Available online at http://scik.org
Adv. Fixed Point Theory, 2024, 14:58
https://doi.org/10.28919/afpt/8878
ISSN: 1927-6303

WEAK FORM OF IDEAL NANO TOPOLOGICAL SPACES FOR SEMI-LOCAL FUNCTIONS

P. GAYATHRI[1], G. SELVI[1,*], I. RAJASEKARAN[2]

[1]Department of Mathematics, Saveetha School of Engineering, Saveetha Institute of Medical and Technical Sciences, Chennai- 602105, Tamil Nadu, India

[2]Department of Mathematics, Tirunelveli Dakshina Mara Nadar Sangam College, T. Kallikulam-627 113, Tirunelveli District, Tamil Nadu, India

Abstract. In this paper we introduced *ns*-$\mathscr{OS}$ in a perfect nano topological space $\mathbb{N}^X_\star$, we define a nano semi-local function $T^\star_n(\mathbb{I},\mathscr{N})$. Several semi-local function characteristics and attributes and the new notions of ($\mathbb{sI}^n_\alpha$, $\mathbb{sI}^n_s$, $\mathbb{sI}^n_p$, $\mathbb{sI}^n_b$ and $\mathbb{sI}^n_\beta$)-open sets, which are straightforward variations of *n*-open sets in $\mathbb{N}^X_\star$, are introduced and studied in this paper. Additionally, we describe the connections between them and the associated attributes.

Keywords: nano topological space; semi-local function; nano idea.

2010 AMS Subject Classification: 54C05, 54C08, 54C10.

1. Introduction

Throughout this paper, using notations open sets we denote by $\mathscr{OS}$(closed sets is denote $\mathscr{CS}$) and $(U,\mathscr{N})$ we denote by $\mathbb{N}^X$, for an ideal nano topological spaces $(U,\mathbb{I},\mathscr{N})$ we denote by $\mathbb{N}^X_\star$.

*Corresponding author

E-mail address: mslalima11@gmail.com

Received August 29, 2024

Parimala et al. [4, 5] offered several novel ideas with the impression of ideal nano topological spaces.

Using Rajasekaran [8] ns-$\mathscr{OS}$ in a perfect nano topological space $\mathbb{N}^X_\star$, we define a nano semi-local function $T^\star_n(\mathbb{I},\mathscr{N})$. We investigate several semi-local function characteristics and attributes and the new notions of ($\mathbb{SI}^n_\alpha$, $\mathbb{SI}^n_s$, $\mathbb{SI}^n_p$, $\mathbb{SI}^n_b$ and $\mathbb{SI}^n_\beta$)-open sets, those are straightforward variations of n-open sets in $\mathbb{N}^X_\star$, are introduced and studied in this paper. Additionally, we describe the connections between them and the associated attributes.

2. Preliminaries

Consequently, the space $(U,\tau_R(X))$ is referred to as the nano topological space, and the topology $\tau_R(X)$ on U is referred to as the nano topology with respect to X, and the space $(U,\tau_R(X))$ is referred to as the nano topological space. Nano-open sets, or n-$\mathscr{OS}$, are the names given to the components of $\tau_R(X)$. A n-$\mathscr{OS}$'s opposite is referred to as a n-$\mathscr{CS}$.

In the remaining sections of the work, a nano topological space is denoted by the symbol $\mathbb{N}^X$, where $\mathscr{N}$ equals $\tau_R(X)$. A subset T of U's nano-interior and nano-closure are identified by the symbols $I_n(T)$ and $C_n(T)$, respectively.

Definition 2.1. [3, 6, 9] *A subset T of $\mathbb{N}^X$ is called $n\alpha$-$\mathscr{OS}$, ns-$\mathscr{OS}$, np-$\mathscr{OS}$, nb-$\mathscr{OS}$, $n\beta$-$\mathscr{OS}$.*

The complements of the aforementioned sets are known as their corresponding closed sets.

Definition 2.2. [8]

Let $\mathbb{N}^X_\star$ and T a subset of U. Then $T^\star_n(\mathbb{I},\mathscr{N}) = \{x \in U : Z \cap K \notin \mathbb{I}$, for every $K \in ns$-$\mathscr{OS}\}$ is called the nano semi-local function of T with respect to $\mathbb{I}$ and $\mathscr{N}$, where n-$SO(U,x) = \{K \in n$-$SO(U) | x \in K\}$. When there is no ambiguity, we shall only use the notation $T^\star_n$ for $T^\star_n(\mathbb{I},\mathscr{N})$.

