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FIXED POINT RESULTS IN SOFT b-FUZZY METRIC SPACES

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Abstract. The main aim of this paper is to investigate some fixed point theorems (FPTs) while considering some constraints on soft b-fuzzy metric. Various findings are demonstrated through the soft b-maps presented here, that includes Ξ-contraction following the continuity of soft t-norm. Moreover, the efficacy of the presented FPTs is verified through suitable illustrations and an application.

Keywords: soft fuzzy metric spaces; fixed points; contraction mapping.

2020 AMS Subject Classification: 47H10, 54H25.

1. Introduction

Zadeh [26] presented fuzzy theory by signifying of concept of uncertainty in real-world problems, which emerged as a solution to the ambiguity issue in control methods. Since then, various authors worked in this direction by using the notion of fuzzy numbers and fuzzification of several classical theories. Fuzzy set theory and applications have established themselves as one of the most active areas of research nowadays. In 1975, Kramosil and Michalek [19] initiated fuzzy metrical theory, and Grabiec provided a standard reformulation of it in [15]. Later, in [14], George and Veeramani somewhat altered this idea in order to redefine the theory of [19],

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which more closely aligned with the fundamental assumptions of a classical metric. They covered a wide range of topological concepts and provided various features of these spaces. Both concepts of fuzzy metric spaces have been extensively investigated theoretically and remain so to this day. However, determining the membership function in each unique situation proved to be challenging.

In order to address uncertainty more effectively, Molodtsov [21] created the notion of soft sets to address this problem. It is possible to think of Zadeh's fuzzy set as a particular instance of the soft set [21]. As a result, soft set theory has useful applications in a wide range of industries. Maji [20] highlighted a few uses for fuzzy soft sets and the theory of soft sets. The idea of soft metric space, which is based on the soft point of soft sets, was then introduced by Das and Samanta ([8]–[10]), who contributed to this field. Beaula and Gunaseli [3] introduced Fuzzy Soft Metric Spaces by employing a fuzzy soft point, while Beaula and Priyanga [2] provided the idea of fuzzy soft normed linear space.

Concept of b-metric spaces was firstly proposed by Bakhtin [5] and Czerwik ([6], [7]) as an extension of metric spaces that mitigate the triangular inequality. Wadkar *et al.* [24] expanded on this idea to define a soft b-metric space. [24] examined certain outcomes meeting generalized contractive requirements with α-monotone characteristic in an ordered soft b-metric space. For additional details over extensions, we advise ([1], [4], [11], [12], [14]–[22], [23]–[25]).

In this article, we give a new definition called soft b-fuzzy metric space ($\widehat{SbFMS}$), by using soft real sets. We introduce soft b-fuzzy contraction with the establishment of some FPTs in $\widehat{SbFMS}$ utilizing Ξ-contraction map.

2. Elementry Interpretations

To establish the primary findings, we offer some basic terminology in this section. The notations $\mathfrak{U}$, $\mathscr{T}$ and $\mathscr{T}(\mathfrak{U})$ stand for the universal set, the collection of parameters, as well as the assembly of all $\mathfrak{U}$-subsets respectively.

Definition 2.1. [21] A pair $(L,\mathscr{T})$ is referred to as a soft set over the universal set $\mathfrak{U}$ where $L:\mathscr{T}\longrightarrow\mathscr{T}(\mathfrak{U})$.

Definition 2.2. [20] If $L(\varsigma)=\mathfrak{U}\ \forall\ \varsigma\in\mathscr{T}$, then a soft set $(L,\mathscr{T})$ over $\mathfrak{U}$ is called an absolute soft set.

Definition 2.3. [8] If $L : \mathscr{T} \to \tilde{B}(\mathbb{R})$ where $\tilde{B}(\mathbb{R})$ signifies the collection of (non-empty) bounded subsets of $\mathbb{R}$, then $(L, \mathscr{T})$ is a soft real set (SRS). A SRS $(L, \mathscr{T})$ is named a soft real number (SRN) if, for $\hbar \in \mathscr{T}$, $L(\hbar)$ is the only element of $\tilde{B}(\mathbb{R})$ and is denoted as $\tilde{p}$. if $\tilde{p}(\hbar) = \{x\}$ for some $x \in \mathbb{R}$ *and SRN* $\tilde{p}$, we name it as $\bar{x}$.

Definition 2.4. [9] A soft set on $\mathfrak{U}$ is called soft point if for a single parameter $\hbar \in \mathscr{T}$, $L(\hbar) = \{p\}$, $p \in \mathfrak{U}$, $L(\nu) = \Omega \; \forall \; \nu \in \mathscr{T} \; \{\hbar\}$. It is named as $\tilde{p}_{\hbar}$.
$\tilde{p}_{\eta} \in (L, \mathscr{T})$ if $\tilde{p}_{\hbar} = \{p\} \subseteq L(\hbar)$ and is also signified as $\tilde{p}_{\hbar} \tilde{\in} (L, \mathscr{T})$. The group with soft points of $(L, \mathscr{T})$ is named with $S\mathscr{T}(L, \mathscr{T})$.

Definition 2.5. [9] A soft element is any map from $\mathscr{T}$ to $\mathfrak{U}$. Stated otherwise, a map $\iota : \mathscr{T} \to \mathfrak{U}$ is a soft element. $S_S(N)$ represents the soft set that results from grouping N of soft elements.

For a parametric set $\mathscr{T}$, the collection of SRS and non-negative SRS are represented by $R(\mathscr{T})$ and $R(\mathscr{T})^*$, respectively. Also, $[s,t](\mathscr{T})$ and $[0,\infty)(\mathscr{T})$, respectively, denote the collection of SRN in $[s,t]$ and $[0,\infty)$.

Definition 2.6. [9] These operations are applied for two SRS $\tilde{q}$ *and* $\tilde{s}$:

(1) $(\tilde{q} \oplus \tilde{s})(\hbar) = \{\tilde{q} + \tilde{s}, \hbar \in \mathscr{T}\}$,

(2) $(\tilde{q} \ominus \tilde{s})(\hbar) = \{\tilde{q} - \tilde{s}, \hbar \in \mathscr{T}\}$,

(3) $(\tilde{q} \odot \tilde{s})(\hbar) = \{\tilde{q}.\tilde{s}, \hbar \in \mathscr{T}\}$.

