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ĆIRIĆ TYPE FIXED POINT THEOREMS UNDER c-DISTANCE ON NON-NORMAL CONE METRIC SPACES OVER BANACH ALGEBRAS

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Abstract. In this paper, we obtain Ćirić type fixed point theorems for continuous or non-continuous mappings under c-distance on mapping-orbitally complete cone metric spaces over Banach algebras without normalities.

Keywords: Ćirić type fixed point; c-distance; cone metric space over Banach algebra; mapping-orbitally complete.

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1. Introduction and Preliminaries

Ćirić[1] introduced and studied the follwoing quasicontraction as one of the most general classes of contrative type mappings:

Let (X,d) is a complete space. $f : X \to X$ is said to be a quasicontracion if, for some $k \in (0,1)$ and for all $x,y \in X$, one has

$$d(fx, fy) \leq k \max\{d(x,y), d(x, fx), d(y, fy), d(x, fy), d(fx, y)\}.$$

He proved that any quasicontration f has a unique fixed point on a complete metric space (X,d). Recently, many researchers discussed and obtained various similar results on metric

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spaces, cone metric spaces and cone metric spaces over Banach algebras, for details, see [2-12]. These conclusions goodly generalize and improve Ćirić's fixed point theorem.

On the other hand, some authors discussed (common) fixed point problems under c-distance on cone metric spaces, see [13-19] and others. Especially, Huang et al[20] and Huang et al[21] discussed and obtained fixed point theorems for mappings under c-distance on cone metric space over Banach algebras without normalities.

In this paper, we will discuss and obtain Ćirić type fixed point problems for continuous or non-continuous mappings under c-distance on mapping-orbitally complete cone metric spaces over Banach algebras without normalities.

Now, we give some known definitions and lemmas:

Let $\mathscr{A}$ always be a Banach algebra, that is, $\mathscr{A}$ is a real Banach space in which an operation of multiplication is defined, subject to the following properties(for all $x,y,z\in\mathscr{A}$, $\alpha\in\mathbb{R}$):

1. $(xy)z = x(yz)$;
2. $x(y+z) = xy+xz$ and $(x+y)z = xz+yz$;
3. $\alpha(xy) = (\alpha x)y = x(\alpha y)$;
4. $\| xy \| \leq \| x \| \| y \|$.

In this paper, we shall assume that a Banach algebra $\mathscr{A}$ has a unit (i.e., a multiplicative identity) e such that $ex = xe = x$ for all $x \in \mathscr{A}$. an element $x\in\mathscr{A}$ is said to be invertible if there is an inverse element $y \in A$ such that $xy = yx = e$. The inverse of x denoted by x^{-1}. For more detail, we refer to [22-24].

A subset P of a Banach algebra $\mathscr{A}$ is called a cone if

1. P is nonempty closed and $\{0,e\}\subset P$;
2. $\alpha P+\beta P\subset P$ for all non-negative real numbers $\alpha.\beta$;
3. $P^2 = PP \subset P$;
4. $P\cap(-P) = \{\mathbf{0}\}$.

Where $\mathbf{0}$ denotes the null of the Banach algebra $\mathscr{A}$.

For a given cone $P\subset\mathscr{A}$, we can define a partial ordering $\leq$ with respect to P by $x\leq y$ if and only if $y-x\in P$. $x<y$ stand for $x\leq y$ and $x\neq y$. While $x\ll y$ sill stand for $y-x\in \operatorname{int}P$, where $\operatorname{int}P$ denotes the interior of P. A cone P is called solid if $\operatorname{int}P\neq\emptyset$.

The cone P is called normal if there is a number $M > 0$ such that for all $x, y \in \mathscr{A}$.

$$0 \leq x \leq y \implies \| x \| \leq M \| y \| .$$

The least positive number satisfying the above is called the normal constant of P.

Here, we always assume that P is a solid and $\leq$ is the partial ordering with respect to P.

Definition 1.1. **[20, 21]** Let X be a non-empty set. Suppose that the mapping $d : X \times X \to \mathscr{A}$ satisfies

1. $0 \leq d(x,y)$ for all $x, y \in X$ and $d(x,y) = 0$ if and only if $x = y$;
2. $d(x,y) = d(y,x)$ for all $x, y \in X$;
3. $d(x,y) \leq d(x,z) + d(z,y)$ for all $x, y, z \in X$.

