_2)^{p+1}}\cdot [d(y_0,y_1)]^{(h_1h_2)^{p+1}h_1}\cdots [d(y_0,y_1)]^{(h_1h_2)^q} \\
& = [d(y_0,y_1)]^{(h_1h_2)^p h_1+(h_1h_2)^{p+1}+(h_1h_2)^{p+1}h_1+\cdots+(h_1h_2)^q} \\
& \leq [d(y_0,y_1)]^{(h_1h_2)^p h_1[1+(h_1h_2)+(h_1h_2)^2+\cdots]+(h_1h_2)^{p+1}[1+(h_1h_2)+(h_1h_2)^2+\cdots]} \\
& = [d(y_0,y_1)]^{\frac{(h_1h_2)^p[h_1+h_1h_2]}{1-(h_1h_2)}} \\
& \leq [d(y_0,y_1)]^{\frac{(h_1h_2)^p[h_1+1]}{1-(h_1h_2)}}\ (\because h_1h_2<1) \\
& \leq [d(y_0,y_1)]^{M(h_1h_2)^p},
\end{aligned} \tag{2.6}$$

where $M=\frac{2}{1-h_1h_2}\max\{h_1,1\}$.

Similarly, we can obtain

$$\begin{aligned}
& d(y_{2p},y_{2q+1})\leq [d(y_0,y_1)]^{M(h_1h_2)^p}; \\
& d(y_{2p},y_{2q})\leq [d(y_0,y_1)]^{M(h_1h_2)^p}; \\
& d(y_{2p+1},y_{2q})\leq [d(y_0,y_1)]^{M(h_1h_2)^p}.
\end{aligned} \tag{2.7}$$

Combining (2.6)-(2.7), we know that for each natural numbers p,q with $q>p$,

$$d(y_p,y_q)\leq [d(y_0,y_1)]^{M(h_1h_2)^{[\frac{p}{2}]}}. \tag{2.8}$$

$h_1h_2<1$ implies that $(h_1h_2)^{[\frac{p}{2}]}\to 0$ as $p\to\infty$, hence $d(y_p,y_q)\to 1$ as $p,q\to\infty$. This completes that $\{y_n\}$ is a multiplicative Cauchy sequence.

Suppose that either SX or BX is complete. Since $y_{2n}\in SX\subset BX$ for all $n=01,2,\cdots)$ and $\{y_{2n}\}$ is also a multiplicative Cauchy sequence, there exist $u,v\in X$ such that $y_{2n}\to u=Bv$ as $n\to\infty$, i.e.,$d(y_{2n},u)\to 1$ as $n\to\infty$. Hence $d(y_{2n+1},u)\to 1$ as $n\to\infty$ since $d(y_{2n+1},u)\leq d(y_{2n+1},y_{2n})\cdot d(y_{2n},u)$, i.e., $y_{2n+1}\to u$ as $n\to\infty$.

By (2.1), for each n,

$$\begin{aligned} d(y_{2n},Tv) &= d(Sx_{2n},Tv) \\ &\le \psi(d(Ax_{2n},Bv),d(Ax_{2n},Sx_{2n}),d(Bv,Tv),d(Ax_{2n},Tv),d(Bv,Sx_{2n})) \qquad (2.9) \\ &= \psi(d(y_{2n-1},u),d(y_{2n-1},y_{2n}),d(u,Tv),d(y_{2n-1},Tv),d(u,y_{2n})). \end{aligned}$$

Letting $n\to\infty$ in (2.9), we obtain

$$d(u,Tv) \le \psi(1,1,d(u,Tv),d(u,Tv),1),$$

hence $d(u,Tv)=1$ by (ψ_3), therefore $Tv=u=Bv$. This shows that u is a point of coincidence of the pair (B,T).

$u=Tv\in TX\subset AX$ implies that there exists $w\in X$ such that $u=Aw$. By (2.1) again,

$$\begin{aligned} d(Sw,y_{2n+1}) &= d(Sw,Tx_{2n+1}) \\ &\le \psi(d(Aw,Bx_{2n+1}),d(Aw,Sw),d(Bx_{2n+1},Tx_{2n+1}),d(Aw,Tx_{2n+1}),d(Bx_{2n+1},Sw)) \qquad (2.10) \\ &= \psi(d(u,y_{2n}),d(u,Sw),d(y_{2n},y_{2n+1}),d(u,y_{2n+1}),d(y_{2n},Sw)). \end{aligned}$$

Let $n\to\infty$ in (2.10), then

$$d(Sw,u) \le \psi(1,d(u,Sw),1,1,d(u,Sw)),$$

hence $d(u,Sw)=1$ by (ψ_3) again, therefore $Sw=u=Aw$. this shows that u is also a point of coincidence of the pair (A,S).

