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FIXED POINT RESULTS IN GENERALIZED b−FUZZY METRIC SPACES

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Abstract. This paper comprises of a few fixed point theorems in the generalized b−fuzzy metric spaces. As a significant outcome, we give an adequate condition for a sequence to be Cauchy in the generalized b−fuzzy metric spaces. In this manner, we proved several fixed point theorems in generalized b−fuzzy metric spaces.

Keywords: fixed point; generalized b− fuzzy metric spaces; symmetric.

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1. INTRODUCTION

The fuzzy set was characterized by Zadeh [18] in 1965 which is a numerical edge to dubiousness or vulnerability in a day by day life. Kramosil and Michalek [8] presented fuzzy metric spaces and this idea was adjusted by George and Veeramani in 1994 [5]. In 2006, S. Sedghi and N. Shobe [14] demonstrated a common fixed point theorem in $\mathscr{M}-$ fuzzy metric spaces. Then again, the idea of b−metric was initiated from the works of Bakhtin [2]. Czerwik [3] gave an axiom which was weaker than the triangular inequality and formally defined a b− metric space

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with a view of generalizing the Banach contraction mapping theorem. Sedghi and Shobe [12] joined the ideas of fuzzy set and b− metric space to present a b− fuzzy metric space.

In this paper, we deal the notation of a countable expansion of the t−norm, we demonstrate a valuable lemma in the generalized b− fuzzy metric space setting that guarantee that a sequence $\{\sigma_n\}$ is a Cauchy sequence. Utilizing this lemma, we improve the evidences of some understand fixed point theorem. We present some of them in the principle part of the paper.

2. Preliminaries

Definition: 2.1 A binary operation $*:[0,1]\to[0,1]$ is a continuous t−norm on the off chance that it fulfills the accompanying conditions:

(i) $*$ is associative and commutative,

(ii) $*$ is continuous,

(iii) $\sigma * 1 = a$ for all $\sigma\in[0,1]$,

(iv) $\sigma*\varsigma\le\alpha*\beta$ for $\sigma,\varsigma,\alpha,\beta\in[0,1]$ with the end goal that $\sigma\le\alpha$ and $\varsigma\le\beta$.

Basic example of a continuous t− norm are $\sigma*\varsigma=\min\{\sigma,\varsigma\}$, $\sigma*\varsigma=\sigma\cdot\varsigma$ and $\sigma*\varsigma=max\{\sigma+\varsigma-1,0\}$.

Definition: 2.2 Let $*$ be a t− norm, and let $*_p;[0,1]\to[0,1]$, $p\in\mathbb{N}$ be deined in the following way

$$*_1(\sigma)=\sigma*\sigma,\ *_{p+1}=*_p(\sigma)*\sigma,\quad p\in\mathbb{N},\ \sigma\in[0,1].$$

t− norm $*$ is said to be $\mathscr{H}-$ type if the family $\{*_p(\sigma)\}_{p\in\mathbb{N}}$ is equicontinuous at $\sigma=1$.

A trivial example of t− norm of $\mathscr{H}-$ type is $\sigma*\varsigma=\min\{\sigma,\varsigma\}$.

Each t− norm $*$ can be extended by associativity in a unique way to an $p-$ ary operation taking for $(\sigma_1,\sigma_2,\ldots,\sigma_p)\in[0,1]^p$ the values

$$*_{\iota=1}^{1}\sigma_\iota=\sigma_1,\ \ *_{\iota=1}^{p}\sigma_\iota=*_{\iota=1}^{p-1}\sigma_\iota*\sigma_p=\sigma_1*\sigma_2*\ldots\sigma_p.$$

A t− norm $*$ can be extended to a countable infinite operation taking for any sequence $(\sigma_p)_{p\in\mathbb{N}}$ from [0,1] the value

$$*_{\iota=1}^{\infty}\sigma_\iota=\lim_{p\to\infty}*_{\iota=1}^{p}\sigma_\iota.$$

The sequence $(*_{\iota=1}^{\infty}\sigma_\iota)_{p\in\mathbb{N}}$ is nonincreasing and bounded from below and hence the limit $*_{\iota=1}^{\infty}\sigma_\iota$ exists.

In the fixed point theory it is of interest to investigate the classes of $\mathsf{t}-$ norms $*$ and sequences (σ_p) from the interval [0,1] with the end goal that $\lim_{p\to\infty}\sigma_p = 1$ and $\lim_{p\to\infty} *_{\iota=p}^{\infty}\sigma_\iota = \lim_{p\to\infty} *_{\iota=1}^{\infty}\sigma_{p+\iota} = 1$.

Definition: 2.3 [6] A quadruple $(\mathscr{G},\mathscr{M},*,\mathsf{b})$ is called a generalized $\mathsf{b}-$fuzzy metric spaces $(Gb-\mathscr{F}\mathscr{M}\mathscr{S})$ with $\mathsf{b}\geq 1$ if $\mathscr{G}$ is an arbitrary non-empty set, $*$ is a continuous $\mathsf{t}-$norm and $\mathscr{M}$ is a fuzzy set on $\mathscr{G}^3\times(0,\infty)$, fulfill the accompanying conditions for each $\sigma,\varsigma,\eta,\mathsf{a}\in\mathscr{G}$ and $\mathsf{t},\mathsf{s}>0$,

($\mathscr{M}$-1) $\mathscr{M}(\sigma,\varsigma,\eta,\mathsf{t})>0$,
($\mathscr{M}$-2) $\mathscr{M}(\sigma,\varsigma,\eta,\mathsf{t})=1$ if and only if $\sigma=\varsigma=\eta$,
($\mathscr{M}$-3) $\mathscr{M}(\sigma,\varsigma,\eta,\mathsf{t})=\mathscr{M}(p(\sigma,\varsigma,\eta,),\mathsf{t})$, where p is a permutation function,
($\mathscr{M}$-4) $\mathscr{M}(\sigma,\varsigma,\eta,\mathsf{t}+\mathsf{s})\geq\mathscr{M}(\sigma,\varsigma,a,\frac{\mathsf{t}}{\mathsf{b}})*\mathscr{M}(a,\eta,\eta,\frac{\mathsf{s}}{\mathsf{b}})$,
($\mathscr{M}$-5) $\mathscr{M}(\sigma,\varsigma,\eta,\cdot):(0,\infty)\to$[0, 1] is continuous.

