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THEOREMS ON COMMON FIXED POINT OF EXPANSIVE SORT OF MAPPING IN DISLOCATED S_b-METRIC SPACE

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Abstract. This write up is mainly focusing on establishing the common fixed point theorem under expansive kind of mapping in dislocated S_b-metric spaces which is a term based on the concept of dislocatedness in S_b metric space. The results are derived for weakly compatible mappings and also by unwinding dS_b-continuity of the mapping.

Keywords: dislocated S_b-metric space; fixed point; coincidence point; weakly compatible.

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1. INTRODUCTION

A well-known fixed point theorem for metric spaces is Banach contraction principle that has been proved by Banach in 1922. There are a lot of extensions of this popular theorem in metric space which are acquiring contractive conditions as a generalizing one and a couple of speculations of it in various distinct spaces that are having metric type structures. Another interesting area of study is expansive mapping in fixed point theory that was developed in the year 1984 by Wang, Li. Gao and Iseki [18] implementing an expansive mapping in complete metric space. Generalization has made in this result by Daffer and Kaneko [3] using pair of

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self-mappings.

In various spaces, enormous authors developed the results in fixed, coincidence and common fixed point using expansive condition. In [12], he authors built up these outcomes in G-metric spaces and in [14], Sahin and Mustafa implemented this broad condition for finding the fixed point outcomes about in cone-metric spaces. In [8],[9],[10],[15],[17] different speculation of fixed point hypotheses were built up utilizing expansive condition in b-metric spaces, dislocated metric spaces and various other generalized spaces on metric.

Now this article focused on the outcomes of fixed point on common for expansive kind of mappings are established for weakly compatible mappings in dislocated S_b-metric space.

2. Preliminaries

Definition 2.1. Let the self-mappings of a metric space (A,ζ) be S and T. Then (S,T) is said to be compatible if there is a sequence $\{t_n\}$ in A such that
$lim_{n\to\infty}\zeta(STt_n, TSt_n) = 0$ whenever $lim_{n\to\infty} = S_n = lim_{n\to\infty}Tt_n = t$ for some $t \in A$

Definition 2.2. Let the self-mappings be S and T defined on a metric space (A,ζ) . Then if $v = St = Tt$ for some $t \in A$ and $v \in A$, the element t in A is called a coincidence point or v is a point of coincidence of S and T.

Definition 2.3. The self-mappings S,T of a metric space (A,ζ) are weakly compatible if whenever $Sd = Td$ then $TSd = STd$ is satisfied for every $d \in A$.

Definition 2.4. Let A be a non empty set with $\zeta : A^3 \to R_0^+$. If ζ satisfies the following conditions

(i) $\zeta(a,d,e) > 0$ for all $a,d,e \in A$ with $a \neq d \neq e$

(ii) $\zeta(a,d,e) = 0 \Longrightarrow a = d = e$

(iii) $\zeta(a,d,e) = \zeta(a,e,d) = \zeta(d,a,e) = \zeta(d,e,a) = \zeta(e,a,d) = \zeta(e,d,a)$

(iv) $\zeta(a,a,d) = \zeta(d,d,a)$ for every $a,d \in A$

(v) $\zeta(a,d,e) \leq b[\zeta(a,a,s)+\zeta(d,d,s)+\zeta(e,e,s)]$ for all $a,d,e,s \in A$ with $b \geq 1$.

then (A,ζ) is a dislocated S_b-metric space with a dS_b-metric. [simply as dS_b-metric space].

Definition 2.5. The sequence $\{t_n\}$ in dS_b-metric space (A,ζ) is termed as dS_b-convergent if for given $\varepsilon > 0$, there occurs a $n_0 \in I$ such that $\zeta(t_n,t_n,t) < \varepsilon$ or $\zeta(t,t,t_n) < \varepsilon; (n \geq n_0)$.It can be noted as $dS_b - lim_{n\to\infty} t_n = t$ and t is a dS_b-limit point of $\{t_n\}$.

Definition 2.6. The dS_b-metric space (A,ζ) is a dS_b-Cauchy if for $t > 0$, there occurs a number $n_0 \in I$ such that $\zeta(t_n,t_m,t_l) < \varepsilon; (n,m,l \geq n_0)$.

