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SEQUENCES OF $\varphi-$CONTRACTIONS AND CONVERGENCE OF FIXED POINTS

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Abstract. Given a metric space (X, d) and, for each $n = 1, 2, ...$, let $T_n : X_n \to X_n$ be a mapping with fixed point x_n, where $\{X_n\}$ is a sequence of nonempty subsets of X. Assume that each mapping T_n is a φ-contraction with respect to a different metric d_n. In this paper conditions are obtained under which the convergence of the sequence $\{T_n\}$ in some general sense to a limit mapping implies the convergence of the sequence of their fixed points $\{x_n\}$. This leads to a number of new stability results which generalize certain well-known results.

Keywords: φ-contraction; fixed points; stability, sequence of metrics

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1. Introduction and preliminaries

The study of the relationship between the convergence of a sequence of self-mappings $\{T_n\}$ and their fixed points $\{x_n\}$ of a metric (resp. topological) space X, known as the stability of fixed points has been of continuing interest. The first result in this direction for contraction mappings is due to Bonsall [3] (see also, [14]). Recently, using some new notions of convergence Barbet and Nachi [2](see also, [1] and [12]) obtained some

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interesting stability results in a metric space which extend the earlier results of Bonsall [3] and Nadler [13] over a variable domain. These results have been further generalized by Mishra et al. [6-11]. In this paper we present a generalization of two classical results of Fraser and Nadler [5] for the class of φ-contractions or nonlinear contractions due to Boyd and Wong [4] using the Barbet - Nachi convergence (cf. [2]). The results so obtained here in compliment the results of Fraser and Nadler [5] and Barbet and Nachi [12].

First, we recall some definitions, notations and preliminary results. Throughout, $\mathbb{N}$ will denote the set of natural numbers and $\overline{\mathbb{N}} = \mathbb{N} \cup \{\infty\}$.

Definition 1.1. Let (X, d) be a metric space. A mapping $T : X \to X$ is called *contraction* (resp. $k-$contraction) if there exists a constant $k \in [0, 1)$ such that

$$d(Tx, Ty) \leq kd(x, y) \text{ for all } x, y \in X. \tag{1.1}$$

If the above condition holds for $k \geq 0$, then T is called Lipschitz (rep. k-Lipschitz).

The mapping $T : X \to X$ is called φ-contraction (resp. *nonlinear contraction)* (see [4]) if

$$d(Tx, Ty) \leq \varphi(d(x, y)) \text{ for all } x, y \in X, \tag{1.2}$$

where $\varphi : [0, \infty) \to [0, \infty)$ is upper semicontinuous from the right and $\varphi(t) < t$ for $t > 0$.

Remark 1.2. Notice that (1.2) includes the well-known Banach contraction (1.1) and $\varphi(0) = 0$.

Definition 1.3. [2] Let $\{X_n\}_{n\in\overline{\mathbb{N}}}$ be a family of nonempty subsets of a metric space (X, d) and $\{T_n : X_n \to X\}_{n\in\overline{\mathbb{N}}}$ a family of mappings. Then:

T_∞ is called a $(G)-$limit of the sequence $\{T_n\}_{n\in\mathbb{N}}$ or, equivalently $\{T_n\}_{n\in\overline{\mathbb{N}}}$ satisfies the property (G), if the following condition holds:

(G) $Gr(T_\infty) \subset \liminf Gr(T_n)$: for every $x \in X_\infty$, there exists a sequence $\{x_n\} \in \Pi_{n\in\mathbb{N}} X_n$ such that:

$$\lim_n d(x_n, x) = 0 \text{ and } \lim_n d(T_n x_n, T_\infty x) = 0,$$

where $Gr(T)$ stands for the graph of T.

