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OCCASIONALLY WEAKLY COMPATIBLE MAPPINGS ON INTUITIONISTIC FUZZY METRIC SPACES IN THE COMMON FIXED POINT

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Abstract: The purpose of this paper is to obtain common fixed point theorem in Intuitionistic fuzzy metric space. While proving this result, we utilize the idea of occasionally weakly compatible maps due to Al Thagafi and N. Shahzad[13]. Thisresult improves a multitude of relevant common fixed point theorems of the existing literature in Intuitionistic fuzzy metric space.

Keywords: occasionally weakly compatible maps; weakly compatible maps; Intuitionistic fuzzy metric space; fixed point.

2000 AMS Subject Classification:47H10; 54H25

1. Introduction

Park [9] introduced the notion of intuitionistic fuzzy metric spaces as a generalization of fuzzy sets introduced by Zadeh [15] while Atanassov [2] introduced the concept of intuitionistic fuzzy sets. Turkogluet. al [14], introduced the concept of compatible maps on intuitionistic fuzzy metric spaces. Turkoglu et al. [14] gave generalization of Jungck's common fixed point theorem [7] to intuitionistic fuzzy metric spaces. However, the study of common fixed points of non-compatible maps is also very interesting and this condition has further been weakened by introducing the notion of weakly compatible mappings by Jungck and Rhoades[7]. Gregory et al. [6], Sadati and Park [10], Y. J. Cho et al. [4] studied the concept of intuitionistic fuzzy metric

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spaces and its applications. In 2008 Al-Thagafi and N. Shahzad [13] introduced the notion of occasionally weakly compatible mappings which is more general than the concept of weakly compatible maps.

In this paper, with the help of occasionally weakly compatible mappings, we prove common fixed point theorem in intuitionistic fuzzy metric space. We extend generalized and improved the corresponding results given by many authors earlier given in intuitionistic fuzzy metric space.

2. Preliminaries

Definition 1 ([11]). A binary operation $*: [0, 1] \times [0, 1] \to [0, 1]$ is a continuous t-norm if it satisfies the following conditions:

(1) $*$ is associative and commutative,

(2) $*$ is continuous,

(3) $a*1 = a$ for all $a \in [0, 1]$,

(4) $a*b \leq c*d$ whenever $a \leq c$ and $b \leq d$, for each $a, b, c, d \in [0, 1]$.

Definition 2 ([11]). A binary operation $\lozenge: [0, 1] \times [0, 1] \to [0, 1]$ is a continuous t-conorm if it satisfies the following conditions:

(1) $\lozenge$ is associative and commutative,

(2) $\lozenge$ is continuous,

(3) $a \lozenge 0 = a$ for all $a \in [0, 1]$,

(4) $a \lozenge b \leq c \lozenge d$ whenever $a \leq c$ and $b \leq d$, for each $a, b, c, d \in [0, 1]$.

Definition 3 ([1]). A 5-tuple $(X, M, N, *, \lozenge)$ is called a intuitionistic fuzzy metric space (Shortly IFM Space) if X is an arbitrary (non-empty) set, $*$ is a continuous t-norm, $\lozenge$ is a continuous t- conorm and M, N are fuzzy sets on $X^2 \times (0,\infty)$, satisfying the following conditions:

(IFM-1) $M(x, y, t) + N(x, y, t) \leq 1$, for all $x, y \in X$ and $t > 0$;

(IFM-2) $M(x, y, 0) = 0$, for all $x, y \in X$;

(IFM-3) $M(x, y, t) = 1$ for all $x, y \in X$ and $t > 0$ if and only if $x = y$;

(IFM-4) $M(x, y, t) = M(y, x, t)$ for all $x, y \in X$ and $t > 0$;

(IFM-5) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$ for all $x, y, z \in X$ and $s, t > 0$;

(IFM-6) for all $x, y \in X$, $M(x, y, .) : [0,\infty) \rightarrow [0, 1]$ is left continuous;

(IFM-7) $\lim_{t\rightarrow\infty} M(x, y, t) = 1$ for all $x, y \in X$ and $t > 0$;

(IFM-8) $N(x, y, 0) = 1$, for all $x, y \in X$;

(IFM-9) $N(x, y, t) = 0$ for all $x, y \in X$ and $t > 0$ if and only if $x = y$;

(IFM-10) $N(x, y, t) = N(y, x, t)$ for all $x, y \in X$ and $t > 0$;

(IFM-11) $N(x, y, t) \lozenge N(y, z, s) \geq N(x, z, t + s)$ for all $x, y, z \in X$ and $s, t > 0$;

(IFM-12) for all $x, y \in X$, $N(x, y, .) : [0,\infty) \rightarrow [0, 1]$ is right continuous;

(IFM-13) $\lim_{t\rightarrow\infty} N(x, y, t) = 0$ for all $x, y \in X$.

