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SOME FIXED POINT RESULTS IN MENGER SPACES

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Abstract. In the present paper, we prove a common fixed point theorem for weakly compatible mappings in Menger space. An example is furnished to support our main result. We also prove a fixed point theorem for six self mappings by using the notion of commuting pairwise. We extend our main result to four finite families of self mappings.

Keywords: t-norm; Menger space; compatible mappings; weakly compatible mappings; fixed point.

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1. Introduction

There have been a number of generalizations of metric spaces, one such generalization is probabilistic metric space (shortly, PM-space) introduced by Karl Menger [8] in 1942. The idea of Menger was to use distribution functions instead of non-negative real numbers as values of the metric. Since then the theory of PM-space was expanded rapidly with the pioneering works of Schweizer and Sklar [12, 13]. It is also of fundamental importance in probabilistic functional analysis, nonlinear analysis and applications (see [1]).

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In 1972, Sehgal and Bharucha-Reid [14] initiated the study of contraction mappings on PM-spaces. In 1986, Jungck [4] introduced the notion of compatible mappings in metric spaces. Mishra [9] extended the notion of compatibility to PM-spaces and proved a common fixed point theorem. This condition has further been weakened by introducing the notion of weakly compatible mappings by Jungck and Rhoades [5]. The concept of weakly compatible mappings is most general as each pair of compatible mappings is weakly compatible but the converse is not true. In 2005, Singh and Jain [15] extended the notion of weakly compatible mappings to PM-space and proved a common fixed point theorem. Several interesting and elegant results have been obtained by various authors in this direction (see [2, 3, 7, 10, 11]). In 2007, Kohli and Vashistha [6] proved common fixed point theorems for variants of R-weakly commuting mappings in PM-spaces.

The aim of this paper is to prove common fixed point theorems for weakly compatible mappings in Menger spaces satisfying ϕ-contractive conditions. We give an example which demonstrates the validity of the hypotheses and degree of generality of our main result. We prove a fixed point theorem for six self mappings in Menger spaces by using the notion of pairwise commuting. As an application, we present a fixed point theorem for four finite families of mappings.

2. Preliminaries

Definition 2.1. [13] A triangular norm (shortly, t-norm) $*$ is a binary operation on the unit interval [0,1] such that for all $a, b, c, d \in [0,1]$ and the following conditions are satisfied:

(1) $a * 1 = a$;

(2) $a * b = b * a$;

(3) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$;

(4) $(a * (b * c)) = ((a * b) * c)$.

Examples of t-norms are $a * b = \min\{a, b\}$, $a * b = ab$ and $a * b = \max\{a + b - 1, 0\}$.

Definition 2.2. [13] A mapping $F : \mathbb{R} \to \mathbb{R}^+$ is said to be a distribution function if it is non-decreasing and left continuous with $\inf\{F(t) : t \in \mathbb{R}\} = 0$ and $\sup\{F(t) : t \in \mathbb{R}\} = 1$.

We shall denote by $\Im$ the set of all distribution functions while H will always denote the specific distribution function defined by

$$H(t) = \begin{cases} 0, & \text{if } t \leq 0; \\ 1, & \text{if } t > 0. \end{cases}$$

If X is a non-empty set, $\mathcal{F} : X \times X \to \Im$ is called a probabilistic distance on X and $\mathcal{F}(x, y)$ is usually denoted by $F_{x,y}$.

Definition 2.3. [13] The ordered pair $(X, \mathcal{F})$ is called a PM-space if X is a nonempty set and $\mathcal{F}$ is a probabilistic distance satisfying the following conditions: for all $x, y, z \in X$ and $t, s > 0$,

(1) $F_{x,y}(t) = 1$ if and only if $x = y$;
(2) $F_{x,y}(0) = 0$;
(3) $F_{x,y}(t) = F_{y,x}(t)$;
(4) $F_{x,z}(t) = 1,\ F_{z,y}(s) = 1 \Rightarrow F_{x,y}(t + s) = 1$.

