Available online at http://scik.org
Adv. Fixed Point Theory, 2023, 13:21
https://doi.org/10.28919/afpt/8218
ISSN: 1927-6303

SOME FIXED POINTS RESULTS USING (ψ,φ) GENERALIZED WEAKLY CONTRACTIVE MAP ON A GENERALIZED 2-METRIC SPACE

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Abstract. The main purpose of this paper is to define a generalized 2-metric and prove the existence and uniqueness of fixed points for (ψ,φ) generalized weakly contractive mappings in a generalized 2-metric space.

Keywords: fixed points; weak contraction; sub-linear.

2020 AMS Subject Classification: 47H10, 54H25.

1. INTRODUCTION

The study of fixed point theorems for self maps satisfying different contraction conditions is the center of rigorous research activities. Dutta et al. introduced (ψ,φ)-weakly contractive maps in 2008 and obtained some fixed point results for such contractions, [4]. Later, G. V. R. Babu et al. introduced (ψ,φ)-almost weakly contractive maps in G-metric space, [1]. Fixed points of contractive maps on S-metric spaces were studied by several authors and since then, several contractions have been considered for proving fixed point theorems, [6, 2, 3, 10]. The authors D. Venkatesh et al. further proved some fixed point outcomes in S_b-metric spaces using (ψ,φ)-generalized weakly contractive maps in S_b-metric spaces, [7].

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Received September 14, 2023

The concept of an area of a triangle in $\mathbb{R}^2$ inspired, Gähler to introduced the concept of a 2-metric, as a generalization of the metric, [5].

Definition 1.1. [5] *Let X be a non-empty set and $d: X \times X \times X \to [0,\infty)$ be a map satisfying the following properties*
(i) If $x,y,z \in X$ then $d(x,y,z) = 0$ only if at least two of the three points are the same.
(ii) For $x,y \in X$ such that $x \neq y$ there exists a point $z \in X$ such that $d(x,y,z) \neq 0$.
(iii) symmetry property: for $x,y,z \in X$,

$$d(x,y,z) = d(x,z,y) = d(y,x,z) = d(y,z,x) = d(z,x,y) = d(z,y,x).$$

(iv) rectangle inequality:

$$d(x,y,z) \leq d(x,y,t) + d(y,z,t) + d(z,x,t)$$

for $x,y,z,t \in X$.
Then d is a 2-metric and (X,d) is a 2-metric space.

Definition 1.2. *Let X be a non-empty set and $d: X \times X \times X \to [0,\infty)$ be a map satisfying the following properties:*
(i) If $x,y,z \in X$ then $d(x,y,z) = 0$ only if at least two of the three points are the same.
(ii) For $x,y \in X$ such that $x \neq y$ there exists a point $z \in X$ such that $d(x,y,z) \neq 0$.
(iii) symmetry property: for $x,y,z \in X$,

$$d(x,y,z) = d(x,z,y) = d(y,x,z) = d(y,z,x) = d(z,x,y) = d(z,y,x).$$

(iv) modified rectangle inequality:there exists $\alpha, \beta, \gamma \geq 1$ such that

$$d(x,y,z) \leq \alpha d(x,y,t) + \beta d(y,z,t) + \gamma d(z,x,t)$$

for $x,y,z,t \in X$.
Then d is a generalized 2-metric and (X,d) is a generalized 2- metric space.

If $\alpha = \beta = \gamma = 1$ then a generalized 2-metric is a 2-metric.

Definition 1.3. *Let (X,d) be a generalized 2-metric space. Let $x,y\in X$ and $\varepsilon>0$. Then the subset*

$$B_\varepsilon(x,y)=\{z\in X; d(x,y,z)<\varepsilon\}$$

of X is called a generalized 2-ball centered at x,y with radius ε. A topology can be generated in X by taking the collection of all generalized 2-balls as a subbasis, which we call the generalized 2-metric topology and is denoted by τ. Thus (X,τ) is a generalized 2-metric topological space. Members of τ are called 2-open sets. From the property of the metric is can easily be seen that $B_\varepsilon(x,y)=B_\varepsilon(y,x)$ for $\varepsilon>0$.