Definition 2.7. A complete dS_b-metric space (A,ζ) is a dS_b-metric space in which every dS_b-Cauchy sequence is dS_b-convergent in A.

Definition 2.8. Let (A,ζ) , (B,ζ') be two dS_b-metric spaces and let $f : A \to B$ be a defined function which is said to be dS_b-continuous at $p \in A$ if for every $\varepsilon > 0$ there is a $\delta > 0$ such that $\zeta^{'}(f(a), f(a), f(p)) < \varepsilon$ whenever $\zeta(a,a,p) < \delta$. If f is a dS_b- continuous in each and every point of a subset L of A, we say that f is dS_b-continuous on L.

Theorem 2.9. *A sequence in a dS_b-metric space is dS_b-convergent to atmost one dS_b-limit point.*

Theorem 2.10. *Let $\{d_n\}$ be a sequence in dS_b-metric space (A,ζ) with $b \geq 1$ such that $\zeta(d_n,d_n,d_{n+1}) \leq l\zeta(d_{n-1},d_{n-1},d_n)$ where $l \in (0,1/b)$ and $n = 1,2,\cdots$. Then $\{d_n\}$ is dS_b-Cauchy sequence in A.*

3. Main Results

Theorem 3.1. *Let (A,ζ) be a dS_b-metric space which is complete. The self mappings on dS_b-metric space f and g are surjective, injective respectively satisfying*

$$\zeta(fs, fd, fv) \geq q\zeta(gs, gd, gv) \tag{3.1}$$

For each $s,d,v \in A$. Then f and g have a unique common fixed point.

Proof. Let s_0 ,an element from A be chosen. Choosing an element $s_1 \in A$ such that $fs_0 = gs_1$. In general, $fs_n = gs_{n+1} = a_n$. From (3.1),

$$\begin{aligned}
\zeta(fs_n, fs_n, fs_{n+1}) &\geq q\zeta(gs_n, gs_n, gs_{n+1}) \\
&= q\zeta(a_{n-1}, a_{n-1}, a_n) \\
&= q\zeta(fs_{n-1}, fs_{n-1}, fs_n) \\
&\geq q^2\zeta(gs_{n-1}, gs_{n-1}, gs_n) \\
&= q^2\zeta(a_{n-2}, a_{n-2}, a_{n-1})
\end{aligned}$$

Continuing in this fashion we get,

$$\begin{aligned}
\zeta(fs_n, fs_n, fs_{n+1}) &\geq q^n\zeta(gs_1, gs_1, gs_2) \\
\Longrightarrow \zeta(gs_1, gs_1, gs_2) &\leq q^{-n}\zeta(a_0, a_0, a_1)
\end{aligned}$$

Now if $n > m$,

$$\begin{aligned}
\zeta(a_n, a_m, a_m) &\leq b[\zeta(a_n, a_n, a_{n+1}) + \zeta(a_{n+1}, a_{n+1}, a_{n+2}) + \cdots + \zeta(a_m, a_m, a_{m+1})] \\
&= b[\zeta(fs_{n+1}, fs_{n+1}, fs_{n+2}) + \zeta(fs_{n+2}, fs_{n+2}, fs_{n+3}) + \cdots + \zeta(fs_{m+1}, fs_{m+1}, fs_{m+2})] \\
&< b[q^{-n}\zeta(a_0, a_0, a_1) + q^{-(n+1)}\zeta(a_0, a_0, a_1) + \cdots + q^{-m}\zeta(a_0, a_0, a_1)] \\
&= bq^{-n}[1 + q^{-1} + q^{-2} + \cdots + q^{-m-n}]\zeta(a_0, a_0, a_1) \\
&= b/q^n[1 - 1/q]^{-1}\zeta(a_0, a_0, a_1) \\
&= bq^{-n}\frac{q}{q-1}\zeta(a_0, a_0, a_1) \\
&= bq^{1-n}\frac{1}{q-1}\zeta(a_0, a_0, a_1)
\end{aligned}$$

Since $q > 1$ and as $n \to \infty$, $q^{1-n} \to 0$, therefore by letting $m, n \to \infty$, we get, $\zeta(a_n, a_m, a_m) \to 0$ and so $\{a_n\}$ is a dS_b-Cauchy sequence. Also by hypothesis, we have a dS_b limit point in A such that

$$lim_{n\to\infty}a_n = a$$

$$\Longrightarrow lim_{n\to\infty}fs_n = a$$

$$\Longrightarrow lim_{n\to\infty}gs_{n+1} = a$$

Since f is surjective we have, $f^{-1}(a) = k \in A$, then $a = f(k)$.