Then d is called a cone metric on X and (X,d) is called a cone metric space(over a Banach algebra $\mathscr{A}$).

Remark 1.1. If $\mathscr{A} = E$ is a Banach space in Definition 1.1, then(X,d) is call a cone metric space.

Definition 1.2. **[21]** Let (X,d) be a cone metric space over a Banach algebra $\mathscr{A}$, $x \in X$ and $\{x_n\}$ a sequence in X. Then:

1. $\{x_n\}$ converges to x whenever for each $c \in \mathscr{A}$ with $0 \ll c$ there is a natural number N such that $d(x_n, x) \ll c$ for all $n \geq N$. We denote this by $\lim_{n\to\infty} x_n = x$ or $x_n \to x$.
2. $\{x_n\}$ is Cauchy sequence whenever for each $c \in \mathscr{A}$ with $0 \ll c$ there is a natural number N such that $d(x_n, x_m) \ll c$ for all $n, m \geq N$.
3. (X,d) is a complete cone metric space if every Cauchy sequence is convergent.

Definition 1.3. **[17, 18, 22]** Let P is a solid cone in a Banach space $\mathscr{A}$. A sequence $\{u_n\} \subset \mathscr{A}$ is a c-sequence if for each $c \gg 0$ there exists $n_0 \in \mathbb{N}$ such that $u_n \ll c$ for all $n \geq n_0$.

Definition 1.4. **[20, 21]** Let (X,d) be a cone metric space over a Banach algebra. A function $q : X \times X \to \mathscr{A}$ is called a c-distance on X. If

(q_1) $\theta \leq q(x,y)$ for all $x, y \in X$;

(q_2) $q(x,z) \leq q(x,y) + q(y,z)$ for all $x, y, z \in X$;

(q_3) If a sequence $\{y_n\}$ in X converges to a point $y \in X$, and for any $x \in X$, there exists $u = u_x \in P$ such that $q(x, y_n) \leq u$ holds for each $n \in \mathbb{N}$, then $q(x,y) \leq u$;

(q_4) For each $c \in \mathscr{A}$ with $\theta \ll c$, there exists $e \in \mathscr{A}$ with $\theta \ll e$, such that $q(z,x) \ll e$ and $q(z,y) \ll e$ implies $d(x,y) \ll c$.

Remark 1.2. [13, 15] Generally, $q(x,y) \neq (y,x)$ for $x,y \in X$, and $q(x,y) = 0$ is not necessarily equivalent to $x = y$.

Definition 1.5. [12] Let (X,d) be a cone metric space over a Banach algebra $\mathscr{A}$, $T : X \to X$ a mapping. For any $x \in X$ and any positive number n, let

$$O_T(x,n) = \{x, Tx, T^2x, \cdots, T^nx\}, O_T(x,+\infty) = \{x, Tx, T^2x, \cdots\}.$$

The set $O_T(x,+\infty)$ is called the T-orbit at x. (X,d) is said to be T-orbitally complete if, every Cauchy sequence in $O_T(x,+\infty)$ is convergent for every $x \in X$.

Lemma 1.1. [22] Let $\mathscr{A}$ be a Banach algebra with a unit e, and $x \in \mathscr{A}$. If the spectral radius $r(x)$ of x is less than 1, i.e.,

$$r(x) = \lim_{n\to\infty} \| x^n \|^{\frac{1}{n}} = \inf_{n\to\infty} \| x^n \|^{\frac{1}{n}} < 1.$$

Then $(e - x)$ is invertible. Actually,

$$(e-x)^{-1} = \sum_{i=0}^{+\infty} x^i.$$

Lemma 1.2. [22] Let P is a solid cone in a Banach algebra $\mathscr{A}$ and $\{u_n\}$ and $\{v_n\}$ be two c-sequences in $\mathscr{A}$. If $k, l \in P$ are two arbitarily given vectors, then $\{ku_n + lv_n\}$ is a c-sequence in $\mathscr{A}$.

