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FIXED POINT RESULTS FOR GERAGHTY QUASI-CONTRACTION TYPE MAPPINGS IN DISLOCATED METRIC SPACES

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Abstract. Fixed point results for a newly introduced Geraghty quasi-contraction type mapping are proved by employing less restrictions on the mapping in a T-orbitally complete dislocated metric space. Geraghty quasi-contraction type mapping generalizes and extends Ciric's quasi-contraction mapping and other Geraghty quasi-contractive type mappings in the literature. An example is given to illustrate results.

Keywords: fixed point; dislocated metric space; Geraghty quasi-contraction type mapping.

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1. Introduction

The Banach contraction principle [2] is one of the most interesting and earliest results in fixed point theory with applications in several disciplines of mathematics. Since then, a good number of authors have improved, extended and generalized this result in different ways [1-21].

In 1973, Geraghty [5] obtained a generalization of the Banach contraction mapping in metric spaces by using an auxiliary function instead of a constant. Later, Amini-Harandi and

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Emami [1] improved the result of Geraghty in the setting of a partially ordered complete metric space. Gorji et al. [6] extended the results of Amini-Harandi and Emami by introducing the notion of ψ-Geraghty contraction. For other results relating to Geraghty contractions, see [3,11,12,16,18,20,21].

In 2000, Hitzler [7] introduced the notion of dislocated metric space in which the self distance of points is not necessarily zero and showed that the popular Banach contraction mapping is also valid in the space. Dislocated metric space is known to have applications in semantic analysis of logical programming, electronic engineering and in topology [8].

Some other results on fixed points for self-mappings with different contraction conditions in dislocated metric spaces are found in the literature [7,9,17].

2. Preliminaries

We recollect some basic definitions and results in the literature.

Definition 2.1 [7]. Let X be a non-empty set and let $d : X \times X \to \mathbb{R}^+$ be a function such that the following are satisfied:

(i) $d(x,y) = d(y,x) = 0$ implies that $x = y$;

(ii) $d(x,y) = d(y,x)$;

(iii) $d(x,y) \leq d(x,z) + d(z,y)$ for all $x,y,z \in X$.

Then d is called dislocated metric on X and the pair (X,d) is called a dislocated metric space.

The following definition by Ciric [4] is also true for dislocated metric spaces.

Definition 2.2. Let $T : X \to X$ be a self mapping on a dislocated metric space. For each $x \in X$ and for any positive whole number n,

$$O_T(x,n) = \{x, Tx, ..., T^n x\} \;\; and \;\; O_T(x,\infty) = \{x, Tx, ..., T^n x, ...\}.$$

The set $O_T(x,\infty)$ is called the orbit of T at x and the dislocated metric space X is called T-orbitally complete if every Cauchy sequence in $O_T(x,\infty)$ is convergent in X.

It is clear that every complete dislocated metric space is T-orbitally complete. But the converse does not hold in general.

As an generalization of α-admissible mappings introduced by Karapinar et al. [10] and Samet et al. [13], Popescu [18] introduced and used the following concepts to prove the existence and uniqueness of fixed point results in a complete metric space.

Definition 2.3 [18]. Let $T : X \to X$ be a self-mapping and $\alpha : X \times X \to \mathbb{R}^+$ be a function. Then T is said to be α-orbital admissible if $\alpha(x, Tx) \geq 1$ implies $\alpha(Tx, T^2x) \geq 1$.

Definition 2.4 [18]. Let $T : X \times X$ be a self-mapping and $\alpha : X \times X \to \mathbb{R}^+$ be a function. Then T is said to be triangular α-orbital admissible if T is α-orbital admissible and $\alpha(x, y) \geq 1$, $\alpha(y, Ty) \geq 1$ imply $\alpha(x, Ty) \geq 1$.

Lemma 2.5 [18]. Let $T : X \to X$ be a triangular α - orbital admissible mapping. Assume that there exists $x_1 \in X$ such that $\alpha(x_1, Tx_1) \geq 1$. Define a sequence $\{x_n\}$ by $x_{n+1} = Tx_n$. Then, we have $\alpha(x_n, x_m) \geq 1$ for all $m, n \in \mathbb{N}$ with $n < m$.

Let F be the family of all functions $\beta : [0, \infty) \to [0, 1)$ which satisfies the condition

$$\lim_{n\to\infty} \beta(t_n) = 1 \implies \lim_{n\to\infty} t_n = 0.$$

Using such a function, Geraghty [5] obtained the following result.

