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A COMMON FIXED POINT THEOREM OF COMPATIBLE MAPPINGS OF TYPE (A) IN MENGER SPACES

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Abstract. In this paper, we establish a common fixed point theorem for four compatible mappings of type (A) in Menger spaces under a new contraction condition. Also, an example is given to justify our results.

Keywords: T-norm; Menger spaces; Common fixed points; Compatible maps.

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1. Introduction

There have been a number of generalizations of metric space. One such generalization is Menger space introduced in 1942 by Menger [6] who was use distribution functions instead of non-negative real numbers as values of the metric. Schweizer and Sklar [11] and [12] studied this concept and gave some fundamental results on this space. The important development of fixed-point theory in Menger spaces was due to Sehgal and Bharucha-Reid [13]. The theory of probabilistic metric spaces is of fundamental importance in probabilistic functional analysis. For the detailed discussions of these spaces and their applications

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we refer to [3],[16],[17],[18],[19].

Sessa [14] introduced weakly commuting maps in metric spaces. Jungck [4] enlarged this concept to compatible maps. The notion of compatible maps in Menger spaces has been introduced by Mishra [7]. In 1993, Jungck, Murthy and Cho [5] generalize the concept of compatible mappings into compatible mappings of type (A). Recently, some fixed point theorems in Menger spaces have been proved by many, authors, Radu [8]-[10], Stojakovic [20],[21], Dedeic and Sarapa [2], Cho, Murthy and Stojakovic [1] and others under various contractive conditions.

In this paper, a common fixed point theorem is proved for four compatible mappings of type (A) in Menger space, which satisfy a new contraction condition and a fixed point theorem in a metric space as a corollary.

2. Preliminaries

Let R denote the set of reals, R^+ the nonnegative reals and N denote the set of all natural numbers. A mappings $F : R^+ \to R^+$ is called a distribution function if it is nondecreasing and left continuous with $\inf F = 0$ and $\sup F = 1$. We will denote $\triangle$ by the set of all distribution functions.

A probabilistic metric space (briefly, PM-space) is a pair (X, ξ) where X is a nonempty set and ξ is a mapping from $X \times X$ to $\triangle$. For $(u, v) \in X \times X$, the distribution function $F(u, v)$ is denoted by $F_{u,v}$. The function $F_{u,v}$. are assumed to satisfy the following conditions:

(P1) $F_{u,v}(x) = 1 \quad$ for every $\quad x > 0 \quad$ iff $\quad u = v$,

(P2) $F_{u,v}(0) = 0 \quad$ for every $\quad u, v \in X$,

(P3) $F_{u,v}(x) = F_{v,u}(x) \quad$ for every $\quad u, v \in X$,

(P4) if $F_{u,w}(x) = 1 \quad$ and $F_{w,v}(y) = 1 \quad$ then $F_{u,v}(x + y) = 1$ for every $\quad u, v, w \in X$.

In a metric space (X, d) the metric d induces a mapping $F : X \times X \to \triangle$ such that

$$F(u, v)(x) = F_{u,v}(x) = H(x - d(u, v)),$$

for every $u, v \in X$ and $x \in R$, where H is a specific distribution function defined by

$$H(x) = \begin{cases} 0, & x \leq 0, \\ \\ 1, & x > 0. \end{cases}$$

The following definitions and lemmas are needed in the sequel.

Definition 2.1[11]. *A T-norm is a function $t : [0,1] \times [0,1] \to [0,1]$ which satisfies:*

(T1) $t(a,1) = a$ *and* $t(0,0) = 0$,

(T2) $t(a,b) = t(b,a)$,

(T3) $t(c,d) \geq t(a,b), \quad c \geq a, d \geq b$,

(T4) $t(t(a,b),c) = t(a,t(b,c))$.

Definition 2.2[11]. *A Menger space is an order triple (X, ξ, t) where (X, ξ) is a probabilistic metric space and t is T-norm satisfying:*

$(P4)'$ $\quad F_{u,v}(x+y) \geq t(F_{u,w}(x), F_{w,v}(y))$ *for all* $u, v, w \in X$ *and* $x, y \geq 0$.

