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ĆIRIĆ-CONTRACTION TYPE VIA WT-DISTANCE

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Abstract. In this paper, among other things, we give the idea of Ćirić-contractions type via wt-distance and then we will show some additional fixed point results for these mappings, which generalize and enhance Ćirić's fixed point theorems. The new results' usefulness is illustrated by an example.

Keywords: Ćirić-contractions type; wt-distance; b-metric space.

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1. Introduction

Fixed point theory begins with the Banach contraction principle. We will highlight some of the fixed point theorems that we wish to generalize in the setting of wt-distance. This theory has been developed in many different directions. Ćirić [1] elaborated on this concept as follows: Suppose that there are nonnegative functions v_1, v_2, v_3, and v_4 that fulfill

$$\sup\{v_1(r,t)+v_2(r,t)+v_3(r,t)+2v_4(r,t) \,:\, r,t \in \Omega\} = \sigma < 1, \tag{1.1}$$

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such that, for each $r,t \in \Omega$,

$$Y(r,t) = v_1(r,t)d(r,t) + v_2(r,t)d(r,hr) + v_3(r,t)d(t,ht) + v_4(r,t)[d(r,ht) + d(hr,t)]. \tag{1.2}$$

The mapping $h : \Omega \to \Omega$ is said to be a σ-generalized contraction if and only if

$$d(hr,ht) \leq \sigma Y(r,t), \tag{1.3}$$

for all $r,t \in \Omega$. Ćirić proved fixed theorem of the σ-generalized contraction of a self-mapping h orbitally complete metric space.

The formula for inequality (1.3) is

$$d(hr,ht) \leq Y(r,t) - (1-\sigma)Y(r,t). \tag{1.4}$$

Currently, among other things, the publications on weakly contractive maps by Alber and Guerre - Delabrieriere [2] and Rhoades [3] provide as motivation for our further work.

We should discuss the history of these concepts and some relationships between them and the ordinary metric since we substitute the notions of wt-distance for the usual metric in our claims.

As a generalization of metric spaces, b-metric spaces were introduced by Bakhtin in [4] and Czerwik in [5] and [6]. Within this framework, the contraction principle was created.

Definition 1.1. *Let Ω be a set and let $d : \Omega \times \Omega \to [0,\infty)$ be a map that satisfies the following:*

(i) $d(r,t) = 0 \Leftrightarrow r = t \quad \forall r,t \in \Omega;$

(ii) $d(r,t) = d(t,r) \quad \forall r,t \in \Omega;$

(iii) $d(r,t) \leq b[d(r,v) + d(v,t)] \quad \forall r,t \in \Omega$ *for some constant* $b \geq 1$.

The function d is called a b-metric with coefficient b and a triplet (Ω, d, b) is called a b-metric space.

Bakhtin and Czerwik provided examples of b-metric spaces that did not satisfy the triangle inequality. It is worth noting that, like with classical metrics, any b-metric produces a topology. In this topology, [7], [8], and [9] demonstrated with appropriate examples that b-metric is not necessarily continuous and that an open ball is not always an open set with respect to b-metric. Convergence in b-metric spaces is defined in [10] as follows:

Definition 1.2. *Let (Ω, d) be a b-metric space.*

(i) *The sequence $\{r_n\}$ converges to $r \in \Omega \Leftrightarrow \lim_{n\to\infty}(r_n, r) = 0$;*

(ii) *The sequence $\{r_n\}$ is Cauchy $\Leftrightarrow \lim_{n\to\infty}(r_n, r_m) = 0$.*

We say that (Ω, d) is complete if and only if any Cauchy sequence in Ω is convergent.

The notion of wt-distance in generalized b-metric spaces was recently proposed by Hussain et al. [11]. They also demonstrated that wtdistance is an extension of w-distance in [12] and used wt-distance to prove some fixed point theorems in a partially ordered b-metric space.

