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OPTIMIZING EMPLOYER BRANDING AND WAGE CONFIDENCE: APPLYING THE MINIMAX THEOREM IN SERVICE COMPANIES

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Abstract. This article explores the strategic application of the Minimax Theorem in the context of employer branding and wage confidence dynamics within service companies. Moreover, "skilled employees have become aware of the abundance of opportunities in the labour market and the possibility of piloting their careers by accumulating experiences in different companies" (Peretti & Swalhi, 2007, p. 278) and organizations are trying to find ways to retain that staff. Thus, developing confidence in the organization can be an important asset in a logic of sustainable social exchange between the employee and the employer [2].

Keywords: fixed point; multivalued mapping; game theory; minmax employer brand; dynamics of wage confidence.

2020 AMS Subject Classification: 90C47, 47H10, 47H08, 91A05, 58E17.

1. Introduction

In today's competitive business landscape, employer branding and wage confidence are critical factors that influence the recruitment, retention, and overall satisfaction of employees.

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For service companies, which rely heavily on their workforce to deliver high-quality services, understanding and managing these aspects are paramount. This article explores the application of the Minimax Theorem in the context of employer branding and wage confidence dynamics in service companies. In the intricate world of human resources, where the quest for talent meets the pursuit of profit, a unique and powerful lens is emerging to decipher the complexities of employer-employee relationships–game theory. This field, which has traditionally found its home in economics and strategic studies, is now being increasingly applied to the realm of organizational dynamics. In this article, we delve into the fascinating interplay between game theory and the employer brand, exploring how this analytical framework can decode the dynamics of wage confidence within service companies. The choice of wage, job satisfaction, and an employee's perception of the employer are all pieces of this intricate puzzle. By adopting a game-theoretical perspective, we aim to shed light on the strategies, incentives, and outcomes that shape the employment landscape of service-based businesses [1].

The concepts and models of game theory become tools favorable to the researchers to give the scientific character to the analysis of the social sciences. This may explain the emergence and expansion of this scientific approach to the point where many researchers have received the Nobel Prize for economics through their contributions to the development and application of this theory.

The article by Ambler and Barrow (1996), laying the foundations of the concept of employer brand, is the starting point of a dynamic and continuous current of research. After more than two decades of work devoted to the measurement of the employer brand (Berthon et al., 2005) and its internal and external effects (Bodderas et al., 2011; Franca and Pahor, 2012), Backhaus (2016) calls for new research focused on what she calls employer brand sustainability. In the context of the employer brand, sustainability can be understood as the brand's ability to deliver on its promises over time, in order to maximize benefits for employees (Backhaus, 2016).[19]

Our work aims at contributing to scientific efforts in the field of the development of the minimax theorem and applying the outcoming results to the analysis of themployer brand and the dynamics of wage confidence in the case of service companies.

Employer branding is the image and reputation that an organization cultivates as an employer.

It encompasses the company's values, culture, and the overall experience it offers to its employees. As part of the management of the employer brand, the branded product would thus refer to the specific employment experience offered by an organization to its current and potential employees (Hanin et al., 2013, the employment offering). As such, the employer brand belongs to the field of HR marketing, whose desire is to use the logic and techniques of marketing and communication to attract candidates, but also retain the best profiles (Liger, 2007; Panczuk and Point, 2011).

According to Lievens (2007), the employer brand management process aims to identify and position this value proposition and then communicate and promote it, and ensure that the speeches are consistent with the reality experienced by employees so that there is no gap between external employer brand (communicated upstream and during the recruitment process) and internal (after recruitment), which would be detrimental and result in the departure or withdrawal of newcomers (Charbonnier-Voirin, Laget and Vignolles, 2014; Mark and Toelken, 2009).[18].