If $z=Ax=Sx$ is also a point of coincidence of the pair (A,S), then by (2.1),

$$\begin{aligned} d(z,u) &= d(Sx,Tv) \\ &\le \psi(d(Ax,Bv),d(Ax,Sx),d(Bv,Tv),d(Ax,Tv),d(Bv,Sx)) \\ &= \psi(d(z,u),1,1,d(z,u),d(z,u)), \end{aligned}$$

hence $d(u,z)=1$ by (ψ_3) again, i.e., $z=u$, which shows that the pair (A,S) have a unique point of coincidence u. Similarly, u is also the unique point of coincidence of the pair (B,T). By (iii) and Lemma 1.5, u is the unique common fixed point of the pair (A,S) and the pair (B,T) respectively, hence u is a common fixed point of $\{A,B,S,T\}$. Obviously, u is the unique common fixed point of $\{A,B,S,T\}$. Similarly, we give the same conclusion for either TX or AX being complete.

Using theorem 2.1, we can give many (common) fixed point theorems. But, we only list two simple fixed point theorems here:

Theorem 2.2. Suppose that (X,d) is a multiplicative metric space and $T : X \to X$ a mapping. If either X or TX is complete and for each $x,y \in X$,

$$d(Tx,Ty) \le \psi(d(x,y),d(x,Tx),d(y,Ty),d(x,Ty),d(y,Tx)).$$

Then T has a unique fixed point.

Proof. Let $S = T$ and $A = B = 1_X$ in theorem 2.1, then the conclusion follows from Theorem 2.1.

Theorem 2.3. Suppose that (X,d) is a complete multiplicative metric space and $A : X \to X$ a surjective mapping. If for each $x,y \in X$,

$$d(x,y) \le \psi(d(Ax,Ay),d(Ax,x),d(Ay,y),d(Ax,y),d(Ay,x)).$$

then A has a unique fixed point.

Proof. Let $S = T = 1_X, A = B$ in Theorem 2.1, then the conclusion follows from theorem 2.1.

Define a function as follows $\psi'(x_1,x_2,x_3,x_4,x_5) = \left[\max\{x_1,x_2,x_3,x_4,x_5\}\right]^{\lambda}$ for all $x_1,x_2,x_3,x_4,x_5 \in [1,\infty)$, where $\lambda \in (0,\frac{1}{2})$. Let $h = \frac{\lambda}{1-\lambda}$, then $0 < h < 1$. Obviously, ψ is non-decreasing and continuous in every coordinate variable. If $u \le \psi'(v,u,v,1,uv)$, then $u \le \left[\max\{v,u,v,1,uv\}\right]^{\lambda} = \left[uv\right]^{\lambda}$, hence $u \le v^h$. Similarly, $u \le \psi'(v,v,u,uv,1) = \left[\max\{v,v,u,uv,1\}\right]^{\lambda}$ implies that $u \le v^h$. For any $t > 1$, $\psi'(t,1,1,t,t) = \psi'(1,t,1,1,t) = \psi'(1,1,t,t,1) = t^{\lambda} < t$. hence $\psi' \in \Psi$.

Using the above $\psi' \in \Psi$ and Theorem 2.2–Theorem 2.3, we obtain the following Ćirić type fixed point theorems for contractive and expansive mappings respectively on multiplicative metric spaces.

Theorem 2.4. Suppose that (X,d) is a multiplicative metric space and $T : X \to X$ a mapping. If either X or TX is complete and for each $x,y \in X$,

$$d(Tx,Ty) \le \left[\max\{d(x,y),d(x,Tx),d(y,Ty),d(x,Ty),d(y,Tx)\}\right]^{\lambda},$$

where $\lambda \in (0,\frac{1}{2})$. Then T has a unique fixed point.

Theorem 2.5. Suppose that (X,d) is a complete multiplicative metric space and $A : X \to X$ a surjective mapping. If for each $x,y \in X$,

$$d(x,y) \le \left[\max\{d(Ax,Ay), d(Ax,x), d(Ay,y), d(Ax,y), d(Ay,x)\}\right]^{\lambda},$$

where $\lambda \in (0, \frac{1}{2})$. Then A has a unique fixed point.

Conflict of Interests

The authors declare that there is no conflict of interests.

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