Definition 1.3. *Let (Ω, d) be a b-metric space with constant $b \geq 1$. Then a function $q : \Omega \times \Omega \to [0, \infty)$ is called wt-distance on Ω if the following conditions are satisfied:*

(i) $q(r,t) \leq b[q(r,v) + q(v,t)] \quad \forall r,t,v \in \Omega$;

(ii) $\forall r \in \Omega$, $q(r, \cdot) : \Omega \to [0, \infty)$ *is b-lower semicontinuous*;

(iii) $\forall \varepsilon > 0, \exists \delta > 0$ *so that* $q(v,r) \leq \delta \wedge q(v,t) \leq \delta \Rightarrow d(r,t) \leq \varepsilon$.

Let us recall that a real-valued function g defined on a b-metric space Ω is said to be b-lower semicontinuous at a point r_0 in Ω if either $\liminf_{r_n \to r_0} g(r_n) = \infty$ or $g(r_0) \leq \liminf_{r_n \to r_0} bg(r_n)$, whenever $r_n \in \Omega$ and $r_n \to r_0$.

Lemma 1.1. [11] *Let $(\Omega, d, b \geq 1)$ be a b-metric space and q be a wt-distance on Ω.*

(i) *If $\{r_n\}$ is a sequence in Ω such that $\lim_{n\to\infty} q(r_n, r) = \lim_{n\to\infty} q(r_n, t) = 0$.*
Then $r = t$. In particular, if $q(v,r) = q(v,t) = 0$, then $r = t$.

(ii) *If $q(r_n, t_n) \leq \alpha_n$ and $q(r_n, t) \leq \beta_n$ for any $n \in \mathbb{N}$, where $\{\alpha_n\}$ and $\{\beta_n\}$ are sequences in $[0, \infty)$ converging to 0, then $\{t_n\}$ converges to t.*

(iii) *Let $\{r_n\}$ be a sequence in Ω such that for each $\varepsilon > 0$, there exists $N_\varepsilon \in \mathbf{N}$ such that $m > n > N_\varepsilon$ implies $q(r_n, r_m) < \varepsilon$ (or $\lim_{n,m\to\infty} q(r_n, r_m) = 0$), then $\{r_n\}$ is a Cauchy sequence.*

(iv) *If $q(t, r_n) \leq \alpha_n$ for any $n \in \mathbb{N}$, then the sequence $\{r_n\}$ Cauchy.*

In [13] and [14] is given the following result:

Theorem 1.1. *Let $(\Omega, d, b \geq 1)$ be a complete b-metric space and define the sequence $\{r_n\}$ in Ω by the recursion*

$$r_n = Sr_{n-1} = S^n r_0.$$

Let $S : \Omega \to \Omega$ be a mapping such that for all $r,t \in \Omega$, where $v_1 + v_2 + v_3 + 2sv_4 < 1$. Then there exists $r^* \in \Omega$ such that $r_n \to r^*$ and r^* is a unique fixed point.

Lakzian et al. [15] generalize this result in the framework of wt-distance.

Let (Ω, d) be a b-metric space with constant $b \geq 1$ and wt-distance q. Consider

$$Y_{q,b}(r,t) = v_1(r,t)q(r,t) + v_2(r,t)q(r,hr) + v_3(r,t)q(t,ht) + v_4(r,t)[q(r,ht) + q(hr,t) - q(t,t)]. \tag{1.5}$$

with

$$\sup\{v_1(r,t) + v_2(r,t) + v_3(r,t) + 2bv_4(r,t) \,:\, r,t \in \Omega\} = \sigma < \frac{1}{b}. \tag{1.6}$$

The notion of weak $(\phi, Y_{q,b})$-contractive mapping, where the function $\phi : [0,\infty)^5 \to [0,\infty)$ satisfying the condition $\phi^{-1}\{0\} = \{(0,0,0,0,0)\}$.