The ordered triple $(X, \mathcal{F}, *)$ is called a Menger space if $(X, \mathcal{F})$ is a PM-space, $*$ is a t-norm and the following inequality holds:

$$F_{x,y}(t + s) \geq F_{x,z}(t) * F_{z,y}(s),$$

for all $x, y, z \in X$ and $t, s > 0$.

Definition 2.4. [13] Let $(X, \mathcal{F}, *)$ be a Menger space and $*$ be a continuous t-norm. A sequence $\{x_n\}$ in X is said to be

(1) convergent to a point x in X iff for every $\epsilon > 0$ and $\lambda > 0$, there exists a positive integer $\mathbb{N}(\epsilon, \lambda)$ such that $F_{x_n,x}(\epsilon) > 1 - \lambda$ for all $n \geq \mathbb{N}(\epsilon, \lambda)$.
(2) A sequence $\{x_n\}$ in X is said to be Cauchy if for every $\epsilon > 0$ and $\lambda > 0$, there exists a positive integer $\mathbb{N}(\epsilon, \lambda)$ such that $F_{x_n,x_m}(\epsilon) > 1 - \lambda$ for all $n, m \geq \mathbb{N}(\epsilon, \lambda)$.

A Menger space in which every Cauchy sequence is convergent is said to be complete.

Definition 2.5. [9] A pair (A, S) of self mappings of a Menger space $(X, \mathcal{F}, *)$ is said to be compatible if $F_{ASx_n,SAx_n}(t) \to 1$ for all $t > 0$, whenever $\{x_n\}$ is a sequence in X such that $Ax_n, Sx_n \to z$ for some $z \in X$ as $n \to \infty$.

Definition 2.6. [5] A pair (A, S) of self mappings of a non-empty set X is said to be weakly compatible (or coincidentally commuting) if they commute at their coincidence points, that is, if $Az = Sz$ for some $z \in X$, then $ASz = SAz$.

If self mappings A and S of a Menger space $(X, \mathcal{F}, *)$ are compatible then they are weakly compatible but the converse need not be true (see [15, Example 1]).

Definition 2.7. [3] Two families of self mappings $\{A_i\}_{i=1}^m$ and $\{S_k\}_{k=1}^n$ are said to be pairwise commuting if

(1) $A_iA_j = A_jA_i$, $i, j \in \{1, 2, \ldots, m\}$,

(2) $S_kS_l = S_lS_k$, $k, l \in \{1, 2, \ldots, n\}$,

(3) $A_iS_k = S_kA_i$, $i \in \{1, 2, \ldots, m\}$, $k \in \{1, 2, \ldots, n\}$.

3. Main results

Theorem 3.1. *Let A, B, S and T be self mappings of a complete Menger space $(X, \mathcal{F}, *)$, where $*$ is a continuous t-norm satisfying the following conditions:*

(1) *$A(X) \subseteq T(X)$ and $B(X) \subseteq S(X)$,*

(2) *one of $T(X)$ and $S(X)$ is a closed subset of X,*

(3) *the pairs (A, S) and (B, T) are weakly compatible,*

(4) *for all $x, y \in X$ and $t > 0$,*

$$F_{Ax,By}(t) \geq \phi\left(F_{Sx,Ty}(t)\right),$$

where $\phi : [0, 1] \to [0, 1]$ is a continuous function such that $\phi(s) > s$ for each $0 < s < 1$, $\phi(0) = 0$ and $\phi(1) = 1$.

Then A, B, S and T have a unique common fixed point in X.

Proof. Let x_0 be an arbitrary point in X. By (1), there exist x_1, $x_2 \in X$ such that $Ax_0 = Tx_1$ and $Bx_1 = Sx_2$. Inductively, we construct sequences $\{x_n\}$ and $\{y_n\}$ in X such that $Ax_{2n} = Tx_{2n+1} = y_{2n}$ and $Bx_{2n+1} = Sx_{2n+2} = y_{2n+1}$, for $n = 0, 1, \ldots$.