B. Schweizer and A. Sklar [11] *introduced particular spaces of PM-spaces which are called simple spaces.*

Definition 2.3 *An PM-space (X, ξ) is said to be a simple space if and only if there exists a metric d on X and a distribution function G satisfying $G(0) = 0$, such that for every x, y in X*

$$F_{x,y}(u) = \begin{cases} G(\frac{u}{d(x,y)}), & x \neq y, \\ \\ H(u), & x = y. \end{cases}$$

Furthermore, we say that (X, ξ) is the simple space generated by the metric space (X, d) and the distribution function G.

Theorem 2.4[11]. *A simple space is a Menger space under any choice of T satisfying* $(T1), (T2), (T3) and (T4)$.

Schweizer and Sklar [11] *pointed out, if T-norm t of Menger space (X, F, t) is continuous, then there exists a topology on X such that X, τ is a Hausdorff topological space in the topology τ induced by the family of neighbourhoods $\{U_x(\epsilon, \lambda) : x \in X, \epsilon > 0, \lambda > 0\}$ where*

$$U_x(\epsilon, \lambda) = \{y \in X; F_{x,y}(\epsilon) > 1 - \lambda\}.$$

Definition 2.5. *A sequnce $\{x_n\}$ in a Menger space X is said to be convergent to a point $x \in X$ if for every $\epsilon > 0$ and $\lambda > 0$, there is an integer $N(\epsilon, \lambda)$ such that*

$$F_{x_n,x}(\epsilon) > 1 - \lambda \quad \textit{for all} \quad n \geq N(\epsilon, \lambda).$$

The sequence $\{x_n\}$ is called a Cauchy sequence if for each $\epsilon > 0$ and $\lambda > 0$, there is an integer $N(\epsilon, \lambda)$ such that $F_{x_n,x_m}(\epsilon) > 1 - \lambda$ for all $n, m \geq N(\epsilon, \lambda)$.

An important $T-$norm is the T-norm $t(a, b) = \min\{a, b\}, a, b \in [0, 1]$ and this is the unique $T-$norm such that $t(a, a) \geq a$ for every $a \in [0, 1]$. Indeed if it satisfies this condition, we have

$$\begin{aligned} \min\{a, b\} &\leq t(\min\{a, b\}, \{a, b\}) \leq t(a, b) \\ &\leq t(\min\{a, b\}, 1) = \min\{a, b\} \end{aligned}$$

Therefore, $t = \min$.

Theorem 2.6 [8]. *Let t be a T-norm defined by $t(a, b) = \min\{a, b\}$. Then an induced Menger space $\{X, \xi, t\}$ is complete if a metric space (X, d) is complete.*

For complete topological preliminaries on Menger spaces see, for example [12].

G. Jungck [4] introduced more generalized commuting mappings, called compatible mappings, which are more general than those of weakly commuting mappings. In general, commuting mappings are weakly commuting mappings and weakly commuting mappings are compatible mappings but the converse is not true.

Recently, G. Jungck et al. [5] defined the concept of compatible mappings of type (A) which is equivalent to the concept of compatible mappings under some conditions and proved a common fixed point theorem for compatible mappings of type (A) in metric spaces.

Further, S. N. Mishra [7] and Y. J. Cho et al. [1] introduced the concept of compatible mappings and compatible mappings of type (A) respectively in Menger spaces as follows.

Definition 2.7 [7]. *Two self mappings S and T of Menger space (X, ξ, t), where t is continuous will be called compatible if and only if $F_{STx_n,TSx_n}(u) \to 1$ for all $u > 0$, whenever $\{x_n\}$ is a sequence in X such that $Sx_n, Tx_n \to z$ for some $z \in X$.*

Definition 2.8 [1]. *Let (X, ξ, t)be a Menger space such that T-norm t is continuous and S, T be mappings from X into itself. S and T are said to be compatible of type (A) if*

$$\lim_{n\to\infty} F_{TSx_n,SSx_n}(u) = 1 \quad \textit{and} \quad \lim_{n\to\infty} F_{STx_n,TTx_n}(u) = 1, \quad \textit{for} \quad u > 0,$$

whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = z$ for some $z \in X$.

The following Proposition 2.9 and 2.10 show that the Definitions 2.7 and 2.8 are equivalent under some condition [1].

Proposition 2.9. *Let (X, ξ, t)be a Menger space such that T-norm t is continuous and $t(x, x) \geq x$ for all $x \in [0, 1]$, and let $S, T : X \to X$ be continuous. If S and T are compatible, then they are compatible of type (A).*

Proposition 2.10. *Let (X, ξ, t)be a Menger space such that T-norm t is continuous and $t(x, x) \geq x$ for all $x \in [0, 1]$, and let $S, T : X \to X$ be compatible of type (A). If one of S and T is continuous, then S and T are compatible.*

As a direct consequence of Proposition 2.9 and 2.10 we have the following [1].