The concept of $(C;\sigma)$ condition has been introduced recently in [16], and it is based on the well-known $(C;1)$ condition that Ćirić introduced and investigated in [1]. It is said that a map $h : \Omega \to \Omega$ on a metric space (Ω, d) satisfies the condition $(C;\sigma)$ if there is a constant $\sigma \geq 0$ such that for every sequence $r_n \in \Omega$,

$$r_n \to r_0 \in \Omega \Rightarrow D(r_0) \leq \sigma \limsup_{n\to\infty} D(r_n)$$

where $D(r) = d(r,hr),\ r \in \Omega$. This condition is more relaxing than continuity.

In this paper, we will show some new fixed point theorems that generalize the work of Ćirić [8] on the notion of weak $(\phi, Y_{q,b})$-contractive mappings and give some applications to nonlinear fractional differential equations.

2. Main Results

Definition 2.1. *Let $h : \Omega \to \Omega$ be a given mapping and let q be a wt-distance on a b-metric space (Ω, d) with constant $b \geq 1$. If f is a weak $(\phi, Y_{q,b})$-contractive mapping, then we say that*

$$q(hr,ht) \leq Y_{q,b}(r,t) - \phi\left(q(r,t), q(r,hr), q(t,ht), \frac{q(r,hr)+q(t,ht)}{2}, \frac{q(r,ht)+q(t,hr)}{2}\right), \tag{2.1}$$

for all $r,t \in \Omega$.

Theorem 2.1. *Given a weak $(\phi, Y_{q,b})$-contractive mapping $h : \Omega \to \Omega$ and a wt-distance q on a complete b-metric space (Ω, d) with constant $b \geq 1$. If h satisfies the condition $(C;\sigma)$, or for every $\omega \in \Omega$ with $\omega \neq S\omega$, where $S : \Omega \to \Omega$ be a mapping such that for all $r, t \in \Omega$, and $v_1 + v_2 + v_3 + 2bv_4 < 1$, we have $\inf\{q(r,\omega) + q(r,Sr) : r \in \Omega\} > 0$, then h has a unique fixed point x and moreover, $q(x,x) = 0$.*

Proof. For every $n \geq 0$, define a sequence $\{r_n\}$ in Ω as follows: $r_{n+1} = hr_n = h^{n+1}r_0$. The proof is finished if there is $n_0 \in \mathbb{N}$ such that $r_{n_0} = r_{n_0+1}$ and $x = r_{n_0}$ is a fixed point of h. Henceforth, we presume that

$$r_n \neq r_{n+1}, \quad \forall n. \tag{2.2}$$

Step 1. We will show that $\lim_{n\to\infty} q(r_n, r_{n+1}) = 0$. Using (2.1) and Definition 1.3, we have

(2.3)

$$\begin{aligned}
q(r_n, r_{n+1}) &= q(hr_{n-1}, hr_n)\\
&\leq Y_{q,b}(r_{n-1}, r_n) - \phi\Big(q(r_{n-1}, r_n), q(r_{n-1}, hr_{n-1}), q(r_n, hr_n),\\
&\quad \frac{q(r_{n-1}, hr_{n-1}) + q(r_n, hr_n)}{2}, \frac{q(r_{n-1}, hr_n) + q(r_n, hr_{n-1})}{2}\Big)\\
&\leq Y_{q,b}(r_{n-1}, r_n)\\
&= v_1(r_{n-1}, r_n)q(r_{n-1}, r_n) + v_2(r_{n-1}, r_n)q(r_{n-1}, hr_{n-1}) + v_3(r_{n-1}, r_n)q(r_n, hr_n)\\
&\quad + v_4(r_{n-1}, r_n)[q(r_{n-1}, hr_n) + q(hr_{n-1}, r_n) - q(r_n, r_n)]\\
&= v_1(r_{n-1}, r_n)q(r_{n-1}, r_n) + v_2(r_{n-1}, r_n)q(r_{n-1}, r_n) + v_3(r_{n-1}, r_n)q(r_n, r_{n+1})\\
&\quad + v_4(r_{n-1}, r_n)[q(r_{n-1}, r_{n+1}) + q(r_n, r_n) - q(r_n, r_n)]\\
&= v_1(r_{n-1}, r_n)q(r_{n-1}, r_n) + v_2(r_{n-1}, r_n)q(r_{n-1}, r_n) + v_3(r_{n-1}, r_n)q(r_n, r_{n+1})\\
&\quad + v_4(r_{n-1}, r_n)q(r_{n-1}, r_{n+1})\\
&\leq [v_1(r_{n-1}, r_n) + v_2(r_{n-1}, r_n)]q(r_{n-1}, r_n) + v_3(r_{n-1}, r_n)q(r_n, r_{n+1})\\
&\quad + bv_4(r_{n-1}, r_n)[q(r_{n-1}, r_n) + q(r_n, r_{n+1})], \quad \forall n \geq 1.
\end{aligned}$$