Lemma 1.3. [22] Let P be a solid cone in Banach algebra $\mathscr{A}$ and $u, v, w \in \mathscr{A}$. If $u \leq v \ll w$, then $u \ll w$.

Lemma 1.4. [11] Let P be a solid cone in a Banach algebra $\mathscr{A}$ and $a, k \in P$ with $r(k) < 1$. If $a \leq ka$, then $a = 0$.

Lemma 1.5. [12] If E is a real Banach space with a solid cone P and if $\| x_n \| \to 0$ as $n \to \infty$, then for any $0 \ll c$, there exists $N \in \mathbb{N}$ such that $x_n \ll c$ for all $n > N$.

Lemma 1.6. [23] If $\mathscr{A}$ is a Banach algebra and $k \in \mathscr{A}$ with $r(k) < 1$, then $\| k^n \| \to 0$ as $n \to \infty$.

Lemma 1.7. [23] Let A be a Banach algebra and $x, y \in \mathscr{A}$. If x and y commute, then the following hold:

(i) $r(xy) \leq r(x)r(y)$;

(ii) $r(x+y) \leq r(x)+r(y)$;

(iii) $| r(x)-r(y) | \leq r(x-y)$.

Lemma 1.8. **[24]** Let (X,d) be a cone metric space over a Banach algebra $\mathscr{A}$, $\{x_n\} \subset X$ a sequence. If $\{x_n\}$ is convergent, then the limits of $\{x_n\}$ is unique.

Lemma 1.9. **[21].** Let (X,d) be a cone metric space over Banach algebra $\mathscr{A}$, q a c-distance on X. Suppose that$\{x_n\}$ is a sequences in X and $y,z \in X$. If $\{u_n\}$ and $\{v_n\}$ are two c-sequences in P, then the following properties hold:

(1) If $q(x_n,y) \leq u_n$ and $q(x_n,z) \leq v_n$, $\forall n \in \mathbb{N}$, then $y=z$. In particular, if $q(x,y)=0$ and $q(x,z)=0$, then $y=z$.

(2) If $q(x_n,x_m) \leq u_n$ for all $m>n>n_0$, then $\{x_n\}$ is a Cauchy sequence in X.

2. Ćirić Type Fixed Point Theorems under c-Distance

Theorem 2.1. Let (X,d) be a cone metric space over a Banach algebra, q be a c-distance on X, $f: X \to X$ be continuous on (X,d), $k \in P$ with $r(k)<1$. Suppose that for each $x,y \in X$,

$$q(fx,fy) \leq k\,v(x,y), \tag{2.1}$$

where

$$v(x,y) \in \{q(x,y), q(x,fx), q(y,fy), q(x,fy)\}. \tag{2.2}$$

If X is f-orbitally complete, then f has a unique fixed point $x^* \in X$ and $q(x^*,x^*)=0$.

Proof. For any $x \in X$, Let $x_n = f^n x$ for all $n=1,2,\cdots$, then $x_n = fx_{n-1}$ for all $n=1,2,\cdots$(Here, set $x_0=x$).

First, we will prove that for each $n \geq 2$ and for all i,j such that $1 \leq i<j \leq n$, one has

$$q(x_i,x_j) \leq k\,(1-k)^{-1}\,q(x_0,x_1). \tag{2.3}$$

If $n=2$, then $i=1, j=2$. Hence

$$q(x_1,x_2) = q(fx_0,fx_1) \leq k\,v(x_0,x_1),$$

where

$$\begin{aligned} &v(x_0,x_1) \\ &\in \{q(x_0,x_1), q(x_0,fx_0), q(x_1,fx_1), q(x_0,fx_1)\} \\ &= \{q(x_0,x_1), q(x_1,x_2), q(x_0,x_2)\}. \end{aligned} \tag{2.4}$$

If $v(x_0,x_1) = q(x_0,x_1)$, then

$$q(x_1,x_2) \le k\,q(x_0,x_1) \le k\,(e-k)^{-1}\,q(x_0,x_1).$$

If $v(x_0,x_1) = q(x_1,x_2)$, then

$$q(x_1,x_2) \le k\,q(x_1,x_2) \Longrightarrow (e-k)q(x_1,x_2) \le 0,$$

therefore

$$q(x_1,x_2) = 0 \le k\,(e-k)^{-1}\,q(x_0,x_1).$$

If $v(x_0,x_1) = q(x_0,x_2)$, then

$$q(x_1,x_2) \le k\,q(x_0,x_2) \le k\,[q(x_0,x_1) + q(x_1,x_2)],$$

hence

$$q(x_1,x_2) \le k\,(e-k)^{-1}\,q(x_0,x_1).$$

Based on the above discussions, (2.3) is set up for $n = 2$.