Note that generalized $\mathsf{b}-$ fuzzy metric spaces are a generalized fuzzy metric spaces if b = 1, but the converse does not hold in general.

Definition: 2.4 A function $\mathscr{J}:\mathscr{R}\to\mathscr{R}$ is called $\mathsf{b}-$non-decreasing if $\alpha>\mathsf{b}\,\beta\Rightarrow\mathscr{J}(\alpha)\geq\mathscr{J}(\beta)$ for all $\alpha,\beta\in\mathscr{R}$.

Definition: 2.5 [6] Let $(\mathscr{G},\mathscr{M},*,\mathsf{b})$ be a $Gb-\mathscr{F}\mathscr{M}\mathscr{S}$, then

(i) A sequence $\{\sigma_p\}$ in $\mathscr{G}$ is said to be convergent to σ if for each $\mathsf{t}>0$, $\mathscr{M}(\sigma,\sigma,\sigma_p,\mathsf{t})\to 1$ as $p\to\infty$.
(ii) A sequence $\{\sigma_p\}$ in $\mathscr{G}$ is said to be a Cauchy sequence if for each $0<\varepsilon<1$ and $\mathsf{t}>0$, there exists $p_0\in\mathbb{N}$ with the end goal that $\mathscr{M}(\sigma_p,\sigma_p,\sigma_q,\mathsf{t})>1-\varepsilon$ for each $p,q\geq p_0$.
(iii) A generalized $\mathsf{b}-$fuzzy metric spaces are said to be complete if every Cauchy sequence is convergent.

Definition: 2.6 A $Gb-\mathscr{F}\mathscr{M}\mathscr{S}$ $(\mathscr{G},\mathscr{M},*,\mathsf{b})$ is said to be symmetric if $\mathscr{M}(\sigma,\sigma,\varsigma,\mathsf{t})=\mathscr{M}(\sigma,\varsigma,\varsigma,\mathsf{t})$ for all $\sigma,\varsigma\in\mathscr{G}$ and for each $\mathsf{t}>0$.

3. Main Results

Lemma 3.1 Let $\{\sigma_p\}$ be a sequence in a symmetric $Gb-\mathscr{FMS}$ $(\mathscr{G},\mathscr{M},*,\mathsf{b})$. Let's keep that there are $\tau \in (0,\frac{1}{\mathsf{b}})$ with the end goal that

$$\mathscr{M}(\sigma_p,\sigma_{p+1},\sigma_{p+2},\mathsf{t}) \geq \mathscr{M}\left(\sigma_{p-1},\sigma_p,\sigma_{p+1},\frac{\mathsf{t}}{\tau}\right), \quad p\in\mathbb{N},\ \mathsf{t}>0, \tag{3.1.1}$$

and there exists $\sigma_0,\sigma_1 \in \mathscr{G}$ and $\rho \in (0,1)$ with the end goal that

$$\lim_{p\to\infty} *_{i=p}^{\infty}\mathscr{M}\left(\sigma_0,\sigma_1,\sigma_1,\frac{\mathsf{t}}{\rho^i}\right) = 1, \quad t>0. \tag{3.1.2}$$

Then $\{\sigma_p\}$ is a Cauchy sequence.

Proof. Let $\vartheta \in (\tau\mathsf{b},1)$. At that point the sum $\sum_{i=1}^{\infty}\vartheta^i$ is convergent, and there exists $p_0\in\mathbb{N}$ with the end goal that $\sum_{i=p}^{\infty}\vartheta^i<1$ for every $p>p_0$. Let $p>s>p_0$. Because of being $\mathscr{M}$ is $\mathsf{b}-$non-decreasing, for each $\mathsf{t}>0$, we have

$$\begin{aligned}
\mathscr{M}(\sigma_p,\sigma_p,\sigma_{p+s},\mathsf{t}) &\geq \mathscr{M}\left(\sigma_p,\sigma_p,\sigma_{p+s},\frac{\mathsf{t}\sum_{i=p}^{p+s-1}\vartheta^i}{\mathsf{b}}\right)\\
&\geq \mathscr{M}\left(\sigma_p,\sigma_p,\sigma_{p+1},\frac{\mathsf{t}\vartheta^p}{\mathsf{b}^2}\right)*\mathscr{M}\left(\sigma_{p+1},\sigma_{p+s},\sigma_{p+s},\frac{\mathsf{t}\sum_{i=p+1}^{p+s-1}\vartheta^i}{\mathsf{b}^2}\right)\\
&= \mathscr{M}\left(\sigma_p,\sigma_p,\sigma_{p+1},\frac{\mathsf{t}\vartheta^p}{\mathsf{b}^2}\right)*\mathscr{M}\left(\sigma_{p+1},\sigma_{p+1},\sigma_{p+s},\frac{\mathsf{t}\sum_{i=p+1}^{p+s-1}\vartheta^i}{\mathsf{b}^2}\right)\\
&\geq \mathscr{M}\left(\sigma_p,\sigma_p,\sigma_{p+1},\frac{\mathsf{t}\vartheta^p}{\mathsf{b}^2}\right)*\mathscr{M}\left(\sigma_{p+1},\sigma_{p+1},\sigma_{p+2},\frac{\mathsf{t}\vartheta^{p+1}}{\mathsf{b}^3}\right)\\
&\qquad *\cdots*\mathscr{M}\left(\sigma_{p+s-1},\sigma_{p+s-1},\sigma_{p+s},\frac{\mathsf{t}\vartheta^{p+s-1}}{\mathsf{b}^{s+1}}\right)
\end{aligned}$$

By (3.1.1) we get,

$$\mathscr{M}(\sigma_p,\sigma_{p+1},\sigma_{p+2},\mathsf{t}) \geq \mathscr{M}\left(\sigma_0,\sigma_1,\sigma_2,\frac{\mathsf{t}}{\tau^p}\right), \quad p\in\mathbb{N},\ \mathsf{t}>0.$$