Proposition 2.11 *Let (X, ξ, t)be a Menger space such that T-norm t is continuous and $t(u, u) \geq u$ for all $u \in [0, 1]$, and let $S, T : X \to X$ be mappings. If S and T are compatible mappings of type (A) and $Sz = Tz$ for some $z \in X$, then $STz = TTz = TSz = SSz$.*

Proposition 2.12.*Let (X, ξ, t) be a Menger space such that T-norm t is continuous and $t(u, u) \geq u$ for all $u \in [0, 1]$, and $S, T : X \to X$ be mappings. Let S and T be compatible mappings of type (A) and $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = z$ for some $z \in X$. Then we have*

(1) $\lim_{n\to\infty} TSx_n = Sz$ *if S is continuous at z.*

(2) $STz = TSz and Sz = Tz$ *if S and T are continuous at z.*

3. A Common Fixed Point Theorem

Before proving our main theorem, we need the following lemma.

Lemma 3.1 [16]. *Let $\{x_n\}$ be a sequence in Menger space (X, ξ, t), where t is continuous and*

$t(u, u) \geq u$ for all $u \in [0, 1]$. If there exists a constant $k \in (0, 1)$ such that

$$F_{x_n,x_{n+1}}(kx) \geq F_{x_{n-1},x_n}(x),$$

for all $x > 0$ and $n \in N$, then $\{x_n\}$ is a Cauchy sequence. Now, we are ready to give our main theorem.

Theorem 3.2. *Let (X, ξ, t) be a complete Menger space with $t(x, y) = \min\{x, y\}$ for all $x, y \in [0, 1]$ and A, B, S, T be mappings from X into itself such that*

(3.1) $A(X) \subset T(X)$ *and* $B(X) \subset S(X)$,

(3.2) the pair $\{A, S\}$ and $\{B, T\}$ are compatible of type (A),

(3.3) one of A, B, S and T is continuous,

(3.4) there exists a constant $k \in (0, 1)$ such that

$$\begin{aligned}(F_{Ax,By}(ku))^2 \geq \min \quad &\{F_{Sx,Ax}(u)F_{Ty,By}(u), F_{Sx,By}(2u)F_{Ty,Ax}(u), F_{Sx,Ax}(u)F_{Sx,By}(2u),\\ &F_{Ty,Ax}(u)F_{Ty,By}(u), F_{Sx,Ax}(u)F_{Ty,Ax}(u), F_{Sx,By}(2u)F_{Ty,By}(u),\\ &(F_{Sx,Ax}(u))^2, (F_{Ty,By}(u))^2, (F_{Sx,Ty}(u))^2\},\end{aligned}$$

for all x, $y \in X$ and $u > 0$. Then A, B, S and T Have a unique common fixed point in X.

Proof. By (3.1), since A(X) ⊂ T(X), for any $x_0 \in X$, there exists a point $x_1 \in X$ such that $Ax_0 = Tx_1$. Since $B(X) \subset S(X)$, for this point x_1, we can choose a point $x_2 \in X$

such that $Bx_1 = Sx_2$ and so on. Inductively, one can define a sequence $\{y_n\}$ such that

$$\begin{aligned} y_{2n} &= Tx_{2n+1} = Ax_{2n} \\ y_{2n+1} &= Sx_{2n+2} = Bx_{2n+1}, \quad n \geq 0. \end{aligned}$$

Now, we prove $F_{y_{2n},y_{2n+1}}(ku) \geq F_{y_{2n-1},y_{2n}}(u)$ for all u > 0, where k $\in (0, 1)$. Suppose that $F_{y_{2n},y_{2n+1}}(ku) < F_{y_{2n-1},y_{2n}}(u)$ for some u > 0. Then by using (3.4) and $F_{y_{2n},y_{2n+1}}(ku) \leq F_{y_{2n-1},y_{2n}}(u)$, we have