Assume that (2.3) is true for $n = m > 2$, that is,

$$q(x_i,x_j) \le k\,(e-k)^{-1}\,q(x_0,x_1),\ 1 \le i < j \le m. \tag{2.5}$$

Now, we will prove that (2.3) also holds for $n = m+1$. If $1 \le i < j \le m$, then (2.3) holds by the assumption (i.e., by (2.5)). Thus, without loss of generality, we assume that $j = m+1$ and $1 \le i \le m$. Denote $i = i_0$. By (2.1),

$$q(x_{i_0}, x_{m+1}) = q(fx_{i_0-1}, fx_m) \le k\,v(x_{i_0-1}, x_m), \tag{2.6}$$

where

$$v(x_{i_0-1},x_m) \in \{q(x_{i_0-1},x_m), q(x_{i_0-1},x_{i_0}), q(x_m,x_{m+1}), q(x_{i_0-1},x_{m+1})\}. \tag{2.7}$$

Firstly, we consider that $i_0 = 1$.

If $v(x_{i_0-1},x_m)=d(x_0,x_m)$, then

$$\begin{aligned}&q(x_{i_0},x_{m+1})\\ \leq&k\,q(x_0,x_m)\\ \leq&k\,[q(x_0,x_1)+q(x_1,x_m)]\\ \leq&k\,[q(x_0,x_1)+k(e-k)^{-1}\,q(x_0,x_1)]\\ =&k\,(e-k)^{-1}q(x_0,x_1),\end{aligned} \tag{2.8}$$

and the statement follows.

If $v(x_{i_0-1},x_m)=q(x_0,x_1)$, then

$$q(x_{i_0},x_{m+1})\leq k\,q(x_0,x_1)\leq k\,(e-k)^{-1}d(x_0,x_1), \tag{2.9}$$

and the statement also holds.

If $v(x_{i_0-1},x_m)=q(x_m,x_{m+1})$, then we let $i_1=m$ and we have

$$q(x_{i_0},x_{m+1})\leq k\,q(x_{i_1},x_{m+1}). \tag{2.10}$$

If $v(x_{i_0-1},x_m)=q(x_0,x_{m+1})$, then

$$q(x_{i_0},x_{m+1})\leq k\,q(x_0,x_{m+1})\leq k\,[d(x_0,x_1)+d(x_{i_0},x_{m+1})],$$

which implies that

$$q(x_{i_0},x_{m+1})\leq k\,(e-k)^{-1}d(x_0,x_1), \tag{2.11}$$

and the statement also holds.

Secondly, we consider that $2\leq i_0\leq m$.

If $v(x_{i_0-1},x_m)=q(x_{i_0-1},x_m)$ or $v(x_{i_0-1},x_m)=q(x_{i_0-1},x_{i_0})$, then by the assumption,

$$q(x_{i_0},x_{m+1})\leq k\,v(x_{i_0-1},x_m)\leq k^2(e-k)^{-1}q(x_0,x_1)\leq k\,(e-k)^{-1}q(x_0,x_1), \tag{2.12}$$

and the statement follows.

If $v(x_{i_0-1},x_m)=q(x_m,x_{m+1})$ or $v(x_{i_0-1},x_m)=q(x_{i_0-1},x_{m+1})$, then we let $i_1=m$ or $i_1=i_0-1\geq 1$, respectively, hence

$$q(x_{i_0},x_{m+1})\leq k\,v(x_{i_0-1},x_{m+1})=k\,q(x_{i_1},x_{m+1}). \tag{2.13}$$