Because of being $p > s$ and $\mathsf{b} > 1$, we have

$$\begin{aligned}\mathscr{M}(\sigma_p,\sigma_p,\sigma_{p+s},\mathsf{t}) &\geq \mathscr{M}\left(\sigma_0,\sigma_0,\sigma_1,\frac{\mathsf{t}\vartheta^p}{\mathsf{b}^2\tau^p}\right)*\mathscr{M}\left(\sigma_0,\sigma_0,\sigma_1,\frac{\mathsf{t}\vartheta^{p+1}}{\mathsf{b}^3\tau^{p+1}}\right)\\ &\qquad *\cdots*\mathscr{M}\left(\sigma_0,\sigma_0,\sigma_1,\frac{\mathsf{t}\vartheta^{p+s-1}}{\mathsf{b}^{s+1}\tau^{p+s-1}}\right)\\ &\geq *_{i=p}^{p+s-1}\mathscr{M}\left(\sigma_0,\sigma_0,\sigma_1,\frac{\mathsf{t}\vartheta^i}{\mathsf{b}^{i-p+2}\tau^i}\right)\\ &\geq *_{i=p}^{p+s-1}\mathscr{M}\left(\sigma_0,\sigma_0,\sigma_1,\frac{\mathsf{t}\vartheta^i}{\mathsf{b}^i\tau^i}\right)\\ \mathscr{M}(\sigma_p,\sigma_p,\sigma_{p+s},\mathsf{t}) &\geq *_{i=p}^{\infty}\mathscr{M}\left(\sigma_0,\sigma_0,\sigma_1,\frac{\mathsf{t}}{\rho^i}\right), \qquad \text{where } \rho=\frac{\mathsf{b}\tau}{\vartheta}.\end{aligned}$$

Because of being $\rho \in (0,1)$, by (3.1.2) for this reason $\{\sigma_p\}$ is a Cauchy sequence. □

Corollary 3.2 Let $\{\sigma_p\}$ be a sequence in a $Gb-\mathscr{F}\mathscr{M}\mathscr{S}$ and let $*$ be of $\mathscr{H}-$ type. If there are $\tau \in (0,\frac{1}{\mathsf{b}})$ with the end goal that

$$\mathscr{M}(\sigma_p,\sigma_{p+1},\sigma_{p+2},\mathsf{t}) \geq \mathscr{M}\left(\sigma_{p-1},\sigma_p,\sigma_{p+1},\frac{\mathsf{t}}{\tau}\right), \quad p\in\mathbb{N},\ \mathsf{t}>0.$$

Then $\{\sigma_p\}$ is a Cauchy sequence.

Lemma 3.3 If for few $\tau \in (0,1)$ with $\sigma,\varsigma,\eta \in \mathscr{G}$,

$$\mathscr{M}(\sigma,\varsigma,\eta,\mathsf{t}) \geq \mathscr{M}(\sigma,\varsigma,\eta,\frac{\mathsf{t}}{\tau}),\ \mathsf{t}>0. \tag{3.3.1}$$

Then $\sigma=\varsigma=\eta$.

Proof. By(3.3.1) we get,

$$\mathscr{M}(\sigma,\varsigma,\eta,\mathsf{t}) \geq \mathscr{M}(\sigma,\varsigma,\eta,\frac{\mathsf{t}}{\tau^p}),\ p\in\mathbb{N},\ \mathsf{t}>0.$$

$$\therefore\ \mathscr{M}(\sigma,\varsigma,\eta,\mathsf{t}) \geq \lim_{p\to\infty}\mathscr{M}(\sigma,\varsigma,\eta,\frac{\mathsf{t}}{\tau^p}) = 1,\ \mathsf{t}>0.$$

Hence $\sigma=\varsigma=\eta$. □

Theorem 3.4 Let $(\mathscr{G},\mathscr{M},*,\mathsf{b})$ be a symmetric complete $Gb-\mathscr{F}\mathscr{M}\mathscr{S}$ with let $\mathscr{J}:\mathscr{G}\to\mathscr{G}$. Let's keep that there are $\tau \in (0,\frac{1}{\mathsf{b}})$ with the end goal that

$$\mathscr{M}(\mathscr{J}\sigma,\mathscr{J}\varsigma,\mathscr{J}\eta,\mathsf{t}) \geq \mathscr{M}\left(\sigma,\varsigma,\eta,\frac{\mathsf{t}}{\tau}\right), \quad \sigma,\varsigma,\eta\in\mathscr{G},\ \mathsf{t}>0, \tag{3.4.1}$$

and there exists $\sigma_0 \in \mathscr{G}$ and $\rho \in (0,1)$ with the end goal that

$$\lim_{p\to\infty} *_{i=p}^{\infty} \mathscr{M}\left(\sigma_0, \sigma_0, \mathscr{J}\sigma_0, \frac{\mathsf{t}}{\rho^i}\right) = 1,\ \mathsf{t} > 0. \tag{3.4.2}$$

Then $\mathscr{J}$ has a unique fixed point on $\mathscr{G}$.

Proof. Let $\sigma_0 \in \mathscr{G}$ with $\sigma_{p+1} = \mathscr{J}\sigma_p,\ p \in \mathbb{N}$. Take $\sigma = \sigma_{p-1},\ \varsigma = \sigma_{p-1}$ and $\eta = \sigma_p$ for every $p \in \mathbb{N}$ with each $\mathsf{t} > 0$ we have

$$\mathscr{M}(\sigma_p, \sigma_p, \sigma_{p+1}, \mathsf{t}) \geq \mathscr{M}\left(\sigma_{p-1}, \sigma_{p-1}, \sigma_p, \frac{\mathsf{t}}{\tau}\right).$$

Using Lemma (3.1) we get $\{\sigma_p\}$ is a Cauchy sequence. Because of being $(\mathscr{G}, \mathscr{M}, *, \mathsf{b})$ is complete, there exists $\sigma \in \mathscr{G}$ with the end goal that

$$\lim_{p\to\infty} \sigma_p = \sigma \quad \text{and} \quad \lim_{p\to\infty} \mathscr{M}(\sigma, \sigma, \sigma_p, \mathsf{t}) = 1,\ \ \mathsf{t} > 0. \tag{3.4.3}$$

Consider,

$$\begin{aligned}
\mathscr{M}(\mathscr{J}\sigma, \mathscr{J}\sigma, \sigma, \mathsf{t}) &\geq \mathscr{M}\left(\mathscr{J}\sigma, \mathscr{J}\sigma, \sigma_p, \frac{\mathsf{t}}{2\mathsf{b}}\right) * \mathscr{M}\left(\sigma_p, \sigma, \sigma, \frac{\mathsf{t}}{2\mathsf{b}}\right) \\
&\geq \mathscr{M}\left(\sigma, \sigma, \sigma_{p-1}, \frac{\mathsf{t}}{2\mathsf{b}\tau}\right) * \mathscr{M}\left(\sigma_p, \sigma, \sigma, \frac{\mathsf{t}}{2\mathsf{b}}\right) \\
&\geq 1 * 1 \quad \text{as} \quad p \to \infty \\
\mathscr{M}(\mathscr{J}\sigma, \mathscr{J}\sigma, \sigma, \mathsf{t}) &\geq 1.
\end{aligned}$$

Hence $\mathscr{J}\sigma = \sigma$.