Therefore,

$$q(r_n, r_{n+1}) \leq \frac{v_1(r_{n-1}, r_n) + v_2(r_{n-1}, r_n) + bv_4(r_{n-1}, r_n)}{1 - v_3(r_{n-1}, r_n) - bv_4(r_{n-1}, r_n)} q(r_{n-1}, r_n), \quad \forall n \geq 1. \tag{2.4}$$

Thus, $q(r_n, r_{n+1}) \leq \sigma q(r_{n-1}, r_n)$ for all $n \geq 1$. From (1.6) and $\sigma < 1$, we obtain that

$$v_1(r,t) + v_2(r,t) + bv_4(r,t) + \sigma v_3(r,t) + \sigma bv_4(r,t) \leq \sigma.$$

and so

$$\frac{v_1(r,t) + v_2(r,t) + bv_4(r,t)}{1 - v_3(r,t) - bv_4(r,t)} \leq \sigma, \quad \forall\, r,t \in \Omega. \tag{2.5}$$

Using (2.4) and (1.6), we have

$$q(r_n, r_{n+1}) \leq \sigma^n q(r_1, r_0), \quad \forall\, n \geq 1. \tag{2.6}$$

Hence,

$$\lim_{n\to\infty} q(r_n, r_{n+1}) = 0. \tag{2.7}$$

Step 2. We will show that

$$\lim_{n,m\to\infty} q(r_n, r_m) = 0. \tag{2.8}$$

For each $m, n \in \mathbb{N}$ with $m > n$, applying (i) of Definition 1.3 and (2.6), we obtain

$$\begin{aligned}
q(r_n, r_m) &\leq b[q(r_n, r_{n+1}) + q(r_{n+1}, r_m)] \\
&\leq bq(r_n, r_{n+1}) + b^2[q(r_{n+1}, r_{n+2}) + q(r_{n+2}, r_m)] \\
&\leq bq(r_n, r_{n+1}) + b^2 q(r_{n+1}, r_{n+2}) + \cdots + b^{m-n} q(r_{m-1}, r_m) \\
&\leq b\sigma^n q(r_0, r_1) + b^2\sigma^{n+1} q(r_0, r_1) + \cdots + b^{m-n}\sigma^{m-1} q(r_0, r_1) \\
&\leq b\sigma^n q(r_0, r_1)[1 + b\sigma + \cdots + (b\sigma)^{m-n-1}] \\
&\leq \frac{b\sigma^n}{1 - b\sigma} q(r_0, r_1) \to 0, \quad \text{as } n \to \infty.
\end{aligned} \tag{2.9}$$

Thus, by Lemma 1.1, the sequence $\{r_n\}$ is Cauchy in (Ω, d). From Ω is a complete b-metric space, there exists $x \in \Omega$ such that $r_n \to x$ as $n \to \infty$.

Step 2. We will show that x is a fixed point of h.