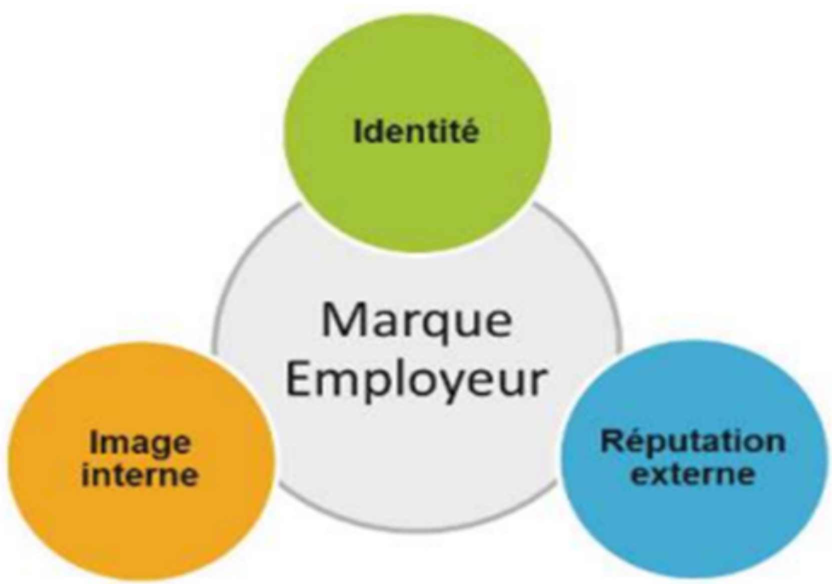

Figure: Image Illustrative des Dimensions de la Marque Employeur

Source: Site Advents

Employer identity: It refers to the DNA of the employer, that is to say all the elements of the company that characterizes it, that make it what it is. Among these elements, we find the values,

the missions, its sectors of activity, its trades, its culture, etc. In other words, the identity of the brand can be defined as the set of characteristics defining the company.
The employer image: Also called the internal image, it represents the image of employees of the company, so people within the company. This image is linked to the interest and motivation of employees to achieve their goals and remain in the company. In order to retain and retain its employees, the company must become an employer of choice by highlighting qualities that are dear to employees. According to Susan Hunt and Robert Landry, there is a direct link between the employee's perception of his employer and his participation in order to promote the brand externally. This means that the employee who believes in his company will recommend it without restraint and in this way will contribute to the development of the brand, making the employee a brand ambassador.
Employer reputation: Referred to as external image, it reflects the perception or opinion of people of a company vis-à-vis a brand, thus having a role in loyalty with customers of the company. It is appropriate to distinguish between reputation and notoriety because often an amalgam is made. Notoriety can be defined as the reputation of a brand or company, its degree of knowledge, its presence in the minds of consumers. In other words, notoriety is linked to the idea of visibility and fame. Reputation is defined as ”the way in which someone, something is known, considered in an audience.”and also as ”the public's favourable or unfavourable opinion for someone, something”.
These are the three pillars of the employer brand giving it an important place in the company, making it possible to attract and charm future employees on the external job market, and to retain the company's skills.[9]
The Employer Brand is part of HR Marketing, which reflects the desire ”to apply the logic and techniques of marketing and communication to attract candidates and retain employees [16, Liger,p.1]. It is a concept born from the desire of companies to assert their difference as providers of employment on the labour market [3] (Backhaus, 2004; Backhaus and Tikoo, 2004; Knox and Freeman, 2006; Lievens, 2007; Lievens, Van Hoye and Anseel, 2007). Facebook is the most concrete example of this with its three main objectives:

- The audience: Being present daily in people's lives through their communication tool.

- The growth: Buy companies equal to theirs, to ensure the stability of the company and obtain business opportunities in new market niches.
- The profitability: Producing value for the various shareholders.

Trust is a transversal notion, certainly in direct connection with a form of calculation and reasoning, but it is also at the heart of a past history, a projection in the future and therefore has an important place in the emotional functioning of the individual. It is certainly for this reason that the notion of trust as an object of study initially and historically emerged in the field of psychology (Deutsh – 1958).