Then (M, N) is called an intuitionistic fuzzy metric in X. The functions $M(x, y, t)$ and $N(x, y, t)$ denote the degree of nearness and the degree of non-nearness between x and y with respect to t, respectively.

Remark 1. Every fuzzy metric space (X, M, *) is an intuitionistic fuzzy metric space of the form $(X, M, 1-M, *, \lozenge)$ such that t-norm * and t-conorm $\lozenge$ are associated i.e.

$x \lozenge y = 1 - ((1 - x) * (1 - y))$ for any $x, y \in X$.

Example 1. Let (X, d) be a metric space. Define t-norm $a * b = \min\{a, b\}$ and the t-conorm $a \lozenge b = \min\{1, a+b\}$ and for all $a, b \in [0, 1]$ and let M_d and N_d be fuzzy sets on $X^2 \times [0, \infty)$,define as follows

$$M_d(x, y, t) = \frac{t}{t+d(x,y)}$$

and

$$N_d(x, y, t) = \frac{d(x,y)}{t+d(x,y)}$$

Then $(X, M, N, *, \lozenge)$ is an intuitionistic fuzzy metric space. We call (M_d, N_d) intuitionistic fuzzy metric induced by a metric ‘d’ the standard intuitionistic fuzzy metric.

Remark 2. In intuitionistic fuzzy metric space $(X, M, N, *, \lozenge)$, $M(x, y, .)$ is non-decreasing and $N(x, y, .)$ is non-increasing for all $x, y \in X$.

Definition 4. Let $(X, M, N, *, \lozenge)$ be a intuitionistic fuzzy metric space. Consider I :

$X \to X$ and $T : X \to CB(X)$. A point $z \in X$ is called a coincidence point of I and T if and only if $Iz \in Tz$. We denote by CB(X) the set of all non-empty bounded and closed subsets of X .

Definition 5. A pair of self mappings (A, S) of a metric space is said to be weakly compatible if they commute at the coincidence points i.e. Ax = Sx for some x in X, then ASx = SAx.

It is easy to see that two compatible maps are weakly compatible but converse is not true.

Definition 6.Two self mappings A and S of Intuitionistic fuzzy metric space (X, M, N, *, ◊) are said to be occasionally weakly compatible (owc) iff there is a point x in X which is coincidence point of A and S at which A and S commute.

Lemma 1. Let (X, M, N, *, ◊) be Intuitionistic fuzzy metric space and for all x, y $\in X$, $t > 0$ and if for a number $k \in (0,1)$,

$M(x, y, kt) \geq M(x, y, t)$

and

$N(x, y, kt) \leq N(x, y, t)$ Then $x = y$.

Lemma 2. Let A and S be the self maps in an Intuitionistic fuzzy metric space (X, M, N, *, ◊) and let A and S have a unique point of coincidence, w = Ax = Sx, then w is the unique common fixed point of A and S.

Proof: Since A and B are owc, there exists a point x in X such that Ax = Sx = w and ASx = SAx. Thus, AAx = ASx = SAx, which says that Ax is also a point of coincidence of A and S. Since the point of coincidence w = Ax is unique by hypothesis, SAx = AAx = Ax, and w = Ax is a common fixed point of A and S. Moreover, if z is any common fixed point of A and S, then z = Az =Sz = w by the uniqueness of the point of coincidence.

Remark 3. Let Φ be the set of all continuous and increasing functions $\phi: [0, 1]^5 \to [0, 1]$ in any coordinate and $\phi(t, t, t, t, t) > t$ for all t in [0, 1]. Also let Ψ be the set of all continuous and decreasing functions
$\psi: [0, 1]^5 \to [0, 1]$ in any coordinate and $\psi(t, t, t, t, t) < t$ for all t in [0, 1].