Example 1.4. *Let $X=[0,1]$ and define*

$$d(x,y,z)=\begin{cases} 0 & , \text{ only if at least two of the three points are the same} \\ e^{|x-y|+|y-z|+|z-x|} & , \text{ otherwise} \end{cases} \tag{1}$$

For $x,y,z\in X$ and using Jensens' inequality, we get

$$\begin{aligned}
&d(x,y,z)\\
&=e^{|x-y|+|y-z|+|z-x|}\\
&=e^{\frac{1}{2}|x-y|+\frac{1}{3}|y-z|+\frac{1}{6}|z-x|}e^{\frac{1}{2}|x-y|+\frac{2}{3}|y-z|+\frac{5}{6}|z-x|}\\
&\le e^2 e^{\frac{1}{2}|x-y|+\frac{1}{3}|y-z|+\frac{1}{6}|z-x|}\\
&\le e^2\left\{\frac{1}{2}e^{|x-y|}+\frac{1}{3}e^{|y-z|}+\frac{1}{6}e^{|z-x|}\right\}\\
&\le e^2\left\{\frac{1}{2}e^{|x-y|+|y-t|+|t-x|}+\frac{1}{3}e^{|z-y|+|y-t|+|t-z|}+\frac{1}{6}e^{|z-x|+|x-t|+|t-z|}\right\}\\
&=\alpha d(x,y,t)+\beta d(z,y,t)+\gamma d(z,x,t)
\end{aligned}$$

where $\alpha=\frac{1}{2}e^2\ge 1$, $\beta=\frac{1}{3}e^2\ge 1$ and $\gamma=\frac{1}{6}e^2\ge 1$. It follows that d is a generalized 2-metric.

Definition 1.5. *Let $\{x_n\}_{n\in\mathbb{N}}$ be a sequence in a generalized 2-metric space (X,d).*
a) the sequence $\{x_n\}_{n\in\mathbb{N}}$ is convergent to $x\in X$ iff for all $\xi\in X$,

$$\lim_{n\to\infty} d(x_n,x,\xi)=0.$$

b) the sequence $\{x_n\}_{n\in\mathbb{N}}$ *is a Cauchy sequence in* X *iff for all* $\xi \in X$,

$$\lim_{n,m\to\infty} d(x_n, x_m, \xi) = 0.$$

2. Main Result

Definition 2.1. [8] *A function* $\psi : [0,\infty) \to [0,\infty)$ *is an altering distance function if it satisfies:*
(i) ψ *is continuous and non-decreasing.*
(ii) $\psi(t) = 0 \iff t = 0$.

To this prove uniqueness and existence of a fixed point the definition was amended to include:
(iii) ψ is sublinear function.
Denote the class of all altering distances functions by Ψ.

Definition 2.2. *Let* (X,d) *be a generalized* $2-$*metric space and* $T : X \to X$ *is a contraction if there exists* $0 \le \lambda < 1$ *such that*

$$d(Tx, Ty, \xi) \le \lambda d(x, y, \xi)$$

for all $x, y, \xi \in X$.

In [9], authors have proved a similar result in a b_2 metric space with the additional property that the set is partially ordered.

Definition 2.3. *Let* (X,d) *be a generalized* 2*-metric space and a mapping* $T : X \to X$ *is a* (ψ,φ) *generalized almost weakly contraction if it satisfies the inequality*

$$\begin{aligned} &\beta\psi(d(Tx,Ty,\xi)) \\ &\le \psi\left(\max\left\{d(x,y,\xi), \tfrac{d(x,Tx,\xi)d(y,Ty,\xi)}{1+d(x,y,\xi)}, \tfrac{d(x,Tx,\xi)d(y,Ty,\xi)}{1+d(Tx,Ty,\xi)}\right\}\right) \\ &-\varphi\left(\max\left\{d(x,y,\xi), \tfrac{d(x,Tx,\xi)d(y,Ty,\xi)}{1+d(x,y,\xi)}, \tfrac{d(x,Tx,\xi)d(y,Ty,\xi)}{1+d(Tx,Ty,\xi)}\right\}\right) \\ &+\mu \min\{d(x,Tx,\xi), d(x,Ty,\xi), d(y,Tx,\xi), d(y,Ty,\xi)\} \end{aligned} \tag{2}$$

where $x, y, z \in X$, $\mu \ge 0$ *and* $\psi, \varphi \in \Psi$.