According to Valérie Neveu [21], the status of trust has evolved according to the periods that have marked the history of philosophy, while supporting the idea that trust and rationality can coexist, so that trust can constitute a particular form of belief in a form of truth that encourages action and builds a social bond. Trust is then defined as ”the willingness of one party to become vulnerable to the actions of the other party, based on the expectation that the other party will perform actions that are important to itself, without any form of control or monitoring being necessary” (Mayer, Davis, Schoorman, 1995: 712). Work on the role of organizational trust in the development of a sustainable employment relationship has shown that organizational trust is linked to its own background and attitudes that are centered on social exchange with the organization itself-same (Aryee, Budhwar and Chen, 2002; Dirks and Ferrin, 2002; Mayer and Gavin, 2005; Wat and Shaffer, 2005).

In the introduction of his book on trust at work, Laurent Karsenty [11] notes the absence of a univocal acceptance of the concept of trust, because it is multidimensional. The author chooses a definition reflecting the main dimensions of trust: affective, cognitive and relational (Orrigi, 2008) and writes: Trust is a feeling of serenity that emanates from the relationship with an actor on whom we rely in a given situation hoping that he will take care of our interests.

The source of the need for trust at work, he believes, lies in the inevitable existence of uncertainty and risk. Laurent Karsenty [12] says that uncertainty is inherent in the social world and especially in the world of work. For him, the need for trust comes from the unexpected inherent in the world of work and the resulting uncertainty: the use of trust is then necessary to ensure that the risk is zero or limited. Concretely, the individual must be able to count on himself, on another, on his group of belonging or his organization.

In the continuation of this vision of trust, we find the definition of Roland Reitter [22] according to which: Trust is not decreed, but constitutes one of the components of the interplay between actors that are intrinsically both rational and affective. Indeed, the author illustrates his point by first presenting game theory, because it makes it possible to understand the issues linking trust and cooperation. Indeed, the game theory shows that when two individuals cooperate, if one betrays, the other also betrays. This theory is complicated if we add a third party guarantor likely to establish a judgment. In this case, each individual continues to calculate, but while taking into account possible sanctions and therefore the exogenous social dimension in which he finds himself. Because of the existence of this social and collective view, his individual behavior may be led to change.

In a recent book entitled. The 5 levers of trust [14], the two authors Laurent Combalbert and Marwan Mery specify that their analysis is based on ten years of experience in supporting teams of executives. The authors then illustrate and demonstrate the existence of a virtuous cycle of trust which, according to the authors, is the means for organizations to strive for excellenc. The study and support of hundreds of teams, companies or organizations facing particularly complex situations have shown that the determining ingredient of their performance is trust. The actual existence of this trust is the result of a delicate alchemy that includes self-confidence, team confidence, hierarchical confidence, trust in mission and confidence in history. In other words, successful organizations are those that have created what the authors call the ”CIRCLER OF TRUST”.

Eric Simon, in ”La confiance dans tous ses états [24]” reports on his research on the place of trust in managerial literature and notes that few authors have made synthesis efforts on empirical work devoted to trust.

Wage confidence refers to an employee’s belief in the fairness and competitiveness of their compensation. It directly affects job satisfaction, motivation, and commitment. In service companies, wage confidence plays a vital role in retaining skilled employees who directly impact service quality.

The Minimax Theorem, a fundamental concept in mathematical optimization, is commonly applied in various decision-making scenarios [4]. It helps to find the best strategy when facing an opponent or uncertainty. In the context of employer branding and wage confidence dynamics. Service companies can utilize the Minimax Theorem to strike a balance between employer branding and wage confidence dynamics. This involves creating a strategy that minimizes the maximum wage-related concerns while maximizing the employer brand's attractiveness.

This article will include case studies from service companies that have successfully applied the Minimax Theorem to optimize their employer branding and wage confidence dynamics. These real-world examples will demonstrate the practicality of this approach and its impact on employee satisfaction and organizational performance.

In an era where service companies rely on a skilled and motivated workforce to deliver exceptional service, mastering employer branding and wage confidence dynamics is crucial.