$$\begin{aligned} (F_{y_{2n},y_{2n+1}}(ku))^2 &\geq (F_{Ax_{2n},Bx_{2n+1}}(ku))^2 \\ &> min\{F_{Sx_{2n},Ax_{2n}}(u)F_{Tx_{2n+1},Bx_{2n+1}}(u), F_{Sx_{2n},Bx_{2n+1}}(2u)F_{Tx_{2n+1},Ax_{2n}}(u), \\ &\quad F_{Sx_{2n},Ax_{2n}}(u)F_{Sx_{2n},Bx_{2n+1}}(2u), F_{Tx_{2n+1},Ax_{2n}}(u)F_{Tx_{2n+1},Bx_{2n+1}}(u), \\ &\quad F_{Sx_{2n},Ax_{2n}}(u)F_{Tx_{2n+1},Ax_{2n}}(u), F_{Sx_{2n},Bx_{2n+1}}(2u)F_{Tx_{2n+1},Bx_{2n+1}}(u), \\ &\quad (F_{Sx_{2n},Ax_{2n}}(u))^2, (F_{Tx_{2n+1},Bx_{2n+1}}(u))^2, (F_{Sx_{2n},Tx_{2n+1}}(u))^2\} \\ &= min\{F_{y_{2n-1},y_{2n}}(u)F_{y_{2n},y_{2n+1}}(u), F_{y_{2n-1},y_{2n+1}}(2u)F_{y_{2n},y_{2n}}(u), \\ &\quad F_{y_{2n-1},y_{2n}}(u)F_{y_{2n-1},y_{2n+1}}(2u), F_{y_{2n},y_{2n}}(u)F_{y_{2n},y_{2n+1}}(u), \\ &\quad F_{y_{2n-1},y_{2n}}(u)F_{y_{2n},y_{2n}}(u), F_{y_{2n-1},y_{2n+1}}(2u)F_{y_{2n},y_{2n+1}}(u), \\ &\quad (F_{y_{2n-1},y_{2n}}(u))^2, (F_{y_{2n},y_{2n+1}}(u))^2, (F_{y_{2n-1},y_{2n}}(u))^2\} \\ &> min\{F_{y_{2n-1},y_{2n}}(u)F_{y_{2n},y_{2n+1}}(u), t(F_{y_{2n-1},y_{2n}}(u), F_{y_{2n},y_{2n+1}}(u)), \\ &\quad F_{y_{2n-1},y_{2n}}(u)t(F_{y_{2n-1},y_{2n}}(u), F_{y_{2n},y_{2n+1}}(u)), F_{y_{2n},y_{2n+1}}(u), \\ &\quad F_{y_{2n+1},y_{2n}}(u)t(F_{y_{2n-1},y_{2n}}(u), F_{y_{2n},y_{2n+1}}(u)), F_{y_{2n},y_{2n+1}}(u), \\ &\quad (F_{y_{2n-1},y_{2n}}(u))^2, (F_{y_{2n},y_{2n+1}}(u))^2, (F_{y_{2n-1},y_{2n}}(u))^2\} \\ &> min\{(F_{y_{2n},y_{2n+1}}(ku))^2, F_{y_{2n},y_{2n+1}}(ku), (F_{y_{2n},y_{2n+1}}(ku))^2, \\ &\quad F_{y_{2n},y_{2n+1}}(ku), F_{y_{2n},y_{2n+1}}(ku), (F_{y_{2n},y_{2n+1}}(ku))^2, \\ &\quad (F_{y_{2n},y_{2n+1}}(u))^2, (F_{y_{2n},y_{2n+1}}(u))^2, (F_{y_{2n},y_{2n+1}}(u))^2\} \\ &= (F_{y_{2n},y_{2n+1}}(u))^2, \end{aligned}$$

which is a contradiction. Thus, we have

$$F_{y_{2n},y_{2n+1}}(ku) \geq F_{y_{2n-1},y_{2n}}(u).$$

Similarly, we obtain

$$F_{y_{2n+1},y_{2n+2}}(ku) \geq F_{y_{2n},y_{2n+1}}(u).$$

Therefore

$$F_{y_n,y_{n+1}}(ku) \geq F_{y_{n-1},y_n}(u),$$

for all n $\in$ N and $u > 0$.

Hence by Lemma 3.1, it follows that $\{y_n\}$ is a Cauchy sequence in X. Since the Menger space (X, ξ, t) is complete, $\{y_n\}$ converges to a point z in X and the subsequences Ax_{2n},Sx_{2n}, Bx_{2n+1} and Tx_{2n-1} also converges to z.