Theorem 2.6 [5]. Let (X, d) be a complete metric space and let T be a self mapping on X. Suppose that there exists $\beta \in F$ such that for all $x, y \in X$,

$$d(Tx, Ty) \leq \beta(d(x, y))d(x, y), \tag{2.1}$$

then T has a unique fixed point $x^* \in X$ and $\{T^n x\}$ converges to x^* for all $x \in X$.

The purpose of this paper is to prove some fixed point results in dislocated metric space using new concepts of Geraghty quasi-contraction type self mappings that the authors just introduced and proved fixed point results in the context of metric spaces. The result is obtained by dropping the restriction of continuity and proving the existence and uniqueness of fixed point in an orbitally complete dislocated metric space which is a relaxation of completeness in the space.

3. Main Results

Let Φ denote the class of the functions $\phi : [0,\infty) \to [0,\infty)$ which satisfy the following conditions:

(i) ϕ is non decreasing;

(ii) ϕ is continuous;

(iii) $\phi(t) = 0 \iff t = 0$.

The following new mapping was introduced by the authors in [16].

Definition 3.1. Let (X,d) be a metric space and $\alpha : X \times X \to \mathbb{R}^+$ be a function. A self mapping $T : X \to X$ is called an α-ϕ-Geraghty quasi-contraction type mapping if there exists $\beta \in F$ such that for all $x,y \in X$,

$$\alpha(x,y)\phi(d(Tx,Ty)) \leq \beta(\phi(M_T(x,y)))\phi(M_T(x,y)) \tag{3.1}$$

where $M_T(x,y) = \max\{d(x,y), d(x,Tx), d(y,Ty), d(x,Ty), d(y,Tx)\}$.

Remark 3.2.

(i) From inequality (3.1) above, suppose $\alpha(x,y) = 1$, $\phi(t) = t$ and $M_T(x,y) = d(x,y)$, then we have the Geraghty [5] contraction mapping defined on a metric space. In addition, if $\beta(t) = q$; where $q \in [0,1)$, we have the Banach contraction mapping. Inequality (3.1) generalizes that of Cho et al. [3], Karapinar [11,12], Popescu [18], among others.

(ii) Definition 3.1 is also true for a dislocated metric space since every metric space is a dislocated metric space but the converse is not necessarily true. An example, which is inspired by that in Hitzler [7], is provided to justify this.

Example 3.3. Let $X = [0,\infty)$ and $d(x,y) = \max\{x,y\}$ for all $x,y \in X$. Let $\beta(t) = \frac{1}{t}$ for all $t > 0$. Then $\beta \in F$. Let $\phi(t) = 2t$ and a mapping $T : X \to X$ be defined by

$$T(x) = \begin{cases} \dfrac{1}{2+x}, & \text{if} \quad x \in [0,1], \\ \\ 1, & \text{if} \quad x > 1. \end{cases}$$

Define a function $\alpha : X \times X \to [0,\infty)$ by

$$\alpha(x,y) = \begin{cases} 1, & \text{if} \quad 0 \leq x,y \leq 1, \\ \\ 0 & otherwise. \end{cases}$$

Then T is an α-ϕ-Geraghty quasi-contraction type mapping defined on a dislocated metric space but not on a metric space.

Theorem 3.4. Let (X,d) be a T-orbitally complete dislocated metric space such that $T : X \to X$ is a self-mapping. Suppose $\alpha : X \times X \to \mathbb{R}^+$ is a function satisfying the following conditions:

(i) T is an α-ϕ-Geraghty quasi-contraction type mapping.

(ii) T is triangular α-orbital admissible mapping.

(iii) There exists $x_1 \in X$ such that $\alpha(x_1, Tx_1) \geq 1$.

Then T has a unique fixed point $x^* \in X$ and $\{T^n x_1\}$ converges to x^*.

Proof

Let $x_1 \in X$ such that $\alpha(x_1, Tx_1) \geq 1$. Define a sequence $\{x_i\}$ by $x_{i+1} = T^i x$, for $1 \leq i \leq n-1$. If $x_i = x_{i+1}$ for some $1 \leq i \leq n-1$, then T has a fixed point. Thus, we assume that $x_i \neq x_{i+1}$, for all $i \geq 1$. By Lemma 2.5, we have

$$\alpha(x_i, x_{i+1}) \geq 1, \quad for \quad all \quad i \geq 1. \tag{3.2}$$

By inequality (3.1), for $1 \leq j \leq n$,

$$\begin{aligned} \phi(d(T^i x, T^j x)) &= \phi(d(TT^{i-1}x, TT^{j-1}x)) \\ &\leq \alpha(T^{i-1}x, T^{j-1}x)\phi(d(TT^{i-1}x, TT^{j-1}x)) \\ &\leq \beta(\phi(M_T(T^{i-1}x, T^{j-1}x)))\phi(M_T(T^{i-1}x, T^{j-1}x)) \end{aligned} \tag{3.3}$$