T_∞ is called an $(H)-$limit of the sequence $\{T_n\}_{n\in\mathbb{N}}$ or, equivalently $\{T_n\}_{n\in\overline{\mathbb{N}}}$ satisfies the property (H) if the following condition holds:

(H) For all sequences $\{x_n\} \in \Pi_{n\in\ \mathbb{N}} X_n$, there exists a sequence $\{y_n\}$ in X_∞ such that:

$$\lim_n d(x_n, y_n) = 0 \text{ and } \lim_n d(T_n x_n, T_\infty y_n) = 0.$$

Remark 1.4. Note that the alternate formulation of a (G)-limit in a sequencial form above is obtained by using the properties of the graph of a function along with the limit of a sequence of sets.

Remark 1.5. For the sake of completeness and an easy reading, we note the following properties of the above limits. For details we refer the reader to Barbet and Nachi [2].

- **(i):** A $(G)-$limit need not be unique. However, if T_n is a $k-$contraction (resp. $k-$Lipschitz) for each $n \in \mathbb{N}$, then it is so.
- **(ii):** An (H)-limit need not be unique.
- **(iii):** When T_∞ is continuous and the condition $X_\infty \subset \liminf X_n$ is satisfied, then the following implication holds [2, Proposition 9]: $(H) \Rightarrow (G)$, whereas a counter example in [2, page 56] shows that a (G)-limit is not necessarily an (H)-limit.
- **(iv):** Pointwise convergence $\Rightarrow$ $(G)-$convergence. However, the above implication is not reversible unless $\{T_n\}_{n\in\mathbb{N}}$ is equicontinuous on a common domain of definition.
- **(v):** The interrelationship between the (H) convergence and uniform convergence is captured in [2, Proposition 10].

The following classical results were obtained by Fraser and Nadler [5].

Theorem 1.6. *[5, Theorem 2] Let (X, d) be a metric space and $\{d_n\}_{n\in\mathbb{N}}$ a sequence of metrics on X converging uniformly to d, where each d_n is equivalent to d. Let $\{T_n : X \to X\}_{n\in\mathbb{N}}$ be a sequence of contractive mappings on (X, d_n) converging pointwise to a mapping $T_\infty : X \to X$. If for each $n \in \mathbb{N}$, x_n is a fixed point of T_n, and if $\{x_n\}_{n\in\mathbb{N}}$ admits a subsequence converging to x_∞, then x_∞ is a fixed point of T_∞.*

Theorem 1.7. *[5, Theorem 3] Let (X,d) be a metric space and $\{d_n\}_{n\in\mathbb{N}}$ a sequence of metrics on X converging uniformly to d. Let $\{T_n : X \to X\}_{n\in\mathbb{N}}$ be a sequence of $k-$contraction mappings on (X,d_n) converging pointwise to a mapping $T_\infty : X \to X$. If for each $n \in \overline{\mathbb{N}}$, x_n is a fixed point of T_n, then the sequence $\{x_n\}_{n\in\mathbb{N}}$ converges to x_∞.*

Following Nachi [12], we have the following convergence properties.

Definition 1.8. Let (X,d) be a metric space, $\{d_n\}_{n\in\mathbb{N}}$ a sequence of metrics on X and $\{X_n\}_{n\in\overline{\mathbb{N}}}$ a family of nonempty subsets of X. Then $\{d_n\}_{n\in\mathbb{N}}$ is said to satisfy condition:

- **(A):** For all $x \in X_\infty$ and $\{x_n\}_{n\in\mathbb{N}} \in \prod_{n\in\mathbb{N}} X_n : \lim_n d_n(x_n,x) = 0 \Leftrightarrow \lim_n d(x_n,x) = 0$.
- **($\mathbf{A}_0$):** For all $x \in X$ and $\{x_n\}_{n\in\mathbb{N}} \subset X : \lim_n d_n(x_n,x) = 0 \Leftrightarrow \lim_n d(x_n,x) = 0$.
- **(B):** For all sequences $\{x_n\}_{n\in\mathbb{N}} \in \prod_{n\in\mathbb{N}} X_n$, there exists a sequence $\{y_n\}$ in X_∞ : $\lim_n d_n(x_n,y_n) = 0 \Leftrightarrow \lim_n d(x_n,y_n) = 0$.
- **($\mathbf{B}_0$):** For all sequences $\{x_n\}_{n\in\mathbb{N}} \subset X$ and $\{y_n\}_{n\in\mathbb{N}} \subset X : \lim_n d_n(x_n,y_n) = 0 \Leftrightarrow \lim_n d(x_n,y_n) = 0$.