Now suppose that T is continuous, Since B and T are compatible of type (A), then by Proposition 2.12, we have

$$BTx_{2n+1}, TTx_{2n+1} \to Tz \text{ } as \text{ } n \to \infty.$$

Putting x = x_{2n} and y = Tx_{2n+1} in (3.4), we have

$$\begin{aligned}
(F_{Ax_{2n},BTx_{2n+1}}(ku))^2 \quad \geq \quad & min\{F_{Sx_{2n},Ax_{2n}}(u)F_{TTx_{2n+1},BTx_{2n+1}}(u), F_{Sx_{2n},BTx_{2n+1}}(2u)F_{TTx_{2n+1},Ax_{2n}}(u), \\
& F_{Sx_{2n},Ax_{2n}}(u)F_{Sx_{2n},BTx_{2n+1}}(2u), F_{TTx_{2n+1},Ax_{2n}}(u)F_{TTx_{2n+1},BTx_{2n+1}}(u), \\
& F_{Sx_{2n},Ax_{2n}}(u)F_{TTx_{2n+1},Ax_{2n}}(u), F_{Sx_{2n},BTx_{2n+1}}(2u)F_{TTx_{2n+1},BTx_{2n+1}}(u), \\
& (F_{Sx_{2n},Ax_{2n}}(u))^2, (F_{TTx_{2n+1},BTx_{2n+1}}(u))^2, (F_{Sx_{2n},TTx_{2n+1}}(u))^2\}
\end{aligned}$$

Taking limit as n $\to \infty$, we have

$$\begin{aligned}
(F_{z,Tz}(ku))^2 \quad \geq \quad & min\{F_{z,z}(u)F_{Tz,Tz}(u), F_{z,Tz}(2u)F_{Tz,z}(u), F_{z,z}(u)F_{z,Tz}(2u), \\
& F_{Tz,z}(u)F_{Tz,Tz}(u), F_{z,z}(u)F_{Tz,z}(u), F_{z,Tz}(2u)F_{Tz,Tz}(u), \\
& (F_{z,z}(u))^2, (F_{Tz,Tz}(u))^2, (F_{z,Tz}(u))^2\} \\
= \quad & (F_{z,Tz}(u))^2,
\end{aligned}$$

which implies that Tz = z. Again replacing x by x_{2n} and y by z in (3.4), we have

$$\begin{aligned}(F_{Ax_{2n},Bz}(ku))^2 \;\; \geq \;\; & min\{F_{Sx_{2n},Ax_{2n}}(u)F_{Tz,Bz}(u), F_{Sx_{2n},Bz}(2u)F_{Tz,Ax_{2n}}(u),\\ & F_{Sx_{2n},Ax_{2n}}(u)F_{Sx_{2n},Bz}(2u), F_{Tz,Ax_{2n}}(u)F_{Tz,Bz}(u),\\ & F_{Sx_{2n},Ax_{2n}}(u)F_{Tz,Ax_{2n}}(u), F_{Sx_{2n},Bz}(2u)F_{Tz,Bz}(u),\\ & (F_{Sx_{2n},Ax_{2n}}(u))^2, (F_{Tz,Bz}(u))^2, (F_{Sx_{2n},Tz}(u))^2\}\end{aligned}$$

Taking limit as n $\to \infty$ and using Tz = z, we have

$$\begin{aligned}(F_{z,Bz}(ku))^2 \;\; \geq \;\; & min\{F_{z,z}(u)F_{z,Bz}(u), F_{z,Bz}(2u)F_{z,z}(u), F_{z,z}(u)F_{z,Bz}(2u),\\ & F_{z,z}(u)F_{z,Bz}(u), F_{z,z}(u)F_{z,z}(u), F_{z,Bz}(2u)F_{z,Bz}(u),\\ & (F_{z,z}(u))^2, (F_{z,Bz}(u))^2, (F_{z,z}(u))^2\}\\ = \;\; & (F_{z,Bz}(u))^2,\end{aligned}$$

which implies that Bz = z. Since B(X) $\subset$ S(X), there exists a point w in X such that Bz = Sw = z.

