Available online at http://scik.org

Advances in Fixed Point Theory, 3 (2013), No. 1, 16-28

ISSN: 1927-6303

RESULTS IN RANDOM FUZZY SPACE

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Abstract: In this paper we prove fixed point theorem with implicit relation in random fuzzy space also another result for rational expression.

Keywords: Random fuzzy space, Cauchy Sequence, Fixed point.

AMS Classification: 47H10, 54H25.

INTRODUCTION AND PRILIMINARIES

Probabilistic functional analysis has emerged as one of the important mathematical disciplines in view of its role in analyzing probabilistic models in the applied sciences. The study of fixed points of random operators forms a central topic in this area. The Prague school of probabilistic initiated its study in the 1950. However, the research in this area flourished after the publication of the survey article of Bharucha-Reid [3]. Since then many interesting random fixed point results and several applications have appeared in the literature; for example the work of Beg and Shahazad [1,2,]and fixed point results in fuzzy space have been done in [8,9,10].

In recent years, the study of random fixed points have attracted much attention some of the recent literatures in random fixed points may be noted in [1,2,3,5,6,7].

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Received July 20, 2012

In particular ,random iteration schemes leading to random fixed point of random operators have been discussed in [5,6,7].

Definition 1. A binary operation * : $[0,1]\times[0,1]\rightarrow[0,1]$ is a continuous t-norm if it satisfies the following conditions

(1) * is associative and commutative,

(2) * is continuous ,

(3) $a*1 = a$ for all $a\in[0,1]$,

(4) $a*b \le c*d$ whenever $a \le c$ and $b \le d$, for each a, b, c, d $\in[0,1]$.

Two typical examples of continuous t-norm are

$a*b = a\, b$ and $a*b = \min(a, b)$.

Definition 2. Let $(\Omega, \sum)$ be a measurable space with $\sum$ a sigma algebra of subsets of Ω and M a non-empty subset of a metric space $X = (X, d)$. Let 2^M be the family of all non-empty subsets of M and $C(M)$ the family of all nonempty closed subsets of M. A mapping $G: \Omega \rightarrow 2^M$ is called measurable if, for each open subset U of $M, G^{-1}(U) \in \sum, where\, G^{-1}(U)=\{w\in\Omega : G(w)\cap U \neq \phi\}$.

Definition 3. A mapping $\xi : \Omega \rightarrow M$ is called a measurable selector of a measurable mapping $G : \Omega \rightarrow 2^M$ if ξ is measurable and $\xi(w) \in G(w)$ for each $w \in \Omega$.

Definition 4. A mapping $T : \Omega \times M \rightarrow X$ is said to be a random operator if, for each fixed $x \in M$, $T(., x) : \Omega \rightarrow X$ is measurable.

Definition 5. A measurable mapping $\xi : \Omega \rightarrow M$ is a random fixed point of a random operator $T : \Omega \times M \rightarrow X$ if $\xi(w) = T\left(w, \xi(w)\right)$ for each $w \in \Omega$.

Definition 6. Let $(\Omega, \sum)$ be a measurable space with $\sum$ a sigma algebra of subsets of Ω . The 3-tuple $(X,M,*)$ is called a Random Fuzzy Metric Space (RFM space) if X is an arbitrary (non empty) set and C is a non empty subset of X = (X,d) also * is a continuous t-norm and M is a fuzzy set in (ΩxCx (ΩxCx (Ωx[0,∞)) satisfying the following conditions for all x(ξ), y(ξ) in (ΩxC),

(1) $M(x(\xi),y(\xi),0(\xi)) = 0$,

(2) $M(x(\xi),y(\xi),t(\xi)) = 1$ for all $t(\xi)>0$ if and only if $x(\xi) = y(\xi)$,

(3) $M(x(\xi),y(\xi),t(\xi)) = M(y(\xi),x(\xi),t(\xi))$,

(4) $M(x(\xi),y(\xi),t(\xi)) * M(x(\xi),z(\xi),s(\xi)) \leq M(x(\xi),z(\xi),t(\xi)+s(\xi))$,

(5) $M(x(\xi),y(\xi),\alpha(\xi)):[0, 1] \to [0,1]$ is left continuous.