3. Main results

Theorem 3.1. Let (X, M, N, *, ◊) be a IFM with continuous t-norm * and continuous t-conorm ◊ defined by $t * t \geq t$ and $(1 - t) \lozenge (1 - t) \leq (1 - t)$ for all $t \in [0, 1]$. Let A, B, S and T be self maps of X. Let Pairs (A, S) and (B, T) are occasionally weakly compatible self mappings on given IFM (X, M, N, *, ◊) satisfying:

If there exits $0 < q < \frac{1}{2}$ such that

(3.1.1) $M(Ax, By, qt) \geq \phi\{M(Sx, Ty, t), M(Ax, Sx, t), M(By, Ty, t), M(Ax, Ty, \alpha t), M(By, Sx, (2-\alpha)t)\}$

and

$N(Ax, By, qt) \leq \psi\{N(Sx, Ty, t), N(Ax, Sx, t), N(By, Ty, t), M(Ax, Ty, \alpha t), N(By, Sx, (2-\alpha)t)\}$

for all $0 < \alpha < 2$ and $x, y \in X$ and $\phi(t) > t$ and $\psi(t) < t$. Then there exist a unique common fixed point of A, B, S, and T.

Proof. As the pairs (A, S) and (B, T) are occasionally weakly compatible, there exist points x and y in X such that Ax = Sx, and By = Ty. Now we show that Ax = By. With the help of inequality (3.1.1)

$M(Ax, By, qt) \geq \phi\{M(Sx, Ty, t), M(Ax, Sx, t), M(By, Ty, t), M(Ax, Ty, \alpha t), M(By, Sx, (2-\alpha)t)\}$

$= \phi\{M(Ax, By, t), M(Ax, Ax, t), M(By, By, t), M(Ax, By, \alpha t), M(By, Ax, (2-\alpha)t)\}$

taking $\alpha = 1 - \beta$ where $\beta \in (0, 1)$ then we have

$= \phi\{M(Ax, By, t), M(Ax, Ax, t), M(By, By, t), M(Ax, By, (1-\beta)t), M(By, Ax, (1+\beta)t)\}$

on taking limit $\beta \to 0$

$= \phi\{M(Ax, By, t), M(Ax, Ax, t), M(By, By, t), M(Ax, By, t), M(By, Ax, t)\}$

$= \phi\{M(Ax, By, t)] > M(Ax, By, t)$

and

$N(Ax, By, qt) \leq \psi\{N(Sx, Ty, t), N(Ax, Sx, t), N(By, Ty, t), N(Ax, Ty, \alpha t), N(By, Sx, (2-\alpha)t)\}$

$= \psi\{N(Ax, By, t), N(Ax, Ax, t), N(By, By, t), N(Ax, By, \alpha t), N(By, Ax, (2-\alpha)t)\}$

taking $\alpha=1-\beta$ where $\beta\in(0, 1)$ then we have

$= \psi\{N(Ax, By, t), N(Ax, Ax, t), N(By, By, t), N(Ax, By, (1-\beta) t), N(By, Ax, (1+\beta)t)\}$

on taking limit $\beta\rightarrow 0$

$= \psi\{N(Ax, By, t), N(Ax, Ax, t), N(By, By, t), N(Ax, By, t), N(By, Ax, t)\}$

$= \psi\{N(Ax, By, t)] < N(Ax, By, t)$

Thus Ax = By. Therefore, Ax = Sx = By = Ty. If there is another point z such that Az = Sz, then again by using inequality (3.1.1), it follows that Az = Sz = By = Ty i.e. Az = Ax. Hence w = Ax = Sx is unique point of coincidence of A and S. By Lemma 2, w is the unique common fixed point of A and S i.e. Aw = Sw = w. Similarly, there is unique point z in X such that z = Bz = Tz. Now, we claim that w = z.