Case I. Assume that h obeys the condition $(C; \sigma)$. It holds that

$$d(x, hx) \leq \sigma \limsup_{n\to\infty} d(r_n, hr_n) = \sigma \limsup_{n\to\infty} d(r_n, r_{n+1}) = 0.$$

From $\{r_n\}$ is a Cauchy sequence, so we conclude that $x = hx$.

Case II. Assume that $\inf\{q(r,\omega)+q(r,Sr) : r\in\Omega\}>0$, for every $\omega\in\Omega$ with $\omega\neq S\omega$. Using (2.8), for each $\varepsilon>0$ there exists an $N_\varepsilon\in\mathbb{N}$ such that $n>N_\varepsilon$ implies $q(r_{N_\varepsilon},r_n)<\varepsilon$. But, $r_n\to x$ and $q(r,\cdot)$ is b-lower semi-continuous, and so using Definition 2.1, we have

$$q(r_{N_\varepsilon},x)\leq \liminf_{n\to\infty} bq(r_{N_\varepsilon},r_n)\leq \varepsilon. \tag{2.10}$$

Letting $\varepsilon=\dfrac{1}{b\sigma}$ and $N_\varepsilon=n_\sigma$, we have

$$\lim_{\sigma\to\infty} q(r_{n_\sigma},x)=0. \tag{2.11}$$

Suppose that $x\neq hx$. Then

$$0<\inf\{q(r,x)+q(r,hr) : r\in\Omega\}\leq \inf\{q(r_n,x)+q(r_n,r_{n+1}) : n\in\mathbb{N}\}. \tag{2.12}$$

Using (2.7) and (2.11), we obtain $\inf\{q(r_n,x)+q(r_n,r_{n+1}) : n\in\mathbb{N}\}=0$, which is a contradiction. Therefore, $x=hx$.

Step 4. For $x\in\Omega$ and $x=hx$, we have

$$\begin{aligned} q(x,x)&=q(hx,hx)\\ &\leq Y_{q,b}(x,x)-\phi\left(q(x,x),q(x,hx),q(x,hx),\frac{q(x,hx)+q(x,hx)}{2},\frac{q(x,hx)+q(x,hx)}{2}\right)\\ &< Y_{q,b}(x,x)\\ &= v_1(x,x)q(x,x)+v_2(x,x)q(x,hx)+v_3(x,x)q(x,hx)+v_4(x,x)[q(x,hx)+q(hx,x)-q(x,x)]\\ &\leq v_1(x,x)q(x,x)+v_2(x,x)q(x,x)+v_3(x,x)q(x,x)+bv_4(x,x)[q(x,x)+q(x,x)-q(x,x)]\\ &= [v_1(x,x)+v_2(x,x)+v_3(x,x)+bv_4(x,x)]q(x,x)\\ &\leq \sigma q(x,x). \end{aligned} \tag{2.13}$$

Thus, we conclude that $q(x,x)=0$.

Step 5. We will show that x is unique. Let x^* be another fixed point of h. We will prove that $x=x^*$.

$$\begin{aligned} q(x,x^*)&=q(hx,hx^*)\\ &\leq Y_{q,b}(x,x^*)-\phi\left(q(x,x^*),q(x,hx),q(x^*,hx^*),\frac{q(x,hx)+q(x^*,hx^*)}{2},\frac{q(x,hx^*)+q(x^*,hx)}{2}\right)\\ &< Y_{q,b}(x,x^*) \end{aligned} \tag{2.14}$$

$$
\begin{aligned}
&= v_1(x,x^*)q(x,x^*) + v_2(x,x^*)q(x,hx) + v_3(x,x^*)q(x^*,hx^*) \\
&\quad + v_4(x,x^*)[q(x,hx^*) + q(hx,x^*) - q(x^*,x^*)] \\
&\leq v_1(x,x^*)q(x,x^*) + v_2(x,x^*)q(x,x) + v_3(x,x^*)q(x^*,x^*) \\
&\quad + bv_4(x,x^*)[q(x,x^*) + q(x,x^*) - q(x^*,x^*)] \\
&= [v_1(x,x^*) + 2bv_4(x,x^*)]q(x,x^*) \\
&\leq \sigma q(x,x^*) \\
&< q(x,x^*),
\end{aligned}
$$

which is a contradiction. Hence, $q(x,x^*) = 0$. From Step 4, $q(x,x) = 0$. Using Lemma 1.1, $x = x^*$. □