Putting $x = x_{2n}$ and $y = x_{2n+1}$ in (4), then we get

$$\begin{aligned} F_{Ax_{2n},Bx_{2n+1}}(t) &\geq \phi\left(F_{Sx_{2n},Tx_{2n+1}}(t)\right) \\ F_{y_{2n},y_{2n+1}}(t) &\geq \phi\left(F_{y_{2n-1},y_{2n}}(t)\right). \end{aligned}$$

Similarly, we get

$$F_{y_{2n+1},y_{2n+2}}(t) \geq \phi\left(F_{y_{2n},y_{2n+1}}(t)\right).$$

In general, we obtain

$$F_{y_n,y_{n+1}}(t) \geq \phi\left(F_{y_{n-1},y_n}(t)\right), \tag{1}$$

for all n.

Case I: If $0 < F_{y_{n-1},y_n}(t) < 1$. Now since $\phi(t) > t$ for $0 < t < 1$. Then inequality (1) implies

$$F_{y_n,y_{n+1}}(t) \geq \phi\left(F_{y_{n-1},y_n}(t)\right) > F_{y_{n-1},y_n}(t), \tag{2}$$

for all n. Thus $\{F_{y_n,y_{n+1}}(t) : n \geq 0\}$ is a bounded strictly increasing sequence of positive real numbers in $[0,1]$ and therefore tends to a limit, say $L(t) \leq 1$. We claim that $L(t) = 1$. For if $L(t_0) < 1$ for some t_0, then letting $n \to \infty$ in inequality (2), we get $L(t_0) \geq \phi\left(L(t_0)\right) > L(t_0)$, a contradiction. Hence $L(t) = 1$, that is, $\lim(n \to \infty)F_{y_n,y_{n+1}}(t) = 1$, for all $t > 0$. Now for any non zero integer p, we obtain

$$F_{y_n,y_{n+p}}(t) \geq F_{y_n,y_{n+1}}\left(\frac{t}{p}\right) * F_{y_{n+1},y_{n+2}}\left(\frac{t}{p}\right) * \ldots * F_{y_{n+p-1},y_{n+p}}\left(\frac{t}{p}\right).$$

Since, $*$ is continuous t-norm and letting $n \to \infty$, we obtain

$$\lim_{n\to\infty} F_{y_n,y_{n+p}}(t) \geq 1 * 1 * \ldots * 1,$$

which shows that $\{y_n\}$ is a Cauchy sequence in X.

Case II: If $F_{y_{n-1},y_n}(t) = 1$. Then inequality (1) implies

$$F_{y_n,y_{n+1}}(t) \geq \phi\left(F_{y_{n-1},y_n}(t)\right) = \phi(1) = 1.$$

So it follows that $F_{y_n,y_{n+1}}(t) = 1$, which in turn implies that $\{y_n\} = \{y_{n+1}\}$, for each n, that is, $\{y_n\}$ is a constant sequence. Thus, in either case $\{y_n\}$ is a Cauchy sequence in X.

From above two cases, it is clear that $\{y_n\}$ is a Cauchy sequence in X. Since the Menger space $(X, \mathcal{F}, *)$ is complete, $\{y_n\}$ converges to a point z in X. That is,

$$\lim_{n\to\infty} y_n = \lim_{n\to\infty} Ax_{2n} = \lim_{n\to\infty} Bx_{2n+1} = \lim_{n\to\infty} Sx_{2n+1} = \lim_{n\to\infty} Tx_{2n} = z.$$

Suppose that $T(X)$ is a closed subset of X. Then for some $v \in X$ we have $z = Tv$. Putting $x = x_{2n}$ and $y = v$ in (4), we have

$$F_{Ax_{2n},Bv}(t) \geq \phi\left(F_{Sx_{2n},Tv}(t)\right),$$

passing limit as $n \to \infty$, we get

$$F_{z,Bv}(t) \geq \phi\left(F_{z,z}(t)\right) = \phi(1) = 1,$$

for $t > 0$, it follows that $z = Bv$. Therefore $z = Bv = Tv$ which shows that v is a coincidence point of the pair (B, T). Since the pair (B, T) is weakly compatible, we have $Bz = BTv = TBv = Tz$. We show that $Bz = Tz = z$. We claim that $z = Bz$. For if $z \neq Bz$, then there exists a positive real number t such that $F_{z,Bz}(t) < 1$. Putting $x = x_{2n}$ and $y = z$ in (4), we get

