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FIXED POINTS OF KANNAN AND REICH INTERPOLATIVE CONTRACTIONS IN CONTROLLED METRIC SPACES

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Abstract: In this paper, we introduce (λ, α)-interpolative Kannan contraction, (λ, α, β)-interpolative Kannan contraction and $(\lambda, \alpha, \beta, \gamma)$-interpolative Reich contraction. Also, we establish some fixed-point theorems in complete controlled metric spaces. Additionally, these theorems expand and apply a number of intriguing findings from metric fixed-point theory to the controlled metric setting.

Keywords: fixed-point; iterative method; interpolative; contraction; controlled metric space.

2010 Mathematics Subject Classification: 46T99, 46N40, 47H10.

1. INTRODUCTION AND PRELIMINARIES

The first fixed point theorem for rational contraction conditions in metric space was established by Dass and Gupta [26].

Theorem 1.1 (see [26]). Let (X, d) be a complete metric space, and let $\mathcal{T}: X \to X$ be a self-mapping. If there exist $\alpha, \beta \in [0, 1)$ with $\alpha + \beta < 1$ such that

$$d(\mathcal{T}x, \mathcal{T}y) \leq \alpha d(x, y) + \beta \frac{[1 + d(x, \mathcal{T}x)]d(y, \mathcal{T}y)}{1 + d(x, y)} \tag{1.1}$$

for all $x, y \in X$, then $\mathcal{T}$ has a unique fixed point $x^* \in X$.

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A genuine generalization of the Dass-Gupta fixed point theorem within the framework of dualistic partial metric spaces was demonstrated by Nazam et al. [27]. As generalizations of metric spaces, Czerwik [1] presented a new class of generalized metric spaces known as b-metric spaces.

Definition 1 (see [1]) Let X be a nonempty set and $s \geq 1$. A function $d_b: X \times X \longrightarrow [0, \infty)$ is said to be a b -metric if for all $x, y, z \in X$,

(b1). $d_b(x, y) = 0$ iff $x = y$

(b2). $d_b(x, y) = d_b(y, x)$ for all $x, y \in X$

(b3). $d_b(x, z) \leq s[d_b(x, y) + d_b(y, z)]$

Then, we refer to the pair (X, d_b) as a b-metric space. Many fixed-point findings on such spaces were subsequently obtained (see to [2–7]).

Extended b-metric spaces are a concept first introduced by Kamran et al. [8].

Definition 2 (see [8]) Let X be a nonempty set and $p: X \times X \longrightarrow [1, \infty)$ be a function. A function $d_e: X \times X \longrightarrow [0, \infty)$ is called an extended b -metric if for all $x, y, z \in X$,

(e1). $d_e(x, y) = 0$ iff $x = y$

(e2). $d_e(x, y) = d_e(y, x)$ for all $x, y \in X$

(e3). $d_e(x, z) \leq p(x, z)[d_e(x, y) + d_e(y, z)]$

The pair (X, d_e) is called an extended b-metric space.

Mlaiki et al. have presented a novel type of generalized b-metric space [9].

Definition 3 (see [9]) Let X be a nonempty set and $p: X \times X \longrightarrow [1, \infty)$ be a function. A function $d_c: X \times X \longrightarrow [0, \infty)$ is called a controlled metric if for all $x, y, z \in X$,

(c1). $d_c(x, y) = 0$ iff $x = y$

(c2). $d_c(x, y) = d_c(y, x)$ for all $x, y \in X$

(c3). $d_c(x, z) \leq p(x, y) d_c(x, y) + p(y, z) d_c(y, z)$

The pair (X, d_c) is called a controlled metric space (see also [10]).

Definition 4 (see [9]) Let (X, d_c) be a controlled metric space and $\{x_n\}_{n \geq 0}$ be a sequence in X. Then,

1. The sequence $\{x_n\}$ converges to some x in X if for every $\varepsilon > 0$, there exists $N = N(\varepsilon) \in \mathbb{N}$ such that $d_c(x_n, x) < \varepsilon$ for all $n \geq N$. In this case, we write $\lim_{n\to\infty} x_n = x$.
2. The sequence $\{x_n\}$ is Cauchy if for every $\varepsilon > 0$, there exists $N = N(\varepsilon) \in \mathbb{N}$ such that $d_c(x_n, x_m) < \varepsilon$ for all $n, m \geq N$.
3. The controlled metric space (X, d_c) is called complete if every Cauchy sequence is convergent.