Again by using (3.4), we have

$$\begin{aligned}(F_{Aw,z}(ku))^2 \;\; = \;\; & (F_{Aw,Bz}(ku))^2\\ \geq \;\; & min\{F_{Sw,Aw}(u)F_{Tz,Bz}(u), F_{Sw,Bz}(2u)F_{Tz,Aw}(u), F_{Sw,Aw}(u)F_{Sw,Bz}(2u),\\ & F_{Tz,Aw}(u)F_{Tz,Bz}(u), F_{Sw,Aw}(u)F_{Tz,Aw}(u), F_{Sw,Bz}(2u)F_{Tz,Bz}(u),\\ & (F_{Sw,Aw}(u))^2, (F_{Tz,Bz}(u))^2, (F_{Sw,Tz}(u))^2\}\\ = \;\; & min\{F_{z,Aw}(u)F_{z,z}(u), F_{z,z}(2u)F_{z,Aw}(u), F_{z,Aw}(u)F_{z,z}(2u),\\ & F_{z,Aw}(u)F_{z,z}(u), F_{z,Aw}(u)F_{z,Aw}(u), F_{z,z}(2u)F_{z,z}(u),\\ & (F_{z,Aw}(u))^2, (F_{z,z}(u))^2, (F_{z,z}(u))^2\}\\ = \;\; & (F_{z,Aw}(u))^2,\end{aligned}$$

which implies that Aw = z. Since A and S are compatible of type (A) and Aw = Sw = z by Proposition 2.12, we have

$$Az = ASw = SSw = Sz.$$

By using (3.4) again, we have Az = z. Therefore Az = Bz = Sz = Tz = z, that is z is a common fixed point of A, B, S and T. For uniqueness, let $\acute{z}$ be another common fixed point such that $\acute{z} \neq z$. Then by (3.4), we have

$$\begin{aligned}
(F_{z,\acute{z}}(ku))^2 &= (F_{Az,B\acute{z}}(ku))^2 \\
&\geq min\{F_{Sz,Az}(u)F_{T\acute{z},B\acute{z}}(u), F_{Sz,B\acute{z}}(2u)F_{T\acute{z},Az}(u), F_{Sz,Az}(u)F_{Sz,B\acute{z}}(2u), \\
&\quad F_{T\acute{z},Az}(u)F_{T\acute{z},B\acute{z}}(u), F_{Sz,Az}(u)F_{T\acute{z},Az}(u), F_{Sz,B\acute{z}}(2u)F_{T\acute{z},B\acute{z}}(u), \\
&\quad (F_{Sz,Az}(u))^2, (F_{T\acute{z},B\acute{z}}(u))^2, (F_{Sz,T\acute{z}}(u))^2\} \\
&= (F_{z,\acute{z}}(u))^2,
\end{aligned}$$

which means that z = $\acute{z}$. Thus z is a unique common fixed point of A, B, S and T.

As a consequence of Theorem 2.6 and Theorem 3.2, we have the following corollary in a metric space.

Corollary 3.3. *Let A, B, S, T be mappings from a complete metric space (X, d) into itself such that*

(3.5) $A(X) \subset T(X)$ *and* $B(X) \subset S(X)$,

(3.6) the pair $\{A, S\}$ *and* $\{B, T\}$ *are compatible of type (A),*

(3.7) one of A, B, S *and* T *is continuous,*

(3.8) there exists a constant $k \in (0, 1)$ *such that*

$$\begin{aligned}
(d(Ax, By))^2 \leq\ & k\ max\{d(Sx, Ax)d(Ty, By), \frac{1}{2}d(Sx, By)d(Ty, Ax), \frac{1}{2}d(Sx, Ax)d(Sx, By), \\
& d(Ty, Ax)d(Ty, By), d(Sx, Ax)d(Ty, Ax), \frac{1}{2}d(Sx, By)d(Ty, By) \\
& (d(Sx, Ax))^2, (d(Ty, By))^2, (d(Sx, Ty))^2\},
\end{aligned}$$

for all x, $y \in X$*. Then A, B, S and T have a unique common fixed point in X.*

Now we give an example of a PM-space and two pairs of mappings which satisfy all axioms of Theorem 3.2.