$$F_{Ax_{2n},Bz}(t) \geq \phi\left(F_{Sx_{2n},Tz}(t)\right).$$

Letting $n \to \infty$, we get

$$F_{z,Bz}(t) \geq \phi\left(F_{z,Bz}(t)\right) > F_{z,Bz}(t),$$

which is a contradiction. It follows that $z = Bz$. Therefore $z = Bz = Tz$.

Since, $B(X) \subseteq S(X)$, there exists $u \in X$ such that $Su = z$. Putting $x = u$ and $y = z$ in (4), we have

$$F_{Au,Bz}(t) \geq \phi\left(F_{Su,Tz}(t)\right),$$

and so

$$F_{Au,z}(t) \geq \phi\left(F_{z,z}(t)\right) = \phi\left(1\right) = 1.$$

for $t > 0$, we get $z = Au$. Therefore $z = Au = Su$ which shows that u is a coincidence point of the pair (A, S). Since the pair (A, S) is weakly compatible, we have $Az = ASu = SAu = Sz$.

Now we claim that $z = Az$. For if $z \neq Az$, then there exists a positive real number t such that $F_{Az,z}(t) < 1$. On using (4) with $x = z$, $y = v$, we get

$$F_{Az,Bv}(t) \geq \phi\left(F_{Sz,Tv}(t)\right),$$

and so

$$F_{Az,z}(t) \geq \phi\left(F_{Az,z}(t)\right) > F_{Az,z}(t),$$

which is a contradiction. Hence, $z = Az = Sz$. Therefore $z = Az = Bz = Sz = Tz$, that is, z is a common fixed point of the self mappings A, B, S and T.

Uniqueness: Let $w(\neq z)$ be another common fixed point of self mappings A, B, S and T. Then there exists a positive real number t such that $F_{z,u}(t) < 1$. On using (4) with $x = z$ and $y = w$, we have

$$F_{Az,Bw}(t) \geq \phi\left(F_{Sz,Tw}(t)\right),$$

or, equivalently,

$$F_{z,w}(t) \geq \phi\left(F_{z,w}(t)\right) > F_{z,w}(t),$$

which is a contradiction. Hence, $z = u$. Therefore the mappings A, B, S and T have a unique common fixed point in X.

Similarly the result follows when $S(X)$ is a closed subset of X.

The following example illustrates Theorem 3.1.

Example 3.2. Let $X = [0, 30]$ with the metric d defined by $d(x, y) =| x - y |$ and for each $t \in [0, 1]$ define

$$F_{x,y}(t) = \begin{cases} \frac{t}{t+|x-y|}, & \text{if } t > 0; \\ 0, & \text{if } t = 0, \end{cases}$$

for all $x, y \in X$. Clearly $(X, \mathcal{F}, *)$ be a complete Menger space, where $*$ is a continuous t-norm. Define the self mappings A, B, S and T by

$$A(x) = \begin{cases} 0, & \text{if } x = 0; \\ 6, & \text{if } 0 < x \leq 30. \end{cases} \quad B(x) = \begin{cases} 0, & \text{if } x = 0; \\ 9, & \text{if } 0 < x \leq 30. \end{cases}$$

$$S(x) = \begin{cases} 0, & \text{if } x = 0; \\ 15 - x, & \text{if } 0 < x \leq 15; \\ x - 9, & \text{if } 15 < x \leq 30. \end{cases} \quad T(x) = \begin{cases} 0, & \text{if } x = 0; \\ 15 - x, & \text{if } 0 < x \leq 15; \\ x - 6, & \text{if } 15 < x \leq 30. \end{cases}$$

