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# STABILITY RESULTS FOR GENERALIZED CONTRACTIONS IN PARTIAL METRIC SPACES

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**Abstract.** In 1994, Mathews [7] introduced the notion of partial metric spaces as a part of his study of denotational semantics of dataflow networks and obtained a generalization of the Banach contraction principle in partial metric spaces. In this paper, we prove stability results in partial metric spaces.

**Keywords**: partial metric space; weak stability; $\varphi$-contraction.

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## 1. Introduction

A partial metric on a nonempty set $X$ is a function $p : X \times X \to \mathbb{R}^+$ such that for all $x, y, z \in X$;

(1) $x = y \Leftrightarrow p(x,x) = p(x,y) = p(y,y)$;
(2) $p(x,x) \leq p(x,y)$;
(3) $p(x,y) = p(y,x)$;
(4) $p(x,z) \leq p(x,y) + p(y,z) - p(y,y)$.

A pair $(X,p)$ is called a partial metric space, where $X$ is a nonempty set and $p$ is a partial metric on $X$.

Each partial metric $p$ on $X$ generates a $T_0$ topology $\tau_p$ on $X$ which has as a base the family of all open $p-$ balls $\{B_p(x,\varepsilon) : x \in X \text{ and } \varepsilon > 0\}$, where $B_p(x,\varepsilon) = \{y \in X : p(x,y) < p(x,x) + \varepsilon\}$ for all $x \in X$ and $\varepsilon > 0$.

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A sequence $\{x_n\}$ in a partial metric space $(X,p)$ is said to be convergent to a point $z \in X$ iff $p(z,z) = \lim_{n\to\infty} p(z,x_n)$.

A sequence $\{x_n\}$ in a partial metric space $(X,p)$ is said to be a Cauchy sequence if $\lim_{n,m\to\infty} p(x_n,x_m)$ exists and finite.

A partial metric space $(X,p)$ is said to be complete if every Cauchy sequence $\{x_n\}$ in $X$ converges with respect to $\tau_p$ to a point $z \in X$ such that $p(z,z) = \lim_{n,m\to\infty} p(x_n,x_m)$.

If $p$ is a partial metric on $X$, then the function $p^s : X \times X \to \mathbb{R}^+$ defined by

$$p^s(x,y) = 2p(x,y) - p(x,x) - p(y,y)$$

is a metric on $X$.

**lemma 1.1** Let $(X,p)$ be a partial metric space,

(1) A sequence $\{x_n\}_{n=0}^{\infty}$ in a partial metric space is a Cauchy sequence if and only if it is a Cauchy sequence in the metric space $(X,p^s)$.

(2) A partial metric space $(X,p)$ is complete if and only if the metric space $(X,p^s)$ is complete. Moreover $\lim_{n\to\infty} p^s(z,x_n) = 0$ iff $\lim_{n\to\infty} p(z,x_n) = \lim_{n,m\to\infty} p(x_n,x_m)\ p(z,z)$.

Let $(X,p)$ be a partial metric space, $T : X \to X$ a mapping with $F_T = \{x \in X : Tx = x\} \neq \phi$, and $\{x_n\}_{n=0}^{\infty}$ a sequence obtained by a certain fixed point iteration procedure defined by

$$x_{n+1} = f(T,x_n), \quad n = 0,1,2,... \tag{1}$$

where $x_0 \in X$, and $f(T,x_n)$ includs all parameters that define the given fixed point iteration. For example if Krasnoselskij iteration prodcedure is used, then $f(T,x_n) = (1-\lambda)x_n + \lambda T x_n$, $n = 0,1,2,....$

Suppose $\{x_n\}_{n=0}^{\infty}$ converges to a fixed point $z^*$of $T$. We usually follow the next steps to compute $\{x_n\}_{n=0}^{\infty}$ :

(1) Choose the intial approximation $x_0 \in X$.