Example 3.4. *Let X = [0, 1] with the Euclidean metric d and let* $\xi : X \times X \to D$ *be defined as*

$$F_{x,y}(u) = \begin{cases} G(\frac{u}{d(x,y)}), & x \neq y, \\ H(u), & x = y \end{cases}$$

for all x, y $\in$ *X, where G(X) is any distribution function such that G(0) = 0. Then, (X,* ξ*) is a simple space and using Theorem 2.4, the space (X,* ξ*, t) will be Menger space with 't = min'.*

Define the mappings A, B, S and T as the following:

$$Ax = \begin{cases} 0, & 0 \leq x < 1, \\ \frac{1}{12}, & x = 1, \end{cases} \quad Sx = \frac{x}{4}, \quad 0 \leq x \leq 1$$

$$Bx = \begin{cases} 0, & 0 \leq x < 1, \\ \frac{1}{4}, & x = 1, \end{cases} \quad Tx = x, \quad 0 \leq x \leq 1.$$

These mappings satisfy the conditions (3.1), (3.2) and (3.3). Moreover, for $k = \frac{3}{4}$ *and* $u \geq 0$*, A, B, S and T satisfying (3.4) as follows:*

(I) If x = y = 1, then

L.H.S. of (3.4) $= (F_{\frac{1}{12},\frac{1}{4}}(\frac{3u}{4}))^2 = (G(\frac{9u}{2}))^2$.

$$\begin{aligned} R.H.S.\ of\ (3.4) &= min\{F_{\frac{1}{4},\frac{1}{12}}(u)F_{\frac{1}{4},\frac{1}{4}}(u), F_{\frac{1}{4},\frac{1}{4}}(2u)F_{\frac{1}{4},\frac{1}{12}}(u), F_{\frac{1}{4},\frac{1}{12}}(u)F_{\frac{1}{4},\frac{1}{4}}(2u), \\ &\quad F_{1,\frac{1}{12}}(u)F_{1,\frac{1}{4}}(u), F_{\frac{1}{4},\frac{1}{12}}(u)F_{1,\frac{1}{12}}(u), F_{\frac{1}{4},\frac{1}{4}}(2u)F_{1,\frac{1}{4}}(u), \\ &\quad (F_{\frac{1}{4},\frac{1}{12}}(u))^2, (F_{1,\frac{1}{12}}(u))^2, (F_{\frac{1}{4},1}(u))^2\} \\ &\leq (F_{1,\frac{1}{4}}(u))^2 = (G(\frac{4u}{3}))^2. \end{aligned}$$

Hence (3.4) is satisfied.

(II) If $0 \leq x, y < 1$*, then (3.4) is trivially satisfied.*

(III) If $x = 1$, $0 \leq y < 1$, then

L.H.S. of (3.4) $= (F_{\frac{1}{12},0}(\frac{3u}{4}))^2 = (G(9u))^2$.

$$\begin{aligned} R.H.S.\ of\ (3.4) &= min\{F_{\frac{1}{4},\frac{1}{12}}(u)F_{y,0}(u), F_{\frac{1}{4},0}(2u)F_{y,\frac{1}{12}}(u), F_{\frac{1}{4},\frac{1}{12}}(u)F_{\frac{1}{4},0}(2u), \\ &\quad F_{y,\frac{1}{12}}(u)F_{y,0}(u), F_{\frac{1}{4},\frac{1}{12}}(u)F_{y,\frac{1}{12}}(u), F_{\frac{1}{4},0}(2u)F_{y,0}(u), \\ &\quad (F_{\frac{1}{4},\frac{1}{12}}(u))^2, (F_{y,0}(u))^2, (F_{\frac{1}{4},y}(u))^2\} \\ &\leq (F_{\frac{1}{4},\frac{1}{12}}(u))^2 = (G(6u))^2. \end{aligned}$$

Hence (3.4) is satisfied.

(IV) If $0 \leq x < 1$, $y = 1$then

L.H.S. of (3.4) $= (F_{0,\frac{1}{4}}(\frac{3u}{4}))^2 = (G(4u))^2$.

$$\begin{aligned} R.H.S.\ of\ (3.4) &= min\{F_{x,0}(u)F_{1,\frac{1}{4}}(u), F_{x,\frac{1}{4}}(2u)F_{1,0}(u), F_{x,0}(u)F_{x,\frac{1}{4}}(2u), \\ &\quad F_{1,0}(u)F_{1,\frac{1}{4}}(u), F_{x,0}(u)F_{1,0}(u), F_{x,\frac{1}{4}}(2u)F_{1,\frac{1}{4}}(u), \\ &\quad (F_{x,0}(u))^2, (F_{1,\frac{1}{4}}(u))^2, (F_{x,1}(u))^2\} \\ &\leq (F_{1,\frac{1}{4}}(u))^2 = (G(\frac{4u}{3}))^2. \end{aligned}$$

Hence (3.4) is satisfied. Note that 0 is the unique common fixed point of A, B, S and T.

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