Definition 2.7. [10] We view $\tilde{\mathfrak{U}}_{\mathscr{T}}$ as a soft set that is absolute on a universal set $\mathfrak{U}$. A map $\mathfrak{F} : S\mathscr{T}(\tilde{\mathfrak{U}}_{\mathscr{T}}) \times S\mathscr{T}(\tilde{\mathfrak{U}}_{\mathscr{T}}) \to \mathbb{R}(\mathscr{T})^*$ is considered as soft metric over $\tilde{\mathfrak{U}}_{\mathscr{T}}$ if the conditions listed below hold true:

(1) $\mathfrak{F}(\tilde{w}_{t_i}, \tilde{z}_{t_j}) \geq 0$, $\forall \; \tilde{w}_{t_i}, \tilde{z}_{t_j} \in \tilde{\mathfrak{U}}_{\mathscr{T}}$,

(2) $\mathfrak{F}(\tilde{w}_{t_i}, \tilde{z}_{t_j}) = 0 \; \Leftrightarrow \; \tilde{w}_{t_i} = \tilde{z}_{t_j}, \forall \; \tilde{w}_{t_i}, \tilde{z}_{t_j} \in \tilde{\mathfrak{U}}_{\mathscr{T}}$,

(3) $\mathfrak{F}(\tilde{w}_{t_i}, \tilde{z}_{t_j}) = \mathfrak{F}(\tilde{z}_{t_j}, \tilde{w}_{t_i})$, $\forall \; \tilde{w}_{t_i}, \tilde{z}_{t_j} \in \tilde{\mathfrak{U}}_{\mathscr{T}}$,

(4)$\mathfrak{F}(\tilde{w}_{t_i}, \tilde{h}_{t_k}) \leq \mathfrak{F}(\tilde{w}_{t_i}, \tilde{z}_{t_j}) + \; \mathfrak{F}(\tilde{z}_{t_j}, \tilde{h}_{t_k}), \forall \; \tilde{w}_{t_i}, \tilde{z}_{t_j}, \tilde{w}_{t_k} \in \tilde{\mathfrak{U}}_{\mathscr{T}}$.

The absolute-soft set $\tilde{\mathfrak{U}}$ with map $\mathfrak{F}$ is called a soft metric space $(\widehat{SMS})$ and is written as $(\tilde{\mathfrak{U}}_{\mathscr{T}}, \mathfrak{F}, \mathscr{T})$.

Definition 2.8. [25] For two $\widehat{SMS}s$ $(\tilde{\mathfrak{U}}_{\mathscr{T}_L}, \mathfrak{F}, \mathscr{T}_L)$, $(\tilde{V}_{\mathscr{T}_M}, \mathfrak{F}, \mathscr{T}_M)$, consider a map (Γ, Δ) : $(\tilde{\mathfrak{U}}_{\mathscr{T}_L}, \mathfrak{F}, \mathscr{T}_L) \to (\tilde{V}_{\mathscr{T}_M}, \mathfrak{F}, \mathscr{T}_M)$. The map (Γ, Δ) is considered a soft map if $\Gamma : \tilde{\mathfrak{U}}_{\mathscr{T}_L} \to \tilde{V}_{\mathscr{T}_M}$ and $\Delta : \mathscr{T}_L \to \mathscr{T}_M$.

Definition 2.9. [13] The set $S_{\mathscr{F}} = \{(\tilde{p}_{t_i}, \mu_{S_{\mathscr{F}}}(\tilde{p}_{t_i})),\ \tilde{p}_{t_i} \in \tilde{\mathfrak{U}}_{\mathscr{T}}, t_i \in \mathscr{T}\}$, is a is called a soft fuzzy (SF) set in $\tilde{\mathfrak{U}}_{\mathscr{T}}$ wherein $\mu_{S_{\mathscr{F}}} : \tilde{\mathfrak{U}}_{\mathscr{T}} \longrightarrow [0,1](\mathscr{T})$ is soft membership map and $\mu_{S_{\mathscr{F}}}(\tilde{p}_{t_i})$ is the grade of soft membership for soft point $\tilde{p}_{t_i} \in S_{\mathscr{F}}$.

Definition 2.10. [13] The map $\tilde{\star} : [0,1](\mathscr{T}) \times [0,1](\mathscr{T}) \to [0,1](\mathscr{T})$ is a continuous soft t-norm if

$(1)\tilde{\star}$ *is commutative as well as associative*;

(2) *continuity of* $\tilde{\star}$ *is justified*;

(3) $\tilde{g} \,\tilde{\star}\, \tilde{1} = \tilde{g},\ \forall\, \tilde{g} \in [0,1](\mathscr{T})$;

(4) $\tilde{g} \,\tilde{\star}\, \tilde{h} \ \leq\ \tilde{e} \,\tilde{\star}\, \tilde{f}\ \forall\, \tilde{g},\ \tilde{h},\ \tilde{e},\ \tilde{f} \in [0,1]\mathscr{T}$.

Definition 2.11. [13] The map $\mathfrak{R}_{FM} : S\mathscr{T}(\tilde{\mathfrak{U}}_{\mathscr{T}}) \times S\mathscr{T}(\tilde{\mathfrak{U}}_{\mathscr{T}}) \times]0,\infty[(\mathscr{T}) \to [0,1](\mathscr{T})$ is considered as a soft fuzzy metric $(\widehat{SFM})$ on $\tilde{\mathfrak{U}}_{\mathscr{T}}$ if $\forall\, \tilde{w}_{t_i},\ \tilde{z}_{t_j},\ \tilde{h}_{t_k} \in \tilde{\mathfrak{U}}_{\mathscr{T}},\ \tilde{x},\ \tilde{l} > 0$, we have

(1) $\mathfrak{R}_{FM}(\tilde{w}_{t_i}, \tilde{z}_{t_j}, \tilde{x}) \geq \tilde{0}$,

(2) $\mathfrak{R}_{FM}(\tilde{w}_{t_i}, \tilde{z}_{t_j}, \tilde{x}) = \tilde{1} \ \Leftrightarrow\ \tilde{w}_{t_i} = \tilde{z}_{t_j}$,

(3) $\mathfrak{R}_{FM}(\tilde{w}_{t_i}, \tilde{z}_{t_j}, \tilde{x}) = \mathfrak{R}_{FM}(\tilde{z}_{t_i}, \tilde{w}_{t_j}, \tilde{x})$,

(4) $\mathfrak{R}_{FM}(\tilde{w}_{t_i}, \tilde{h}_{t_k}, \tilde{x} \oplus \tilde{l}) \geq \mathfrak{R}_{FM}(\tilde{w}_{t_i}, \tilde{z}_{t_j}, \tilde{x}) \,\tilde{\star}\, \mathfrak{R}_{FM}(\tilde{z}_{t_j}, \tilde{h}_{t_k}, \tilde{l})$,

(5) $\mathfrak{R}_{FM}(\tilde{w}_{t_i}, \tilde{z}_{t_j}, .) :]0,\infty[(\mathscr{T}) \to [0,1](\mathscr{T})$ *is continous*.