(2) Compute $x_1 = f(T,x_0)$, due to various errors that occur during the computations, we do not get the exact value of $x_1$, but we get another one, say $y_1$, which is however close enough to $x_1$, i.e $y_1 \approx x_1$.

(3) When computing $x_2 = f\ (T,x_1)$, we will actually compute $x_1$ as $x_2 = f(T,y_1)$, and so, instead of $x_2$, we will obtain different value, say $y_2$, which is close enough to $x_2$, i.e, $y_2 \approx x_2, ...$ and so on.

In this way, we will obtain a sequence $\{y_n\}_{n=0}^{\infty}$ instead of the theoretical sequence $\{x_n\}_{n=0}^{\infty}$.

The fixed point iteration method is said to be numerically stable if and only if for $y_n$ close enough to $x_n$ at each stage, the approximation sequence $\{y_n\}_{n=0}^{\infty}$ still converges to the fixed point $z^*$of $T$.

Following this idea, the concept of stability, due to Harder and Hicks [3], was used. For details, we refer to [2, 8]. Fixed point theory for partial metric spaces was discussed in [1, 4, 5, 6, 9].

## 2. Main Results

We now give the following definition in partial metric space.

**Definition 2.1** Let $(X, p)$ be a partial metric space, $T : X \to X$ a mapping, $x_0 \in X$. Assume that the iteration procedure (1) converges to a fixed point $z^*$of $T$, with $p(z^*, z^*) = 0$. Let $\{y_n\}_{n=0}^{\infty}$ be an arbitrary sequence in $X$ and put

$$\varepsilon_n = p(y_{n+1}, f(T, y_n)), \text{ for } \ n = 0, 1, 2, ... \tag{2}$$

The fixed point iteration (2) is called $T-$ stable if and only if

$$\lim_{n\to\infty} \varepsilon_n = 0 \Longleftrightarrow \lim_{n\to\infty} p(y_n, z^*) = 0. \tag{3}$$

We use the following lemma from [2] to prove the next theorem.

**Lemma 2.2** Let $\{a_n\}_{n=0}^{\infty}$ and $\{b_n\}_{n=0}^{\infty}$ be sequences of nonnegative numbers, and $0 \le q < 1$, so that

$$a_{n+1} \le qa_n + b_n \quad \text{ for all } n \ge 0.$$

(a) If $\lim\limits_{n\to\infty} b_n = 0$, then $\lim\limits_{n\to\infty} a_n = 0$.
(b) If ${}_{n=0}^{\infty}b_n < \infty$, then ${}_{n=0}^{\infty}a_n < \infty$.

**Theorem 2.3** Let $(X, p)$ be a partial metric space and $T : X \to X$ a $k-$ contraction mapping i.e.

$$p(Tx, Ty) \le kp(x, y), \text{ for every } x, y \in X, \tag{4}$$

where $k \in [0, 1)$. Suppose $T$ has a fixed point $z^*, x_0 \in X$ and $x_{n+1} = Tx_n, \ n \ge 0$. Then $\{x_n\}_{n=0}^{\infty}$ converges strongly to $z^*$ and is stable with respect to $T$.

**Proof** Let $\varepsilon_n = p(y_{n+1}, Ty_n)$.Then we have

$$\begin{aligned} p(y_{n+1}, z^*) &\le p(y_{n+1}, Ty_n) + p(Ty_n, z^*) - p(Ty_n, Ty_n) \\ &\le p(y_{n+1}, Ty_n) + p(Ty_n, z^*) \\ &\le \varepsilon_n + k\, p(y_n, z^*). \end{aligned}$$

Assume that $\lim\limits_{n\to\infty} \varepsilon_n = 0$.Then, since $k \in [0, 1)$, it follows by Lemma 1.1 that $\lim\limits_{n\to\infty} p(y_n, z^*) = 0$. Furthermore, $p(x_{n+1}, z^*)) \le k^n p(x_0, z^*) \longrightarrow 0$ as $n \longrightarrow \infty$, that is $\{x_n\}_{n=0}^{\infty}$ converges to $z^*$. Conversely, if