where

$$\begin{aligned}\phi(M_T(T^{i-1}x,T^{j-1}x)) &\leq \phi(\max\{d(T^{i-1}x,T^{j-1}x),d(T^{i-1}x,T^{i}x),d(T^{j-1}x,T^{j}x),\\ &\quad d(T^{i-1}x,T^{j}x),d(T^{j-1}x,T^{i}x)\})\\ &\leq \phi(\delta[O_T(x,n)]).\end{aligned}$$

The assertion $\phi(M_T(T^{i-1}x,T^{j-1}x)) = \phi(d(T^{i}x,T^{j}x))$ is not true. This is because,

$$\begin{aligned}\phi(d(T^{i}x,T^{j}x)) &\leq \beta(\phi(M_T(T^{i-1}x,T^{j-1}x)))\phi(M_T(T^{i-1}x,T^{j-1}x))\\ &\leq \beta(\phi(d(T^{i}x,T^{j}x)))\phi(d(T^{i}x,T^{j}x))\\ &< \phi(d(T^{i}x,T^{j}x)),\end{aligned}$$

is a contradiction. Thus, $\phi(d(T^{i}x,T^{j}x)) < \phi(d(T^{i-1}x,T^{j-1}x))$ for all $0 \leq i,j \leq n$. Note that $\beta(\phi(M_T(T^{i-1}x,T^{j-1}x))) \leq d(T^{i}x,T^{j}x)$ for all $0 \leq i,j \leq n$. Thus, the sequence $\{T^{i}x\}$ is positive and decreasing. Consequently, there exists r such that

$$\lim_{i,j\to\infty} d(T^{i}x,T^{j}x) = r.$$

We claim that $r = 0$. Suppose, on the contrary, that $r > 0$. Then, from (3.3)

$$\frac{\phi(d(T^{i}x,T^{j}x))}{\phi(M_T(T^{i-1}x,T^{j-1}x))} \leq \beta(\phi(M_T(T^{i-1}x,T^{j-1}x))) < 1$$

which yields that

$$\lim_{i,j\to\infty} \beta(\phi(M_T(T^{i-1}x,T^{j-1}x))) = 1.$$

Since $\beta \in F$, it implies that

$$\lim_{i,j\to\infty} \phi(M_T(T^{i-1}x,T^{j-1}x)) = 0 \tag{3.4}$$

and so

$$r = \lim_{i,j\to\infty} (d(T^{i}x,T^{j}x)) = 0,$$

which is a contradiction.

Next, we show that the sequence $\{T^{i}x\}$ is Cauchy. Let n and m be any positive integers. On the contrary, suppose that $\{T^i x\}$ is not Cauchy. Then there exists $\varepsilon > 0$, such that

$$\varepsilon = \lim_{m,n\to\infty} d(T^{n-1}x, T^{m-1}x) > 0, \quad n \geq m. \tag{3.5}$$

Using triangle inequality,

$$d(T^{n-1}x, T^{m-1}x) \leq d(T^{n-1}x, T^n x) + d(T^n x, T^m x) + d(T^m x, T^{m-1}x)$$

implies

$$d(T^{n-1}x, T^{m-1}x) - d(T^{n-1}x, T^n x) - d(T^m x, T^{m-1}x) \leq d(T^n x, T^m x).$$

Applying ϕ, we have

$$\begin{aligned}
\phi(d(T^{n-1}x, T^{m-1}x) - d(T^{n-1}x, T^n x) - d(T^m x, T^{m-1}x)) &\leq \phi(d(T^n x, T^m x)) \\
&\leq \alpha(x_n, x_m)\phi(d(T^{n-1}x, T^{m-1}x) \\
&\leq \beta(\phi(M_T(T^{n-1}x, T^{m-1}x))) \\
&\quad \phi(M_T(T^{n-1}x, T^{m-1}x)).
\end{aligned}$$

Taking the limits and using (3.5) we get,

$$\begin{aligned}
\phi(\varepsilon) &\leq \lim_{m,n\to\infty} \beta(\phi(M_T(T^{n-1}x, T^{m-1}x)))\phi(\varepsilon) \\
1 &\leq \lim_{m,n\to\infty} \beta(\phi(M_T(T^{n-1}x, T^{m-1}x))).
\end{aligned}$$