Let's keep that σ and $\mathfrak{v}$ are fixed point for $\mathscr{J}$. Now,

$$\mathscr{M}(\sigma, \sigma, \mathfrak{v}, \mathsf{t}) = \mathscr{M}(\mathscr{J}\sigma, \mathscr{J}\sigma, \mathscr{J}\mathfrak{v}, \mathsf{t}) \geq \mathscr{M}\left(\sigma, \sigma, \mathfrak{v}, \frac{\mathsf{t}}{\tau}\right) \quad \text{by} \ \ (3.4.1)$$

Using Lemma (3.3) implies that $\sigma = \mathfrak{v}$. □

Example 3.5 Let $\mathscr{G} = [0,1]$ and $\mathscr{M}(\sigma, \varsigma, \eta, \mathsf{t}) = e^{-\frac{[(\sigma-\varsigma)^2+(\varsigma-\eta)^2+(\eta-\sigma)^2]^{\frac{1}{2}}}{\mathsf{t}}}$, $\sigma, \varsigma, \eta \in \mathscr{G},\ \mathsf{t} > 0$ is symmetric $Gb - \mathscr{F}\mathscr{M}\mathscr{S}$ with $\mathsf{b} = 2$.

Let $\mathscr{J}\sigma = k^2\sigma,\ k < \frac{1}{\sqrt{2}}, \sigma \in \mathscr{G}$.

$$\mathscr{M}(\mathscr{J}\sigma, \mathscr{J}\varsigma, \mathscr{J}\eta, \mathsf{t}) = e^{-\frac{[k^2\,(\sigma-\varsigma)^2+k^2\,(\varsigma-\eta)^2+k^2\,(\eta-\sigma)^2]^{\frac{1}{2}}}{\mathsf{t}}}$$

$$\geq e^{-\frac{[\tau\,(\sigma-\varsigma)^2+\tau\,(\varsigma-\eta)^2+\tau\,(\eta-\sigma)^2]^{\frac{1}{2}}}{\mathsf{t}}}$$

$$\mathscr{M}(\mathscr{J}\sigma, \mathscr{J}\varsigma, \mathscr{J}\eta, \mathsf{t}) \geq \mathscr{M}(\sigma, \varsigma, \eta, \frac{\mathsf{t}}{\tau}), \qquad \sigma, \varsigma, \eta \in \mathscr{G},\ \mathsf{t} > 0$$

for $\frac{1}{\mathsf{b}} > \tau > k^2$. So condition (3.4.1) of Theorem 3.4 is satisfied. Therefore $\mathscr{J}$ has a unique fixed point in $\mathscr{G}$.

Theorem 3.6 Let $(\mathscr{G}, \mathscr{M}, *, \mathsf{b})$ be a symmetric complete $Gb-\mathscr{F}\mathscr{M}\mathscr{S}$ and let $\mathscr{J}: \mathscr{G} \to \mathscr{G}$. Let's keep that there are $\tau \in (0, \frac{1}{\mathsf{b}})$ with the end goal that

(3.6.1)

$$\mathscr{M}(\mathscr{J}\sigma, \mathscr{J}\varsigma, \mathscr{J}\eta, \mathsf{t}) \geq \min\Big\{\mathscr{M}\Big(\sigma, \varsigma, \eta, \frac{\mathsf{t}}{\tau}\Big), \mathscr{M}\Big(\sigma, \sigma, \mathscr{J}\sigma, \frac{\mathsf{t}}{\tau}\Big), \mathscr{M}(\varsigma, \varsigma, \mathscr{J}\varsigma, \frac{\mathsf{t}}{\tau}\Big), \mathscr{M}(\eta, \eta, \mathscr{J}\eta, \frac{\mathsf{t}}{\tau}\Big)\Big\}$$

for all $\sigma, \varsigma, \eta \in \mathscr{G}$, $\mathsf{t} > 0$, and there exists $\sigma_0 \in \mathscr{G}$ and $\rho \in (0,1)$ with the end goal that

$$\lim_{p\to\infty} *_{i=p}^{\infty} \mathscr{M}\Big(\sigma_0, \sigma_0, \mathscr{J}\sigma_0, \frac{\mathsf{t}}{\rho^i}\Big) = 1,\ \mathsf{t} > 0. \tag{3.6.2}$$

Then $\mathscr{J}$ has a unique fixed point in $\mathscr{G}$.

Proof. Let $\sigma_0 \in \mathscr{G}$ with $\sigma_{p+1} = \mathscr{J}\sigma_p,\ p \in \mathbb{N}$. Take $\sigma = \sigma_{p-1},\ \varsigma = \sigma_{p-1}$ and $\eta = \sigma_p$ for every $p \in \mathbb{N}$ and each $\mathsf{t} > 0$ we have

$$\begin{aligned}\mathscr{M}(\sigma_p, \sigma_p, \sigma_{p+1}, \mathsf{t}) &= \mathscr{M}(\mathscr{J}\sigma_{p-1}, \mathscr{J}\sigma_{p-1}, \mathscr{J}\sigma_p, \mathsf{t})\\ &\geq \min\Big\{\mathscr{M}\Big(\sigma_{p-1}, \sigma_{p-1}, \sigma_p, \frac{\mathsf{t}}{\tau}\Big), \mathscr{M}\Big(\sigma_{p-1}, \sigma_{p-1}, \mathscr{J}\sigma_{p-1}, \frac{\mathsf{t}}{\tau}\Big),\\ &\qquad \mathscr{M}\Big(\sigma_{p-1}, \sigma_{p-1}, \mathscr{J}\sigma_{p-1}, \frac{\mathsf{t}}{\tau}\Big), \mathscr{M}\Big(\sigma_p, \sigma_p, \mathscr{J}\sigma_p, \frac{\mathsf{t}}{\tau}\Big)\Big\}\\ &\geq \min\Big\{\mathscr{M}\Big(\sigma_{p-1}, \sigma_{p-1}, \sigma_p, \frac{\mathsf{t}}{\tau}\Big), \mathscr{M}\Big(\sigma_{p-1}, \sigma_{p-1}, \sigma_p, \frac{\mathsf{t}}{\tau}\Big),\\ &\qquad \mathscr{M}\Big(\sigma_{p-1}, \sigma_{p-1}, \sigma_p, \frac{\mathsf{t}}{\tau}\Big), \mathscr{M}\Big(\sigma_p, \sigma_p, \sigma_{p+1}, \frac{\mathsf{t}}{\tau}\Big)\Big\}\end{aligned}$$

$$\mathscr{M}(\sigma_p, \sigma_p, \sigma_{p+1}, \mathsf{t}) \geq \min\Big\{\mathscr{M}\Big(\sigma_{p-1}, \sigma_{p-1}, \sigma_p, \frac{\mathsf{t}}{\tau}\Big), \mathscr{M}\Big(\sigma_p, \sigma_p, \sigma_{p+1}, \frac{\mathsf{t}}{\tau}\Big)\Big\}.$$