$\lim_{n\to\infty} p(y_n, z^*) = 0$, then

$$\begin{aligned}\varepsilon_n &= p(y_{n+1}, Ty_n) \leq p(y_{n+1}, z^*) + p(z^*, Ty_n) - p(z^*, z^*)\\ &\leq p(y_{n+1}, z^*) + k\ p(z^*, y_n) \longrightarrow 0 \text{ as } n \longrightarrow \infty.\end{aligned}$$

Hence $\lim_{n\to\infty} \varepsilon_n = 0$.

We use the following definition of weak stability due to Berinde [2] on partial metric spaces.

**Definition 2.4** Let $\{x_n\}_{n=0}^{\infty}$ be a sequence in a partial metric space $(X, p)$. We say that $\{y_n\}_{n=0}^{\infty}$ is an approximate sequence of $\{x_n\}_{n=0}^{\infty}$ if, for each $k \in \mathbb{N}$, there exists $\eta = \eta(k)$ such that

$$p(x_n, y_n) \leq \eta, \quad \text{for all } n \geq k.$$

The proof of the following lemma has been taken from [2]. It works in our case too.

**Lemma 2.5** Let $(X, p)$ be a partial metric space. A sequence $\{y_n\}_{n=0}^{\infty}$ is an approximate sequence of $\{x_n\}_{n=0}^{\infty}$ if and only if there exists a decreasing sequence of positive numbers $\{\epsilon_n\}_{n=0}^{\infty}$ convergent to some $\eta \geq 0$ such that

$$p(x_n, y_n) \leq \epsilon_n, \quad \text{for any } n \geq k.$$

**Proof** Suppose that the sequence $\{y_n\}_{n=0}^{\infty}$ is an approximate sequence of $\{x_n\}_{n=0}^{\infty}$. For $k = 1$ there exists $\eta_1$ such that $p(x_n, y_n) \leq \eta_1$, for $n = 1, 2, 3, ....$ Take $\varepsilon_1 = \eta_1$. For $k = 2$ there exists $\eta_2$ such that $p(x_n, y_n) \leq \eta_2$, for $n = 2, 3, 4, ....$ Put $\varepsilon_2 = \min\{\eta_1, \eta_2\}$. In this way we obtain a decreasing sequence of positive numbers $\{\epsilon_n\}_{n=0}^{\infty}$ which is convergent to some $\eta \geq 0$. For sufficient condition take $\eta(k) = \epsilon_k, k = 0, 1, 2, ....$

Let $(X, p)$ be a partial metric space and $T : X \longrightarrow X$ a mapping such that $F_T \neq \phi$ and there exists a certain fixed point iteration procedure which converges to some fixed point $z^*$. When computing $z^*$ we use a certain approximate mapping $S$ of $T$, i.e, a mapping $S : X \longrightarrow X$, such that for $\eta > 0$ we have

$$p(Tx, Sx) \leq \eta, \text{ for every } x \in X.$$

Assume that $q \in F_S$. It is natural to ask the following question: Is $q$ an approximation of $z^*$, and if yes how can we estimate $p(z^*, q)$?

The next theorem provides an answer to the prevoius question. The following statement due to Berinde [2] is used to prove the theorem.