Definition 5 (see [9]) Let (X, d_c) be a controlled metric space. Let $x \in X$ and $\varepsilon > 0$.

1. The open ball $B(x, \varepsilon)$ is

$$B(x, \varepsilon) = \{y \in X: d_c(y, x) < \varepsilon\}.$$

2. The mapping $F: X \longrightarrow X$ is said to be continuous at $x \in X$ if for all $\varepsilon > 0$, there exists $\delta > 0$ such that $F\big(B(x, \varepsilon)\big) \subseteq B(Fx, \varepsilon)$.

This study aims to introduce a fixed-point theorem for (λ, α)-interpolative Kannan contraction, (λ, α, β)-interpolative Kannan contraction and $(\lambda, \alpha, \beta, \gamma)$-interpolative Reich contraction in the context of complete controlled metric spaces. These theorems also extend and apply to the controlled metric environment several interesting results from metric fixed-point theory. Our result generalizes and extends some well-known results in the literature.

2. MAIN RESULT

We begin by defining the terms below.

Definition 2.1 Let (X, d_c) be a controlled metric space. Let $F: X \longrightarrow X$ be a self-map. We shall call F a (λ, α)-interpolative Kannan contraction, if there exist $\lambda \in [0,1), \alpha \in (0,1)$ such that

$$d_c(\mathrm{F}x, Fy) \leq \lambda\big(d_c(x, \mathrm{F}x)\big)^{\alpha}\big(d_c(y, Fy)\big)^{1-\alpha} \tag{2.1}$$

for all $x, y \in X$, with $x \neq y$.

Definition 2.2 Let (X, d_c) be a controlled metric space. Let $F: X \longrightarrow X$ be a self-map. We shall call F a (λ, α, β)-interpolative Kannan contraction, if there exist $\lambda \in [0,1), \alpha, \beta \in (0,1)$, $\alpha + \beta < 1$ such that

$$d_c(\mathrm{F}x, Fy) \le \lambda\big(d_c(x, \mathrm{F}x)\big)^{\alpha}\big(d_c(y, Fy)\big)^{\beta} \tag{2.2}$$

for all $x, y \in X$, with $x \neq y$.

Definition 2.3 Let (X, d_c) be a controlled metric space. Let $F: X \longrightarrow X$ be a self-map. We shall call F a $(\lambda, \alpha, \beta, \gamma)$-interpolative Reich contraction, if there exist $\lambda \in [0,1), \alpha, \beta, \gamma \in (0,1)$, $\alpha + \beta + \gamma < 1$ such that

$$d_c(\mathrm{F}x, Fy) \le \lambda\big(d_c(x, y)\big)^{\alpha}\big(d_c(x, \mathrm{F}x)\big)^{\beta}\big(d_c(y, Fy)\big)^{\gamma} \tag{2.3}$$

for all $x, y \in X$, with $x \neq y$.

Our first main result as follows.

Theorem 2.4 Let (X, d_c) be a complete controlled metric space. Let $F: X \longrightarrow X$ be a (λ, α)-interpolative Kannan contraction. For $x_0 \in X$, take $x_n = F^n x_0$. Assume that

$$\sup_{m \ge 1} \lim_{i \to \infty} \frac{p(x_{i+1}, x_{i+2}) p(x_{i+1}, x_m)}{p(x_i, x_{i+1})} < \frac{1}{\lambda} \tag{2.4}$$

Then F has a unique fixed point.

Proof. Let $x_0 \in X$ be initial point. Define a sequence $\{x_n\}$ as $x_{n+1} = F x_n, \forall\, n \in \mathbb{N}$. Obviously, if $\exists\, n_0 \in \mathbb{N}$ for which $x_{n_0+1} = x_{n_0}$, then $F x_{n_0} = x_{n_0}$, and the proof is finished. Thus, we suppose that $x_{n+1} \neq x_n$ for each $n \in \mathbb{N}$. Thus, by (2.1), we have

$$\begin{aligned} d_c(x_n, x_{n+1}) &= d_c(F x_{n-1}, F x_n) \\ &\le \lambda\big(d_c(x_{n-1}, \mathrm{F}x_{n-1})\big)^{\alpha}\big(d_c(x_n, F x_n)\big)^{1-\alpha} \\ &= \lambda\big(d_c(x_{n-1}, x_n)\big)^{\alpha}\big(d_c(x_n, x_{n+1})\big)^{1-\alpha} \end{aligned}$$