Case (i) $\mathscr{M}(\sigma_p,\sigma_p,\sigma_{p+1},\mathsf{t}) \geq \mathscr{M}\left(\sigma_p,\sigma_p,\sigma_{p+1},\frac{\mathsf{t}}{\tau}\right)$

By Lemma (3.1) we get $\sigma_p = \sigma_{p+1}$.

Case (ii) $\mathscr{M}(\sigma_p,\sigma_p,\sigma_{p+1},\mathsf{t}) \geq \mathscr{M}\left(\sigma_{p-1},\sigma_{p-1},\sigma_p,\frac{\mathsf{t}}{\tau}\right)$

Using Lemma (3.1) we get $\{\sigma_p\}$ is a Cauchy sequence. Because of being $(\mathscr{G},\mathscr{M},*)$ is complete, there are $\sigma \in \mathscr{G}$ with the end goal that

$$\lim_{p\to\infty}\sigma_p = \sigma \quad \text{and} \quad \lim_{p\to\infty}\mathscr{M}(\sigma,\sigma,\sigma_p,\mathsf{t}) = 1,\ \ \mathsf{t}>0. \tag{3.6.3}$$

Let $\vartheta_1 \in (\tau\mathsf{b},1)$ and $\vartheta_2 = 1-\vartheta_1$.

$$\begin{aligned}
\mathscr{M}(\sigma,\sigma,\mathscr{J}\sigma,\mathsf{t}) &\geq \mathscr{M}\left(\sigma,\sigma,\mathscr{J}\sigma_p,\frac{\vartheta_1\,\mathsf{t}}{\mathsf{b}}\right) * \mathscr{M}\left(\mathscr{J}\sigma_p,\mathscr{J}\sigma,\mathscr{J}\sigma,\frac{\vartheta_2\,\mathsf{t}}{\mathsf{b}}\right)\\
&\geq \mathscr{M}\left(\sigma,\sigma,\mathscr{J}\sigma_p,\frac{\vartheta_1\,\mathsf{t}}{\mathsf{b}}\right) * \min\Bigg\{\mathscr{M}\left(\sigma_p,\sigma,\sigma,\frac{\vartheta_2\,\mathsf{t}}{\mathsf{b}\,\tau}\right),\\
&\qquad \mathscr{M}\left(\sigma_p,\sigma_p,\mathscr{J}\sigma_p,\frac{\vartheta_2\,\mathsf{t}}{\mathsf{b}\,\tau}\right),\mathscr{M}\left(\sigma,\sigma,\mathscr{J}\sigma,\frac{\vartheta_2\,\mathsf{t}}{\mathsf{b}\,\tau}\right),\\
&\qquad\qquad \mathscr{M}\left(\sigma,\sigma,\mathscr{J}\sigma,\frac{\vartheta_2\,\mathsf{t}}{\mathsf{b}\,\tau}\right)\Bigg\}\\
&\geq 1 * \min\left\{1,1,\mathscr{M}\left(\sigma,\sigma,\mathscr{J}\sigma,\frac{\vartheta_2\,\mathsf{t}}{\mathsf{b}\,\tau}\right)\right\}, \qquad \text{as} \quad p\to\infty\\
&\geq \mathscr{M}\left(\sigma,\sigma,\mathscr{J}\sigma,\frac{\vartheta_2\,\mathsf{t}}{\mathsf{b}\,\tau}\right)\\
\mathscr{M}(\sigma,\sigma,\mathscr{J}\sigma,\mathsf{t}) &\geq \mathscr{M}\left(\sigma,\sigma,\mathscr{J}\sigma,\frac{\mathsf{t}}{\rho}\right), \qquad \text{where} \quad \rho = \frac{\mathsf{b}\,\tau}{\vartheta_2}.
\end{aligned}$$

Using Lemma (3.3) we get, σ is a fixed point of $\mathscr{J}$. Let's keep that σ and $\mathfrak{v}$ are fixed point of $\mathscr{J}$. Now

$$\begin{aligned}
\mathscr{M}(\sigma,\sigma,\mathfrak{v},\mathsf{t}) &= \mathscr{M}(\mathscr{J}\sigma,\mathscr{J}\sigma,\mathscr{J}\mathfrak{v},\mathsf{t})\\
&\geq \min\Bigg\{\mathscr{M}\left(\sigma,\sigma,\mathfrak{v},\frac{\mathsf{t}}{\tau}\right),\mathscr{M}\left(\sigma,\sigma,\mathscr{J}\sigma,\frac{\mathsf{t}}{\tau}\right),\\
&\qquad\qquad \mathscr{M}\left(\sigma,\sigma,\mathscr{J}\sigma,\frac{\mathsf{t}}{\tau}\right),\mathscr{M}\left(\mathfrak{v},\mathfrak{v},\mathscr{J}\mathfrak{v},\frac{\mathsf{t}}{\tau}\right)\Bigg\}\\
&\geq \min\left\{\mathscr{M}\left(\sigma,\sigma,\mathfrak{v},\frac{\mathsf{t}}{\tau}\right),1,1,1\right\}\\
\mathscr{M}(\sigma,\sigma,\mathfrak{v},\mathsf{t}) &\geq \mathscr{M}\left(\sigma,\sigma,\mathfrak{v},\frac{\mathsf{t}}{\tau}\right)
\end{aligned}$$