2. Convergence of fixed points

In this section we present some generalizations of Theorems 1.6 and 1.7 for a sequence $\{T_n\}_{n\in\mathbb{N}}$ of φ-contraction mappings by weakening the hypotheses of the above theorems. The domain of definition being different for each T_n, the convergence of $\{T_n\}_{n\in\mathbb{N}}$ under consideration will be in the sense of (G) (resp. (H)).

First we note the following result which ensures the existence of a unique (G)-limit.

Proposition 2.1. *[7, Proposition 3.1] Let (X,d) be a metric space, $\{X_n\}_{n\in\overline{\mathbb{N}}}$ a family of nonempty subsets of X and $\{T_n : X_n \to X\}_{n\in\mathbb{N}}$ a sequence of φ-contraction mappings. If $T_\infty : X_\infty \to X$ is a (G)-limit of $\{T_n\}$, then T_∞ is unique.*

When $\varphi(t) = kt$, $k \in [0,1)$ we have the following result in [2, Proposition 1] as a direct consequence of Proposition 2.1.

Corollary 2.2. *Let (X, d) be a metric space, $\{X_n\}_{n\in\overline{\mathbb{N}}}$ a family of nonempty subset of X and $\{T_n : X_n \to X\}_{n\in\mathbb{N}}$ a sequence of k-contraction mappings. If $T_\infty : X_\infty \to X$ is a (G)-limit of $\{T_n\}_{n\in\mathbb{N}}$, then T_∞ is unique.*

The following result presents a generalization of Theorem 1.6.

Theorem 2.3. *Let (X, d) be a metric space and $\{d_n\}_{n\in\mathbb{N}}$ a sequence of metrics on X satisfying the property (A). Let $\{X_n\}_{n\in\overline{\mathbb{N}}}$ be a family of nonempty subsets o f X and $\{T_n : X_n \to X_n\}_{n\in\mathbb{N}}$ a sequence of φ-contraction mappings on (X_n, d_n) converging in the sense of (G) to a mapping $T_\infty : X_\infty \to X$. If for each $n \in \overline{\mathbb{N}}$, x_n is a fixed point of T_n and if the sequence $\{x_n\}_{n\in\mathbb{N}}$ admits a subsequence converging to a point $x_\infty \in X_\infty$, then x_∞ is a fixed point of T_∞.*

Proof. Let $\{x_{n_j}\}$ be a subsequence of $\{x_n\}$ converging to $x_\infty \in X_\infty$. Then by the property (G) there is a sequence $\{y_n\} \in \prod_{n\in\mathbb{N}} X_n$ such that:

$$\lim_n d(y_n, x_\infty) = 0 \text{ and } \lim_n \; d(T_n y_n, T_\infty x_\infty) = 0.$$

Therefore by (A),

$$\lim_n d_n(y_n, x_\infty) = 0 \text{ and } d_n(T_n y_n, T_\infty x_\infty) = 0. \tag{2.1}$$

Now define a sequence $\{z_n\}$ such that

$$\begin{aligned} z_{n_j} &= x_{n_j} \text{ for all } j \in \mathbb{N}, \\ z_n &= y_n \text{ if } n \neq n_j, \text{ for any } j \in \mathbb{N}. \end{aligned}$$