Consider, $\zeta(a_n, a_n, a) = \zeta(fs_n, fs_n, fk) \geq q\zeta(gs_n, gs_n, gk)$ Letting $n \to \infty$, we get,

$$\zeta(a,a,a) \geq q\zeta(a,a,gk)$$

$$0 \geq q\zeta(a,a,gk)$$

Since $q > 1$ so that $\zeta(a,a,gk) = 0 \Longrightarrow gk = a$. Thus $fk = gk = a$ for some $k \in A$ and hence a is a point of coincidence Now,

$\zeta(k,k,a) \leq b[2\zeta(k,k,fs_n) + \zeta(a,a,fs_n)]$

Letting $n \to \infty$,

$$\begin{aligned} \zeta(k,k,a) &\leq b[2\zeta(k,k,a) + \zeta(a,a,a)] \\ \Longrightarrow 2b\zeta(k,k,a) - \zeta(k,k,a) &\geq 0 \\ \Longrightarrow (2b-1)\zeta(k,k,a) &\geq 0 \\ \Longrightarrow \zeta(k,k,a) &= 0 \\ \therefore k &= a \\ \Longrightarrow fk = gk &= k \end{aligned}$$

Hence the common fixed point for f and g is obtained as k. Now for individuality of the point, let k, j be different two common fixed points. We have, $\zeta(k,k,j) > 0$. Then

$$\begin{aligned} \zeta(k,k,j) &= \zeta(fk,fk,fj) \\ &\geq q\zeta(gk,gk,gj) \\ \Longrightarrow \zeta(k,k,j) &\geq \zeta(gk,gk,j) \end{aligned}$$

(3.2)

Since $q > 1$. This is a conflict term. Hence k is a unique common fixed point. □

Proposition 3.2. *Let S and Tbe self-maps that are weakly compatible well-defined on dS_b-metric space A. If S and T have a sole point of coincidence then S and T have precisely one common fixed point.*

Proof. Let the point of coincidence be y. Then $St = Tt = y$ for some $t \in A$. Since S and T are weakly compatible, it gives that $STt = TSt \implies Sy = Ty$. Now we assert that $St = Tt = t$. Suppose that $St \neq t$, then $\zeta(St,t,t) > 0$. Now,

$$\begin{aligned}
\zeta(St,St,Sy) &\leq b[2\zeta(St,St,Ty) + \zeta(Sy,Sy,Ty)] \\
&= b[2\zeta(St,St,Sy) + \zeta(Sy,Sy,Sy)] \\
\implies \zeta(St,St,t) &\leq b[2\zeta(St,St,t)] \\
\implies \zeta(St,St,t) &< \zeta(St,St,t)
\end{aligned}$$

This contradicts our hypothesis terms. $\therefore Tt = St = t$. and so t is a common fixed point. Now choose two different common fixed points t,y. Then $St = Tt = t, Sy = Ty = y$. Consider,

$$\begin{aligned}
\zeta(t,t,y) &= \zeta(St,St,Sy) \\
&\leq b[2\zeta(St,St,Tt) + \zeta(Sy,Sy,Tt)] \\
&= b[2\zeta(t,t,t) + \zeta(y,y,t)] \\
&= b\zeta(y,y,t) \\
\implies \zeta(t,t,y) &\leq \zeta(t,t,y)
\end{aligned}$$

This is again a contradiction. Hence $t = y$ is unique. □

Theorem 3.3. *Let the dS_b-metric space be (A,ζ) with $b \geq 1$ and f,g are self-mappings sustaining the conditions below:*

(i) $g(A) \subseteq f(A)$

(ii) $\zeta(fs,fd,fe) \geq \alpha\zeta(gs,gs,fd)\frac{[1+\zeta(gd,gd,fe)]}{[1+\zeta(ge,ge,fs)]} + \beta\zeta(gs,gd,gv)$

For all $s,d,e \in A$ and $s \neq d \neq e$, where $\alpha,\beta \geq 0$ with $\alpha + b\beta > b$ and $\alpha > 1$. In the event that one of the range subspaces is dS_b-complete, at that point f and g have a coincidence point. Also on the off chance that the pair (f,g) is weakly compatible then f and g have exactly one common fixed point in A.