Note that $M(x(\xi),y(\xi),t(\xi))$ can be thought as degree of nearness between (x(ξ) and y(ξ) with respect to t(ξ)

We identify $x(\xi) = y(\xi)$ with $M(x(\xi), y(\xi), t(\xi)) = 1$ for all measurable $t(\xi) > 0$ and $M(x(\xi), y(\xi), t(\xi)) = 0$ with ∞. In the following example, we know that every metric induces a fuzzy metric.

Example (1) Let (X, d) be a metric space.

Define $a * b = a\, b$, or $a * b = \min\{a, b\}$) and $\forall\ x(\xi), y(\xi) \in \Omega x C$ and $t(\xi) > 0$,

$$M\left(x(\xi), y(\xi), t(\xi)\right) = \frac{t(\xi)}{t(\xi)+d\left(x(\xi), y(\xi)\right)} \qquad \ldots\ldots\ldots..(1)$$

Then (X, M,*) is a fuzzy metric space. We call this fuzzy metric M induced by the metric d the standard fuzzy metric.

Definition (7): Let (X, M,*) is a random fuzzy metric space.

(i) A sequence $\{x_n(\xi)\}$ in X is said to be convergent to a point $x \in X$,

$$\lim_{n\to\infty} M(x_n(\xi), x(\xi), t(\xi)) = 1$$

(ii) A sequence $\{x_n(\xi)\}$ in X is called a Cauchy sequence if

$$\lim_{n\to\infty} M(x_{n+p}(\xi), x_n(\xi), t(\xi)) = 1, \forall\, t(\xi) \succ 0 \; and \; p \succ 0$$

(iii) A random fuzzy metric space in which every Cauchy sequence is convergent is said to be complete.

Let (X.M,*) is a random fuzzy metric space with the following condition.

(FM-6) $\lim\limits_{t\to\infty} M(x(\xi), y(\xi), t(\xi)) = 1, \forall x, y \in X$

Definition (8): A function M is continuous in random fuzzy metric space iff whenever

$$x_n(\xi) \to x(\xi), y_n(\xi) \to y(\xi) \Rightarrow \lim_{n\to\infty} M(x_n(\xi), y_n(\xi), t(\xi)) \to M(x(\xi), y(\xi), t(\xi))$$

Definition (9): Two mappings A and S on random fuzzy metric space X are weakly commuting iff M (ASu(ξ), SAu(ξ), t(ξ)) $\geq$ M (Au(ξ),Su(ξ),t(ξ))

Lemma (i) For all x(ξ), y(ξ) $\in$ X, M(x(ξ), y(ξ), t(ξ)) is non -decreasing.

Lemma (ii) Let {y_n}(ξ) }be a sequence in a random fuzzy metric space (X, M,*) with the condition

(FM -7) If there exists a number q $\in$ (0,1) such that

$$M(y_{n+2}(\xi), y_{n+1}(\xi), qt(\xi)) \geq M(y_{n+1}(\xi), y_n(\xi), t(\xi)) \;, \forall t(\xi) \succ 0 \; and \; n = 1, 2, 3......,$$
$$then \; \{y_n(\xi)\} \; is \; a \; cauchy \; sequence \; in \; X.$$

Lemma (iii) If, for all x(ξ), y(ξ) $\in$ X, t (ξ) > 0 and for a number q $\in$ (0,1),

$$M(x(\xi), y(\xi), qt(\xi)) \geq M(x(\xi), y(\xi), t(\xi)), then \; x(\xi) = y(\xi)$$

Main results

THEOREM (1.1): Let (X, M, *) be a complete random fuzzy metric space with the condition (FM-6) and let S and T be continuous mappings of X , then S and T have a common fixed point in X if there exists continuous mappings A of X into S (X) $\cap$ T (X) which commute weakly with S and T and