The space $(\tilde{\mathfrak{U}}_{\mathscr{T}}, \mathfrak{R}_{FM},\ \tilde{\star})$ is called soft fuzzy metric space $(\widehat{SFMS})$.

Definition 2.12. ([5]–[7], [24]) For a set $\mathfrak{U}$ and a map $\mathfrak{B} : \mathfrak{U} \times \mathfrak{U} \to [0,+\infty[$, if the following axioms hold true for $w, h, l \in \mathscr{W}$:

(b_1) $\mathfrak{B}(w,h) = 0 \Leftrightarrow w = h$;

(b_2) $\mathfrak{B}(w,h) = \mathfrak{B}(h,w)$;

(b_3) $\mathfrak{B}(w,h) \leq b(\mathfrak{B}(w,l) + \mathfrak{B}(l,h))$ *where* $k \geq 1$.

Then $\mathfrak{B}$ named as b-metric space and is symbolized by $(\mathfrak{U}, \mathfrak{B})$.

3. Main Results

Definition 3.1. The map $\mathfrak{R}_{FM} : S\mathscr{T}(\tilde{\mathfrak{U}}_{\mathscr{T}}) \times S\mathscr{T}(\tilde{\mathfrak{U}}_{\mathscr{T}}) \times]0,\infty[(\mathscr{T}) \to [0,1](\mathscr{T})$ is considered as a soft b-fuzzy metric $(\widehat{SbFM})$ on $\tilde{\mathfrak{U}}_{\mathscr{T}}$ if $\forall\ \tilde{w}_{t_i},\ \tilde{z}_{t_j},\ \tilde{h}_{t_k} \in \tilde{\mathfrak{U}}_{\mathscr{T}},\ \tilde{x},\ \tilde{l} > 0$, we have

(1) $\mathfrak{R}_{FM}(\tilde{w}_{t_i}, \tilde{z}_{t_j}, \tilde{x}) \geq \tilde{0}$,

(2) $\mathfrak{R}_{FM}(\tilde{w}_{t_i}, \tilde{z}_{t_j}, \tilde{x}) = \tilde{1} \ \Leftrightarrow\ \tilde{w}_{t_i} = \tilde{z}_{t_j}$,

(3) $\mathfrak{R}_{FM}(\tilde{w}_{t_i}, \tilde{z}_{t_j}, \tilde{x}) = \mathfrak{R}_{FM}(\tilde{z}_{t_i}, \tilde{w}_{t_j}, \tilde{x})$,

(4) $\mathfrak{R}_{FM}(\tilde{w}_{t_i}, \tilde{h}_{t_k}, b(\tilde{x} \oplus \tilde{l})) \geq \mathfrak{R}_{FM}(\tilde{w}_{t_i}, \tilde{z}_{t_j}, \tilde{x}) \,\tilde{\star}\, \mathfrak{R}_{FM}(\tilde{z}_{t_j}, \tilde{h}_{t_k}, b\tilde{l})$ *where* $b \geq 1$,

(5) $\mathfrak{R}_{FM}(\tilde{w}_{t_i}, \tilde{z}_{t_j}, .) :]0,\infty[(\mathscr{T}) \to [0,1](\mathscr{T})$ *is continous*.

The space $(\tilde{\mathfrak{U}}_{\mathscr{T}}, \mathfrak{R}_{FM},\ \tilde{\star},\ b \geq 1)$ is called soft b-fuzzy metric space $(\widehat{SbFMS})$.

Definition 3.2. A (soft) b-sequence $\{w^p_{t_i}\}$ in $\widehat{SbFMS}$ $(\tilde{\mathfrak{U}}_{\mathscr{T}}, \mathfrak{R}_{FM},\ \tilde{\star},\ b \geq 1)$ converges to a (soft) b-point $\tilde{z}_{t_j} \in \tilde{\mathfrak{U}}_{\mathscr{T}}$ if

$$\lim_{p\to\infty} \mathfrak{R}_{FM}(\tilde{w}^p_{t_i}, \tilde{z}_{t_j}, b\tilde{x}) = 1\ \forall\ x\ >\ 0.$$

Alternatively, $\exists\ \tilde{n}_0$ in $\mathbb{Z}^+$ and $\tilde{\varepsilon} \in]0,1[$ such that,

$$\mathfrak{R}_{FM}(\tilde{w}^p_{t_i},\ \tilde{z}_{t_j}, b\tilde{x}) > 1 \ominus \tilde{\varepsilon}\ \forall\ \tilde{x}\ >\ 0,\ p \geq \tilde{n}_0.$$

Definition 3.3. A (soft) b-sequence $\{w^p_{t_i}\}$ in $\widehat{SbFMS}$ $(\tilde{\mathfrak{U}}_{\mathscr{T}}, \mathfrak{R}_{FM},\ \tilde{\star},\ b \geq 1)$ is Cauchy if

$$\lim_{p,q\to\infty} \mathfrak{R}_{FM}(\tilde{w}^p_{t_i},\ \tilde{w}^q_{t_i}, b\tilde{x}) = 1,\ \forall \tilde{x}\ >\ 0.$$

Alternatively, $\exists\ \tilde{n}_0$ in $\mathbb{Z}^+$ and $\tilde{\varepsilon} \in]0,1[$ such that,

$$\mathfrak{R}_{FM}(\tilde{w}^p_{t_i},\ \tilde{w}^q_{t_i}, b\tilde{x}) > 1 \ominus \tilde{\varepsilon}\ \forall\ \tilde{x}\ >\ 0,\ p,q \geq \tilde{n}_0.$$

Definition 3.4. The $\widehat{SbFMS}$ owns completeness if each Cauchy sequence from $\widehat{SbFMS}$ is convergent in $\widehat{SbFMS}$.

Definition 3.5. If every SF sequence in $\tilde{\mathfrak{U}}_{\mathscr{T}}$ admits a convergent soft subsequence, then $\widehat{SbFMS}$ $(\tilde{\mathfrak{U}}_{\mathscr{T}}, \mathfrak{R}_{FM}, \tilde{\star}, b \geq 1)$ is compact.