Example 1. *Let $\Omega = [0,4]$ and d be a function $d : \Omega \times \Omega \to [0,+\infty)$ defined by $d(r,t) = (r-t)^2$. Then d is a b-metric with coefficient $b = 2$. We define a function $q : \Omega \times \Omega \to [0,+\infty)$ on (Ω,d) with $q(r,t) = t^2$. Then q is a wt-distance on (Ω,d).*

Let $h : \Omega \to \Omega$ be a mapping such that $h(r) = \dfrac{r}{20}$ for $r \in [0,2]$ and $h(r) = \dfrac{r}{40}$ for $r \in (2,4]$. Notice that h is not continuous and h is not a contraction with respect to b-metric d.

On the other hand, if we let $v_1 = v_2 = v_3 = v_4 = \dfrac{1}{60}$ and define a function $\phi : [0,+\infty)^5 \to [0,+\infty)$ with $\phi(u_1,u_2,u_3,u_4,u_5) = \dfrac{1}{180}u_1 + \dfrac{1}{360}u_2 + \dfrac{1}{72}u_3 + \dfrac{1}{36}u_4 + \dfrac{1}{90}u_5$, then we obtain

$$Y_{q,b}(r,t) = \frac{1}{60}t^2 + \frac{1}{60}(hr)^2 + \frac{1}{60}(ht)^2 + \frac{1}{60}(ht)^2 = \frac{1}{60}t^2 + \frac{1}{60}(hr)^2 + \frac{1}{30}(ht)^2$$

and

$$
\begin{aligned}
&\phi\left(t^2,(hr)^2,(ht)^2,\frac{(hr)^2+(ht)^2}{2},\frac{t^2+(ht)^2}{2}\right) \\
&= \frac{1}{180}t^2 + \frac{1}{360}(hr)^2 + \frac{1}{72}(ht)^2 + \frac{1}{72}[(hr)^2+(ht)^2] + \frac{1}{180}[t^2+(ht)^2].
\end{aligned}
$$

Thus,

$$Y_{q,b}(r,t) - \phi\left(t^2,(hr)^2,(ht)^2,\frac{(hr)^2+(ht)^2}{2},\frac{t^2+(ht)^2}{2}\right) = \frac{1}{60}t^2 - \frac{1}{90}t^2 = \frac{1}{180}t^2.$$

We conclude that

$$q(hr,ht) \leq (ht)^2 \leq \frac{1}{180}t^2.$$

Therefore, h is a weak $(\phi, Y_{q,b})$-contractive mapping.

Corollary 2.1. *Let q be a wt-distance on a complete b-metric space (Ω, d) with constant $b \geq 1$. Let $h : \Omega \to \Omega$ be a self-mapping satisfying*

$$q(hr, ht) \leq \sigma Y_{q,b}(r,t), \tag{2.15}$$

for all $r, t \in \Omega$, where $\sigma \in (0,1)$. Suppose either $\inf\{q(r,\omega) + q(r,hr) : r \in \Omega\} > 0$ *for every $\omega \in \Omega$ with $\omega \neq h\omega$, or the mapping h is continuous. Then h has a unique fixed point u and moreover $q(x,x) = 0$.*

Proof. Letting $\phi(u_1, u_2, u_3, u_4, u_5) = 1 - \sigma(\sum_{i=1}^{5} u_i)$ in Theorem 2.1 and follows proof of Theorem 2.1. □

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CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests.

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