The application of the Minimax Theorem offers a strategic framework for service companies to navigate these challenges, ultimately enhancing their competitive advantage and reputation as employers of choice, the foundations of modern game theory were indeed significantly developed by John von Neumann, but the initial groundbreaking work was published a bit later. In 1944, John von Neumann and Oskar Morgenstern published their seminal book "Theory of Games and Economic Behavior," which is often considered the starting point of modern game theory. This book laid the groundwork for the formal study of strategic interactions and decision-making in various fields, including economics, political science, and social sciences. While von Neumann made earlier contributions to the field, this book is widely recognized as the foundational text of game theory as we know it today. [26, 27, 13].

By balancing the elements of employer branding and wage confidence, companies can create an environment that attracts and retains top talent while fostering a high-performance culture.

2. Preliminaries

Our work aims at contributing to scientific efforts in the field of the development of the minimax theorem and applying the outcoming results to the employer branding and wage confidence dynamics within service companies.

A two-player, zero-sum game is a triple (X,Y,f), where X,Y are nonempty sets ([4, page 326]), whose elements are called strategies, and $f : X\times Y \to \mathbb{R}$ is the gain function. There are two players, α and β, and $f(x,y)$ represents the gain of the player α when he chooses the strategy $x\in X$ and the player β chooses the strategy $y\in Y$. The quantity $-f(x,y)$ represents the gain of the player β in the same situation. The target of the player α is to maximize his gain when the player β chooses a strategy that is the worst for α, that is, to choose $x_0\in X$ such that :

$$\inf_{y\in Y} f(x_0,y) = \max_{x\in X}\inf_{y\in Y} f(x,y). \tag{2.1}$$

Similarly, the player β chooses $y_0\in Y$ such that:

$$\sup_{x\in X} f(x,y_0) = \min_{y\in Y}\sup_{x\in X} f(x,y). \tag{2.2}$$

It follows

$$\sup_{x\in X}\inf_{y\in Y} f(x,y) = \inf_{y\in Y} f(x_0,y) \le f(x_0,y_0) \le \sup_{x\in X} f(x,y_0) \le \inf_{y\in Y}\sup_{x\in X} f(x,y). \tag{2.3}$$

Note that in general

$$\sup_{x\in X}\inf_{y\in Y} f(x,y) \le \inf_{y\in Y}\sup_{x\in X} f(x,y). \tag{2.4}$$

If the equality holds in (4.3), then, by (4.4),

$$\sup_{x\in X}\inf_{y\in Y} f(x,y) = f(x_0,y_0) = \inf_{y\in Y}\sup_{x\in X} f(x,y). \tag{2.5}$$

The common value in (1.5) is called the value of the game,
$(x_0,y_0)\in X\times Y$ a solution of the game and x_0 and y_0 winning strategies. It follows that to prove the existence of a solution of a game we have to prove equality (1.4).
For more details on game theory and minimax theorems, we refer to the books of Aubin, J.P [1] and Carl, S, Heikkilä, S [4].

Von Neumann's Minimax Theorem: [26]

Let X and Y be two non-empty, compact, and convex sets in finite-dimensional Euclidean spaces, and let $f : X\times Y\to\mathbb{R}$ be a continuous function. Then, for Player 1 (the Minimizer) and Player 2 (the Maximizer):

Player 1 seeks to find a strategy $x^* \in X$ that minimizes their loss:

$$x^* = \arg\min_{x\in X}\max_{y\in Y} f(x,y)$$

Player 2 seeks to find a strategy $y^* \in Y$ that maximizes their gain:

$$y^* = \arg\max_{y\in Y}\min_{x\in X} f(x,y)$$

The Minimax Theorem states that there exist optimal strategies x^* and y^*, and the following equality holds:

$$\max_{y\in Y}\min_{x\in X} f(x^*,y) = \min_{x\in X}\max_{y\in Y} f(x,y^*)$$

In other words, both players achieve their best outcomes simultaneously in equilibrium.

In other words:

In a two-player, zero-sum game, there exists a pair of strategies (x^*,y^*) such that:

$$\min_x \max_y f(x,y) = \max_y \min_x f(x,y) = f(x^*,y^*)$$

where:

- $f(x,y)$ represents the utility or payoff function of the game.
- x is a strategy for the first player.
- y is a strategy for the second player.
- x^* and y^* are the optimal strategies that achieve the equilibrium.