Definition 3.6. For $\widehat{SbFMS}$ $(\tilde{\mathfrak{U}}_{\mathscr{T}}, \mathfrak{R}_{FM}, \tilde{\star}, b \geq 1)$, the map $(\Gamma, \Delta) : (\tilde{\mathfrak{U}}_{\mathscr{T}}, \mathfrak{R}_{FM}, \tilde{\star}) \to (\tilde{\mathfrak{U}}_{\mathscr{T}}, \mathfrak{R}_{FM}, \tilde{\star})$ is called a fuzzy soft contraction if $\exists\ \tilde{\beta} \in [0,1[$ with

$$\mathfrak{R}_{FM}((\Gamma,\Delta)\tilde{w}_{t_i}, (\Gamma,\Delta)\tilde{z}_{t_j}, b\tilde{x}) \geq \mathfrak{R}_{FM}(\tilde{w}_{t_i}, \tilde{z}_{t_j}, \frac{\tilde{x}}{\tilde{\beta}}),\ \forall\ \tilde{w}_{t_i}, \tilde{z}_{t_j} \in \tilde{\mathfrak{U}}_{\mathscr{T}},\ \tilde{x} > 0.$$

Definition 3.7. The map $\chi : \mathbb{R}(\mathscr{T}) \to [0,\infty[(\mathscr{T})$, if meets the following requirements; is referred to as Ξ-map:

$$(1)\ \chi(\tilde{x}) = 0 \Leftrightarrow \tilde{x} = 0;$$

$$(2)\ \chi(\tilde{h}) < \chi(\tilde{w})\ \Leftrightarrow\ \tilde{h} < \tilde{w};$$

$$(3)\ \chi\ is\ left\ continuous\ for\ \tilde{x} > 0;$$

$$(4)\ \chi\ follows\ continuity\ at\ \tilde{x} = 0;$$

$$(5)\ \chi(\tilde{x}) \to \infty,\ whenever\ x \to \infty.$$

Definition 3.8. For $\widehat{SbFMS}$ $(\tilde{\mathfrak{U}}_{\mathscr{T}}, \mathfrak{R}_{FM}, \tilde{\star}, b \geq 1)$, the map $(\Gamma, \Delta) : (\tilde{\mathfrak{U}}_{\mathscr{T}}, \mathfrak{R}_{FM}, \tilde{\star}, b \geq 1) \to (\tilde{\mathfrak{U}}_{\mathscr{T}}, \mathfrak{R}_{FM}, \tilde{\star}, b \geq 1)$ is a Ξ-contractive map if $\exists$ a b-soft real number $\tilde{\beta} \in [0,1[$ and $\chi \in \Xi$ with

$$\mathfrak{R}_{FM}((\Gamma,\Delta)\tilde{w}_{t_i}, (\Gamma,\Delta)\tilde{z}_{t_j}, b\chi(\tilde{x})) \geq \mathfrak{R}_{FM}\left(\tilde{w}_{t_i}, \tilde{z}_{t_j}, \chi\left(\frac{\tilde{x}}{\tilde{\beta}}\right)\right),\ \forall\ \tilde{w}_{t_i}, \tilde{z}_{t_j} \in \tilde{\mathfrak{U}}_{\mathscr{T}},\ \tilde{x} > 0.$$

Remark 3.9. This is the implication diagram that we see:

$Metric\ space \implies Soft\ metric\ space \implies Soft\ Fuzzy\ metric\ space$

$\Downarrow$

$b-metric\ space \implies Soft\ b-Fuzzy\ metric\ space$

OR

$\widehat{MS} \implies \widehat{SMS} \implies \widehat{SFMS}$

$\Downarrow$

$\widehat{bMS} \implies \widehat{SbFMS}$

Theorem 3.10. *Consider a complete* $\widetilde{SbFMS}$ $(\tilde{\mathfrak{U}}_{\mathscr{T}}, \mathfrak{R}_F, \tilde{\star})$ *with*

$$\lim_{p\to\infty} \mathfrak{R}_{FM}(\tilde{w}_{t_i}, \tilde{z}_{t_j}, b\tilde{x}) = 1,\ \forall \tilde{w}_{t_i}, \tilde{z}_{t_j} \in \tilde{\mathfrak{U}}_{\mathscr{T}}, \tilde{x} > 0. \tag{1}$$

Then, a single soft fixed point (SFP) is admitted by the soft fuzzy contraction (SFC) map (Γ,Δ) *on* $\tilde{K}_P$.

Proof. Let $\tilde{w}^0_{t_i} \in \tilde{\mathfrak{U}}_{\mathscr{T}}$ be a soft point and $\{\tilde{w}^p_{t_i}\}$ be a soft sequence where $\tilde{w}^p_{t_i} = (\Gamma,\Delta)^p \tilde{w}^0_{t_i}$.

By induction procedure, we acquire

$$\mathfrak{R}_{FM}(\tilde{w}^p_{t_i}, \tilde{w}^{p+1}_{t_i}, b\tilde{x}) \geq \mathfrak{R}_{FM}(\tilde{w}^0_{t_i}, \tilde{w}^1_{t_i}, \frac{b\tilde{x}}{\tilde{\beta}^p}). \tag{2}$$

By equation (2) and axiom (4) of $\widetilde{SbFMS}$, we obtain for $d \in \mathbb{Z}^+$,

$$\begin{aligned}\mathfrak{R}_{FM}(\tilde{w}^p_{t_i}, \tilde{w}^{p+d}_{t_i}, b\tilde{x}) &\geq \mathfrak{R}_{FM}(\tilde{w}^p_{t_i}, \tilde{w}^{p+1}_{t_i}, \frac{b\tilde{x}}{\tilde{d}}) \underbrace{\tilde{\star} \ldots \tilde{\star}}_{d\text{-times}} \mathfrak{R}_{FM}(\tilde{w}^{p+d-1}_{t_i}, \tilde{w}^{p+d}_{t_i}, \frac{b\tilde{x}}{\tilde{d}}) \\ &\geq \mathfrak{R}_{FM}(\tilde{w}^0_{t_i}, \tilde{w}^1_{t_i}, \frac{b\tilde{x}}{\tilde{d}}\beta^p) \underbrace{\tilde{\star} \ldots \tilde{\star}}_{d\text{-times}} \mathfrak{R}_{FM}(\tilde{w}^0_{t_i}, \tilde{w}^1_{t_i}, \frac{b\tilde{x}}{\tilde{d}\beta^{p+d-1}}).\end{aligned}$$

Using (1), we have

$$\lim_{p\to\infty} \mathfrak{R}_{FM}(\tilde{w}^p_{t_i}, \tilde{w}^{p+d}_{t_i}, b\tilde{x}) \geq \underbrace{1 \tilde{\star} 1 \tilde{\star} 1 \tilde{\star} \ldots \tilde{\star} 1}_{d\text{-times}} = 1.$$

Hence, the sequence $\{\tilde{w}^p_{t_i}\}$ is Cauchy $(\tilde{\mathfrak{U}}_{\mathscr{T}}, \mathfrak{R}_{FM}, \tilde{\star}, b \geq 1)$ and thus convergent due to completeness of $\widetilde{SbFMS}$ $(\tilde{\mathfrak{U}}_{\mathscr{T}}, \mathfrak{R}_{FM}, \tilde{\star}, b \geq 1)$. Now, assume that $\{\tilde{w}^p_{t_i}\} \to \tilde{z}_{t_j}$ *i.e.*

$$\lim_{p\to\infty} \mathfrak{R}_{FM}(\tilde{w}^p_{t_i}, \tilde{z}^q_{t_j}, b\tilde{x}) = 1. \tag{3}$$