For this, put x = w and y = z in (3.1.1), we have

$M(w, z, qt) = M(Ax, By, qt) \geq \phi\{M(Sx, Ty, t), M(w, Sx, t), M(By, Ty, t), M(w, Ty, \alpha t), M(By, Sx, (2-\alpha)t)\}$

$= \phi\{M(w, z, t), M(w, w, t), M(z, z, t), M(w, z, \alpha t), M(z, w, (2-\alpha)t)\}$

taking $\alpha=1-\beta$ where $\beta\in(0, 1)$ then we have

$= \phi\{M(w, z, t), M(w, w, t), M(z, z, t), M(w, z, (1-\beta)t), M(z, w, (1+\beta)t)\}$

on taking limit $\beta\rightarrow 0$

$= \phi\{M(w, z, t), M(w, w, t), M(z, z, t), M(w, z, t), M(z, w, t)\}$

$= \phi\{M(w, z, t)\} > M(w, z, t)$

and

$N(w, z, qt) = N(Ax, By, qt) \leq \psi\{N(Sx, Ty, t), N(Ax, Sx, t), N(By, Ty, t), N(Ax, Ty, \alpha t), N(By, Sx, (2-\alpha)t)\}$

$= \psi\{N(w, z, t), N(w, w, t), N(z, z, t), N(w, z, \alpha t), N(z, w, (2-\alpha)t)\}$

taking $\alpha=1-\beta$ where $\beta\in(0, 1)$ then we have

$= \psi\{N(w, z, t), N(w, w, t), N(z, z, t), N(w, z, (1-\beta) t), N(z, w, (1+\beta)t)\}$

on taking limit $\beta\rightarrow 0$

$= \psi\{N(w, z, t), N(w, w, t), N(z, z, t), N(w, z, t), N(z, w, t)\}$

$= \psi\{N(w, z, t)\} < N(w, z, t)$

This gives, w = z. Hence, w is unique common fixed point of A, B, S and T in X. This completes the proof.

Theorem 3.2. Let (X, M, N, *, ◊) be intuitionistic fuzzy metric space and let A, B, S, and T be self maps of X. Pairs (A, S) and (B, T) be occasionally weakly compatible self mappings on (X, M, N, *, ◊). If there exits $0 < q < \frac{1}{2}$ such that

(3.1.2) $M(Ax, By, qt) \geq [\alpha_1(t)M(Sx, Ty, t) + \alpha_2(t)M(Ax, Sx, t) + \alpha_3(t)M(By, Ty, t) + \alpha_4(t)M(Ax, Ty, \alpha t) + \alpha_5(t)M(By, Sx, (2-\alpha)t)]^{1/2}$

and

$N(Ax, By, qt) \leq [\alpha_1(t)N(Sx, Ty, t) + \alpha_2(t)N(Ax, Sx, t) + \alpha_3(t)N(By, Ty, t) + \alpha_4(t)N(Ax, Ty, \alpha t) + \alpha_5(t)N(By, Sx, (2-\alpha)t)]^{1/2}$

for all $x, y\in X$, $0 < \alpha < 2$ and $\alpha_i : R^+\rightarrow (0, 1]$ such that $\sum\alpha_i(t) = 1$ where i=1 to5. Then A, B, S, and T have a unique common fixed point in X.

Proof. By Theorem 3.1, if we define

$\phi (x_1, x_2, x_3, x_4, x_5) \geq [\alpha_1(t) x_1 + \alpha_2(t) x_2 + \alpha_3(t) x_3 + \alpha_4(t) x_4 + \alpha_5(t) x_5]^{1/2}$

and

$\psi (x_1, x_2, x_3, x_4, x_5) \leq [\alpha_1(t) x_1 + \alpha_2(t) x_2 + \alpha_3(t) x_3 + \alpha_4(t) x_4 + \alpha_5(t) x_5]^{1/2}$

then we have the conclusion.

Theorem 3.3. Let (X, M, N, ⋆, ◊)be a IFM. Let A, B, S, T, P and Q be self maps of X.Let Pairs (P, ST) and (Q, AB) are occasionally weakly compatible self mappings on given IFM (X, M, N, *, ◊) satisfying: If there exits $0 < q < \frac{1}{2}$ such that

(3.1.3) $M(Px, Qy, qt) \geq \phi\{M(STx, ABy, t), M(Px, STx, t), M(Qy, ABy, t), M(Px,$

$ABy, \alpha t), M(Qy, STx, (2-\alpha)t)\}$

and

$N(Px, Qy, qt) \leq \psi\{N(STx, ABy, t), N(Px, STx, t), N(Qy, ABy, t), N(Px, ABy, \alpha t), N(Qy, STx, (2-\alpha)t)\}$

for all $x, y \in X$, $0 < \alpha < 2$ and $\phi(t) > t$, $\psi(t) < t$. Then there exist a unique common fixed point of A, B, S, T, P, and Q.