In conclusion from discussion of both cases, it results that either the proof is complete, that is

$$q(x_{i_0}, x_{m+1}) \leq k\,(e-k)^{-1} q(x_0, x_1), \tag{2.14),}$$

or there exists an integer i_1 such that

$$q(x_{i_0}, x_{m+1}) \leq k\,d(x_{i_1}, x_{m+1}),\;\; 1 \leq i_1 \leq m. \tag{2.15}$$

As for the latter situation, we continue in a similar way, and come to the result that either

$$q(x_{i_1}, x_{m+1}) \leq k\,(e-k)^{-1} q(x_0, x_1), \tag{2.16),}$$

which implies that

$$q(x_{i_0}, x_{m+1}) \leq k\,q(x_{i_1}, x_{m+1}) \leq k^2\,(e-k)^{-1} q(x_0, x_1) \leq k\,(e-k)^{-1} q(x_0, x_1), \tag{2.17),}$$

and the proof is complete, or there exists integer i_2 such that

$$q(x_{i_1}, x_{m+1}) \leq k\,q(x_{i_2}, x_{m+1}), \exists 1 \leq i_2 \leq m, \tag{2.18}$$

which implies that

$$q(x_{i_0}, x_{m+1}) \leq k^2\,q(x_{i_2}, x_{m+1}), \exists 1 \leq i_2 \leq m. \tag{2.19}$$

Generally, if the procedure ends by the l-th step with $l \leq m-1$, that is, there exist $l+1$ integers

$$i_0, i_1, \cdots, i_l \in \{1, 2, \cdots, m\} \tag{2.20}$$

such that

$$q(x_{i_0}, x_{m+1}) \leq k\,q(x_{i_1}, x_{m+1}) \leq \cdots \leq k^l\,q(x_{i_l}, x_{m+1}), \tag{2.21}$$

and

$$q(x_{i_l}, x_{m+1}) \leq k\,(e-k)^{-1} q(x_0, x_1), \tag{2.22}$$

then

$$q(x_{i_0}, x_{m+1}) \leq k^l\,q(x_{i_l}, x_{m+1}) \leq k^{l+1}\,(e-k)^{-1} q(x_0, x_1) \leq k\,(e-k)^{-1} q(x_0, x_1). \tag{2.23}$$

Hence, the proof is complete.

If the procedure continues more than m steps, then exist $(m+1)$ integers

$$i_0, i_1, \cdots, i_m \in \{1, 2, \cdots, m\} \tag{2.24}$$

such that

$$q(x_{i_0},x_{m+1}) \preceq k\,q(x_{i_1},x_{m+1}) \preceq \cdots \preceq k^m q(x_{i_m},x_{m+1}), \tag{2.25}$$

From (2.24), there must exist integers p and q such that

$$0 \leq p < q \leq m,\ \ i_p = i_q. \tag{2.26}$$

Hence by (2.25) and (2.26),

$$q(x_{i_p},x_{m+1}) \preceq k^{q-p} q(x_{i_q},x_{m+1}) = k^{q-p} d(x_{i_p},x_{m+1}), \tag{2.27}$$

which implies that

$$(e-k^{q-p})q(x_{i_p},x_{m+1}) \preceq 0.$$

Hence $d(x_{i_q},x_{m+1}) = 0$ since $r(k^{q-p}) \leq (r(k))^{q-p} < 1$ implies that $(e-k^{q-p})$ is invertible. From (2.25) again,

$$q(x_{i_0},x_{m+1}) \preceq k^p q(x_{i_p},x_{m+1}) = 0 \preceq k(e-k)^{-1}q(x_0,x_1). \tag{2.28}$$

Therefore, by induction, (2.3) holds.

For any $1 < m < n$, denote that

$$C(m,n) = \{q(x_i,x_j) | m \leq i < j \leq n\}. \tag{2.29}$$

From (2.1) and (2.2), for each $u \in C(m,n)$, there exists $v \in C(m-1,n)$ such that

$$u \preceq kv. \tag{2.30}$$

Consequently, using (2.3) and (2.30), we obtain that

$$q(x_m,x_n) \preceq k u_1 \preceq k^2 u_2 \preceq k^{m-1} u_{m-1} \preceq k^m (e-k)^{-1} q(x_0,x_1), \tag{2.31}$$

where

$$u_1 \in C(m-1,n), u_2 \in C(m-2,n) \cdots, u_{m-1} \in C(1,n), u_{m-1} \preceq k(e-k)^{-1} q(x_0,x_1). \tag{2.32}$$

Since $r(k) < 1$, $k^m(e-k)^{-1}q(x_0,x_1)$ is a c-sequence by Lemma 1.2 and Lemma 1.5 - Lemma 1.6, which implies that $\{x_n\}$ is a Cauchy sequence by Lemma 1.9 and (2.31). Thus there exsits $x^* \in X$ such that $x_n \to x^*$ as $n \to \infty$ by the f-orbitally completeness of X.