(1.1)

$$M\left(A(\xi, x(\xi)), A(\xi, y(\xi)), q\xi(t)\right) \geq \inf \left\{ \begin{array}{l} M\left(T(\xi, y(\xi)), A(\xi, y(\xi)), t(\xi)\right), M\left(S(\xi, x(\xi)), A(\xi, x(\xi)), t(\xi)\right), \\ M\left(S(\xi, x(\xi)), T(\xi, y(\xi)), t(\xi)\right), M\left(A(\xi, x(\xi)), T(\xi, y(\xi)), t(\xi)\right), \\ M\left(S(\xi, x(\xi)), A(\xi, y(\xi)), t(\xi)\right) \end{array} \right\}$$

for all $x(\xi), y(\xi) \in X$, $t(\xi) > 0$ and $0 < q < 1$. Then S, T and A have a unique common fixed point.

PROOF: We define a sequence $\{x_n(\xi)\}$ such that $A(\xi, x_{2n}(\xi)) = S(\xi, x_{2n-1}(\xi))$ and $A(\xi, x_{2n-1}(\xi)) = T(\xi, x_{2n}, (\xi)$ for n = 1, 2,….. .We shall prove that $\{A(\xi, x_n(\xi))\}$is a Cauchy sequence.

$$M\left(A(\xi,x_{2n}(\xi)),A(\xi,x_{2n+1}(\xi),qt(\xi)\right)\geq \inf\begin{Bmatrix} M\left(T(\xi,x_{2n+1}(\xi)),A(\xi,x_{2n+1}(\xi)),t(\xi)\right), \\ M\left(S(\xi,x_{2n}(\xi)),A(\xi,x_{2n}(\xi)),t(\xi)\right), \\ M\left(S(\xi,x_{2n}(\xi)),T(\xi,x_{2n+1}(\xi)),t(\xi)\right), \\ M\left(A(\xi,x_{2n}(\xi)),T(\xi,x_{2n+1}(\xi)),t(\xi)\right), \\ M\left(S(\xi,x_{2n}(\xi)),A(\xi,x_{2n+1}(\xi)),t(\xi)\right) \\ \end{Bmatrix}$$

$$M\left(A(\xi,x_{2n}(\xi)),A(\xi,x_{2n+1}(\xi)),qt(\xi)\right)\geq \inf\begin{Bmatrix} M\left(A(\xi,x_{2n}(\xi)),A(\xi,x_{2n+1}(\xi),t(\xi)\right), \\ M\left(A(\xi,x_{2n+1}(\xi)),A(\xi,x_{2n}(\xi),t(\xi)\right), \\ M\left(A(\xi,x_{2n+1}(\xi)),A(\xi,x_{2n}(\xi)),t(\xi)\right), \\ M\left(A(\xi,x_{2n}(\xi)),A(\xi,x_{2n}(\xi)),t(\xi)\right), \\ M\left(A(\xi,x_{2n+1}(\xi)),A(\xi,x_{2n+1}(\xi)),t(\xi)\right) \\ \end{Bmatrix}$$

$$M\left(A(\xi,x_{2n}(\xi)),A(\xi,x_{2n+1}(\xi)),qt(\xi)\right)\geq \inf\begin{Bmatrix} M(A(\xi,x_{2n}(\xi)),A(\xi,x_{2n+1}(\xi),t(\xi)), \\ M(A(\xi,x_{2n+1}(\xi)),A(\xi,x_{2n}(\xi)),t(\xi)), \\ M(A(\xi,x_{2n+1}(\xi)),A(\xi,x_{2n}(\xi)),t(\xi)),1,1 \end{Bmatrix}$$

$$M\left(A(\xi,x_{2n}(\xi)),A(\xi,x_{2n+1}(\xi)),qt(\xi)\right)\geq \inf\begin{Bmatrix} M(A(\xi,x_{2n-1}(\xi)),A(\xi,x_{2n}(\xi)),\dfrac{t(\xi)}{q}), \\ M(A(\xi,x_{2n}(\xi)),A(\xi,x_{2n-1}(\xi)),\dfrac{t(\xi)}{q}), \\ M(A(\xi,x_{2n}(\xi)),A(\xi,x_{2n-1}(\xi)),\dfrac{t(\xi)}{q}),1,1 \end{Bmatrix}$$

$$\Rightarrow M\left(A(\xi,x_{2n}(\xi)),A(\xi,x_{2n+1}(\xi)),qt(\xi)\right)\geq\ M\left(\ A(\xi,x_{2n-1}(\xi)),A(\xi,x_{2n}(\xi),\frac{t(\xi)}{q}\right)$$