The mathematician John Forbes Nash used the Kakutani fixed point theorem to demonstrate a major theorem of game theory, a consequence of which is the existence of a Nash equilibrium in any mixed strategy infni game

3. Main Results

We all know the power of fixed-point theorems to prove the existence of equilibrium points in various mathematical contexts, including game theory.

Definition 3.1. *Let X be a Banach space, and let C be a nonempty convex subset of X. The application $T : C \to C$ is said to be* affine *if, for all $x,y \in X$ and for all $\lambda \in [0,1]$, the following property holds:*

$$T(\lambda x+(1-\lambda)y)=\lambda T(x)+(1-\lambda)T(y).$$

Definition 3.2. *Two applications T_1 and T_2 are said to be commutative if the following property holds:*

$$T_1\circ T_2=T_1\circ T_1$$

Definition 3.3. *Two applications $T_1,T_2:C\times C\to C$ are said to be symmetrical if the following property holds:*

$$T_1(x,y)=T_2(y,x)$$

We will use a fixed point theorem different from the others used to find a equilibrium between two players, this theorem ensures the existence of a common fixed point between a finite family of commutative, affine and continuous applications, this theorem is due to Markoff (1936)-Kakutani(1938).

Theorem 3.4. [17] *Let X be a Banach space, and let K be a nonempty convex and compact subset of X. Let $I\subset\mathbb{N}$ and $\Gamma=(T_i)_{i\in I}$ is a family of commutative, affine and continuous applications such that $T_i:K\to K$, Then Γ have a common fixed point.*

Theorem 3.5. [28] *Let X be a Banach space, and let K be a nonempty convex and compact subset of X. Let $I\subset\mathbb{N}$ and $\Gamma=(T_i)_{i\in I}$ is a family have the closed graph, commutative, affine multivalued applications such that $T_i:K\to 2^K$ and $T(x)$ is nonempty closed and convex for every $x\in K$, Then Γ have a common fixed point.*

Remark 1. *Γ is commuting in the sense that if T_1,T_2 belong to Γ, then $T_1(T_2(x))=T_2(T_1(x))$ for all $x\in K$ and $T_1(T_2(x))=\bigcup\limits_{y\in T_2(x)}T_1(y)$*

Theorem 3.6. *Let E be a Banach space, and X,Y be a nonempty convex and compact subsets of E. Let $(f_i)_{i\in I}$ is a family of commutative, affine and continuous applications such that $f_i:X\times Y\to\mathbb{R}$.*
Suppose that :

1. *The functions $f_i:X\times Y\to\mathbb{R}$ are continuous.*

(2) $\forall x \in X$, $f_i(x,.)$ are convex,

(3) $\forall y \in Y$, $f_i(.,y)$ are concave.

(4) $\forall i \in I$, f_i and f_{i+1} are symmetrical.

Consider a two-player, zero-sum game with sets of strategies X and Y for Player 1 and Player 2, respectively. Let $f_i : X \times Y \to \mathbb{R}$ be the payoff function. Then, there exist optimal strategies $x^* \in X$ for Player 1 and $y^* \in Y$ for Player 2 such that:

$$\max_{y\in Y} \min_{x\in X} f_i(x,y^*) = \min_{x\in X} \max_{y\in Y} f_i(x^*,y)$$

Proof. We set that :

$$\phi_i(x) = \min_{y\in Y} f_i(x,y) = \min f_i(x \times Y), x \in X$$

and

$$\psi_i(y) = \max_{x\in X} f_i(x,y) = \max f_i(X \times y), y \in Y$$

$$N_y = \{x \in X : f_i(x,y) = \psi_i(y)\} \text{ and } M_x = \{y \in Y : f_i(x,y) = \phi_i(x)\}$$

and

$$N_{D'} = \cup_{y\in D'} N_y, \;\; M_D = \cup_{x\in D} M_x$$

for any set $(D \times D')$ of $X \times Y$, by the compactness of X and Y, ϕ_i and ψ_i are continuous too.
We pose : $C = X \times Y$ and $c = (x,y)$.
the product set C is convex and compact,(product of two convex and two compact)
therefore, the following two mapping can be defined by:

$$F_i : C \to 2^C$$
$$c \mapsto N_y \times M_x$$

whith $c = (x,y) \in X \times Y$.
First, we will show that F_i have the closed graph.
Indeed, Let (x_n,y_n) be a sequence in C such that $(x_n,y_n) \to (x,y) \in C$, let (u_n,v_n) be a sequence

such that $(u_n, v_n) \in F(x_n, y_n)$ and $(u_n, v_n) \to (u, v)$, We shall show that $(u, v) \in F_i(x, y)$, we have:

$$\begin{aligned}(u_n, v_n) \in F(x_n, y_n) \quad &\Leftrightarrow \quad (u_n, v_n) \in N_{y_n} \times M_{x_n} \\ &\Leftrightarrow \quad f_i(u_n, y_n) = \psi(y_n) \text{ and } f_i(x_n, v_n) = \phi(x_n)\end{aligned}$$

Since f_i and ϕ are continuous, for $n \in \mathbb{N}$, we will have that :

$$f_i(u, y) = \psi(y) \text{ and } f_i(x, v) = \phi(x)$$

So, $(u, v) \in F_i(x, y)$, which implies that F_i has a closed graph.

It is obvious to see that the sets M_x and N_y are nonempty, closed and convex.

Using hypothesis (4), it is clear to see that F_i are commutative.

Thus, by theorem 3.5, F_i have a common fixed point $c^\star = (x^\star, y^\star)$.

So, we have $c^\star \in F_i c^\star = N_{y^\star} \times M_{x^\star}$.

in other words,

$$x^\star \in N_{y^\star} \Leftrightarrow f_i(x^\star, y^\star) = \max_{x \in X} f_i(x, y^\star) \geq \inf_{y \in Y} \max_{x \in X} f_i(x, y)$$

$$y^\star \in M_{x^\star} \Leftrightarrow f_i(x^\star, y^\star) = \min_{y \in Y} f_i(x^\star, y) \leq \sup_{x \in X} \min_{y \in Y} f_i(x, y)$$

Taking into account these last two inequalities and (1.4), we get

$$f_i(x^\star, y^\star) \leq \sup_{y \in Y} \min_{x \in X} f_i(x, y) \leq \inf_{x \in X} \max_{y \in Y} K(x, y) \leq f_i(x^\star, y^\star)$$

implying

$$\max_{x \in X} \min_{y \in Y} f_i(x, y) = f_i(x^\star, y^\star) = \min_{y \in Y} \max_{x \in X} f_i(x, y)$$

This completes the proof. □

Comparison between Von Neumann's Minimax theorem and our Minimax theorem:

- **Von Neumann's Minimax Theorem:**

  1. **Setup:** - Consider a two-player, zero-sum game with payoff function $f : X \times Y \to \mathbb{R}$, where X and Y are compact and convex sets in Euclidean spaces.

  2. **Continuous Bilinear Form:** - Represent the payoff function as a continuous bilinear form on the product space $X \times Y$.

3. **Saddle Point Existence:** - Use the Brouwer Fixed-Point Theorem and Kakutani's Fixed-Point Theorem to show the existence of a saddle point in the game. This saddle point corresponds to a pair of optimal strategies (x^*, y^*).

4. **Saddle Point Property:** - Show that the optimal strategies (x^*, y^*) satisfy the minimax property:

$$\max_{y\in Y}\min_{x\in X} f(x^*, y) = \min_{x\in X}\max_{y\in Y} f(x, y^*)$$

- **Our Minimax Theorem:**

1. **Setup:** - Consider a two-player, zero-sum game with family of commutative, affine and continuous payoff functions $f_i : X \times Y \to \mathbb{R}$, where X and Y are compact and convex sets in Banach spaces.

2. **Continuous Bilinear Form:** - Represent the family of payoff functions as a continuous bilinear form on the product space $X \times Y$.

3. **Saddle Point Existence:** - Use the Markoff Fixed-Point Theorem and to show the existence of a saddle point in the game. This saddle point corresponds to a pair of optimal strategies (x^*, y^*).