The last inequality gives

$$d_c(x_n, x_{n+1})^{\alpha} \le \lambda d_c(x_{n-1}, x_n)^{\alpha} \tag{2.5}$$

Since $\alpha < 1$, we have

$$d_c(x_n, x_{n+1}) \le \lambda^{\frac{1}{\alpha}} d_c(x_{n-1}, x_n) \le \lambda d_c(x_{n-1}, x_n)$$

and then

$$d_c(x_n, x_{n+1}) \le \lambda d_c(x_{n-1}, x_n) \le \lambda^2 d_c(x_{n-2}, x_{n-1}) \le \cdots \le \lambda^n d_c(x_0, x_1) \tag{2.6}$$

For all $n, m \in \mathbb{N}$ and $n < m$, we have

$$
\begin{aligned}
d_c(x_n, x_m) &\leq p(x_n, x_{n+1})d_c(x_n, x_{n+1}) + p(x_{n+1}, x_m)d_c(x_{n+1}, x_m) \\
&\leq p(x_n, x_{n+1})d_c(x_n, x_{n+1}) + p(x_{n+1}, x_m)p(x_{n+1}, x_{n+2})d_c(x_{n+1}, x_{n+2}) \\
&+p(x_{n+1}, x_m)p(x_{n+2}, x_m)d_c(x_{n+2}, x_m) \\
&\leq p(x_n, x_{n+1})d_c(x_n, x_{n+1}) + p(x_{n+1}, x_m)p(x_{n+1}, x_{n+2})d_c(x_{n+1}, x_{n+2}) \\
&+p(x_{n+1}, x_m)p(x_{n+2}, x_m)p(x_{n+2}, x_{n+3})d_c(x_{n+2}, x_{n+3}) \\
&+p(x_{n+1}, x_m)p(x_{n+2}, x_m)p(x_{n+3}, x_m)d_c(x_{n+3}, x_m) \\
&\leq p(x_n, x_{n+1})d_c(x_n, x_{n+1}) \\
&+ \sum_{i=n+1}^{m-2}\left(\prod_{j=n+1}^{i} p\left(x_j, x_m\right)\right) p(x_i, x_{i+1})d_c(x_i, x_{i+1}) \\
&+ \prod_{i=n+1}^{m-1} p\left(x_j, x_m\right) d_c(x_{m-1}, x_m) \qquad (2.7)
\end{aligned}
$$

This implies that

$$
\begin{aligned}
d_c(x_n, x_m) &\leq p(x_n, x_{n+1})d_c(x_n, x_{n+1}) \\
&+ \sum_{i=n+1}^{m-2}\left(\prod_{j=n+1}^{i} p\left(x_j, x_m\right)\right) p(x_i, x_{i+1})d_c(x_i, x_{i+1}) \\
&+ \prod_{i=n+1}^{m-1} p\left(x_j, x_m\right) d_c(x_{m-1}, x_m) \\
&\leq p(x_n, x_{n+1})\lambda^n d_c(x_0, x_1) \\
&+ \sum_{i=n+1}^{m-2}\left(\prod_{j=n+1}^{i} p\left(x_j, x_m\right)\right) p(x_i, x_{i+1})\lambda^i d_c(x_0, x_1) \\
&+ \prod_{i=n+1}^{m-1} p\left(x_j, x_m\right) \lambda^{m-1} d_c(x_0, x_1) \\
&\leq p(x_n, x_{n+1})\lambda^n d_c(x_0, x_1) \\
&+ \sum_{i=n+1}^{m-1}\left(\prod_{j=n+1}^{i} p\left(x_j, x_m\right)\right) p(x_i, x_{i+1})\lambda^i d_c(x_0, x_1) \qquad (2.8)
\end{aligned}
$$

Let

$$\eta_r = \sum_{i=0}^{r}\left(\prod_{j=0}^{i} p\left(x_j, x_m\right)\right) p(x_i, x_{i+1})\lambda^i d_c(x_0, x_1) \qquad (2.9)$$