By induction

$$M\left(A(\xi,x_{2k}(\xi)),A(\xi,x_{2m+1}(\xi)),qt(\xi)\right)\geq M\left(A(\xi,x_{2m}(\xi)),A(\xi,x_{2k-1}(\xi)),\frac{t(\xi)}{q}\right)$$

For every k and m in N, Further if 2m +1 > 2k, then

$$M\left(A(\xi,x_{2k}(\xi)),A(\xi,x_{2m+1}(\xi)),qt(\xi)\right)\geq M\left(A(\xi,x_{2k-1}(\xi)),A(\xi,x_{2m}(\xi)),\frac{t(\xi)}{q}\right)\ldots\ldots\ldots\ldots$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\geq M\left(A(\xi,x_{0}(\xi)),A(\xi,x_{2m+1-2k}(\xi),\frac{t(\xi)}{q^{2k}}\right)\text{---- (1.1b)}$$

If 2k > 2m+1, then

$$M\left(A(\xi,x_{2k}(\xi)),A(\xi,x_{2m+1}(\xi)),qt(\xi)\right)\geq M\left(A(\xi,x_{2k-1}(\xi)),A(\xi,x_{2m}(\xi)),\frac{t(\xi)}{q}\right)\ldots\ldots\ldots\ldots$$

$$\ldots\ldots\ldots\ldots\geq M\left(A(\xi,x_{2k-(2m+1)}(\xi)),A(\xi,x_{0}(\xi)),\frac{t(\xi)}{q^{2m+1}}\right)\text{----- (1.1 c)}$$

By simple induction with (1.1 b) and (1.1c) we have

$$M\left(A(\xi,x_{n}(\xi)),A(\xi,x_{n+p}(\xi),qt(\xi)\right)\geq M\left(A(\xi,x_{0}(\xi,)A(\xi,x_{p}(\xi)),\frac{t(\xi)}{q^{n}}\right).$$

For n = 2k, p = 2m+1 or n = 2k+1, p = 2m +1 and by (FM-4)

$$M\left(A(\xi,x_{n}(\xi)),A(\xi,x_{n+p}(\xi)),qt(\xi)\right)\geq M\left(A(\xi,x_{0}(\xi)),A(\xi,x_{1}(\xi)),\frac{t(\xi)}{2q^{n}}\right).*$$

$$M\left(A(\xi,x_{1}(\xi)),A(\xi,x_{p}(\xi)),\frac{t(\xi)}{q^{n}}\right).---(1.1\text{ d})$$

If n = 2k, p = 2m or n = 2k+1, p= 2m

For every positive integer p and n in N, by nothing that

$$M\left(A(\xi,x_{0}(\xi)),A(\xi,x_{p}(\xi)),\frac{t(\xi)}{q^{n}}\right).\rightarrow 1\,as\,n\rightarrow\infty$$

Thus $\{Ax_n\}$ is a Cauchy sequence. Since the space X is complete $\exists$ z $\in$ X, such that

$$\lim_{n\to\infty} A(\xi, x_n(\xi)) = \lim_{n\to\infty} S(\xi, x_{2n-1}(\xi)) = \lim_{n\to\infty} T(\xi, x_{2n}(\xi)) = z(\xi) \quad \textit{which is measurable}$$