4. **Saddle Point Property:** - Show that the optimal strategies (x^*, y^*) satisfy the minimax property:

$$\max_{y\in Y}\min_{x\in X} f_i(x^*, y) = \min_{x\in X}\max_{y\in Y} f_i(x, y^*)$$

Remark 2. *The difference between the two results above is that each player has a* ***single*** *gain function in the case of the Neumann's Minimax Theorem, however in our Minimax Theorem, each player has a* ***family*** *of gain functions.*

Application and Modeling:

In the context of service companies, employer branding plays a crucial role in attracting and retaining talented employees. The dynamics of wage confidence involve the interplay between what employees expect in terms of compensation and how employers strategize to manage these expectations, it might imply that employers are making decisions to minimize the maximum potential negative impact on their branding and wage confidence.

- **Family of gain functions:** This suggests that there is a set or family of gain functions involved in the model. Gain functions could represent the benefits or positive outcomes that employers aim to maximize or minimize in the decision-making process.
- **Employer Branding:** This refers to the way in which an employer is perceived by employees, potential employees, and other stakeholders. It involves elements such as company culture, values, and reputation.

  For example, Employees, on the other hand, want to maximize their expected wage satisfaction, anticipating the best possible compensation outcome. Each employee can be seen as playing a strategy (negotiating for a higher wage) to maximize their potential satisfaction.
- **Wage Confidence Dynamics :** This implies that the model is considering the changing and dynamic nature of wage confidence. Wage confidence could be influenced by factors such as economic conditions, industry trends, and company performance.

  The company's objective is to find a wage policy that minimizes the maximum dissatisfaction among employees, considering the potential negotiations and expectations of each individual.
- **Service Companies:** The context of the model is within service companies, suggesting that the dynamics of employer branding and wage confidence are specific to the service industry.
- **Equilibrium Point:** An equilibrium point could be reached when the company establishes a wage policy and each employee sets their wage expectations in a way that no one has a strong incentive to deviate from their chosen strategy.
- **Mathematical Representation:** Let W be the wage offered by the company and E_i be the wage expectation of employee i. The company's objective is to find W such that $\min_i(W - E_i)$, Each employee's objective is to maximize their wage satisfaction $\max_W(W - E_i)$,

  At equilibrium, the company's choice of W and the employees' expectations E_i are such that no one has an incentive to deviate.

The family of gain functions $f_i : X \times Y \to \mathbb{R}$ is characterized by its values $f_i(x_{\alpha_i}, y_{\beta_j})$ when $\alpha_i \in I$ and $\beta_j \in J$ such that I and J are a nonempty convex and compact subsets. This is the reason why such games are called matrix game.

$$\begin{array}{c|c} W \backslash E_i & Y \\ \hline X & \begin{pmatrix} f_i(x_{\alpha_1}, y_{\beta_1}) & f_i(x_{\alpha_1}, y_{\beta_2}) & \cdots \\ f_i(x_{\alpha_2}, y_{\beta_1}) & f_i(x_{\alpha_2}, y_{\beta_2}) & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix} \end{array}$$

Once the game is represented by a matrix as above, we identify the i^{th} strategy of W with the i^{th} row of the matrix and the j^{th} strategy of E_i with the j^{th} column.

Conclusion:

Putting it all together, our model exploring how employers in service companies can strategically make decisions regarding employer branding and wage levels by using the Minimax Theorem and a family of gain functions. The goal may be to find optimal strategies that minimize the potential negative impact on employer branding and wage confidence dynamics.

While this application is conceptual and simplified, it demonstrates how the Minimax Theorem can be adapted to model strategic decision-making in the context of employer branding and wage dynamics within service companies. The actual implementation would require more detailed data and a sophisticated modeling approach.

Conflict of Interests

The authors declare that there is no conflict of interests.

References

[1] J.P. Aubin, Mathematical methods of games and economic theory, North-Holland, Amsterdam, 1982.

[2] A. Charbonnier-Voirin, M. Lissillour, La marque employeur comme outil de fidélisation organisationnelle, Rev. Rech. Sci. Gest. 125 (2018), 97–119.