Let $T : X \longrightarrow X$ be a $\varphi-$ contraction, i.e, $p(Tx, Ty) \leq \varphi(p(x, y))$ for every $x, y \in X$, where $\varphi : \mathbb{R}^+ \rightarrow \mathbb{R}^+$ is a monotone nondecreasing with $\lim_{n\to\infty} \varphi^n(t) = 0$ for every $t \geq 0$. Denote

$$t_\eta = \sup\{t \in \mathbb{R}^+ \mid t - \varphi(t) \le \eta\}, \quad \eta > 0. \tag{5}$$

Note that

$$\lim_{\eta \to 0} t_\eta = 0. \tag{6}$$

**Theorem 2.6** Let $(X, p)$ be a complete partial metric space and $T, S : X \to X$ two mappings satisfying:

(a) $T$ is a $\varphi-$ contraction,

(b) $q \in F_S$,

(c) there exists $\eta > 0$ such that

$$p(Tx, Sx) \le \eta, \quad \text{for all } x \in X.$$

Then

$$p(z^*, q) \le t_\eta,$$

where $F_T = \{z^*\}$.

**Proof** By Theorem 3.2 in [9] we get that $\{T^n x_0\}$ converges to $z^*$, for any $x_\circ \in X$. By using (a), (b) and (c) we have

$$\begin{aligned} p(z^*, q) &= p(Tz^*, Sq) \le p(Tz^*, Tq) + p(Tq, Sq) - p(Tq, Tq) \\ &\le \varphi(p(z^*, q)) + \eta. \end{aligned}$$

Hence

$$p(z^*, q) - \varphi(p(z^*, q)) \le \eta.$$

Using (6), we get

$$p(z^*, q) < t_\eta.$$

**Theorem 2.7** Let $(X, p)$ be a complete partial metric space and $T : X \to X$ a $\varphi$ - contraction mapping. Furthermore $\varphi$ is subadditive and $\sum_{n=0}^{\infty} \varphi^n(t)$ converges for every $t > 0$ . Let $S : X \to X$ be an approximate mapping of $T$ and $\{x_n\}_{n=0}^{\infty}, \{y_n\}_{n=0}^{\infty}$ be the Picard iterations associated to $T$ and $S$ respectively, satisfying from $x_0$. Then $F_T = \{z^*\}$ . If $q \in F_S$, then

(a) $p(y_n, z^*) < s(\eta) + s(p(x_n, x_{n+1}))$,

(b) $p(z^*, q) < s(\eta)$,

where $s(t)$ denotes the sum of the comparison series $\underset{k=0}{\overset{\infty}{}}\varphi^k(t)$.

**Proof**

(a) By Theorem 3.2 [9], $F_T = \{z^*\}$, $p(z^*, z^*) = 0$ and $\lim_{n\to\infty} p(x_n, z^*) = 0$, for any $x_0 \in X$. Since $y_1 = Sx_0, y_2 = Sy_1, ..., y_n = Sy_{n-1},\ \ n > 1$,

$$\begin{aligned} p(y_n, z^*) &\leq p(y_n, x_n) + p(x_n, z^*) - p(x_n, x_n) \\ &< p(y_n, x_n) + p(x_n, z^*). \quad (7) \end{aligned}$$

Now, we compute $p(y_n, x_n)$.

$$\begin{aligned} p(y_n, x_n) &= p(Sy_{n-1}, Tx_{n-1}) \\ &\leq p(Sy_{n-1}, Ty_{n-1}) + p(Ty_{n-1}, Tx_{n-1}) - p(Ty_{n-1}, Ty_{n-1}) \\ &\leq p(Sy_{n-1}, Ty_{n-1}) + p(Ty_{n-1}, Tx_{n-1}) \\ &\leq \eta + \varphi(p(y_{n-1}, x_{n-1})) \\ &= \eta + \varphi(p(Sy_{n-2}, Tx_{n-2})) \\ &\leq \eta + \varphi(p(Sy_{n-2}, Ty_{n-2}) + p(Ty_{n-2}, Tx_{n-2}) + \\ &\quad -p(Ty_{n-2}, Ty_{n-2})) \\ &\leq \eta + \varphi(p(Sy_{n-2}, Ty_{n-2}) + p(Ty_{n-2}, Tx_{n-2})) \\ &\leq \eta + \varphi(\eta + \varphi(p(y_{n-2}, x_{n-2})) \\ &\leq \eta + \varphi(\eta) + \varphi^2(p(Sy_{n-3}, Tx_{n-3})) \\ &< \eta + \varphi(\eta) + \varphi^2(\eta + \varphi(y_{n-3}, x_{n-3})) \\ &\leq \eta + \varphi(\eta) + \varphi^2(\eta) + \varphi^3(p(Sy_{n-4}, Tx_{n-4})) \\ &\leq \ldots\ldots\ldots\ldots\ldots\ldots \\ &\leq \ldots\ldots\ldots\ldots\ldots\ldots \\ &\leq \eta + \varphi(\eta) + \varphi^2(\eta) + ... + \varphi^{n-1}(\eta) \\ &\leq s(\eta). \qquad (8) \end{aligned}$$