It follows that A(ξ ,x(ξ)) = S(ξ ,z (ξ))= T(ξ ,z(ξ)) and so

$$M\left(A(\xi, z(\xi)), A(\xi, A(\xi, z(\xi))), qt(\xi)\right) \geq \inf \begin{Bmatrix} M\left(T(\xi, A(\xi, z(\xi))), A(\xi, A(\xi, z(\xi))), t(\xi)\right), \\ M\left(S(\xi, z(\xi)), A(\xi, z(\xi)), t(\xi)\right), \\ M\left(S(\xi, z(\xi)), T(\xi, A(\xi, z(\xi))), t(\xi)\right), \\ M\left(A(\xi, z(\xi)), T(\xi, A(\xi, z(\xi))), t(\xi)\right), \\ M\left(S(\xi, z(\xi)), A(\xi, A(\xi, z(\xi))), t(\xi)\right) \end{Bmatrix}$$

$$M(A(\xi, z(\xi)), A(\xi, A(\xi, z(\xi))), qt(\xi)) \geq M(S(\xi, z(\xi)), T(\xi, A(\xi, z(\xi))), t(\xi))$$

$$\geq M(S(\xi, z(\xi)), A(\xi, T(\xi, z(\xi))), t(\xi)) \geq M(A(\xi, z(\xi)), A(\xi, A(\xi, z(\xi))), t(\xi)) \ldots\ldots\ldots\ldots$$
$$\geq M(A(\xi, z(\xi)), A(\xi, A(\xi, z(\xi))), \frac{t(\xi)}{q^n})$$

Since $\lim_{n\to\infty} \; M(A(\xi, z(\xi)), A(\xi, A(\xi, z(\xi))), \frac{t(\xi)}{q^n}) = 1$, so $A(\xi, z(\xi)) = A(\xi, A(\xi, z(\xi)))$.

Thus z is common fixed point of A, S and T.

For **uniqueness**, let v (ξ)(v(ξ) $\neq$ z(ξ)) be another common fixed point of S, T and A.

By (1.1 a) we write

$$M\left(A(\xi, z(\xi)), A(\xi, v(\xi)), qt(\xi)\right) \geq \inf \begin{Bmatrix} M\left(T(\xi, v(\xi)), A(\xi, v(\xi)), t(\xi)\right), \\ M\left(S(\xi, z(\xi)), A(\xi, z(\xi)), t(\xi)\right), \\ M\left(S(\xi, z(\xi)), T(\xi, v(\xi)), t(\xi)\right), \\ M\left(A(\xi, z(\xi)), T(\xi, v(\xi)), t(\xi)\right), \\ M\left(S(\xi, z(\xi)), A(\xi, v(\xi)), t(\xi)\right) \end{Bmatrix}$$

$$M\left(A(\xi,z(\xi)),A(\xi,v(\xi)),qt(\xi)\right)\geq \inf\begin{Bmatrix}M\left(v(\xi),v(\xi),t(\xi)\right),M\left(z(\xi),z(\xi),t(\xi)\right),M\left(z(\xi),v(\xi),t(\xi)\right),\\ M\left(z(\xi),v(\xi),t(\xi)\right),M\left(z(\xi),v(\xi),t(\xi)\right)\end{Bmatrix}$$

$$M\left(A(\xi,z(\xi)),A(\xi,v(\xi)),qt(\xi)\right)\geq M\left(z(\xi),v(\xi),t(\xi)\right)$$

This implies that $M\left(z(\xi),v(\xi),qt(\xi)\right)\geq M\left(z(\xi),v(\xi),t(\xi)\right)$

Therefore **by lemma iii,** we write $z(\xi)=v(\xi)$. .

THEOREM (1.2): Let (X, M, *) be a complete fuzzy metric space with the condition (FM-6) and let S and T be continuous mappings of X in X, then S and T have a common fixed point in X if there exists continuous mappings A of X into S (X) $\cap$ T (X) which commute weakly with S and T and

1.2(a)

$$M\left(A(\xi,x(\xi),A(\xi,y(\xi),qt(\xi)\right)\geq \inf\begin{Bmatrix}M\left(T(\xi,y(\xi)),A(\xi,y(\xi)),t(\xi)\right),\\ M\left(S(\xi,x(\xi)),A(\xi,x(\xi)),t(\xi)\right),\\ M\left(S(\xi,x(\xi)),T(\xi,y(\xi)),t(\xi)\right),\\ \dfrac{M\left(S(\xi,x(\xi)),T(\xi,y(\xi)),t(\xi)\right)}{M\left(A(\xi,x(\xi)),T(\xi,y(\xi)),t(\xi)\right)},\\ \dfrac{M\left(T(\xi,y(\xi)),A(\xi,y(\xi)),t(\xi)\right)}{M\left(S(\xi,x(\xi)),A(\xi,x(\xi)),t(\xi)\right)},\\ \dfrac{M\left(S(\xi,x(\xi)),A(\xi,x(\xi)),t(\xi)\right)}{M\left(T(\xi,y(\xi)),A(\xi,y(\xi)),t(\xi)\right)}\end{Bmatrix}$$

for all x,y $\in$ X, t > 0,and 0 < q < 1. Then S, T and A have a unique common fixed point.