Consider

$$\mu_i = \sum_{i=0}^{r}\left(\prod_{j=0}^{i} p\left(x_j, x_m\right)\right) p(x_i, x_{i+1})\lambda^i d_c(x_0, x_1) \qquad (2.10)$$

In view of condition (2.4) and the ratio test, we ensure that the series $\sum_i \mu_i$ converges. Thus, $\lim_{n\to\infty} \eta_n$ exists. Hence, the real sequence $\{\eta_n\}$ is Cauchy. Now, using (2.6), we get

$$d_c(x_n, x_m) \le d_c(x_0, x_1)[\lambda^n p(x_n, x_{n+1}) + (\eta_{m-1} - \eta_n)] \tag{2.11}$$

Above, we used $p(x, y) \ge 1$. Letting $n, m \longrightarrow \infty$ in (2.11), we obtain

$$\lim_{n,m\to\infty} d_c(x_n, x_m) = 0 \tag{2.12}$$

Thus, the sequence $\{x_n\}$ is Cauchy in the complete controlled metric space (X, d_c). So, there is some $x^* \in X$.So that

$$\lim_{n\to\infty} d_c(x_n, x^*) = 0; \tag{2.13}$$

that is, $x_n \longrightarrow x^*$ as $n \longrightarrow \infty$. Now, we will prove that x^* is a fixed point of F. By (2.1) and condition (c3), we get

$$\begin{aligned} d_c(x^*, Fx^*) &\le p(x^*, x_{n+1}) d_c(x^*, x_{n+1}) + p(x_{n+1}, Fx^*) d_c(x_{n+1}, Fx^*) \\ &= p(x^*, x_{n+1}) d_c(x^*, x_{n+1}) + p(x_{n+1}, Fx^*) d_c(Fx_n, Fx^*) \\ &\le p(x^*, x_{n+1}) d_c(x^*, x_{n+1}) \\ &\quad + p(x_{n+1}, Fx^*) \left[\lambda\big(d_c(x_n, Fx_n)\big)^{\alpha} \big(d_c(x^*, Fx^*)\big)^{1-\alpha}\right] \\ &\le p(x^*, x_{n+1}) d_c(x^*, x_{n+1}) \\ &\quad + p(x_{n+1}, Fx^*) \left[\lambda\big(d_c(x_n, x_{n+1})\big)^{\alpha} \big(d_c(x^*, Fx^*)\big)^{1-\alpha}\right] \end{aligned} \tag{2.14}$$

Taking the limit as $n \longrightarrow \infty$ and using (2.10), (2.11) we obtain that

$$d_c(x^*, Fx^*) = 0 \tag{2.15}$$

This yields that $x^* = Fx^*$. Now, we prove the uniqueness of x^*. Let y^* be another fixed point of F in X, then $Fy^* = y^*$. Now, by (2.1), we have

$$\begin{aligned} d_c(x^*, y^*) &= d_c(Fx^*, Fy^*) \\ &\le \lambda\big(d_c(x^*, x^*)\big)^{\alpha} \big(d_c(y^*, y^*)\big)^{1-\alpha} = 0 \end{aligned} \tag{2.16}$$

This yields that $x^* = y^*$. It completes the proof.

Theorem 2.5 Let (X,d_c) be a complete controlled metric space. Let $F: X \longrightarrow X$ be a (λ,α,β)-interpolative Kannan contraction with (2.4) and for $x_0 \in X$, $x_n = F^n x_0$. Then F has a unique fixed point.

Proof. Following the steps of proof of Theorem 2.4, we construct the sequence $\{x_n\}$ by iterating

$$x_{n+1} = Fx_n, \forall\, n \in \mathbb{N},$$

where $x_0 \in X$ is arbitrary starting point. Then, by (2.2), we have

$$\begin{aligned} d_c(x_n, x_{n+1}) &= d_c(Fx_{n-1}, Fx_n) \\ &\leq \lambda\big(d_c(x_{n-1}, \mathrm{F}x_{n-1})\big)^{\alpha}\big(d_c(x_n, Fx_n)\big)^{\beta} \\ &= \lambda\big(d_c(x_{n-1}, x_n)\big)^{\alpha}\big(d_c(x_n, x_{n+1})\big)^{\beta} \end{aligned}$$