Therefore $\lim_{m,n\to\infty} \beta(\phi(M_T(T^{n-1}x, T^{m-1}x))) = 1$ and so $\lim_{m,n\to\infty} \phi(M_T(T^{n-1}x, T^{m-1}x)) = 0$. Thus $\lim_{m,n\to\infty} d(T^{n-1}x, T^{m-1}x) = 0$, which is a contradiction. Thus the sequence is Cauchy. Since X is T-orbitally complete, there exists $x^* \in X$ such that $\lim_{n\to\infty} T^n x = x^*$. To show that $Tx^* = x^*$, consider

$$\begin{aligned}
\phi(d(x^*, Tx^*)) &\leq \phi(d(x^*, T^{k+1}x) + d(T^{k+1}x, Tx^*)) \qquad 1 \leq k < n \\
&\leq \phi(d(x^*, T^{k+1}x)) + \phi(d(TT^k x, Tx^*)) \\
&\leq \phi(d(x^*, T^{k+1}x)) + \alpha(T^k x, x^*)\phi(d(TT^k x, Tx^*)) \\
&\leq \phi(d(x^*, T^{k+1}x)) + \beta(\phi(M_T(T^k x, x^*)))\phi(M_T(T^k x, x^*)),
\end{aligned}$$

where,

$$M_T(T^kx,x^*) \quad = \quad \max\{d(T^kx,x^*),d(T^kx,T^{k+1}x),d(x^*,T^*x),d(T^kx,T^*x),d(x^*,T^{k+1}x)\}.$$

Taking the limit as $n\to\infty$ above, gives $d(x^*,Tx^*)\le d(Tx^*,x^*)=0$, implying that $x^*=Tx^*$ and so the fixed point of T is x^*.

For the uniqueness of a fixed point, consider the following hypothesis:

(J): For all $x\neq y\in Fix(T)$ there exists $w\in X$ such that $\alpha(x,w)\ge 1,\alpha(y,w)\ge 1$ and $\alpha(w,Tw)\ge 1$. $Fix(T)$ denotes the set of fixed points of T.

Theorem 3.5. Adding condition (J) to the conditions of Theorem 3.4, we obtain that x^* is a unique fixed point of T.

Proof. From the proof of Theorem 3.4, x^* is a fixed point of T. Assume that x_1^* and x_2^* are distinct fixed points of T. By assumption, there exists $w\in X$ such that $\alpha(x_1^*,w)\ge 1,\alpha(x_2^*,w)\ge 1$ and $\alpha(w,Tw)\ge 1$. Since T is triangular α-orbital admissible, $\alpha(x_1^*,T^nw)\ge 1$ and $\alpha(x_2^*,T^nw)\ge 1$ for all $n\ge 1$. So

$$\phi(d(x_1^*,T^{n+1}w))\le\alpha(x_1^*,T^nw)\phi(d(Tx_1^*,T^{n+1}w))\le\beta(\phi(M_T(x_1^*,T^nw)))\phi(M_T(x_1^*,T^nw))$$

for all $n\ge 1$, where

$$M_T(x_1^*,T^nw) \quad = \quad \max\left\{d(x_1^*,T^nw),d(T^nw,T^{n+1}w),d(x_1^*,Tx_1^*),d(Tx_1^*,T^nw),d(x_1^*,T^{n+1}w)\right\}.$$

We deduce, by Theorem 3.4, that the sequence $\{T^nw\}$ converges to a fixed point z (say). Letting $n\to\infty$ in the above inequality, we obtain $\lim_{n\to\infty}M_T(x_1^*,T^nw)=d(x_1^*,z)$. If $x_1^*\neq z$, then

$$\frac{\phi(d(x_1^*,T^{n+1}w))}{\phi(M_T(x_1^*,T^nw))}\le\beta(\phi((M_T(x_1^*,T^nw)))<1$$

and as $n\to\infty$, $\lim_{n\to\infty}\beta(\phi(M_T(x_1^*,T^nw)))=1$ implies that $\lim_{n\to\infty}\phi(M_T(x_1^*,T^nw))=0$. Thus $d(x_1^*,z)=0$. Similarly, $d(x_2^*,z)=0$ is a contradiction. Therefore, $d(x_1^*,z)=d(x_2^*,z)=0$ implies that $x_1^*=x_2^*=z$. Hence, T has a unique fixed point.