Thus,

$$\begin{aligned}\mathfrak{R}_{FM}((\Gamma,\Delta)\tilde{z}_{t_j}, \tilde{z}_{t_j}, b\tilde{x}) &\geq \mathfrak{R}_{FM}((\Gamma,\Delta)\tilde{z}_{t_j}, (\Gamma,\Delta)\tilde{w}^p_{t_i}, \frac{b\tilde{x}}{2}) \tilde{\star} \mathfrak{R}_{FM}((\Gamma,\Delta)\tilde{w}^p_{t_i}, (\Gamma,\Delta)\tilde{z}_{t_j}, \frac{b\tilde{x}}{2}) \\ &\geq \mathfrak{R}_{FM}(\tilde{z}_{t_j}, \tilde{w}^p_{t_i}, \frac{b\tilde{x}}{2\tilde{\beta}}) \tilde{\star} \mathfrak{R}_{FM}(\tilde{w}^{p+1}_{t_i}, \tilde{z}_{t_j}, \frac{b\tilde{x}}{2}).\end{aligned}$$

By (3), we get

$$\begin{aligned}\lim_{p\to\infty}(\mathfrak{R}_{FM}(\Gamma,\Delta)\tilde{z}_{t_j}, \tilde{z}_{t_j}, b\tilde{x}) &\geq 1 \tilde{\star} 1 = 1, \\ \Rightarrow \lim_{p\to\infty}(\mathfrak{R}_{FM}(\Gamma,\Delta)\tilde{z}_{t_j}, \tilde{z}_{t_j}, b\tilde{x}) &= 1.\end{aligned}$$

Thus, $(\Gamma,\Delta)\tilde{z}_{t_j} = \tilde{z}_{t_j}$ and hence $\tilde{z}_{t_j}$ is SFP of (Γ,Δ).

It is simple to confirm the uniqueness of a SFP of the SFC-map (Γ,Δ). □

Theorem 3.11. *Consider a complete* $\overbrace{SbFMS}$ $(\tilde{\mathfrak{U}}_{\mathscr{T}}, \mathfrak{R}_{FM}, \tilde{\star}, b \geq 1)$ *with*

$$\lim_{p\to\infty} \mathfrak{R}_{FM}(\tilde{w}_{t_i}, \tilde{z}_{t_j}, b\tilde{x}) = 1 \;\forall\; p \;>\; 0. \tag{4}$$

Then, the Ξ*-contractive map* $(\Gamma,\Delta) : (\tilde{\mathfrak{U}}_{\mathscr{T}}, \mathfrak{R}_{FM}, \tilde{\star}, b \geq 1) \to (\tilde{\mathfrak{U}}_{\mathscr{T}}, \mathfrak{R}_{FM}, \tilde{\star}, b \geq 1)$ *admits a single SFP.*

Proof. Let $\tilde{w}^0_{t_i} \in \tilde{\mathfrak{U}}_{\mathscr{T}}$ be a soft point and $\{\tilde{w}^m_{t_i}\}$ be a soft sequence where $\tilde{w}^p_{t_i} = (\Gamma,\Delta)^p \tilde{w}^0_{t_i}$. According to criteria (1) and (4) of definition 3.7, for $\tilde{x} > 0$, $\exists$ a $\tilde{k} > 0$ for which $\tilde{x} > \chi(\tilde{k})$.

By induction procedure, we acquire

$$\mathfrak{R}_{FM}(\tilde{w}^p_{t_i}, \tilde{w}^{p+1}_{t_i}, b\tilde{x}) \geq \mathfrak{R}_{FM}\left(\tilde{w}^0_{t_i}, \tilde{w}^1_{t_i}, \frac{(\chi(bk))}{\tilde{\beta}^p}\right). \tag{5}$$

For $d \in \mathbb{Z}^+$, using conditions (5) and axiom (4) of $\overbrace{SbFMS}$, we obtain

$$\begin{aligned}
\mathfrak{R}_{FM}(\tilde{w}^p_{t_i}, \tilde{w}^{p+d}_{t_i}, b\tilde{x}) &\geq \mathfrak{R}_{FM}(\tilde{w}^p_{t_i}, \tilde{w}^{p+d}_{t_i}, \chi(k)) \\
&\geq \mathfrak{R}_{FM}\left(\tilde{w}^p_{t_i}, \tilde{w}^{p+1}_{t_i}, \chi\left(\frac{bk}{\tilde{d}}\right)\right) \underbrace{\tilde{\star} \ldots \tilde{\star}}_{d\text{-times}} \mathfrak{R}_{FM}\left(\tilde{w}^{p+d-1}_{t_i}, \tilde{w}^{p+d}_{t_i}, \chi\left(\frac{bk}{\tilde{d}}\right)\right) \\
&\geq \mathfrak{R}_{FM}\left(\tilde{w}^0_{t_i}, \tilde{w}^1_{p_i}, \chi\left(\frac{bk}{\tilde{d}\tilde{\beta}^p}\right)\right) \underbrace{\tilde{\star} \ldots \tilde{\star}}_{d\text{-times}} \mathfrak{R}_{FM}\left(\tilde{w}^0_{t_i}, \tilde{w}^1_{t_i}, \chi\left(\frac{bk}{\tilde{d}\tilde{\beta}^{p+d-1}}\right)\right).
\end{aligned}$$

Taking $\tilde{x} \to \infty$ and by (4), we get

$$\lim_{\tilde{x}\to\infty} \mathfrak{R}_{FM}(\tilde{w}^p_{t_i}, \tilde{w}^{p+d}_{t_i}, b\tilde{x}) \geq \underbrace{1 \,\tilde{\star}\, 1 \,\tilde{\star}\, 1 \,\tilde{\star} \ldots \tilde{\star}\, 1}_{d\text{-times}} \;=\; 1.$$

Hence, the SF sequence $\{\tilde{w}^m_{t_i}\}$ is Cauchy in $(\tilde{\mathfrak{U}}_{\mathscr{T}}, \mathfrak{R}_{FM}, \tilde{\star}, b \geq 1)$ and thus convergent due to completeness of space. Now, consider $\{\tilde{w}^p_{w_i}\}$ tending to $\tilde{z}_{t_j} \in (\tilde{\mathfrak{U}}_{\mathscr{T}})$, *i.e.*

$$\lim_{p\to\infty} \mathfrak{R}_{FM}(\tilde{w}^p_{t_i}, \tilde{z}^q_{t_j}, b\tilde{x}) = 1. \tag{6}$$