From (7) and (8) we get,

$$p(y_n, z^*) < s(\eta) + p(x_n, z^*). \tag{9}$$

Notice that

$$p(x_{n+k}, x_{n+k+1}) \leq \varphi^k(p(x_n, x_{n+1})),\ \ n = 0, 1, 2, ..., k \geq 1$$

and

$$\begin{aligned} p(x_n, x_{n+p}) &\leq p(x_n, x_{n+1}) + p(x_{n+1}, x_{n+2}) + ... + p(x_{n+p-1}, x_{n+p}) + \\ &\quad -p(x_{n-1}, x_{n-1}) - p(x_{n-2}, x_{n-2}) - ... - p(x_{n+p-1}, x_{n+p-1}) \\ &\leq p(x_n, x_{n+1}) + p(x_{n+1}, x_{n+2}) + ... + p(x_{n+p-1}, x_{n+p}) \\ &\leq p(x_n, x_{n+1}) + \varphi(p(x_n, x_{n+1})) + ... + \varphi^{p-1}(p(x_n, x_{n+1})) \\ &= {}_{k=0}^{p-1}\varphi^k(p(x_n, x_{n+1})). \end{aligned}$$

Letting $p \to \infty$, we get,

$$\begin{aligned} p(x_n, z^*) &\leq {}_{k=0}^{\infty}\varphi^k(p(x_n, x_{n+1})) \\ &= s(p(x_n, x_{n+1})). \qquad (10) \end{aligned}$$

From (9) and (10), we have,

$$p(y_n, z^*) < s(\eta) + s(p(x_n, x_{n+1})).$$

(b) Take $x_0 = q$ This implies that $y_n = q$ for each $n \geq 1$, and let $n \to \infty$. Then from (a) we get

$$\begin{aligned} p(z^*, q) &< s(\eta) + s(p(z^*, z^*)) \\ &= s(\eta). \end{aligned}$$

We next define the uniformly convergence and pointwise convergence of a sequence $\{T_n\}_{n=0}^{\infty}$ of self-mappings defined on a partial metric space $X$.

**Definition 2.8** Let $(X, p)$ be a partial metric space. A sequence of mappings, $\{T_n\}_{n=0}^{\infty}$, $T_n : X \to X$, is said to be uniformly convergent to a mapping $T : X \to X$, if for every $\varepsilon > 0$, there exists a natural number $N = N(\varepsilon)$, such that $n \geq N$ implies that

$$p(T_n x, Tx) < \varepsilon + p(Tx, Tx), \text{ for every } x \in X.$$

**Definition 2.9** Let $(X, p)$ be a partial metric space. A sequence of mappings, $\{T_n\}_{n=0}^{\infty}$, $T_n : X \to X$, is said to be pointwise convergent to a mapping $T : X \to X$, if for every $\varepsilon > 0$, and $x \in X$ there exists a natural number $N = N(\varepsilon, x)$, such that

$$p(T_n x, Tx) < \varepsilon + p(Tx, Tx), \quad \text{for every } \; n \geq N.$$

Due to Berinde [2], there is a possible method to approximate the fixed point $z^*$ of $T$ as seen in the following theorems.