Since $\alpha < 1-\beta$, the last inequality gives

$$d_c(x_n, x_{n+1})^{1-\beta} \leq \lambda d_c(x_{n-1}, x_n)^{\alpha} \leq \lambda d_c(x_{n-1}, x_n)^{1-\beta} \tag{2.17}$$

Hence

$$d_c(x_n, x_{n+1}) \leq \lambda^{\frac{1}{1-\beta}} d_c(x_{n-1}, x_n) \leq \lambda d_c(x_{n-1}, x_n)$$

and then

$$d_c(x_n, x_{n+1}) \leq \lambda d_c(x_{n-1}, x_n) \leq \lambda^2 d_c(x_{n-2}, x_{n-1}) \leq \cdots \leq \lambda^n d_c(x_0, x_1) \tag{2.18}$$

As already elaborated in the proof of Theorem 2.4, the classical procedure leads to the existence of a fixed-point $x^* \in X$. Now, we prove the uniqueness of x^*. Let y^* be another fixed point of F in X, then $Fy^* = y^*$. Now, by (2.2), we have

$$\begin{aligned} d_c(x^*, y^*) &= d_c(Fx^*, Fy^*) \\ &\leq \lambda\big(d_c(x^*, x^*)\big)^{\alpha}\big(d_c(y^*, y^*)\big)^{\beta} = 0 \end{aligned} \tag{2.19}$$

This yields that $x^* = y^*$. This completes the proof.

Theorem 2.6 Let (X,d_c) be a complete controlled metric space. Let $F: X \longrightarrow X$ be a $(\lambda,\alpha,\beta,\gamma)$-interpolative Reich contraction and assume that (2.4) hold for $x_0 \in X$ and $x_n = F^n x_0$. Then F has a unique fixed point.

Proof. Following the steps of proof of Theorem 2.4, we construct the sequence$\{x_n\}$ by iterating

$$x_{n+1} = Fx_n, \forall\, n \in \mathbb{N},$$

where $x_0 \in X$ is arbitrary starting point. Then, by (2.2), we have

$$\begin{aligned} d_c(x_n, x_{n+1}) &= d_c(Fx_{n-1}, Fx_n) \\ &\leq \lambda\big(d_c(x_{n-1}, x_n)\big)^{\alpha}\big(d_c(x_{n-1}, \mathrm{F}x_{n-1})\big)^{\beta}\big(d_c(x_n, Fx_n)\big)^{\gamma} \\ &= \lambda\big(d_c(x_{n-1}, x_n)\big)^{\alpha+\beta}\big(d_c(x_n, x_{n+1})\big)^{\gamma} \end{aligned}$$

Since $\alpha + \beta < 1 - \gamma$, the last inequality gives

$$d_c(x_n, x_{n+1})^{1-\gamma} \leq \lambda d_c(x_{n-1}, x_n)^{\alpha+\beta} \leq \lambda d_c(x_{n-1}, x_n)^{1-\gamma} \tag{2.20}$$

Hence

$$d_c(x_n, x_{n+1}) \leq \lambda^{\frac{1}{1-\gamma}} d_c(x_{n-1}, x_n) \leq \lambda d_c(x_{n-1}, x_n)$$

and then

$$d_c(x_n, x_{n+1}) \leq \lambda d_c(x_{n-1}, x_n) \leq \lambda^2 d_c(x_{n-2}, x_{n-1}) \leq \cdots \leq \lambda^n d_c(x_0, x_1) \tag{2.21}$$

As already elaborated in the proof of Theorem 2.4, the classical procedure leads to the existence of a fixed-point $x^* \in X$. Now, we prove the uniqueness of x^*. Let y^* be another fixed point of F in X, then $Fy^* = y^*$. Now, by (2.3), we have

$$\begin{aligned} d_c(x^*, y^*) &= d_c(Fx^*, Fy^*) \\ &\leq \lambda\big(d_c(x^*, y^*)\big)^{\alpha}\big(d_c(x^*, x^*)\big)^{\beta}\big(d_c(y^*, y^*)\big)^{\gamma} = 0 \end{aligned} \tag{2.22}$$

This yields that $x^* = y^*$. This completes the proof.

AUTHORS' CONTRIBUTIONS

All authors contributed equally and significantly in writing this article. All authors read and approved the final manuscript.

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CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests.

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