Also,

$$\begin{aligned}
\mathfrak{R}_{FM}((\Gamma,\Delta)\tilde{z}_{t_j}, \tilde{z}_{t_j}, b\tilde{x}) &\geq \mathfrak{R}_{FM}\left((\Gamma,\Delta)\tilde{z}_{t_j}, (\Gamma,\Delta)\tilde{w}^p_{t_i}, \frac{b\tilde{x}}{2}\right) \tilde{\star}\, \mathfrak{R}_{FM}\left((\Gamma,\Delta)\tilde{w}^p_{t_i}, \tilde{z}_{t_j}, \frac{b\tilde{x}}{2}\right) \\
&\geq \mathfrak{R}_{FM}\left(\tilde{z}_{t_j}, \tilde{w}^p_{t_i}, \chi\left(\frac{b\tilde{k}}{2\tilde{\beta}}\right)\right) \tilde{\star}\, \mathfrak{R}_{FM}\left(\tilde{w}^{p+1}_{t_i}, \tilde{z}_{t_j}, \frac{b\tilde{x}}{2}\right).
\end{aligned}$$

Following continuity of $\tilde{\star}$ and using (6), we get

$$\mathfrak{R}_{FM}((\Gamma,\Delta)\tilde{z}_{t_j},\tilde{z}_{t_j},b\tilde{x}) \to 1 \ as\ p \to \infty.$$

Thus, $\tilde{z}_{t_j}$ is a soft fixed point (SFP) of (Γ,Δ).

It is simple to demonstrate the uniqueness of a SFP of the Ξ-contractive map (Γ,Δ) on $\widehat{SbFMS}$ $(\tilde{\mathfrak{U}}_{\mathscr{T}},\mathfrak{R}_{FM},\ \tilde{\star},\ b\geq 1)$. □

4. ILLUSTRATIONS

Let $\mathfrak{U}=\{w,z,h\}$, $\mathscr{T}=\{p,q\}$ and soft t-norm described as $d\ \tilde{\star}\ h=\min\{d,h\}$. Then, $S\mathscr{T}(\tilde{\mathfrak{U}}_{\mathscr{T}})=\{\tilde{w}_p,\tilde{w}_q,\tilde{z}_p,\tilde{z}_q,\tilde{h}_p,\tilde{h}_q\}$. Fix $b=2$.

Let $\mathfrak{R}_{FM}: S\mathscr{T}(\tilde{\mathfrak{U}}_{\mathscr{T}})\times S\mathscr{T}(\tilde{\mathfrak{U}}_{\mathscr{T}})\times]0,\infty[(\mathscr{T})\to[0,1](\mathscr{T})$ be defined as:

$$\mathfrak{R}_{FM}(\tilde{w}_{t_i},\tilde{z}_{t_j},\tilde{x})=\mathfrak{R}_{FM}(\tilde{z}_{t_j},\tilde{w}_{t_i},\tilde{x})=\begin{cases}0, & \tilde{x}=0\\ \frac{4}{5}, & 0<\tilde{x}\leq 2\\ 1, & \tilde{x}>2,\end{cases}$$

To prove axiom 4 from Definition 3.1, let us consider different possibilities:

Case 1:

Let $\tilde{x}=\tilde{l}=0$.

$$\begin{aligned}\mathfrak{R}_{FM}(\tilde{w}_{t_i},\tilde{h}_{t_k},b(\tilde{x}\oplus\tilde{l})) &= \mathfrak{R}_{FM}(\tilde{w}_{t_i},\tilde{h}_{t_k},2(0\oplus 0))\\ &= \mathfrak{R}_{FM}(\tilde{w}_{t_i},\tilde{h}_{t_k},0)\\ &= 0.\end{aligned}$$

and

$$\begin{aligned}\mathfrak{R}_{FM}(\tilde{w}_{t_i},\tilde{z}_{t_j},\tilde{x})\ \tilde{\star}\ \mathfrak{R}_{FM}(\tilde{z}_{t_j},\tilde{h}_{t_k},b\tilde{l}) &= \mathfrak{R}_{FM}(\tilde{w}_{t_i},\tilde{z}_{t_j},0)\ \tilde{\star}\ \mathfrak{R}_{FM}(\tilde{z}_{t_j},\tilde{h}_{t_k},0)\\ &= 0\ \tilde{\star}\ 0\\ &= \min\{0,0\}\\ &= 0.\end{aligned}$$

Hence,

$$\Re_{FM}(\tilde{w}_{t_i},\tilde{h}_{t_k},b(\tilde{x}\oplus\tilde{l}))\geq\Re_{FM}(\tilde{w}_{t_i},\tilde{z}_{t_j},\tilde{x})\,\tilde{\star}\,\Re_{FM}(\tilde{z}_{t_j},\tilde{h}_{t_k},b\tilde{l}).$$

Case 2:

Let $\tilde{x}=\ \tilde{l}=1$.

$$\begin{aligned}\Re_{FM}(\tilde{w}_{t_i},\tilde{h}_{t_k},b(\tilde{x}\oplus\tilde{l})) &=\Re_{FM}(\tilde{w}_{t_i},\tilde{h}_{t_k},2(1\oplus 1))\\ &=\Re_{FM}(\tilde{w}_{t_i},\tilde{h}_{t_k},4)\\ &=1.\end{aligned}$$

and

$$\begin{aligned}\Re_{FM}(\tilde{w}_{t_i},\tilde{z}_{t_j},\tilde{x})\,\tilde{\star}\,\Re_{FM}(\tilde{z}_{t_j},\tilde{h}_{t_k},b\tilde{l}) &=\Re_{FM}(\tilde{w}_{t_i},\tilde{z}_{t_j},1)\,\tilde{\star}\,\Re_{FM}(\tilde{z}_{t_j},\tilde{h}_{t_k},2)\\ &=0.8\,\tilde{\star}\,0.8\\ &=\min\{0.8,0.8\}\\ &=0.8.\end{aligned}$$

Hence,

$$\Re_{FM}(\tilde{w}_{t_i},\tilde{h}_{t_k},b(\tilde{x}\oplus\tilde{l}))\geq\Re_{FM}(\tilde{w}_{t_i},\tilde{z}_{t_j},\tilde{x})\,\tilde{\star}\,\Re_{FM}(\tilde{z}_{t_j},\tilde{h}_{t_k},b\tilde{l}).$$

Case 3

Let $\tilde{x}=3,\ \tilde{l}=4$.