Therefore $\lim_n d(z_n, x_\infty) = 0$ and so $\lim_n d_n(z_n, x_\infty) = 0$, by (A). Hence

$$d(z_n, y_n) \leq d(z_n, x_\infty) + d(x_\infty, y_n) \to 0 \text{ as } n \to \infty,$$

and thus

$$\lim_n d_n(z_n, y_n) = 0. \tag{2.2}$$

Further, since T_{n_j} is a φ-contraction on (X_{n_j}, d_{n_j}) for each $j \in \mathbb{N}$, we have

$$\begin{aligned} d_{n_j}(T_{n_j}z_{n_j}, T_\infty x_\infty) &\leq d_{n_j}(T_{n_j}z_{n_j}, T_{n_j}y_{n_j}) + d_{n_j}(T_{n_j}y_{n_j}, T_\infty x_\infty) \\ &\leq \varphi(d_{n_j}(z_{n_j}, y_{n_j})) + d_{n_j}(T_{n_j}y_{n_j}, T_\infty x_\infty). \end{aligned}$$

Now by (2.1), (2.2) and the above ineqality, we obtain

$$d_{n_j}(T_{n_j}z_{n_j}, T_\infty x_\infty) \leq \varphi(d_{n_j}(z_{n_j}, y_{n_j})) + d_{n_j}(T_{n_j}y_{n_j}, T_\infty x_\infty) \to 0 \text{ as } j \to \infty.$$

Since $T_{n_j}x_{n_j} = x_{n_j}(= z_{n_j})$ and $x_{n_j} \to x_\infty$ as $j \to \infty$, we conclude that $T_\infty x_\infty = x_\infty$ and the conclusion holds. □

Corollary 2.4. *[12, Theorem 8.4] Let (X, d) be a metric space and $\{d_n\}_{n\in\mathbb{N}}$ a sequence of metrics on X satisfying the property (A). Let $\{X_n\}_{n\in\overline{\mathbb{N}}}$ be a family of nonempty subsets of X and $\{T_n : X_n \to X_n\}_{n\in\mathbb{N}}$ a sequence of k-contraction mappings on (X_n, d_n) converging in the sense of (G) to a mapping $T_\infty : X_\infty \to X$. If for each $n \in \mathbb{N}$, x_n is a fixed point of T_n and if the sequence $\{x_n\}_{n\in\mathbb{N}}$ admits a subsequence converging to a point $x_\infty \in X_\infty$, then x_∞ is a fixed point of T_∞.*

Proof. It comes from Theorem 2.3 with $\varphi(t) = kt$ and $k \in [0, 1)$. □

When $X_n = X$ for all $n \in \overline{\mathbb{N}}$ in Theorem 2.3 we have the following corollary.

Corollary 2.5. *Let (X, d) be a metric space and $\{d_n\}_{n\in\mathbb{N}}$ a sequence of metrics on X satisfying the property (A_0). Let $\{T_n : X \to X\}_{n\in\mathbb{N}}$ be a sequence of φ -contraction mappings on (X, d_n) converging pointwise to a mapping $T_\infty : X \to X$. If for each $n \in \mathbb{N}$, x_n is a fixed point of T_n and if the sequence $\{x_n\}_{n\in\mathbb{N}}$ admits a subsequence converging to a point $x_\infty \in X_\infty$, then x_∞ is a fixed point of T_∞.*

In view of Remark 1.2, we have the following result as a direct consequence of the above corollary.

Corollary 2.6. *Corollary 2.5 with φ-contraction replaced by k-contraction.*

The following theorem, which generalizes Theorem 1.7 is our first stability result.