Since $x_{n+1} = fx_n$ for all n and f is continuous about the metric d,

$$x^* = \lim_{n\to\infty} x_{n+1} = \lim_{n\to\infty} fx_n = fx^*,$$

that is, x^* is a fixed point of f. By (2.1) again,

$$q(x^*,x^*) = q(fx^*,fx^*) \leq k\,v(x^*,x^*) = k\,q(x^*,x^*),$$

hence $q(x^*,x^*) = 0$ since $r(k) < 1$ implies that $(e-k)$ is invertible.

If y^* is also a fixed point of f, then $fy^* = y^*$ and $q(y^*,y^*) = 0$ by the above discussion. By (2.1) again,

$$q(x^*,y^*) = q(fx^*,fy^*) \leq k\,v(x^*,y^*),$$

where

$$v(x^*,y^*) \in \{q(x^*,y^*),0\}.$$

Hence $q(x^*,y^*) = 0$ for any one of two cases, therefore $x^* = y^*$ by Lemma 1.9. So f has a unique fixed point.

Now, we once give another version of Theorem 2.1 under removing the continuity of f:

Theorem 2.2. Let (X,d) be a cone metric space over Banach algebra, q be a c-distance on X, $f : X \to X$ a mapping, $k \in P$ with $r(k) < 1$. Suppose that for each $x,y \in X$,

$$q(fx,fy) \leq k\,u(x,y), \tag{2.33}$$

where

$$u(x,y) \in \{q(x,y),q(x,fx),q(x,fy)\}. \tag{2.34}$$

If X is f-orbitally complete, then f has a unique fixed point $x^* \in X$ and $q(x^*,x^*) = 0$.

Proof. Repeating the proof of Theorem 2.1, we know that there exists a sequence $\{x_n\}$ in X(Here, $\{x_n\}$ satisfies $x_n = fx_{n-1}$ for all $n = 1,2,\cdots$) converging to a point $x^* \in X$. For any n, by (2.33),

$$q(x_n,fx^*) = q(fx_{n-1},fx^*) \leq k\,u(x_{n-1},x^*),$$

where

$$u(x_{n-1},x^*) \in \{q(x_{n-1},x^*),q(x_{n-1},x_n),q(x_{n-1},fx^*)\}.$$

From (2.31) and Definition 1.4(q_3), we have

$$q(x_m, x^*) \le k^m (1-k)^{-1} d(x_0, x_1), \forall m \ge 1. \tag{2.35}$$

If $u(x_{n-1}, x^*) = q(x_{n-1}, x^*)$, then

$$q(x_n, fx^*) \le k\, q(x_{n-1}, x^*). \tag{2.36}$$

If $u(x_{n-1}, x^*) = q(x_{n-1}, x_n)$, then

$$q(x_n, fx^*) \le k\, q(x_{n-1}, x_n). \tag{2.37}$$

If $u(x_{n-1}, x^*) = q(x_{n-1}, fx^*)$, then

$$q(x_n, fx^*) \le k\, q(x_{n-1}, fx^*) \le k\, [q(x_{n-1}, x_n) + q(x_n, fx^*)],$$

hence

$$q(x_n, fx^*) \le k\, (e-k)^{-1} q(x_{n-1}, x_n). \tag{2.38}$$

$\{q(x_m, x_n)\}_{n>m}$ and $\{q(x_n, x^*)\}$ are both c-sequences by (2.31) and (2.35) and Lemma 1.5–Lemma 1.6, hence the right sides of inequalities in (2.35)-(2.38) are all c-sequences. Therefore $x^* = fx^*$ by Lemma 1.9(1). The rest is similar to the proof of Theorem 2.1.

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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