Using Lemma (3.3) we get, $\sigma = \mathfrak{v}$. □

Theorem 3.7 Let $(\mathscr{G},\mathscr{M},*,\mathsf{b})$ be a symmetric complete $Gb-\mathscr{F}\mathscr{M}\mathscr{S}$ with let $\mathscr{J}:\mathscr{G}\to\mathscr{G}$. Let's keep that that there are $\tau\in\left(0,\frac{1}{\mathsf{b}^2}\right)$ with the end goal that

$$\begin{aligned}(3.7.1)\quad \mathscr{M}(\mathscr{J}\sigma,\mathscr{J}\varsigma,\mathscr{J}\eta,\mathsf{t}) \geq \min\Big\{&\mathscr{M}\Big(\sigma,\varsigma,\eta,\frac{\mathsf{t}}{\tau}\Big),\mathscr{M}\Big(\sigma,\sigma,\mathscr{J}\sigma,\frac{\mathsf{t}}{\tau}\Big),\\ &\mathscr{M}\Big(\varsigma,\varsigma,\mathscr{J}\varsigma,\frac{\mathsf{t}}{\tau}\Big),\mathscr{M}\Big(\eta,\eta,\mathscr{J}\eta,\frac{\mathsf{t}}{\tau}\Big),\mathscr{M}\Big(\sigma,\varsigma,\mathscr{J}\eta,\frac{2\,\mathsf{t}}{\tau}\Big),\mathscr{M}\Big(\mathscr{J}\sigma,\mathscr{J}\varsigma,\eta,\frac{\mathsf{t}}{\tau}\Big)\Big\}\end{aligned}$$

for all $\sigma,\varsigma,\eta\in\mathscr{G}$, $\mathsf{t}>0$. Then $\mathscr{J}$ has a unique fixed point in $\mathscr{G}$.

Proof. Let $\sigma_0\in\mathscr{G}$ with $\sigma_{p+1}=\mathscr{J}\sigma_p,\ p\in\mathbb{N}$. Let $\sigma=\sigma_{p-1},\ \varsigma=\sigma_{p-1}$ and $\eta=\sigma_p$. in (3.7.1) and assume that $\alpha*\beta=\min\{\alpha,\beta\}$, we get

$$\begin{aligned}\mathscr{M}(\sigma_p,\sigma_p,\sigma_{p+1},\mathsf{t}) &= \mathscr{M}(\mathscr{J}\sigma_{p-1},\mathscr{J}\sigma_{p-1},\mathscr{J}\sigma_p,\mathsf{t})\\
&\geq \min\Big\{\mathscr{M}\Big(\sigma_{p-1},\sigma_{p-1},\sigma_p,\frac{\mathsf{t}}{\tau}\Big),\mathscr{M}\Big(\sigma_{p-1},\sigma_{p-1},\mathscr{J}\sigma_{p-1},\frac{\mathsf{t}}{\tau}\Big),\\
&\qquad \mathscr{M}\Big(\sigma_{p-1},\sigma_{p-1},\mathscr{J}\sigma_{p-1},\frac{\mathsf{t}}{\tau}\Big),\mathscr{M}\Big(\sigma_p,\sigma_p,\mathscr{J}\sigma_p,\frac{\mathsf{t}}{\tau}\Big),\\
&\qquad \mathscr{M}\Big(\sigma_{p-1},\sigma_{p-1},\mathscr{J}\sigma_p,\frac{2\,\mathsf{t}}{\tau}\Big),\mathscr{M}\Big(\mathscr{J}\sigma_{p-1},\mathscr{J}\sigma_{p-1},\sigma_p,\frac{\mathsf{t}}{\tau}\Big)\Big\}\\
&\geq \min\Big\{\mathscr{M}\Big(\sigma_{p-1},\sigma_{p-1},\sigma_p,\frac{\mathsf{t}}{\tau}\Big),\mathscr{M}\Big(\sigma_{p-1},\sigma_{p-1},\sigma_p,\frac{\mathsf{t}}{\tau}\Big),\\
&\qquad \mathscr{M}\Big(\sigma_{p-1},\sigma_{p-1},\sigma_p,\frac{\mathsf{t}}{\tau}\Big),\mathscr{M}\Big(\sigma_p,\sigma_p,\sigma_{p+1},\frac{\mathsf{t}}{\tau}\Big),\\
&\qquad \mathscr{M}\Big(\sigma_{p-1},\sigma_{p-1},\sigma_{p+1},\frac{2\,\mathsf{t}}{\tau}\Big),\mathscr{M}\Big(\sigma_p,\sigma_p,\sigma_p,\frac{\mathsf{t}}{\tau}\Big)\Big\}\\
&\geq \min\Big\{\mathscr{M}\Big(\sigma_{p-1},\sigma_{p-1},\sigma_p,\frac{\mathsf{t}}{\tau}\Big),\mathscr{M}\Big(\sigma_p,\sigma_p,\sigma_{p+1},\frac{\mathsf{t}}{\tau}\Big),\\
&\qquad \mathscr{M}\Big(\sigma_{p-1},\sigma_{p-1},\sigma_{p+1},\frac{2\,\mathsf{t}}{\tau}\Big)\Big\}\\
&\geq \min\Big\{\mathscr{M}\Big(\sigma_{p-1},\sigma_{p-1},\sigma_p,\frac{\mathsf{t}}{\tau}\Big),\mathscr{M}\Big(\sigma_p,\sigma_p,\sigma_{p+1},\frac{\mathsf{t}}{\tau}\Big),\\
&\qquad \mathscr{M}\Big(\sigma_{p-1},\sigma_{p-1},\sigma_p,\frac{\mathsf{t}}{\mathsf{b}\,\tau}\Big)*\mathscr{M}\Big(\sigma_p,\sigma_{p+1},\sigma_{p+1},\frac{\mathsf{t}}{\mathsf{b}\,\tau}\Big)\Big\}\\
&\geq \min\Big\{\mathscr{M}\Big(\sigma_{p-1},\sigma_{p-1},\sigma_p,\frac{\mathsf{t}}{\tau}\Big),\mathscr{M}\Big(\sigma_p,\sigma_p,\sigma_{p+1},\frac{\mathsf{t}}{\tau}\Big),\\
&\qquad \mathscr{M}\Big(\sigma_{p-1},\sigma_{p-1},\sigma_p,\frac{\mathsf{t}}{\mathsf{b}\,\tau}\Big),\mathscr{M}\Big(\sigma_p,\sigma_{p+1},\sigma_{p+1},\frac{\mathsf{t}}{\mathsf{b}\,\tau}\Big)\Big\}\\
\mathscr{M}(\sigma_p,\sigma_p,\sigma_{p+1},\mathsf{t}) &\geq \min\Big\{\mathscr{M}\Big(\sigma_{p-1},\sigma_{p-1},\sigma_p,\frac{\mathsf{t}}{\mathsf{b}\,\tau}\Big),\mathscr{M}\Big(\sigma_p,\sigma_p,\sigma_{p+1},\frac{\mathsf{t}}{\mathsf{b}\,\tau}\Big)\Big\},\\
&\qquad\qquad p\in\mathbb{N},\ \mathsf{t}>0.\end{aligned}$$