Theorem 2.4. *Let* (X,d) *be a generalized complete* $2-$*metric space and* $T : X \to X$ *be a* (ψ,φ) *generalized almost weakly contractive mapping. Then* T *has a unique fixed point.*

Proof. Let $x_0 \in X$ and define a sequence $\{x_n\}_{n\in\mathbb{N}}$ in X by

$$x_n = Tx_{n-1},$$

for all $n \in \mathbb{N}$. If $x_n = x_{n+1}$ for some $n \in \mathbb{N}$ then we have a fixed point. We assume that $x_n \neq x_{n+1}$ and we shall show that the sequence $\{d(x_n,x_{n+1},\xi)\}_{n\in\mathbb{N}}$ is a decreasing sequence of real numbers. By (2), we get

$$\begin{aligned}
&\psi(d(x_n,x_{n+1},\xi))\\
&= \psi(d(Tx_{n-1},Tx_n,\xi))\\
&\leq \frac{1}{\beta}\Big[\psi\Big(\max\Big\{d(x_{n-1},x_n,\xi),\tfrac{d(x_{n-1},Tx_{n-1},\xi)d(x_n,Tx_n,\xi)}{1+d(Tx_{n-1},Tx_n,\xi)},\tfrac{d(x_{n-1},Tx_{n-1},\xi)d(x_n,Tx_n,\xi)}{1+d(x_{n-1},x_n,\xi)}\Big\}\Big)\\
&-\varphi\Big(\max\Big\{d(x_{n-1},x_n,\xi),\tfrac{d(x_{n-1},Tx_{n-1},\xi)d(x_n,Tx_n,\xi)}{1+d(Tx_{n-1},Tx_n,\xi)},\tfrac{d(x_{n-1},Tx_{n-1},\xi)d(x_n,Tx_n,\xi)}{1+d(x_{n-1},x_n,\xi)}\Big\}\Big)\\
&+\mu\min\{d(x_{n-1},Tx_n,\xi),d(x_n,Tx_n,\xi),d(x_{n-1},Tx_{n-1},\xi),d(x_n,Tx_{n-1},\xi)\}\Big] \qquad (3)
\end{aligned}$$

since $\frac{1}{\beta} < 1$, we get

$$\begin{aligned}
&\psi(d(x_n,x_{n+1},\xi))\\
&= \psi(d(Tx_{n-1},Tx_n,\xi))\\
&\leq \psi\Big(\max\Big\{d(x_{n-1},x_n,\xi),\tfrac{d(x_{n-1},Tx_{n-1},\xi)d(x_n,Tx_n,\xi)}{1+d(Tx_{n-1},Tx_n,\xi)},\tfrac{d(x_{n-1},Tx_{n-1},\xi)d(x_n,Tx_n,\xi)}{1+d(x_{n-1},x_n,\xi)}\Big\}\Big)\\
&-\varphi\Big(\max\Big\{d(x_{n-1},x_n,\xi),\tfrac{d(x_{n-1},Tx_{n-1},\xi)d(x_n,Tx_n,\xi)}{1+d(Tx_{n-1},Tx_n,\xi)},\tfrac{d(x_{n-1},Tx_{n-1},\xi)d(x_n,Tx_n,\xi)}{1+d(x_{n-1},x_n,\xi)}\Big\}\Big)\\
&+\mu\min\{d(x_{n-1},Tx_n,\xi),d(x_n,Tx_n,\xi),d(x_{n-1},Tx_{n-1},\xi),d(x_n,Tx_{n-1},\xi)\} \qquad (4)
\end{aligned}$$

Inequality (4), can be reduced since

$$\begin{aligned}
&\min\{d(x_{n-1},Tx_n,\xi),d(x_n,Tx_n,\xi),d(x_{n-1},Tx_{n-1},\xi),d(x_n,Tx_{n-1},\xi)\}\\
&= \min\{d(x_{n-1},x_{n+1},\xi),d(x_n,x_{n+1},\xi),d(x_{n-1},x_n,\xi),d(x_n,x_n,\xi)\}\\
&= 0. \qquad (5)
\end{aligned}$$

Using (5), inequality (4) reduces to

$$\begin{aligned}
&\psi(d(x_n,x_{n+1},\xi))\\
&\leq \psi(\max\{d(x_{n-1},x_n,\xi),d(x_n,x_{n+1},\xi)\}) - \varphi(\max\{d(x_{n-1},x_n,\xi),d(x_n,x_{n+1},\xi)\}) \qquad (6)
\end{aligned}$$