Proof. Suppose that $s_0 \in A$ and choose $s_1 \in A$ such that $gs_0 = fs_1$. This can be done because of condition (i). Proceeding in this fashion, a sequence $\{s_n\}$ can be constructed in A with $gs_{n-1} = fs_n$ for all $n = 1,2,\cdots$. Now we assert that $\{s_n\}$ is dS_b-Cauchy in A. By condition (ii),

$$\begin{aligned}
\zeta(gs_{n-1},gs_n,gs_{n+1}) &\geq \alpha\zeta(gs_{n-1},gs_{n-1},fs_{n+1})\frac{[1+\zeta(gs_n,gs_n,fs_{n+2})]}{[1+\zeta(gs_{n+1},gs_{n+1},gs_n)]} \\
&+ \beta\zeta(gs_{n-1},gs_n,gs_{n+1}) \\
&= \alpha\zeta(gs_{n-1},gs_{n-1},gs_n)\frac{[1+\zeta(gs_n,gs_n,gs_{n+1})]}{[1+\zeta(gsn+1,gs_{n+1},gs_n)]} \\
&+ \beta\zeta(gs_{n-1},gs_n,gs_{n+1}) \\
&= \alpha\zeta(gs_{n-1},gs_{n-1},gs_n) + \beta\zeta(gs_{n-1},gs_n,gs_{n+1}) \\
\Longrightarrow (1-\beta)\zeta(gs_{n-1},gs_n,gs_{n+1}) &\geq \alpha\zeta(gs_{n-1},gs_{n-1},gs_n) \\
\Longrightarrow \zeta(gs_{n-1},gs_{n-1},gs_n) &\leq \frac{(1-\beta)}{\alpha}\zeta(gs_{n-1},gs_n,gs_{n+1}) \\
\Longrightarrow \zeta(gs_{n-1},gs_{n-1},gs_n) &\leq \lambda\zeta(gs_{n-1},gs_n,gs_{n+1})
\end{aligned}$$

where $\lambda = ((1-\beta)/\alpha), \lambda < 1/b$ and $\alpha + b\beta > b$.

By Theorem (2.10), $\{s_n\}$ is a dS_b-Cauchy sequence.

Through the process of induction we get, $\zeta(gs_{n-1},gs_{n-1},gs_n) \leq \lambda^n\zeta(gs_0,gs_1,gs_2)$ for $n \geq 0$.

Now for $l,m,n \in N$ and $l > m > n$, we have

$$\begin{aligned}
\zeta(gs_n,gs_m,gs_l) &\leq b[\zeta(gs_n,gs_n,gs_{n+1}) + \zeta(gs_m,gs_m,gs_{n+1}) + \zeta(gs_l,gs_l,gs_{n+1})] \\
&\leq b\zeta(gs_n,gs_n,gs_{n+1}) + b[2\zeta(gs_m,gs_m,gs_{n+2}) + \zeta(gs_{n+1},gs_{n+1},gs_{n+2})] \\
&+ b[2\zeta(gs_l,gs_l,gs_{n+3}) + \zeta(gs_{n+1},gs_{n+1},gs_{n+3})] \\
&\leq [b\lambda^n + b^2\lambda^{n-1} + \cdots + b^{l-m-n-2}\lambda^{l-2}b^{l-m-(n-1)}\lambda^{l-1}]\zeta(gs_0,gs_1,gs_2) \\
&= b\lambda^n\frac{1}{(1-b\lambda)}\zeta(gs_0,gs_1,gs_2)
\end{aligned}$$