PROOF: We define a sequence $\{x_n(\xi)\}$ such that

$A(\xi,x_{2n}(\xi))=S(\xi,x_{2n-1}(\xi))$ and $A(\xi,x_{2n-1}(\xi)) = T(\xi,x_{2n}(\xi))$ n = 1, 2,--

We shall prove that $\{A(\xi,x_n(\xi)\}$ is a Cauchy sequence. By (1.2a), we have

$$M\left(A(\xi,x_{2n}(\xi)),A(\xi,x_{2n+1}(\xi)),qt(\xi)\right)\geq \inf\left\{\begin{array}{l} M\left(T(\xi,x_{2n+1}(\xi)),A(\xi,x_{2n+1}(\xi),t(\xi))\right),\\ M\left(S(\xi,x_{2n}(\xi)),A(\xi,x_{2n}(\xi)),t(\xi)\right),\\ M\left(S(\xi,x_{2n}(\xi)),T(\xi,x_{2n+1}(\xi)),t(\xi)\right),\\ \dfrac{M\left(S(\xi,x_{2n}(\xi)),T(\xi,x_{2n+1}(\xi)),t(\xi)\right)}{M\left(A(\xi,x_{2n}(\xi)),T(\xi,x_{2n+1}(\xi)),t(\xi)\right)},\\ \dfrac{M\left(T(\xi,x_{2n+1}(\xi)),A(\xi,x_{2n+1}(\xi)),t(\xi)\right)}{M\left(S(\xi,x_{2n}(\xi)),A(\xi,x_{2n}(\xi)),t(\xi)\right)},\\ \dfrac{M\left(S(\xi,x_{2n}(\xi)),A(\xi,x_{2n}(\xi)),t(\xi)\right)}{M\left(T(\xi,x_{2n+1}(\xi)),A(\xi,x_{2n+1}(\xi)),t(\xi)\right)} \end{array}\right\}$$

$$M\left(A(\xi,x_{2n}(\xi)),A(\xi,x_{2n+1}(\xi)),qt(\xi)\right)\geq \inf\left\{\begin{array}{l} M\left(A(\xi,x_{2n}(\xi)),A(\xi,x_{2n+1}(\xi)),t(\xi)\right),\\ M\left(A(\xi,x_{2n+1}(\xi)),A(\xi,x_{2n}(\xi)),t(\xi)\right),\\ M\left(A(\xi,x_{2n+1}(\xi)),A(\xi,x_{2n}(\xi)),t(\xi)\right),\\ \dfrac{M\left(A(\xi,x_{2n+1}(\xi)),A(\xi,x_{2n}(\xi)),t(\xi)\right)}{M\left(A(\xi,x_{2n}(\xi)),A(\xi,x_{2n}(\xi)),t(\xi)\right)},\\ \dfrac{M\left(A(\xi,x_{2n}(\xi)),A(\xi,x_{2n+1}(\xi)),t(\xi)\right)}{M\left(A(\xi,x_{2n+1}(\xi)),A(\xi,x_{2n}(\xi)),t(\xi)\right)},\\ \dfrac{M\left(A(\xi,x_{2n+1}(\xi)),A(\xi,x_{2n}(\xi)),t(\xi)\right)}{M\left(A(\xi,x_{2n}(\xi)),A(\xi,x_{2n+1}(\xi)),t(\xi)\right)} \end{array}\right\}$$