[3] K. Backhaus, S. Tikoo, Conceptualizing and researching employer branding, Career Dev. Int. 9 (2004), 501–517. https://doi.org/10.1108/13620430410550754.

[4] S. Carl, S. Heikkilä, Fixed point theory in ordered sets and applications: from differential and integral equations to game theory, Springer, New York, 2010.

[5] G.W. Dean, M.C. Wells, eds., Forerunners of realizable values accounting in financial reporting, 1st ed., Routledge, 1982. https://doi.org/10.4324/9781003051091.

[6] G. Demange, J.P. Ponssard, Théorie des jeux et analyse économique, Presses Universitaires de France, (1994).

[7] Eric Denécé (2011). Diplomatie économique et compétition des États, Géoéconomie, 2011/1 (n 56), p. 71-78., p.71.

[8] E. Nicolas, Théorie des jeux, Dunod, Paris, 2013.

[9] E.G.D. Maganga, Contribution à l'analyse de la perception de la Marque employeur au Gabon: cas du groupe bancaire BGFIBank Gabon, Université Paul Valéry - Montpellier III, 2021.

[10] A. Gethin, Apports et limites de la théorie des jeux, Regards Croisés Econ. 22 (2018), 68–71.

[11] K. Laurent, Le management par la confiance, Ergo-Management, Toulouse, 2017.

[12] K. Laurent, La confiance au travail, introduction, Octares Editions, Paris, 2013.

[13] K. Fan, Fixed-point and minimax theorems in locally convex topological linear spaces, Proc. Natl. Acad. Sci. U.S.A. 38 (1952), 121–126. https://doi.org/10.1073/pnas.38.2.121.

[14] L. Combalbert, M. Méry, Les 5 leviers de la confiance: aidez vos collaborateurs à se dépasser, Eyrolles, Paris, 2016.

[15] K. Leyton-Brown, Y. Shoham, Essentials of game theory: a concise, multidisciplinary introduction, Springer, Cham, 2008. https://doi.org/10.1007/978-3-031-01545-8.

[16] P. Liger, Marketing RH attirer, intégrer et fidéliser les salariés, Dunod, Paris, 2013.

[17] A. Markoff, Quelques théorèmes sur les ensembles abéliens, C. R. (Doklady) Acad. Sci. URSS. 1 (1936), 311–313.

[18] A. Charbonnier-Voirin, M. Lissillour, La marque employeur comme outil de fidélisation organisationnelle, Audrey Charbonnier-Voirin, Maureen Lissillour, Rech. Sci. Gest. 125 (2018), 97–119.

[19] L. Benraïss-Noailles, O. Herrbach, C. Viot, Marque employeur et management responsable des ressources humaines, Rev. Manag. Avenir. 131 (2022), 103-105.

[20] J.F. Nash Jr., Equilibrium points in n-person games, Proc. Natl. Acad. Sci. U.S.A. 36 (1950), 48–49. https://doi.org/10.1073/pnas.36.1.48.

[21] V. Neveu, La confiance organisationnelle: définition et mesure, Université Paris 1, 2004.

[22] R. Reitter, B. Ramanantsoa, Confiance et défiance dans les organisations, Trust Management Institute, Paris, 2012.

[23] T.C. Schelling, The strategy of conflict, Harvard University Press, 1960.

[24] E. Simon, La confiance dans tous ses états, Rev. Fr. Gest. 175 (2007), 83–94.

[25] S.B. Nadler Jr, Multivalued contraction mappings, Pac. J. Math. 30 (1969), 475–488.

[26] J. von Neumann, O. Morgenstern, Theory of games and economic behavior, Princeton University Press, Princeton, 1944.

[27] J. von Neumann, Uber ein okonomisches gleichungssystemund eine verallgemeinerung des brouwerschen fixpunktsatzes, Erge. Math. Kolloq. 8 (1937), 73–83.

[28] X. Dai,A fixed point theorem of Markov-Kakutani type for commuting family of convex multivalued maps, Fixed Point Theory, 18 (2017), 155–166.