$$\begin{aligned}\Re_{FM}(\tilde{w}_{t_i},\tilde{h}_{t_k},b(\tilde{x}\oplus\tilde{l})) &=\Re_{FM}(\tilde{w}_{t_i},\tilde{h}_{t_k},2(3\oplus 4))\\ &=\Re_{FM}(\tilde{w}_{t_i},\tilde{h}_{t_k},14)\\ &=1.\end{aligned}$$

and

$$\begin{aligned}\Re_{FM}(\tilde{w}_{t_i},\tilde{z}_{t_j},\tilde{x})\,\tilde{\star}\,\Re_{FM}(\tilde{z}_{t_j},\tilde{h}_{t_k},b\tilde{l}) &=\Re_{FM}(\tilde{w}_{t_i},\tilde{z}_{t_j},3)\,\tilde{\star}\,\Re_{FM}(\tilde{z}_{t_j},\tilde{h}_{t_k},8)\\ &=1\,\tilde{\star}\,1\\ &=\min\{1,1\}\end{aligned}$$

$$= 1.$$

Hence,

$$\Re_{FM}(\tilde{w}_{t_i}, \tilde{h}_{t_k}, b(\tilde{x} \oplus \tilde{l})) \geq \Re_{FM}(\tilde{w}_{t_i}, \tilde{z}_{t_j}, \tilde{x}) \tilde{\star} \Re_{FM}(\tilde{z}_{t_j}, \tilde{h}_{t_k}, b\tilde{l}).$$

Case 4:

Let $\tilde{x} = 0$ and $\tilde{l} = 4$.

$$\begin{aligned}\Re_{FM}(\tilde{w}_{t_i}, \tilde{h}_{t_k}, b(\tilde{x} \oplus \tilde{l})) =& \Re_{FM}(\tilde{w}_{t_i}, \tilde{h}_{t_k}, 8) \\ =& 1.\end{aligned}$$

and

$$\begin{aligned}\Re_{FM}(\tilde{w}_{t_i}, \tilde{z}_{t_j}, \tilde{x}) \tilde{\star} \Re_{FM}(\tilde{z}_{t_j}, \tilde{h}_{t_k}, b\tilde{l}) &= \Re_{FM}(\tilde{w}_{t_i}, \tilde{z}_{t_j}, 0) \tilde{\star} \Re_{FM}(\tilde{z}_{t_j}, \tilde{h}_{t_k}, 8) \\ &= 0 \tilde{\star} 1 \\ &= \min\{0, 1\} \\ &= 0.\end{aligned}$$

Hence,

$$\Re_{FM}(\tilde{w}_{t_i}, \tilde{h}_{t_k}, b(\tilde{x} \oplus \tilde{l})) \geq \Re_{FM}(\tilde{w}_{t_i}, \tilde{z}_{t_j}, \tilde{x}) \tilde{\star} \Re_{FM}(\tilde{z}_{t_j}, \tilde{h}_{t_k}, b\tilde{l}).$$

Case 5:

Let $\tilde{x} = 0$ and $\tilde{l} = 2$.

$$\begin{aligned}\Re_{FM}(\tilde{w}_{t_i}, \tilde{h}_{t_k}, b(\tilde{x} \oplus \tilde{l})) &= \Re_{FM}(\tilde{w}_{t_i}, \tilde{h}_{t_k}, 2(0 \oplus 2)) \\ &= \Re_{FM}(\tilde{w}_{t_i}, \tilde{h}_{t_k}, 4) \\ &= 1.\end{aligned}$$

and

$$\begin{aligned}\Re_{FM}(\tilde{w}_{t_i}, \tilde{z}_{t_j}, \tilde{x}) \tilde{\star} \Re_{FM}(\tilde{z}_{t_j}, \tilde{h}_{t_k}, b\tilde{l}) &= \Re_{FM}(\tilde{w}_{t_i}, \tilde{z}_{t_j}, 0) \tilde{\star} \Re_{FM}(\tilde{z}_{t_j}, \tilde{h}_{t_k}, 4) \\ &= 0 \tilde{\star} 1\end{aligned}$$

$$= \min\{0,1\}$$

$$= 0.$$

Hence,

$$\Re_{FM}(\tilde{w}_{t_i}, \tilde{h}_{t_k}, b(\tilde{x} \oplus \tilde{l})) \geq \Re_{FM}(\tilde{w}_{t_i}, \tilde{z}_{t_j}, \tilde{x}) \,\tilde{\star}\, \Re_{FM}(\tilde{z}_{t_j}, \tilde{h}_{t_k}, b\tilde{l}).$$

Case 6

Let $\tilde{x} = 2$, and $\tilde{l} = 3$.

$$\begin{aligned}\Re_{FM}(\tilde{w}_{t_i}, \tilde{h}_{t_k}, b(\tilde{x} \oplus \tilde{l})) &= \Re_{FM}(\tilde{w}_{t_i}, \tilde{h}_{t_k}, 2(2 \oplus 3)) \\ &= \Re_{FM}(\tilde{w}_{t_i}, \tilde{h}_{t_k}, 10) \\ &= 1.\end{aligned}$$

and

$$\begin{aligned}\Re_{FM}(\tilde{w}_{t_i}, \tilde{z}_{t_j}, \tilde{x}) \,\tilde{\star}\, \Re_{FM}(\tilde{z}_{t_j}, \tilde{h}_{t_k}, b\tilde{l}) &= \Re_{FM}(\tilde{w}_{t_i}, \tilde{z}_{t_j}, 2) \,\tilde{\star}\, \Re_{FM}(\tilde{z}_{t_j}, \tilde{h}_{t_k}, 6) \\ &= 0.8 \,\tilde{\star}\, 1 \\ &= \min\{0.8, 1\} \\ &= 0.8\ .\end{aligned}$$

Hence,

$$\Re_{FM}(\tilde{w}_{t_i}, \tilde{h}_{t_k}, b(\tilde{x} \oplus \tilde{l})) \geq \Re_{FM}(\tilde{w}_{t_i}, \tilde{z}_{t_j}, \tilde{x}) \,\tilde{\star}\, \Re_{FM}(\tilde{z}_{t_j}, \tilde{h}_{t_k}, b\tilde{l}).$$

Also, $\Re_{FM}(\tilde{w}_{t_i}, \tilde{z}_{t_j}, \tilde{x}) = 1 \Leftrightarrow \tilde{w}_{t_i} = \tilde{z}_{t_j}\ \forall\ \tilde{w}_{t_i}, \tilde{z}_{t_j} \in \tilde{\mathfrak{U}}_{\mathscr{T}}$ and $\tilde{x} > 0$. It is easy to validate that $(\tilde{\mathfrak{U}}_{\mathscr{T}}, \Re_{FM},\ \tilde{\star},\ b \geq 1)$ is complete $\overbrace{SbFMS}$.