$$\Rightarrow M\left(A(\xi,x_{2n}(\xi)),A(\xi,x_{2n+1}(\xi)),qt(\xi)\right)\geq \inf\left\{\begin{array}{l} M(A(\xi,x_{2n}(\xi)),A(\xi,x_{2n+1}(\xi)),t(\xi)),\\ M(A(\xi,x_{2n+1}(\xi)),A(\xi,x_{2n}(\xi)),t(\xi)),\\ M(A(\xi,x_{2n+1}(\xi)),A(\xi,x_{2n}(\xi)),t(\xi)),\\ 1,1,1 \end{array}\right\}$$

$$\Rightarrow M\left(A(\xi,x_{2n}(\xi)),A(\xi,x_{2n+1}(\xi)),qt(\xi)\right)\geq M\left(A(\xi,x_{2n-1}(\xi)),A(\xi,x_{2n}(\xi)),\frac{t(\xi)}{q}\right)$$

By induction

$$M\left(A(\xi,x_{2k}(\xi)),A(\xi,x_{2m+1}(\xi)),qt(\xi)\right)\geq M\left(A(\xi,x_{2m}(\xi)),A(\xi,x_{2k-1}(\xi)),\frac{t(\xi)}{q}\right)$$

For every k and m in N, Further if 2m +1 > 2k, then

$$M\left(A(\xi,x_{2k}(\xi)),A(\xi,x_{2m+1}(\xi)),qt(\xi)\right)\geq M\left(A(\xi,x_{2k-1}(\xi)),A(\xi,x_{2m}(\xi)),\frac{t(\xi)}{q}\right).......$$

$$..........\geq M\left(A(\xi,x_{0}(\xi)),A(\xi,x_{2m+1-2k}(\xi)),\frac{t(\xi)}{q^{2k}}\right) \text{----- (1.2b)}$$

If 2k > 2m+1, then

$$M\left(A(\xi,x_{2k}(\xi)),A(\xi,x_{2m+1}(\xi)),qt(\xi)\right)\geq M\left(A(\xi,x_{2k-1}(\xi)),A(\xi,x_{2m}(\xi)),\frac{t(\xi)}{q}\right).........$$

$$..........\geq M\left(A(\xi,x_{2k-(2m+1)}(\xi)),A(\xi,x_{0}(\xi)),\frac{t(\xi)}{q^{2m+1}}\right) \text{-- (1.2c)}$$

By simple induction with (1.2b) and (1.2c)

We have

$$M\left(A(\xi,x_{n}(\xi)),A(\xi,x_{n+p}(\xi)),qt(\xi)\right)\geq M\left(A(\xi,x_{0}(\xi)),A(\xi,x_{p}(\xi)),\frac{t(\xi)}{q^{n}}\right).$$

For n = 2k, p = 2m+1 or n = 2k+1, p = 2m +1 and by (FM-4)

$$M\left(A(\xi,x_{n}(\xi)),A(\xi,x_{n+p}(\xi)),qt(\xi)\right)$$
$$\geq M\left(A(\xi,x_{0}(\xi)),A(\xi,x_{1}(\xi)),\frac{t(\xi)}{2q^{n}}\right)*M\left(A(\xi,x_{1}(\xi)),A(\xi,x_{p}(\xi)),\frac{t(\xi)}{q^{n}}\right).---(1.2d)$$

If n = 2k, p = 2m or n = 2k+1, p= 2m

For every positive integer p and n in N, by nothing that

$$M\left(A(\xi,x_{0}(\xi)),A(\xi,x_{p}(\xi)),\frac{t(\xi)}{q^{n}}\right).\rightarrow 1\, as\, n\rightarrow\infty$$

Thus { $A(\xi,x_n(\xi))$ } is a Cauchy sequence.