**Theorem 2.10** Let $(X,p)$ be a complete partial metric space and $\{T_n\}_{n=0}^{\infty}$ a sequence of mappings, $T_n : X \to X$, with $F_{T_n} = \{z_n^*\}$ and $p(z_n^*, z_n^*) = 0$, for every $n = 0, 1, 2, ....$If the sequence $\{T_n\}$ converges uniformly to a $k$-contraction $T : X \to X$, such that $F_T = \{z^*\}$ and $p(z^*, z^*) = 0$, then

$$\lim_{n\to\infty} p(z_n^*, z^*) = 0 \ .$$

**Proof** First note that $p(Tz_n^*, Tz_n^*) \leq k\ p(z_n^*, z_n^*) = 0$ for every $n = 0, 1, 2, ....$ Hence $p(Tz_n^*, Tz_n^*) = 0$ for every $n = 0, 1, 2, ....$ Therefore for any $\varepsilon > 0$ choose a natural number $N$ such that for every $n \geq N$, we have

$$p(T_n z_n^*, Tz_n^*) < \varepsilon\ (1-k), \quad \text{for every } n \geq N,$$

where $k$ is the contraction constant. It follows that, for $n \geq N$, we get

$$\begin{aligned} p(z_n^*, z^*) &= p(T_n z_n^*, Tz^*) \\ &\leq p(T_n z_n^*, Tz_n^*) + p(Tz_n^*, Tz^*) - p(Tz_n^*, Tz_n^*) \\ &< \varepsilon\ (1-k) + k\ p(z_n^*, z^*). \end{aligned}$$

This implies that $p(z_n^*, z^*) < \varepsilon$, for all $n \geq N$, i.e., $\lim_{n\to\infty} p(z_n^*, z^*) = 0$ .

In the next theorem as in [2], if the uniform convergence of $\{T_n\}_{n=0}^{\infty}$ is replaced by the pointwise convergence, then $T_n$ must be a $k-$contraction for every $n \geq 0$ to get that $\lim_{n\to\infty} p(z_n^*, z^*) = 0$.

**Theorem 2.11** Let $(X,p)$ be a complete partial metric space and $T_n, T : X \to X$ be mappings such that

(a) $T_n$ is a $k-$contraction for every $n \geq 0$, with $F_{T_n} = \{z_n^*\}$,

(b) $\{T_n\}_{n=0}^{\infty}$ converges pointwisely to $T$.

Then

$$p(Tx, Ty) \leq p(Tx, Tx) + p(Ty, Ty) + k\ p(x, y) \text{ for every } x, y \in X.$$

If, in addition $z^* \in F_T$ with $p(z^*, z^*) = 0$, then $\lim_{n\to\infty} p(z_n^*, z^*) = 0$.

**Proof** For any $x, y \in X$, we have

$$\begin{aligned} p(Tx, Ty) &\leq p(Tx, T_n x) + p(T_n x, T_n y) + p(T_n y, Ty) - p(T_n x, T_n x) - p(T_n y, T_n y) \\ &\leq p(Tx, T_n x) + k\ p(x, y) + p(T_n y, Ty). \end{aligned}$$

Letting $n \to \infty$, from (b), we get

$$p(Tx, Ty) \leq p(Tx, Tx) + p(Ty, Ty) + k\ p(x, y) \quad \text{for every } x, y \in X.$$

Now we prove $\lim_{n\to\infty} p(z_n^*, z^*) = 0$. Let $\varepsilon > 0$ choose a natural number $N$ such that for every $n \geq N$ we have

$$\begin{aligned} p(T_n z^*, Tz^*) &< \varepsilon\ (1-k) + p(Tz^*, Tz^*) \\ &= \varepsilon\ (1-k). \end{aligned}$$