Corollary 3.6. Let (X,d) be a T-orbitally complete dislocated metric space such that $T : X \to X$ is a self-mapping. Suppose $\alpha : X \times X \to R^+$ is a function satisfying the following conditions:

(i) T is an α-Geraghty quasi-contraction type mapping.
(ii) T is triangular α-orbital admissible mapping.
(iii) There exists $x_1 \in X$ such that $\alpha(x_1, Tx_1) \geq 1$.

Then T has a unique fixed point $x^* \in X$ and $\{T^n x_1\}$ converges to x^*. Further, if for all $x \neq y \in Fix(T)$ there exists $w \in X$ such that $\alpha(x,w) \geq 1, \alpha(y,w) \geq 1$ and $\alpha(w,Tw) \geq 1$, then x^* is a unique fixed point of T.

Proof: Take $\phi(t) = t$ in Theorem 3.4 and Theorem 3.5 and the proof follows.

Remark 3.7.

(i) Suppose the α-ϕ-Geraghty quasi-contraction type mapping is defined on a metric space, then Theorem 3.4 reduces to the result obtained by the authors in [15].
(ii) Moreover, suppose continuity condition is imposed on the mapping T, if it is defined on a complete metric space, which is a stronger restriction than orbital completeness, $\phi(t) = t$ and $M_T(x,y) = \{d(x,y), d(x,Tx), d(y,Ty), \frac{d(x,Ty)+d(y,Tx)}{2}\}$ then the result in Popescu [5] is obtained.
(iii) The results in Karapinar [4,7] and Cho et al. [3] are also corollaries to our result. Therefore, Theorem 3.4 is an improvement and a generalization of other related work in the literature.

The following example illustrates obtained results.

Example 3.8. Let $X = [0,\infty)$ and $d(x,y) = \max\{x,y\}$ for all $x,y \in X$. Let $\beta(t) = \frac{1}{1+t}$. Then $\beta \in F$. Let $\phi(t) = 2t$ and a mapping $T : X \to X$ be defined by

$$T(x) = \begin{cases} \frac{1}{3}x, & \text{if} \quad x \in [0,1), \\ \\ \frac{1}{10}x, & \text{if} \quad x = 1, \\ \\ 2x, & \text{if} \quad x > 1. \end{cases}$$

We define a function $\alpha : X \times X \to [0,\infty)$ by

$$\alpha(x,y) = \begin{cases} 1, & \text{if} \quad (0 \leq x,y \leq 1), \\ \\ 0, & \quad otherwise. \end{cases}$$

One can easily see that X is a dislocated metric space but not a metric space since the self distance is not zero. Also, the self mapping T is not continuous at $x = 1$.

Condition (iii) of Theorem 3.4 is satisfied with $x_1 = 1$.

Obviously, condition (ii) is satisfied. Let x,y be such that $\alpha(x,y) \geq 1$. Then, $x,y \in [0,1]$, and so $Tx,Ty \in [0,1]$. Moreover, $\alpha(y,Ty) = \alpha(x,Tx) = 1$ and $\alpha(Tx,T^2x) = 1$. Thus, T is triangular α-orbital admissible and hence (ii) is satisfied. Finally, to prove that condition (i) is satisfied. If $0 \leq x,y \leq 1$, then $\alpha(x,y) = 1$, and

$$\begin{aligned} & \beta(\phi(M_T(x,y)))\phi(M_T(x,y)) - \alpha(x,y)\phi(M_T(Tx,Ty)) \\ = \; & \beta(\phi(M_T(x,y)))\phi(M_T(x,y)) - \phi(M_T(Tx,Ty)) \\ = \; & \frac{2M_T(x,y)}{1+2M_T(x,y))} - 2\max\{Tx,Ty\} \\ \geq \; & 0. \end{aligned}$$

Therefore, $\alpha(x,y)\phi(d(Tx,Ty)) \leq \beta(\phi(M_T(x,y)))\phi(M_T(x,y))$ for $0 \leq x,y \leq 1$.

Similarly, for $\phi(t) = t$, $\alpha(x,y)d(Tx,Ty) \leq \beta(M_T(x,y))M_T(x,y)$ for $0 \leq x,y \leq 1$.

Now, if $x \in [0,1]$ and $y > 1$ or vice versa then, obviously, $\alpha(x,y) = 0$ and we have

$$\alpha(x,y)\phi(d(Tx,Ty)) \leq \beta(\phi(M_T(x,y)))\phi(M_T(x,y)),$$

and

$$\alpha(x,y)(d(Tx,Ty)) \leq \beta(M_T(x,y))M_T(x,y).$$

Consequently, all assumptions of Theorem 3.4 and Corollary 3.6 are satisfied, and hence T has a unique fixed point $x^* = 0$.

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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