Theorem 2.7. *Let (X, d) be a metric space and $\{d_n\}_{n\in\mathbb{N}}$ a sequence of metrics on X satisfying the property (A). Let $\{X_n\}_{n\in\overline{\mathbb{N}}}$ be a family of nonempty subsets of X and $\{T_n : X_n \to X_n\}_{n\in\mathbb{N}}$ a sequence of φ -contraction mappings on (X_n, d_n) converging in the sense of (G) to a mapping $T_\infty : X_\infty \to X$, where $\varphi-$is nondecreasing. If for each $n \in \overline{\mathbb{N}}$, x_n is a fixed point of T_n, then the sequence $\{x_n\}_{n\in\mathbb{N}}$ converges to x_∞.*

Proof. Since $x_\infty \in X_\infty$, by the property (G) there exists a sequence $\{y_n\}$ in $\prod\limits_{n\in\mathbb{N}} X_n$ such that:

$$\lim_n d(y_n, x_\infty) = 0 \text{ and } \lim_n d(T_n y_n, T_\infty x_\infty) = 0.$$

By (A), we deduce that:

$$\lim_n d_n(y_n, x_\infty) = 0 \text{ and } \lim_n d_n(T_n y_n, T_\infty x_\infty) = 0. \tag{2.3}$$

On the other hand, since $\varphi-$is nondecreasing, for any $n \in \mathbb{N}$,

$$\begin{aligned} d_n(x_n, x_\infty) &\leq d_n(T_n x_n, T_\infty x_\infty) \\ &\leq d_n(T_n x_n, T_n y_n) + d_n(T_n y_n, T_\infty x_\infty) \\ &\leq \varphi(d_n(x_n, y_n)) + d_n(T_n y_n, T_\infty x_\infty) \\ &\leq \varphi(d_n(x_n, x_\infty) + d_n(x_\infty, y_n)) + d_n(T_n y_n, T_\infty x_\infty). \end{aligned}$$

Let $\lim_n d(x_n, x_\infty) = r$. If $r = 0$, then there is nothing to prove. So, assume that $r > 0$. Now, making $n \to \infty$ in the above inequality and using (2.3), we obtain

$$r \leq \varphi(r) < r,$$

a contradiction. Hence $\lim_n d(x_n, x_\infty) = 0$ and the conclusion follows. □

When $X_n = X$ for all $n \in \overline{\mathbb{N}}$ in Theorem 2.7, we have the following

Corollary 2.8. *[12, Theorem 8.5] Let (X, d) be a metric space and $\{d_n\}_{n\in\mathbb{N}}$ a sequence of metrics on X satisfying the property (A_0). Let $\{T_n : X \to X\}_{n\in\mathbb{N}}$ be a sequence of φ-contraction mappings on (X, d_n) converging pointwise to a mapping $T_\infty : X_\infty \to X$, where φ is nondecreasing. If for each $n \in \overline{\mathbb{N}}$, x_n is a fixed point of T_n, then the sequence $\{x_n\}_{n\in\mathbb{N}}$ converges to x_∞.*

Corollary 2.9. *Let (X,d) be a metric space and $\{d_n\}_{n\in\mathbb{N}}$ a sequence of metrics on X satisfying the property (A). Let $\{X_n\}_{n\in\overline{\mathbb{N}}}$ be a family of nonempty subsets of X and $\{T_n : X_n \to X_n\}_{n\in\mathbb{N}}$ a sequence of k-contraction mappings on (X_n, d_n) converging in the sense of (G) to a mapping $T_\infty : X_\infty \to X$. If for each $n \in \overline{\mathbb{N}}$, x_n is a fixed point of T_n, then the sequence $\{x_n\}_{n\in\mathbb{N}}$ converges to x_∞.*

Proof. It comes from Theorem 2.7 when $\varphi(t) = kt$ and $k \in [0,1)$. □

The following result can be compared with Theorem 1.7.

Corollary 2.10. *Let (X,d) be a metric space and $\{d_n\}_{n\in\mathbb{N}}$ a sequence of metrics on X satisfying the property (A_0). Let $\{T_n : X \to X\}_{n\in\mathbb{N}}$ be a sequence of k-contraction mappings on (X, d_n) converging pointwise to a mapping $T_\infty : X \to X$. If for each $n \in \overline{\mathbb{N}}$, x_n is a fixed point of T_n, then the sequence $\{x_n\}_{n\in\mathbb{N}}$ converges to x_∞.*

The following theorem is our second stability result.