As in the proof of Theorem (3.6) by Lemma (3.3) and Corollary (3.2) we get,

$$\mathscr{M}(\sigma_p,\sigma_p,\sigma_{p+1},\mathsf{t}) \geq \mathscr{M}\Big(\sigma_{p-1},\sigma_{p-1},\sigma_p,\frac{\mathsf{t}}{\mathsf{b}\,\tau}\Big), \qquad p\in\mathbb{N},\ \mathsf{t}>0$$

and $\{\sigma_p\}$ is a Cauchy sequence. So there are $\sigma\in\mathscr{G}$ with the end goal that $\lim_{p\to\infty}\sigma_p=\sigma$ and $\lim_{p\to\infty}\mathscr{M}(\sigma,\sigma,\sigma_p,\mathsf{t})=1$, $\mathsf{t}>0$. Let $\vartheta_1\in(\tau\mathsf{b}^2,1)$ and $\vartheta_2=1-\vartheta_1$.
Consider,

$$\begin{aligned}
\mathscr{M}(\sigma,\sigma,\mathscr{J}\sigma,\mathsf{t}) &\geq \mathscr{M}\Big(\sigma,\sigma,\mathscr{J}\sigma_p,\frac{\vartheta_1\,\mathsf{t}}{\mathsf{b}}\Big)*\mathscr{M}\Big(\mathscr{J}\sigma_p,\mathscr{J}\sigma,\mathscr{J}\sigma,\frac{\vartheta_2\,\mathsf{t}}{\mathsf{b}}\Big)\\
&\geq \mathscr{M}\Big(\sigma,\sigma,\mathscr{J}\sigma_p,\frac{\vartheta_1\,\mathsf{t}}{\mathsf{b}}\Big)*\min\Big\{\mathscr{M}\Big(\sigma_p,\sigma,\sigma,\frac{\vartheta_2\,\mathsf{t}}{\mathsf{b}\,\tau}\Big),\mathscr{M}\Big(\sigma_p,\sigma_p,\mathscr{J}\sigma_p,\frac{\vartheta_2\,\mathsf{t}}{\mathsf{b}\,\tau}\Big),\\
&\qquad \mathscr{M}\Big(\sigma,\sigma,\mathscr{J}\sigma,\frac{\vartheta_2\,\mathsf{t}}{\mathsf{b}\,\tau}\Big),\mathscr{M}\Big(\sigma,\sigma,\mathscr{J}\sigma,\frac{\vartheta_2\,\mathsf{t}}{\mathsf{b}\,\tau}\Big),\\
&\qquad \mathscr{M}\Big(\sigma_p,\sigma,\mathscr{J}\sigma,\frac{2\,\vartheta_2\,\mathsf{t}}{\mathsf{b}\,\tau}\Big),\mathscr{M}\Big(\mathscr{J}\sigma_p,\mathscr{J}\sigma,\sigma,\frac{\vartheta_2\,\mathsf{t}}{\mathsf{b}\,\tau}\Big)\Big\}\\
&\geq \min\Big\{\mathscr{M}\Big(\sigma,\sigma,\mathscr{J}\sigma_p,\frac{\vartheta_1\,\mathsf{t}}{\mathsf{b}}\Big),\mathscr{M}\Big(\sigma_p,\sigma,\sigma,\frac{\vartheta_2\,\mathsf{t}}{\mathsf{b}\,\tau}\Big),\mathscr{M}\Big(\sigma_p,\sigma_p,\mathscr{J}\sigma_p,\frac{\vartheta_2\,\mathsf{t}}{\mathsf{b}\,\tau}\Big),\\
&\qquad \mathscr{M}\Big(\sigma,\sigma,\mathscr{J}\sigma,\frac{\vartheta_2\,\mathsf{t}}{\mathsf{b}\,\tau}\Big),\mathscr{M}\Big(\sigma,\sigma,\mathscr{J}\sigma,\frac{\vartheta_2\,\mathsf{t}}{\mathsf{b}\,\tau}\Big),\\
&\qquad \mathscr{M}\Big(\sigma_p,\sigma,\mathscr{J}\sigma,\frac{2\,\vartheta_2\,\mathsf{t}}{\mathsf{b}\,\tau}\Big),\mathscr{M}\Big(\mathscr{J}\sigma_p,\mathscr{J}\sigma,\sigma,\frac{\vartheta_2\,\mathsf{t}}{\mathsf{b}\,\tau}\Big)\Big\}\\
&\geq \min\Big\{\mathscr{M}\Big(\sigma,\sigma,\mathscr{J}\sigma_p,\frac{\vartheta_1\,\mathsf{t}}{\mathsf{b}}\Big),\mathscr{M}\Big(\sigma_p,\sigma,\sigma,\frac{\vartheta_2\,\mathsf{t}}{\mathsf{b}\,\tau}\Big),\mathscr{M}\Big(\sigma_p,\sigma_p,\mathscr{J}\sigma_p,\frac{\vartheta_2\,\mathsf{t}}{\mathsf{b}\,\tau}\Big),\\
&\qquad \mathscr{M}\Big(\sigma,\sigma,\mathscr{J}\sigma,\frac{\vartheta_2\,\mathsf{t}}{\mathsf{b}\,\tau}\Big),\mathscr{M}\Big(\sigma,\sigma,\mathscr{J}\sigma,\frac{\vartheta_2\,\mathsf{t}}{\mathsf{b}\,\tau}\Big),\min\Big\{\mathscr{M}\Big(\sigma_p,\sigma,\sigma,\frac{\vartheta_2\,\mathsf{t}}{\mathsf{b}^2\,\tau}\Big),\\
&\qquad \mathscr{M}\Big(\sigma,\mathscr{J}\sigma,\mathscr{J}\sigma,\frac{\vartheta_2\,\mathsf{t}}{\mathsf{b}^2\,\tau}\Big)\Big\}\mathscr{M}\Big(\mathscr{J}\sigma_p,\mathscr{J}\sigma,\sigma,\frac{\vartheta_2\,\mathsf{t}}{\mathsf{b}\,\tau}\Big)\Big\}\\
&\geq \min\Big\{\mathscr{M}\Big(\sigma,\sigma,\sigma,\frac{\vartheta_1\,\mathsf{t}}{\mathsf{b}}\Big),\mathscr{M}\Big(\sigma,\sigma,\sigma,\frac{\vartheta_2\,\mathsf{t}}{\mathsf{b}\,\tau}\Big),\mathscr{M}\Big(\sigma,\sigma,\sigma,\frac{\vartheta_2\,\mathsf{t}}{\mathsf{b}\,\tau}\Big),\\
&\qquad \mathscr{M}\Big(\sigma,\sigma,\mathscr{J}\sigma,\frac{\vartheta_2\,\mathsf{t}}{\mathsf{b}\,\tau}\Big),\mathscr{M}\Big(\sigma,\sigma,\mathscr{J}\sigma,\frac{\vartheta_2\,\mathsf{t}}{\mathsf{b}\,\tau}\Big),\min\Big\{\mathscr{M}\Big(\sigma,\sigma,\sigma,\frac{\vartheta_2\,\mathsf{t}}{\mathsf{b}^2\,\tau}\Big),\\
&\qquad \mathscr{M}\Big(\sigma,\mathscr{J}\sigma,\mathscr{J}\sigma,\frac{\vartheta_2\,\mathsf{t}}{\mathsf{b}^2\,\tau}\Big)\Big\}\mathscr{M}\Big(\sigma,\mathscr{J}\sigma,\sigma,\frac{\vartheta_2\,\mathsf{t}}{\mathsf{b}\,\tau}\Big)\Big\} \qquad \text{as} \quad p\to\infty\\
&\geq \min\Big\{1,1,1,\mathscr{M}\Big(\sigma,\sigma,\mathscr{J}\sigma,\frac{\vartheta_2\,\mathsf{t}}{\mathsf{b}\,\tau}\Big),\\
&\qquad \min\Big\{1,\mathscr{M}\Big(\sigma,\mathscr{J}\sigma,\mathscr{J}\sigma,\frac{\vartheta_2\,\mathsf{t}}{\mathsf{b}^2\,\tau}\Big)\Big\}\mathscr{M}\Big(\sigma,\mathscr{J}\sigma,\sigma,\frac{\vartheta_2\,\mathsf{t}}{\mathsf{b}\,\tau}\Big)\Big\}\\
&\geq \mathscr{M}\Big(\sigma,\sigma,\mathscr{J}\sigma,\frac{\vartheta_2\,\mathsf{t}}{\mathsf{b}^2\,\tau}\Big)=\mathscr{M}\Big(\sigma,\sigma,\mathscr{J}\sigma,\frac{\mathsf{t}}{\rho}\Big),\ \text{where } \rho=\frac{\mathsf{b}^2\,\tau}{\vartheta_2}\in(0,1).
\end{aligned}$$