Proof. As the pairs (P, ST) and (Q, AB) are occasionally weakly compatible, there exist points x and y in X such that Px = STx, and Qy = ABy. Now we show that Px = Qy. With the help of inequality (3.1.3)

$M(Px, Qy, qt) \geq \phi\{M(STx, ABy, t), M(Px, STx, t), M(Qy, ABy, t), M(Px, ABy, \alpha t), M(Qy, STx, (2-\alpha)t)\}$

$= \phi\{M(Px, Qy, t), M(Px, Px, t), M(Qy, Qy, t), M(Px, Qy, \alpha t), M(Qy, Px, (2-\alpha)t)\}$

taking $\alpha = 1$

$= \phi\{M(Px, Qy,t)\} > M(Px, Qy, t)$

and

$N(Px, Qy, qt) \leq \psi\{N(STx, ABy, t), N(Px, STx, t), N(Qy, ABy, t), N(Px, ABy, \alpha t), N(Qy, STx, (2-\alpha)t)\}$

$= \psi\{N(Px, Qy, t), N(Px, Px, t), N(Qy, Qy, t), N(Px, Qy, \alpha t), N(Qy, Px, (2-\alpha)t)\}$

taking $\alpha = 1$

$= \psi\{N(Px, Qy,t)\} > N(Px, Qy, t)$

Thus Px = Qy. Therefore, Px = STx = Qy = ABy. If there is another point z such that Pz = STz, then again by using inequality (3.1.3), it follows that Pz = STz = Qy = ABy i.e. Pz = Px. Hence w = Px = STx is unique point of coincidence of P and ST. By Lemma 2, w is the unique common fixed point of P and ST i.e. Pw = STw = w. Similarly, there is unique point z in X such that z = Qz = ABz. Now, we claim that w = z.For this, put x = w and y = z in (3.1.3), we have

$M(w, z, qt) = M(Pw, Qz, qt) \geq \phi\{M(STx, ABy, t), M(Px, STx, t), M(Qy, ABy, t), M(Px, ABy, \alpha t), M(Qy, STx, (2-\alpha)t)\}$

taking α =1

= ϕ\{M(w, z, t), M(w, w, t), M(z, z, t), M(w, z, t), M(z, w, t)\}

= ϕ\{M(w, z, t)\}> M(w, z, t)

and

N(w, z, qt) = N(Pw, Qz, qt) $\leq \psi$\{N(STx, ABy, t), N(Px, STx, t), N(Qy, ABy, t), N(Px, ABy, αt), N(Qy, STx, (2-α)t)\}

= ψ\{N(w, z, t), N(w, w, t)\}, N(z, z, t), N(w, z, t), N(z, w, t)\}

= ψ\{N(w, z, t)\}< N (w, z, t).

This gives, w = z. Hence, w is unique common fixed point of A, S, B, T, P and Q in X. This completes the proof.

Theorem 3.4. Let (X, M, N, *, ◊)be a intuitionistic fuzzy metric space. Let S and T be self maps of X. Let Pairs (S, T) is occasionally weakly compatible self mappings on given IFM (X, M, N, *, ◊) satisfying: If there exits $0 < q < \frac{1}{2}$ such that

(3.1.4) M(x, y, qt) $\geq \phi$\{M(Sx, Ty, t), M(x, Sx, t), M(y, Ty, t), M(x, Ty, αt), M(y, Sx, (2-α)t)\}

and

N(x, y, qt) $\leq \psi$\{N(Sx, Ty, t), N(x, Sx, t), N(y, Ty, t), N(x, Ty, αt), N(y, Sx, (2-α)t)\}

for all x, y$\in$X, $0 < \alpha < 2$ and ϕ (t) > t, ψ (t) < t. Then S and T have a unique common fixed point in X.

Proof. If we set A = B = I (the identity mapping) in Theorem 3.1, then it is easy to check that the pairs (I, S) and (I, T) are owc. Hence by Theorem 3.1, S and T have a unique common fixed point in X.

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