Let $\phi : [0,1] \to [0,1]$ be defined by $\phi(s) = \sqrt{s}$ for $0 < s \leq 1$. Then $\phi(s) > s$ for each $0 < s < 1$ and $F_{Ax,By}(t) \geq \phi\left(F_{Sx,Ty}(t)\right)$ for all $x, y \in X$. Then $A(X) = \{0, 6\} \subseteq [0, 24] = T(X)$ and $B(X) = \{0, 9\} \subseteq [0, 21] = S(X)$. Therefore the mappings A, B, S and T satisfy all the conditions of Theorem 3.1 and have a unique common fixed point 0. Notice that the mappings A and S commute at coincidence point 0 and so the pair (A, S) is weakly compatible. Similarly, the pair (B, T) commutes at coincidence point 0 and weakly compatible also. To see the pairs (A, S) and (B, T) are not compatible, let us consider a sequence $\{x_n\}$ defined as $x_n = \{15 + \frac{1}{n}\}_{n \in \mathbb{N}}$, $n \geq 1$, then $x_n \to 15$ as $n \to \infty$. Then Ax_n, $Sx_n \to 6$ as $n \to \infty$ but $\lim\limits_{n\to\infty} F_{ASx_n,SAx_n}(t) = \dfrac{t}{t+ \mid 6 - 9 \mid} \neq 1$. Thus the pair (A, S) is not compatible. Also, Bx_n, $Tx_n \to 9$ as $n \to \infty$ but $\lim\limits_{n\to\infty} F_{BTx_n,TBx_n}(t) = \dfrac{t}{t+ \mid 9 - 6 \mid} \neq 1$. Hence the pair (B, T) is not compatible. All the mappings involved in this example are discontinuous even at the common fixed point $x = 0$.

By choosing A, B, S and T suitably, we can deduce corollaries for two or three self mappings. As a sample, we deduce the following natural result for a pair of self mappings.

Corollary 3.3. *Let A and S be self mappings of a complete Menger space $(X, \mathcal{F}, *)$, where $*$ is a continuous t-norm satisfying the following conditions:*

(1) $A(X) \subseteq S(X)$,

(2) $S(X)$ *is a closed subset of* X,

(3) *the pair* (A, S) *is weakly compatible,*

(4) *for all* $x, y \in X$ *and* $t > 0$,

$$F_{Ax,Ay}(t) \geq \phi\left(F_{Sx,Sy}(t)\right),$$

where $\phi : [0,1] \to [0,1]$ is a continuous function such that $\phi(s) > s$ for each $0 < s < 1$, $\phi(0) = 0$ and $\phi(1) = 1$.

Then A and S have a unique common fixed point in X.

Now we utilize the notion of commuting pairwise and prove a common fixed point theorem for six self mappings.

Theorem 3.4. Let A, B, S, R, T and H be self mappings of a complete Menger space $(X, \mathcal{F}, *)$, where $*$ is a continuous t-norm satisfying the following conditions:

(1) $A(X) \subseteq TH(X)$ and $B(X) \subseteq SR(X)$,

(2) one of $TH(X)$ and $SR(X)$ is a closed subset of X,

(3) the pairs (A, SR) and (B, TH) commute pairwise (that is, $AS = SA$, $AR = RA$, $SR = RS$, $BT = TB$, $BH = HB$ and $TH = HT$),

(4) for all $x, y \in X$ and $t > 0$,

$$F_{Ax,By}(t) \geq \phi\left(F_{SRx,THy}(t)\right),$$

where $\phi : [0,1] \to [0,1]$ is a continuous function such that $\phi(s) > s$ for each $0 < s < 1$, $\phi(0) = 0$ and $\phi(1) = 1$.

Then A, B, S, R, T and H have a unique common fixed point in X.