By Lemma (3.3) we get $\mathscr{J}\sigma = \sigma$.

Let's keep that σ and $\mathfrak{v}$ are fixed point of $\mathscr{J}$. Now

$$\begin{aligned}
\mathscr{M}(\sigma,\sigma,\mathfrak{v},\mathsf{t}) &= \mathscr{M}(\mathscr{J}\sigma,\mathscr{J}\sigma,\mathscr{J}\mathfrak{v},\mathsf{t}) \\
&\geq \min\Big\{\mathscr{M}\Big(\sigma,\sigma,\mathfrak{v},\frac{\mathsf{t}}{\tau}\Big),\mathscr{M}\Big(\sigma,\sigma,\mathscr{J}\sigma,\frac{\mathsf{t}}{\tau}\Big),\mathscr{M}\Big(\sigma,\sigma,\mathscr{J}\sigma,\frac{\mathsf{t}}{\tau}\Big), \\
&\qquad \mathscr{M}\Big(\mathfrak{v},\mathfrak{v},\mathscr{J}\mathfrak{v},\frac{\mathsf{t}}{\tau}\Big),\mathscr{M}\Big(\sigma,\sigma,\mathscr{J}\mathfrak{v},\frac{2\,\mathsf{t}}{\tau}\Big),\mathscr{M}\Big(\mathscr{J}\sigma,\mathscr{J}\sigma,\mathfrak{v},\frac{\mathsf{t}}{\tau}\Big)\Big\} \\
&\geq \min\Big\{\mathscr{M}\Big(\sigma,\sigma,\mathfrak{v},\frac{\mathsf{t}}{\tau}\Big),1,1,1,\mathscr{M}\Big(\sigma,\sigma,\mathfrak{v},\frac{2\,\mathsf{t}}{\tau}\Big),\mathscr{M}\Big(\sigma,\sigma,\mathfrak{v},\frac{\mathsf{t}}{\tau}\Big)\Big\} \\
&\geq \mathscr{M}\Big(\sigma,\sigma,\mathfrak{v},\frac{2\,\mathsf{t}}{\tau}\Big) \\
&\geq \mathscr{M}\Big(\sigma,\sigma,\sigma,\frac{\mathsf{t}}{\mathsf{b}\,\tau}\Big)*\mathscr{M}\Big(\sigma,\mathfrak{v},\mathfrak{v},\frac{\mathsf{t}}{\mathsf{b}\,\tau}\Big) \\
&\geq \mathscr{M}\Big(\sigma,\mathfrak{v},\mathfrak{v},\frac{\mathsf{t}}{\mathsf{b}\,\tau}\Big) \\
\mathscr{M}(\sigma,\sigma,\mathfrak{v},\mathsf{t}) &\geq \mathscr{M}\Big(\sigma,\sigma,\mathfrak{v},\frac{\mathsf{t}}{\mathsf{b}\,\tau}\Big)
\end{aligned}$$

Using Lemma (3.3) we get, $\sigma = \mathfrak{v}$. □

4. CONCLUSION

In this paper, we utilize a countable extension of the $\mathsf{t}-$ norm . Utilizing this development and contraction condition to demonstrate the sequence in generalized fuzzy $\mathsf{b}-$ metric spaces is a Cauchy sequence. Some fixed point theorems are additionally demonstrated by applying our outcomes.

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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