This shows that $\{gs_n\}$, a dS_b-Cauchy sequence in $g(A)$. Now assume that $g(A)$ is a dS_b-complete subspace of A. Then there exists $a \in g(A) \subseteq f(A)$ such that $gs_n \to a$ and also $fs_n \to a$. (i.e) $\{gs_n\}$ is dS_b-convergent in $g(A)$. Also suppose if $f(A)$ is dS_b-complete this will hold for $a \in f(A)$ and so $\{gs_n\}$ is dS_b-convergent to a in $f(A)$. Now let $u \in A$ such that $fu = a$. If $gu \neq a$ then by using $s = s_n, d = s_{n+1}, e = u$ in (ii) we have

$$\zeta(fs_n, fs_{n+1}, fu) \geq \alpha\zeta(gs_n, gs_n, fs_{n+1})\frac{[1+\zeta(gs_{n+1}, gs_{n+1}, fu)]}{(1+\zeta(gu, gu, gs_{n+1}))} + \beta\zeta(gs_n, gs_{n+1}, gu).$$

Letting $n \to \infty$ we get,

$$\begin{aligned} \zeta(a,a,a) &\geq \alpha\zeta(a,a,a)\frac{[1+\zeta(a,a,a)]}{[1+\zeta(gu,gu,a)]} + \beta\zeta(a,a,gu) \\ 0 &\geq \beta\zeta(a,a,gu) \\ \implies gu &= a = fu \end{aligned}$$

$\therefore a$ is the point of coincidence of f and g. Let b, a be distinct points of coincidence of f and g. Then so $fd = gd = b$ for some $d \in A$. Now from (ii) using $s = u, d = e = l$ we have

$$\begin{aligned} \zeta(fu, fl, fl) &\geq \alpha\zeta(gu, gu, fl)\frac{[1+\zeta(gl, gl, fl)]}{[1+\zeta(gl, gl, fu)]} + \beta\zeta(gu, gl, gl) \\ &= \alpha\zeta(gu, gu, b)\frac{[1+\zeta(b,b,b)]}{[1+\zeta(b,b,fu)]} + \beta\zeta(gu, b, b) \\ \zeta(a,b,b) &\geq \beta\zeta(a,b,b) \\ \implies \zeta(a,b,b) &\geq \zeta(a,b,b). \end{aligned}$$

This is a contradiction. Therefore coincidence point for f and g is unique in A. Furthermore, f and g are weakly compatible and from Proposition (3.2) f and g have exactly one common fixed point in A. □

Theorem 3.4. *Let (A, ζ) be a dS_b-metric space which is complete. Let the self-mappings characterized on A be f and g which are surjective, injective individually fulfilling the accompanying conditions:*

1. $g(A) \subseteq f(A)$.

2. $\zeta(fs,fd,fe)+\alpha max\{\zeta(gs,fe,fe),\zeta(fs,gd,fe),\zeta(fs,fs,ge)\}$
$\geq \beta\zeta(fs,gs,gs)\frac{(1+\zeta(ge,fe,ge))}{(1+\zeta(gs,ge,ge))}+\gamma\zeta(gs,gd,ge)$
for all $s,d,e\in A$ *with* $s\neq d\neq e$ *where* $\alpha,\beta,\gamma\geq 0$ *are real constants such that* $b\beta+\gamma>(1+\alpha)b+2b^2\alpha$ *and* $\gamma>1+\alpha$.

3. *Either* $f(A)$ *or* $g(A)$ *is* dS_b*-complete.*

Suppose that the couple of mappings (f,g) *is weakly compatible then* f *and* g *possess a unique common fixed point in* A.

Proof. Assume that $s_0\in A$ and pick an element $s_1\in A$ such that $fs_1=gs_0$. This is possible by (1).