Inequality (6) further reduces, if we assume that

$$\max\{d(x_{n-1},x_n,\xi),d(x_n,x_{n+1},\xi)\}=d(x_{n-1},x_n,\xi)$$

for otherwise, we assume that

$$\max\{d(x_{n-1},x_n,\xi),d(x_n,x_{n+1},\xi)\}=d(x_n,x_{n+1},\xi).$$

In the latter case, inequality (6), reduces to

$$\psi(d(x_n,x_{n+1},\xi))\leq\psi(d(x_n,x_{n+1},\xi))-\varphi(d(x_n,x_{n+1},\xi))$$

(7)

It follows that $0\leq-\varphi(d(x_n,x_{n+1},\xi))$ which leads to a contradiction. Thus $\max\{d(x_{n-1},x_n,\xi),d(x_n,x_{n+1},\xi)\}=d(x_{n-1},x_n,\xi)$. Hence, we have

$$\begin{aligned}\psi(d(x_n,x_{n+1},\xi))&\leq\psi(d(x_{n-1},x_n,\xi))-\varphi(d(x_{n-1},x_n,\xi))\\&\leq\psi(d(x_{n-1},x_n,\xi))\end{aligned}\tag{8}$$

It follows that $\{d(x_n,x_{n+1},\xi)\}_{n\in\mathbb{N}}$ is a decreasing sequence.

We next shall show that $\lim_{n\to\infty}d(x_n,x_{n+1},\xi)=0$. Suppose that $\lim_{n\to\infty}d(x_n,x_{n+1},\xi)=r$ where $r>0$ then taking limit as $n\to\infty$ in inequality (7) we get

$$\psi(r)\leq\psi(r)-\varphi(r)\tag{9}$$

which is a contradiction unless we have that $r=0$ thus $\lim_{n\to\infty}d(x_n,x_{n+1},\xi)=0$.

We next shall prove that $\{x_n\}_{n\in\mathbb{N}}$ is a Cauchy sequence in X. From the modified rectangular inequality we obtain,

$$\begin{aligned}d(x_n,x_m,\xi)&\leq\alpha d(x_n,x_m,x_{n+1})+\beta d(x_m,\xi,x_{n+1})+\gamma d(\xi,x_n,x_{n+1})\\&\leq\alpha d(x_n,x_{n+1},x_m)+\beta\alpha d(x_m,x_{m+1},\xi)+\beta^2 d(x_{n+1},x_{m+1},\xi)\\&+\beta\gamma d(x_m,x_{m+1},x_{n+1})+\gamma d(x_n,x_{n+1},\xi)\end{aligned}$$

(10)

Using properties of the altering distance functions we get,

$$\psi(d(x_n,x_m,\xi))$$
$$\leq \alpha\psi(d(x_n,x_{n+1},x_m))+\beta\alpha\psi(d(x_m,x_{m+1},\xi))+\beta^2\psi(d(x_{n+1},x_{m+1},\xi))$$
$$+\beta\gamma\psi(d(x_m,x_{m+1},x_{n+1}))+\gamma\psi(d(x_n,x_{n+1},\xi))$$

(11)

Using inequality (2) in (11) we get

$$\psi(d(x_n,x_m,\xi))$$
$$\leq \alpha\psi(d(x_n,x_{n+1},x_m))+\beta\alpha\psi(d(x_m,x_{m+1},\xi))$$
$$+\beta\psi\left(\max\left\{d(x_n,x_m,\xi),\frac{d(x_n,Tx_n,\xi)d(x_m,Tx_m,\xi)}{1+d(Tx_n,Tx_m,\xi)},\frac{d(x_n,Tx_n,\xi)d(x_m,Tx_m,\xi)}{1+d(x_n,x_m,\xi)}\right\}\right)$$
$$-\beta\varphi\left(\max\left\{d(x_n,x_m,\xi),\frac{d(x_n,Tx_n,\xi)d(x_m,Tx_m,\xi)}{1+d(Tx_n,Tx_m,\xi)},\frac{d(x_n,Tx_n,\xi)d(x_m,Tx_m,\xi)}{1+d(x_n,x_m,\xi)}\right\}\right)$$
$$+\beta\mu\min\{d(x_n,Tx_n,\xi),d(x_n,Tx_m,\xi),d(x_m,Tx_n,\xi),d(x_m,Tx_m,\xi)\}$$
$$+\beta\gamma\psi(d(x_m,x_{m+1},x_{n+1}))+\gamma\psi(d(x_n,x_{n+1},\xi)) \tag{12}$$