This implies that for every $n \geq N$

$$\begin{aligned} p(z_n^*, z^*) &= p(T_n z_n^*, Tz^*) \leq p(T_n z_n^*, T_n z^*) + p(T_n z^*, Tz^*) - p(T_n z^*, T_n z^*) \\ &< k\ p(z_n^*, z^*) + \varepsilon\ (1-k). \end{aligned}$$

This implies that

$$p(z_n^*, z^*) < \varepsilon, \text{ for every } n \geq N$$

We conclude that

$$\lim_{n\to\infty} p(z_n^*, z^*) = 0.$$

**Theorem 2.12** Let $(X, p)$ be a complete partial metric space and $T_n, T : X \to X$ be mappings such that

(a) $T$ is a $\varphi-$contraction and $\lim_{t\to\infty}(t - \varphi(t)) = \infty$,

(b) $\{T_n\}_{n=0}^{\infty}$ converges uniformly to $T$,

(c) $z_n^* \in F_{T_n}$ with $p(z_n^*, z_n^*) = 0$ for $n \geq 0$.

Then $\lim_{n\to\infty} p(z_n^*, z^*) = 0$, where $F_T = \{z^*\}$ and $p(z^*, z^*) = 0$

**Proof** Note that $p(Tz_n^*, Tz_n^*) \leq \varphi(\ p(z_n^*, z_n^*)) = \varphi(0) = 0$ for every $n = 0, 1, 2, ....$ Hence $p(Tz_n^*, Tz_n^*) = 0$ for every $n = 0, 1, 2, ....$ Therefore for any $\varepsilon > 0$ choose a natural number $N$ such that for every $n \geq N$, we have

$$p(T_n z_n^*, Tz_n^*) < \varepsilon, \quad \text{for every } n \geq N,$$

This implies that

$$\begin{aligned} p(z_n^*, z^*) &= p(T_n z_n^*, Tz^*) \leq p(T_n z_n^*, Tz_n^*) + p(Tz_n^*, Tz^*) - p(Tz_n^*, Tz_n^*) \\ &< \varepsilon + \varphi(p(z_n^*, z^*)) \ \text{ for every } n \geq N. \end{aligned}$$

It follows that

$$p(z_n^*, z^*) - \varphi(p(z_n^*, z^*)) < \varepsilon, \text{ for every } n \geq N.$$

By using (5) we get

$$\lim_{n\to\infty} p(z_n^*, z^*) = 0.$$

## References


[1] I. Alton, F. Sola, H. Simsek, Generalized contractions on partial metric spaces, Topology and its Applications, 157 (2010), 2778 - 2785

[2] V. Berinde, Iterative approximation of fixed points, Lecture Notes in Mathematics, 1912, Springer, Berlin, 2007.

[3] A. M. Harder, T. L. Hicks, Stability results for fixed point iteration procedures. Math. Japonica, 33, No. 5, 693- 706 (1988).

[4] D. Ilic, V. Pavlovic, V. Rakocevic, Some new extensions of Banach's contraction principal to partial metric space, Applied Math. Letters, (2011), (in press).

[5] E. Karapinar, I. M. Erhan, Fixed point theorems for operators on partial metric spaces, Applied Math Letters, 24 (2011), 1894-1899.

[6] E. Karapinar,Generalizations of Caristi-Kirk's theorem on partial metric spaces, Fixed Point Theory and Applications, (2011), (in press).

[7] S. G. Matthews, Partial metric topology, in: Proc, 8th Summer Conference on General Topology and Applications, Ann. New York Acad. Sci. 728 (1994), 183- 197.

[8] N. Shahzad and H. Zegeye, On stability results for phi -strongly pseudocontractive mappings, Nonlinear Anal. 64 (2006), no. 12, 2619 - 2630.

[9] O. Valero, On Banach fixed point theorems for partial metric spaces, Appl. Gen. Topol. 6 (2) (2005) 229- 240.