Proof. Since (A, SR) and (B, TH) are commuting pairwise, obviously both the pairs are weakly compatible. By Theorem 3.1, A, B, SR and TH have a unique common fixed point z in X. We show that $z = Rz$. For if $z \neq Rz$, then there exists a positive real number t such that $F_{Rz,z}(t) < 1$. Putting $x = Rz$ and $y = z$ in (4), we get

$$F_{A(Rz),Bz}(t) \geq \phi\left(F_{SR(Rz),THz}(t)\right),$$

and so

$$F_{Rz,z}(t) \geq \phi\left(F_{Rz,z}(t)\right) > F_{Rz,z}(t),$$

which is a contradiction. Thus $z = Rz$. Hence, $S(Rz) = Sz = z$. Now we prove that $z = Hz$. For if $z \neq Hz$, then there exists a positive real number t such that $F_{z,Hz}(t) < 1$. Putting $x = z$ and $y = Hz$ in (4), we get

$$F_{Az,B(Hz)}(t) \geq \phi\left(F_{SRz,TH(Hz)}(t)\right),$$

or, equivalently,

$$F_{z,Hz}(t) \geq \phi\left(F_{z,Hz}(t)\right) > F_{z,Hz}(t),$$

which is a contradiction. Thus $z = Hz$. Hence, $T(Hz) = Tz = z$. Therefore the mappings A, B, R, S, H and T have a unique common fixed point in X.

As an application of Theorem 3.1, we present a fixed point theorem for four finite families of self mappings.

Theorem 3.5. *Let* $\{A_i\}_{i=1}^m$, $\{B_r\}_{r=1}^n$, $\{S_k\}_{k=1}^p$ *and* $\{T_g\}_{g=1}^q$ *be four finite families of self mappings of a complete Menger space* $(X, \mathcal{F}, *)$, *where* $*$ *is a continuous t-norm such that* $A = A_1A_2\dots A_m$, $B = B_1B_2\dots B_n$, $S = S_1S_2\dots S_p$ *and* $T = T_1T_2\dots T_q$ *satisfying conditions (1), (2) and (4) of Theorem 3.1.*

Moreover, if the family $\{A_i\}_{i=1}^m$ *commutes pairwise with the family* $\{S_k\}_{k=1}^p$ *whereas the family* $\{B_r\}_{r=1}^n$ *commutes pairwise with the family* $\{T_g\}_{g=1}^q$, *then (for all* $i \in \{1, 2, \dots, m\}$, $r \in \{1, 2, \dots, n\}$, $k \in \{1, 2, \dots, p\}$ *and* $g \in \{1, 2, \dots, q\}$*)* A_i, B_r, S_k *and* T_g *have a unique common fixed point in* X.

Proof. The proof of this theorem is similar to that of Theorem 3.1 contained in Imdad et al. [3], hence the details are avoided.

Corollary 3.6. *Let* A, B, S *and* T *be self mappings of a complete Menger space* $(X, \mathcal{F}, *)$, *where* $*$ *is a continuous t-norm satisfying the following conditions:*

(1) $A^m(X) \subseteq T^q(X)$ *and* $B^n(X) \subseteq S^p(X)$,

(2) *one of* $T^q(X)$ *and* $S^p(X)$ *is a closed subset of* X,

(3) $AS = SA$ *and* $BT = TB$,

(4) *for all* $x, y \in X$ *and* $t > 0$,

$$F_{A^m x, B^n y}(t) \geq \phi\left(F_{S^p x, T^q y}(t)\right),$$

where m, n, p, q *are fixed positive integers and* $\phi : [0, 1] \to [0, 1]$ *is a continuous function such that* $\phi(s) > s$ *for each* $0 < s < 1$, $\phi(0) = 0$, $\phi(1) = 1$.

Then A, B, S *and* T *have a unique common fixed point in* X.

Conclusion. Theorem 3.1 is proved for two pairs of weakly compatible mappings in Menger space which improves the results of Kohli and Vashistha [6, Theorem 4.7, Theorem 4.8] in the sense that the notion of weakly compatibility is most general among all the commutativity concepts. Example 3.1 is defined in support of Theorem 3.1. Inspired by Imdad et al. [3], Theorem 3.4 is proved for six self mappings by using the notion of commuting pairwise. As an application to our main result, Theorem 3.5 and Corollary 3.6 is furnished for four finite families of mappings.

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