Theorem 2.11. *Let (X,d) be a metric space and $\{d_n\}_{n\in\mathbb{N}}$ a sequence of metrics on X satisfying the property (B). Let $\{X_n\}_{n\in\overline{\mathbb{N}}}$ be a family of nonempty subset of X and $\{T_n : X_n \to X_n\}_{n\in\mathbb{N}}$ a sequence of mappings on (X_n, d_n) converging in the sense of (H) to a φ-contraction mapping $T_\infty : X_\infty \to X$, where φ is nondecreasing. If for each $n \in \overline{\mathbb{N}}$, x_n is a fixed point of T_n, then the sequence $\{x_n\}_{n\in\mathbb{N}}$ converges to x_∞.*

Proof. By the property (H), there exists a sequence $\{y_n\}$ in X_∞ such that:

$$\lim_n d(x_n, y_n) = 0 \text{ and } \lim_n d(T_n x_n, T_\infty y_n) = 0.$$

Therefore by (B),

$$\lim_n d_n(x_n, y_n) = 0 \text{ and } \lim_n d_n(T_n x_n, T_\infty y_n) = 0. \tag{2.4}$$

Since T_∞ is a φ-contraction and φ is monotonic non-decreasing, we have

$$\begin{aligned} d_n(x_n, x_\infty) &\leq d_n(T_n x_n, T_\infty y_n) + d_n(T_\infty y_n, T_\infty x_\infty) \\ &\leq d_n(T_n x_n, T_\infty y_n) + \varphi(d_n(y_n, x_\infty)) \\ &\leq d_n(T_n x_n, T_\infty y_n) + \varphi(d_n(y_n, x_n) + d_n(x_n, x_\infty)). \end{aligned}$$

Let $\lim_n d(x_n, x_\infty) = r$. If $r = 0$, then we are done. Assume that $r > 0$. Now, making $n \to \infty$ in the above inequality and using (2.4), we obtain

$$r \leq \varphi(r) < r,$$

a contradiction. Hence $\lim_n d(x_n, x_\infty) = 0$ and the conclusion holds. □

When $X_n = X$ for all $n \in \overline{\mathbb{N}}$ in Theorem 2.11, we obtain the following.

Corollary 2.12. *Let (X, d) be a metric space and $\{d_n\}_{n\in\mathbb{N}}$ a sequence of metrics on X satisfying the property (B_0). Let $\{T_n : X \to X\}$ be a sequence of mappings on (X, d_n) converging uniformly to a φ-contraction mapping $T_\infty : X \to X$, where φ is nondecreasing. If for each $n \in \overline{\mathbb{N}}$, x_n is a fixed point of T_n, then the sequence $\{x_n\}_{n\in\mathbb{N}}$ converges to x_∞.*

Corollary 2.13. *Let (X, d) be a metric space and $\{d_n\}_{n\in\mathbb{N}}$ a sequence of metrics on X satisfying the property (B). Let $\{X_n\}_{n\in\overline{\mathbb{N}}}$ be a family of nonempty subset of X and $\{T_n : X_n \to X_n\}_{n\in\mathbb{N}}$ a sequence of mappings on (X_n, d_n) converging in the sense of (H) to a k-contraction mapping $T_\infty : X_\infty \to X$. If for each $n \in \overline{\mathbb{N}}$, x_n is a fixed point of T_n, then the sequence $\{x_n\}_{n\in\mathbb{N}}$ converges to x_∞.*

Proof. It comes from Theorem 2.11 when $\varphi(t) = kt$ and $k \in [0, 1)$. □

Corollary 2.14. *Corollary 2.12 with φ-contraction replaced by k-contraction.*

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