Enduring in this manner, a sequence $\{t_n\}$ in A can be constructed such that $fs_n=gs_{n-1}=t_n$. To prove that $\{t_n\}$ is a ds_b-Cauchy sequence, let $s=t_n,d=t_n,e=t_{n+1}$. Then by condition (2)

$$\zeta(ft_n,ft_n,ft_{n+1})+\alpha max\{\zeta(gt_n,ft_{n+1},ft_{n+1}),\zeta(ft_n,gt_{n+1},ft_{n+1}),\zeta(ft_n,ft_n,gt_{n+1})\}$$
$$\geq \beta\zeta(ft_n,gt_n,gt_n)\frac{(1+\zeta(gt_{n+1},ft_{n+1},gt_{n+1}))}{(1+\zeta(gt_n,gt_{n+1},gt_{n+1}))}$$
$$+\gamma\zeta(gt_n,gt_{n+1},gt_{n+1})$$

$$\Longrightarrow \zeta(gt_{n-1},gt_{n-1},gt_n)+\alpha max\{\zeta(gt_n,gt_n,gt_n),\zeta(gt_{n-1},gt_{n+1},gt_n),\zeta(gt_{n-1},gt_{n-1},gt_{n+1})\}$$
$$\geq \beta\zeta(gt_{n-1},gt_n,gt_n)\frac{(1+\zeta(gt_{n+1},gt_n,gt_{n+1}))}{(1+\zeta(gt_n,gt_{n+1},gt_{n+1}))}$$
$$+\gamma\zeta(gt_n,gt_{n+1},gt_{n+1})$$

$$\Longrightarrow \zeta(gt_{n-1},gt_{n-1},gt_n)+\alpha max\{0,\zeta(gt_{n-1},gt_{n+1},gt_n),\zeta(gt_{n-1},gt_{n-1},gt_{n+1})\}$$
$$\geq \beta\zeta(gt_{n-1},gt_n,gt_n)1+\gamma\zeta(gt_n,gt_{n+1},gt_{n+1})$$

$$\Longrightarrow \zeta(gt_{n-1},gt_{n-1},gt_n)+\alpha\zeta(gt_{n-1},gt_{n-1},gt_{n+1})\geq \beta\zeta(gt_{n-1},gt_n,gt_n)+\gamma\zeta(gt_n,gt_{n+1},gt_{n+1})$$

if $\zeta(gt_{n-1},gt_{n-1},gt_{n+1})$ is chosen as maximum.

$$\Longrightarrow \zeta(gt_{n-1},gt_{n-1},gt_n)+b\alpha 2\zeta(gt_{n-1},gt_{n-1},gt_n)+b\alpha\zeta(gt_{n+1},gt_{n+1},gt_n)$$
$$\geq \beta\zeta(gt_{n-1},gt_n,gt_n)+\gamma\zeta(gt_n,gt_{n+1},gt_{n+1})$$
$$\Longrightarrow (1+2\alpha b-\beta)\zeta(gt_{n-1},gt_{n-1},gt_n)\geq(\gamma-b\alpha)\zeta(gt_{n+1},gt_{n+1},gt_n)$$

$$\implies \zeta(gt_n, gt_{n+1}, gt_{n+1}) \leq \frac{(1+2\alpha b-\beta)}{(\gamma - b\alpha)}\zeta(gt_{n-1}, gt_{n-1}, gt_n)$$
$$= \mu\zeta(gt_{n-1}, gt_{n-1}, gt_n)$$

where $\mu = \frac{(1+2\alpha b-\beta)}{(\gamma-b\alpha)} < \frac{1}{b}$ as $(1+\alpha)b+2\alpha b^2 < \gamma + b\beta$. Also $\gamma > 1+\alpha$
Clearly $\mu \in (0, \frac{1}{b})$
Thus $\zeta(gt_n, gt_{n+1}, gt_{n+1}) \leq \mu\zeta(gt_{n-1}, gt_{n-1}, gt_n)$, where $\mu \in (0, \frac{1}{b})$.
By induction process we get, $\zeta(gt_n, gt_{n+1}, gt_{n+1}) \leq \mu^n\zeta(gt_0, gt_0, gt_1), n \geq 0$.