Taking $m,n\to\infty$ we get,

$$\lim_{m,n\to\infty}\max\left\{d(x_n,x_m,\xi),\frac{d(x_n,Tx_n,\xi)d(x_m,Tx_m,\xi)}{1+d(Tx_n,Tx_m,\xi)},\frac{d(x_n,Tx_n,\xi)d(x_m,Tx_m,\xi)}{1+d(x_n,x_m,\xi)}\right\}$$
$$=\lim_{m,n\to\infty}\max\left\{d(x_n,x_m,\xi),\frac{d(x_n,x_{n+1},\xi)d(x_m,x_{m+1},\xi)}{1+d(x_{n+1},x_{m+1},\xi)},\frac{d(x_n,x_{n+1},\xi)d(x_m,x_{m+1},\xi)}{1+d(x_n,x_m,\xi)}\right\}$$
$$=\lim_{m,n\to\infty}d(x_n,x_m,\xi) \tag{13}$$

and

$$\lim_{m,n\to\infty}\min\{d(x_n,Tx_n,\xi),d(x_n,Tx_m,\xi),d(x_m,Tx_n,\xi),d(x_m,Tx_m,\xi)\}$$
$$=\lim_{m,n\to\infty}\min\{d(x_n,x_{n+1},\xi),d(x_n,x_{m+1},\xi),d(x_m,x_{n+1},\xi),d(x_m,x_{m+1},\xi)\}$$
$$=0 \tag{14}$$

Taking $m,n\to\infty$ in (12), using (13) and (14) we get

$$\psi\left(\lim_{m,n\to\infty}d(x_n,x_m,\xi)\right)\leq\beta\psi\left(\lim_{m,n\to\infty}d(x_n,x_m,\xi)\right)-\beta\varphi\left(\lim_{m,n\to\infty}d(x_n,x_m,\xi)\right) \tag{15}$$

Inequality (15) is only true if $\lim_{m,n\to\infty} d(x_n,x_m,\xi) = 0$. Thus we conclude that $\{x_n\}_{n\in\mathbb{N}}$ is a Cauchy sequence in X. Since (X,d) is complete there exists $x' \in X$ such that $\lim_{m,n\to\infty} d(x_n,x',\xi) = 0$.

We now show that $Tx' = x'$. Replacing $x_n = x_{n+1}$, $x_m = Tx'$ in inequality (15), we get

$$\psi\left(\lim_{n\to\infty} d(x_{n+1},Tx',\xi)\right) \le \beta\psi\left(\lim_{n\to\infty} d(x_{n+1},Tx',\xi)\right) - \beta\varphi\left(\lim_{n\to\infty} d(x_{n+1},Tx',\xi)\right) \tag{16}$$

It follows that

$$\psi\left(d(x',Tx',\xi)\right) \le \beta\psi\left(d(x',Tx',\xi)\right) - \beta\varphi\left(d(x',Tx',\xi)\right) \tag{17}$$

which leads to a contradiction, unless we have $d(x',Tx',\xi) = 0$ i.e., $Tx' = x'$. To prove uniqueness of x', we assume that x'' is a fixed point of T such that $x' \neq x''$. From inequality (2),

$$\begin{aligned}
&\beta\psi(d(x',x'',\xi))\\
&\beta\psi(d(Tx',Tx'',\xi))\\
&\le \psi\left(\max\left\{d(x',x'',\xi), \tfrac{d(x',Tx',\xi)d(x'',Tx'',\xi)}{1+d(x',x'',\xi)}, \tfrac{d(x',Tx',\xi)d(x'',Tx'',\xi)}{1+d(Tx',Tx'',\xi)}\right)\right)\\
&-\varphi\left(\max\left\{d(x',x'',\xi), \tfrac{d(x',Tx',\xi)d(x'',Tx'',\xi)}{1+d(x',x'',\xi)}, \tfrac{d(x',Tx',\xi)d(x'',Tx'',\xi)}{1+d(Tx',Tx'',\xi)}\right)\right)\\
&+\mu\min\left\{d(x',Tx',\xi), d(x',Tx'',\xi), d(x'',Tx',\xi), d(x'',Tx'',\xi)\right\}
\end{aligned} \tag{18}$$

It follows that

$$\beta\psi(d(x',x'',\xi)) \le \psi\left(d(x',x'',\xi)\right) - \varphi\left(d(x',x'',\xi)\right)$$