Now consider (Γ, Δ) defined as

$$(\Gamma,\Delta)(\tilde{w}_p) = \tilde{z}_p,\ (\Gamma,\Delta)(\tilde{w}_q) = \tilde{z}_q,\ (\Gamma,\Delta)(\tilde{z}_p) = \tilde{z}_q,$$

$$(\Gamma,\Delta)(\tilde{z}_q) = \tilde{z}_q,\ (\Gamma,\Delta)(\tilde{h}_p) = \tilde{w}_q,\ (\Gamma,\Delta)(\tilde{h}_q) = \tilde{w}_p.$$

Thus, (Γ, Δ) is a SC map on $\overbrace{SbFMS}$ $(\tilde{\mathfrak{U}}_{\mathscr{T}}, \Re_{FM},\ \tilde{\star},\ b \geq 1)$ and all of the requirements given in Theorem 1 are hence followed and z_q is the single fixed point here.

5. Application

Consider the integral equation:

$$(\Gamma,\Delta)\tilde{w}_{t_i}(q) = \int_0^q K(q,s,(\Gamma,\Delta)(\tilde{z}_{t_j}(q)))dq,\ \forall\ \tilde{w}_{t_i} \in \tilde{\mathfrak{U}}_{\mathscr{T}},$$

with (Γ,Δ) defined on $\widetilde{SbFMS}$ and $K \in ([0,I] \times [0,I] \times R, R)$ where $q,s \in [0,I]$ and $I > 0$. Let $\tilde{\mathfrak{U}}_{\mathscr{T}} = \mathscr{C}([0,1],R)$. Also, let

$$\begin{aligned}\mathfrak{R}_{FM}((\Gamma,\Delta)\tilde{w}_{t_i}(q),(\Gamma,\Delta)\tilde{z}_{t_j}(s),b\tilde{x}) &= \frac{\tilde{x}}{\tilde{x} \bigoplus \mathfrak{F}((\Gamma,\Delta)\tilde{w}_{t_i}(q),(\Gamma,\Delta)\tilde{z}_{t_j}(s))} \\ &= \frac{\tilde{x}}{\tilde{x} \bigoplus \lambda |(\Gamma,\Delta)\tilde{w}_{t_i}(q) - (\Gamma,\Delta)\tilde{z}_{t_j}(s)|}\end{aligned}$$

Clearly, $(\tilde{\mathfrak{U}}_{\mathscr{T}}, \mathfrak{R}_{FM},\ \tilde{\star},\ b \geq 1)$ is complete $\widetilde{SbFMS}$. Now, let

(1) $K(q,s,(\Gamma,\Delta)(\tilde{z}_{t_j})(q)) \geq 0$,

(2) *There exist* $\lambda > 0$ *such that*

$$|K(q,s,(\Gamma,\Delta)(\tilde{w}_{t_i})(q)) - K(s,q,(\Gamma,\Delta)(\tilde{z}_{t_j})(s))| \leq \lambda |(\Gamma,\Delta)(\tilde{w}_{t_i})q - (\Gamma,\Delta)(\tilde{z}_{t_j})s|,$$

(3) *There exist* $h,q \in (0,1);\ (q > h)$ *such that* $\lambda q < h$.

(4) $|(\Gamma,\Delta)(\tilde{w}_{t_j})(q) - (\Gamma,\Delta)(\tilde{z}_{t_j})(s)| \leq |\tilde{w}_{t_j}(q) - \tilde{z}_{t_j}(s)|$,

(5) $\lim_{x\to\infty}(\mathfrak{R}_{FM}(\Gamma,\Delta)\tilde{z}_{t_j},\tilde{z}_{t_j},b\tilde{x}) = 1$.

Define $S : \mathfrak{U} \to \mathfrak{U}$ as

$$\begin{aligned}S(\Gamma,\Delta)\tilde{w}_{t_i}(q) &= \int_0^q K(q,s,(\Gamma,\Delta)(\tilde{z}_{t_j})(q))dq \text{ and} \\ S(\Gamma,\Delta)\tilde{z}_{t_i}(s) &= \int_0^q K(s,q,(\Gamma,\Delta)(\tilde{z}_{t_j})(s))dq.\end{aligned}$$

$$\begin{aligned}|S(\Gamma,\Delta)\tilde{w}_{t_i}(q) - S(\Gamma,\Delta)\tilde{z}_{t_j}(s)| &= \left| \int_0^q [K(q,s,(\Gamma,\Delta)(\tilde{z}_{t_j})(q)) - K(s,q,(\Gamma,\Delta)(\tilde{z}_{t_j})(s))]dq \right| \\ &\leq \int_0^q \left| K(q,s,(\Gamma,\Delta)(\tilde{z}_{t_j})(q)) - K(s,q,(\Gamma,\Delta)(\tilde{z}_{t_j})(s)) \right| dq \\ &\leq \lambda |(\Gamma,\Delta)\tilde{w}_{t_i}(q) - (\Gamma,\Delta)\tilde{z}_{t_j}(s)| \\ &\leq \frac{h}{q} |(\Gamma,\Delta)\tilde{w}_{t_i}(q) - (\Gamma,\Delta)\tilde{z}_{t_j}(s)|.\end{aligned}$$

Now,

$$\begin{aligned}\Re_{FM}((\Gamma,\Delta)\tilde{w}_{t_i}(q),(\Gamma,\Delta)\tilde{z}_{t_j}(s),b\tilde{x}) &= \frac{\tilde{x}}{\tilde{x}\bigoplus\lambda|(\Gamma,\Delta)\tilde{w}_{t_i}(q)-(\Gamma,\Delta)\tilde{z}_{t_j}(s)|}\\ &\geq \frac{\tilde{x}}{\tilde{x}\bigoplus\frac{h}{q}|(\Gamma,\Delta)\tilde{w}_{t_i}(q)-(\Gamma,\Delta)\tilde{z}_{t_j}(s)|}\\ &\geq \frac{\tilde{x}}{\tilde{x}\bigoplus\frac{h}{q}|\tilde{w}_{t_j}(q)-\tilde{z}_{t_j}(s)|}\\ &= \Re_{FM}(\tilde{w}_{t_i}(q),\tilde{z}_{t_j}(s),\frac{b\tilde{x}}{\tilde{\beta}})\ (where\ \tilde{\beta}=\frac{q}{h}).\end{aligned}$$

Note that $\frac{b}{\beta}>1$. This shows the existence of SF b-contraction. Thus, all the requirements of Theorem 3.10 are satisfied. Therefore, (Γ,Δ) admits a single fixed point.

Conflict of Interests

The authors declare that there is no conflict of interests.

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