Since the space X is complete there exists z $\in$ X, such that

$$\lim_{n\to\infty} A(\xi, x_n(\xi)) = \lim_{n\to\infty} S(\xi, x_{2n-1}(\xi)) = \lim_{n\to\infty} T(\xi, x_{2n}(\xi)) = z$$

It follows that $A(\xi, x(\xi)) = S(\xi, z(\xi)) = T(\xi, z(\xi))$ and so

$$M\left(A(\xi, z(\xi), A(\xi, A(\xi, A(\xi, z(\xi))), qt(\xi)\right) \geq \inf \left\{ \begin{array}{l} M\left(T(\xi, A(\xi, z(\xi))), A(\xi, A(\xi, z(\xi))), t(\xi)\right), \\ M\left(S(\xi, z(\xi)), A(\xi, z(\xi)), t(\xi)\right), \\ M\left(S(\xi, z(\xi)), T(\xi, A(\xi, z(\xi))), t(\xi)\right), \\ \dfrac{M\left(S(\xi, z(\xi)), T(\xi, A(\xi, z(\xi))), t(\xi)\right)}{M\left(A(\xi, z(\xi)), T(\xi, A(\xi, z(\xi))), t(\xi)\right)}, \\ \dfrac{M\left(T(\xi, A(\xi, z(\xi))), A(\xi, A(\xi, z(\xi))), t(\xi)\right)}{M\left(S(\xi, z(\xi)), A(\xi, z(\xi)), t(\xi)\right)}, \\ \dfrac{M\left(S(\xi, z(\xi)), A(\xi, z(\xi)), t(\xi)\right)}{M\left(T(\xi, A(\xi, z(\xi)), A(\xi, A(\xi, z(\xi))), t(\xi)\right)}, \end{array} \right\}$$

$$M(A(\xi, z(\xi)), A(\xi, A(\xi, z(\xi))), qt(\xi)) \geq M(S(\xi, z(\xi)), T(\xi, A(\xi, z(\xi))), t(\xi))$$

$$\geq M(S(\xi, z(\xi)), A(\xi, T(\xi, z(\xi))), t(\xi)) \geq M(A(\xi, z(\xi)), A(\xi, A(\xi, z(\xi))), t(\xi)) \ldots\ldots\ldots\ldots$$
$$\ldots\ldots\ldots\ldots \geq M(A(\xi, z(\xi)), A(\xi, A(\xi, z(\xi))), \frac{t(\xi)}{q^n})$$

$$Since \lim_{n\to\infty} M(A(\xi, z(\xi)), A(\xi, A(\xi, z(\xi))), \frac{t(\xi)}{q^n}) = 1$$
$$\Rightarrow A(\xi, z(\xi)) = A(\xi, A(\xi, z(\xi)))$$

Thus z (ξ) is common fixed point of A, S and T.

For uniqueness, let v((ξ)(v(ξ)≠ z(ξ)) be another common fixed point of S, T and A.

By (2.1), we write

$$M\left(A(\xi,z(\xi)),A(\xi,v(\xi)),qt(\xi)\right)\geq \inf\left\{\begin{array}{l} M\left(T(\xi,v(\xi)),A(\xi,v(\xi)),t(\xi)\right),\\ M\left(S(\xi,z(\xi)),A(\xi,z(\xi)),t(\xi)\right),\\ M\left(S(\xi,z(\xi)),T(\xi,v(\xi)),t(\xi)\right),\\ \dfrac{M\left(S(\xi,z(\xi)),T(\xi,v(\xi)),t(\xi)\right)}{M\left(A(\xi,z(\xi)),T(\xi,v(\xi)),t(\xi)\right)},\\ \dfrac{M\left(T(\xi,v(\xi)),A(\xi,v(\xi)),t(\xi)\right)}{M\left(S(\xi,z(\xi)),A(\xi,z(\xi)),t(\xi)\right)},\\ \dfrac{M\left(S(\xi,z(\xi)),A(\xi,z(\xi)),t(\xi)\right)}{M\left(T(\xi,v(\xi)),A(\xi,v(\xi)),t(\xi)\right)} \end{array}\right\}$$

$$M\left(A(\xi,z(\xi)),A(\xi,v(\xi)),qt(\xi)\right)\geq M\left(z(\xi),v(\xi),t(\xi)\right)$$

This implies that

$$M\left(z(\xi),v(\xi),qt(\xi)\right)\geq M\left(z(\xi,)v(\xi),t(\xi)\right)$$

Therefore by **lemma iii**, we write z(ξ)=v(ξ).

This completes the proof of Theorem .

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