(19)

is a contradiction unless $d(x',x'',\xi) = 0$ which implies that $x' = x''$. ☐

Corollary 2.5. *Let (X,d) be a generalized complete 2-metric space and a mapping $T : X \to X$ be a self mapping. If there exists $\psi,\varphi \in \Psi$ such that*

$$\begin{aligned}
\beta\psi(d(Tx,Ty,\xi)) \le \psi&\left(\max\left\{d(x,y,\xi), \tfrac{d(x,Tx,\xi)d(y,Ty,\xi)}{1+d(x,y,\xi)}, \tfrac{d(x,Tx,\xi)d(y,Ty,\xi)}{1+d(Tx,Ty,\xi)}\right\}\right)\\
&-\varphi\left(\max\left\{d(x,y,\xi), \tfrac{d(x,Tx,\xi)d(y,Ty,\xi)}{1+d(x,y,\xi)}, \tfrac{d(x,Tx,\xi)d(y,Ty,\xi)}{1+d(Tx,Ty,\xi)}\right\}\right)
\end{aligned}$$

(20)

where $x,y,z \in X$ and $\psi,\varphi \in \Psi$. Then T has a unique fixed point.

Proof. Follows from theorem 2.4 by taking $\mu = 0$. □

Corollary 2.6. *Let (X,d) be a generalized complete 2-metric space and a mapping $T : X \to X$ be a self mapping. If there exists $\varphi \in \Psi$ such that*

$$\begin{aligned}&\beta(d(Tx,Ty,\xi))\\ &\leq \left(\max\left\{d(x,y,\xi), \tfrac{d(x,Tx,\xi)d(y,Ty,\xi)}{1+d(x,y,\xi)}, \tfrac{d(x,Tx,\xi)d(y,Ty,\xi)}{1+d(Tx,Ty,\xi)}\right\}\right)\\ &-\varphi\left(\max\left\{d(x,y,\xi), \tfrac{d(x,Tx,\xi)d(y,Ty,\xi)}{1+d(x,y,\xi)}, \tfrac{d(x,Tx,\xi)d(y,Ty,\xi)}{1+d(Tx,Ty,\xi)}\right\}\right)\end{aligned}$$

(21)

where $x,y,z \in X$ and $\varphi \in \Psi$. Then T has a unique fixed point.

Proof. Follows from theorem 2.4 by taking ψ- the identity mapping □

Example 2.7. *Let $X = [0,1]$ and define*

$$d(x,y,z) = \begin{cases} 0 & , \textit{ if at least two of the three points are the same} \\ e^{|x-y|+|y-z|+|z-x|} & , \textit{ otherwise} \end{cases} \tag{22}$$

It can be shown that d is a generalized 2-metric.

d is a generalized 2-metric.

Let $T : X \to X$ be defined by

$$T(x) = \sin x$$

then for $x \neq y \neq z \in X$,

$$\begin{aligned}&|Tx - Ty| + |Ty - z| + |z - Tx|\\ &= |\sin x - \sin y| + \left|\sin y - \sin(\sin^{-1} z)\right| + \left|\sin x - \sin(\sin^{-1} z)\right|\\ &\leq |x-y| + \left|y - \sin^{-1} z\right| + \left|x - \sin^{-1} z\right|\\ &\leq |x-y| + |y-z| + |x-z|\end{aligned}$$

Since the exponential function is increasing, it follows that

$$e^{|Tx-Ty|+|Ty-z|+|z-Tx|} \leq e^{|x-y|+|y-z)|+|x-z|} \tag{23}$$

Let $\psi(t) = t$ *then it follows that for some* $\beta \geq 1$

$$\begin{aligned}
&\beta\psi(d(Tx,Ty,z)) \\
&\leq \psi(d(x,y,z)) \\
&\leq \psi\left(\max\left\{d(x,y,z), \frac{d(x,Tx,z)d(y,Ty,z)}{1+d(x,y,z)}, \frac{d(x,Tx,z)d(y,Ty,z)}{1+d(Tx,Ty,z)}\right\}\right)
\end{aligned}$$

It follows from theorem (2.4)*, that* T *has a unique fixed point in* X*.*

3. Conclusion

In this paper, we proved the existence and uniqueness of a fixed point for a (ψ,φ)-weakly contractive mapping in a generalized 2-metric space by further imposing a sublinearity property on the class of all altering distance functions.

Conflict of Interests

The authors declare that there is no conflict of interests.

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