For $m, n, \in I$ with $m > n$ we have,

$$\begin{aligned}
(3.3) \qquad & \zeta(gt_n, gt_n, gt_m) \\
\leq\ & b[2\zeta(gt_n, gt_n, gt_{n+1}) + \zeta(g_m, gt_m, gt_{n+1})] \\
\leq\ & 2b\mu^n\zeta(gt_0, gt_0, gt_1) + 2b^2\mu^{n+1}\zeta(gt_0, gt_0, gt_1) + \cdots + 2b^{m-n}\mu^{m-1}\zeta(gt_0, gt_0, gt_1) \\
=\ & 2b\mu^n[1 + b\mu + \cdots + (b\mu)^{m-n-1}\zeta(gt_0, gt_0, gt_1) \\
=\ & \frac{2b\mu^n}{1-b\mu}\zeta(gt_0, gt_0, gt_1)
\end{aligned}$$

and so $\{t_n\}$ is dS_b-Cauchy sequence in $g(A)$. Assume that $g(A)$ is dS_b-complete subspace of A. At that point, there exists $t \in g(A) \subseteq f(A)$ with the end goal that $gt_n \to t$ and furthermore $ft_n \to t$. If $f(A)$ is dS_b-complete it holds together with $t \in f(A)$. Hence $gt_n \to t$ in $f(A)$. Let $d \in A$ such that $f(d) = t$. If $gd \neq t$ then use $s = t_n, d = t_n, e = d$ in (ii) we get,

$$\zeta(ft_n, ft_n, fd) + \alpha max\{\zeta(gt_n, fd, fd), \zeta(ft_n, gd, fd), \zeta(ft_n, ft_n, gd)\}$$
$$\geq \beta\zeta(ft_n, gt_n, gt_n)\frac{(1+\zeta(gt_n, fd, fd))}{(1+\zeta(gt_n, gd, gd))} + \gamma\zeta(gt_n, gd, gd)$$

Letting $n \to \infty$ we get,

$$\zeta(t,t,t) + \alpha max\{\zeta(t, fd, fd), \zeta(t, gd, fd), \zeta(t,t,gd)\}$$
$$\geq \beta\zeta(t,t,t)\frac{(1+\zeta(t, fd, fd))}{(1+\zeta(t, gd, gd))} + \gamma\zeta(t, gd, gd)$$
$$\implies \alpha max\{0, \zeta(t, gd, t), \zeta(t,t,gd)\} \geq \gamma\zeta(t, gd, gd)$$
$$\implies \alpha\zeta(t,t,gd) \geq \gamma\zeta(t, gd, gd)$$

which is a contradiction, since $\gamma > 1+\alpha, \alpha, \gamma \geq 0$.

$\therefore gd = t$ so that $gd = fd = t$ and so d is a coincidence point f and g.

Uniqueness can be done by assuming the contradictory part that another point of coincidence v exists for f and g. (i.e) $fs = gs = v$ for some $s \in A$.

Now from condition (ii) we get,

$$\begin{aligned}
\zeta(fd,fd,fs) + \alpha max\{\zeta(gd,fs,fs),\zeta(fd,gs,fs),\zeta(fd,fd,gs)\} \\
\geq \beta\zeta(fd,gd,gd)\frac{(1+\zeta(gd,fs,fs))}{(1+\zeta(gd,gs,gs))} + \gamma\zeta(gd,gs,gs) \\
\Longrightarrow \zeta(t,t,v) + \alpha max\{\zeta(t,v,v),\zeta(t,v,v),\zeta(t,t,v)\} \\
\geq \beta\zeta(t,t,t)\frac{(1+\zeta(t,v,v))}{(1+\zeta(t,v,v))} + \gamma\zeta(t,v,v) \\
\Longrightarrow \zeta(t,t,v) + \alpha\zeta(t,t,v) \geq \gamma\zeta(t,v,v) \\
\Longrightarrow (1+\alpha)\zeta(t,t,v) \geq \gamma\zeta(t,v,v)
\end{aligned}$$

This is a contradiction, since $1+\alpha < \gamma; \alpha, \gamma \geq 0$. Therefore $t = v$. Thus f and g ensure a single coincidence point. Moreover in the event that f and g are weakly compatible, at that point by recommendation of the Proposition(3.2) f and g have an exceptional common fixed point in A. □

4. CONCLUSION

As a summary, the continuity of the mapping has been expelled and the completeness of the entire space has been limited to one of the range subspaces. The theorems have been demonstrated for a couple of mappings which were weakly compatible as opposed to utilizing a single mapping.

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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