Theorem 2.1. [8] *Let $\mathbb{N}^X_\star$ and $T,P \subseteq U$. Then*

(1) $T \subseteq P \Rightarrow T^\star_n \subseteq P^\star_n$,
(2) $T^\star_n = C_n(T^\star_n) \subseteq C_n(T)$ ($T^\star_n$ *is a* n-$\mathscr{OS} \subseteq C_n(T)$),
(3) $(T^\star_n)^\star_n \subseteq T^\star_n$,
(4) $(T \cup P)^\star_n = T^\star_n \cup P^\star_n$,

(5) $K \in \mathscr{N} \Rightarrow K \cap T_n^\star = K \cap (K \cap T)_n^\star \subseteq (K \cap T)_n^\star$,

(6) $S \in \mathbb{I} \Rightarrow (T \cup S)_n^\star = T_n^\star = (T - S)_n^\star$.

Theorem 2.2. [8] *Let* $\mathbb{N}_\star^X$ *with* $\mathbb{I}$ *and* $T \subseteq T_n^\star$, *then* $T_n^\star = C_n(T_n^\star) = C_n(T)$.

Definition 2.3. [8] *Let the space* $\mathbb{N}_\star^X$. *The set operator* $C_n^\star$ *called a n⋆-closure is defined by* $C_n^\star(T) = T \cup T_n^\star$ *for* $T \subseteq U$.

It can be readily noted that $C_n^\star(T) \subseteq C_n(T)$.

3. Weak Form of Nano Open Sets

Definition 3.1. *A subset* T *of space* $\mathbb{N}_\star^X$ *is called a nano*

1. α-$\mathbb{S}\mathbb{I}$-$\mathscr{O}\mathscr{S}$ *(resp.* $\mathbb{S}\mathbb{I}_\alpha^n$-$\mathscr{O}\mathscr{S}$*) if* $T \subseteq I_n(C_n^\star(I_n(T)))$,
2. *semi-*$\mathbb{S}\mathbb{I}$-$\mathscr{O}\mathscr{S}$ *(resp.* $\mathbb{S}\mathbb{I}_s^n$-$\mathscr{O}\mathscr{S}$*) if* $T \subseteq C_n^\star(I_n(T))$,
3. *pre-*$\mathbb{S}\mathbb{I}$-$\mathscr{O}\mathscr{S}$ *(resp.* $\mathbb{S}\mathbb{I}_p^n$-$\mathscr{O}\mathscr{S}$*) if* $T \subseteq I_n(C_n^\star(T))$,
4. *b-*$\mathbb{S}\mathbb{I}$-$\mathscr{O}\mathscr{S}$ *(resp.* $\mathbb{S}\mathbb{I}_b^n$-$\mathscr{O}\mathscr{S}$*) if* $T \subseteq I_n(C_n^\star(T)) \cup C_n^\star(I_n(T))$,
5. β-$\mathbb{S}\mathbb{I}$-$\mathscr{O}\mathscr{S}$ *(resp.* $\mathbb{S}\mathbb{I}_\beta^n$-$\mathscr{O}\mathscr{S}$*) if* $T \subseteq C_n^\star(I_n(C_n^\star(T)))$.

The complements of the aforementioned sets are known as their corresponding closed sets.

Theorem 3.1. *The relationship shown below hold for a subset* T *in* $\mathbb{N}_\star^X$.

1. *if* M *is n-*$\mathscr{O}\mathscr{S} \Longrightarrow \mathbb{S}\mathbb{I}_\alpha^n$-$\mathscr{O}\mathscr{S}$.
2. *if* W *is* $\mathbb{S}\mathbb{I}_\alpha^n$-$\mathscr{O}\mathscr{S} \Longrightarrow \mathbb{S}\mathbb{I}_s^n$-$\mathscr{O}\mathscr{S}$.
3. *if* W *is* $\mathbb{S}\mathbb{I}_\alpha^n$-$\mathscr{O}\mathscr{S} \Longrightarrow \mathbb{S}\mathbb{I}_p^n$-$\mathscr{O}\mathscr{S}$.
4. *if* R *is* $\mathbb{S}\mathbb{I}_s^n$-$\mathscr{O}\mathscr{S} \Longrightarrow \mathbb{S}\mathbb{I}_b^n$-$\mathscr{O}\mathscr{S}$.
5. *if* R *is* $\mathbb{S}\mathbb{I}_p^n$-$\mathscr{O}\mathscr{S} \Longrightarrow \mathbb{S}\mathbb{I}_b^n$-$\mathscr{O}\mathscr{S}$.
6. *if* X *is* $\mathbb{S}\mathbb{I}_b^n$-$\mathscr{O}\mathscr{S} \Longrightarrow \mathbb{S}\mathbb{I}_\beta^n$-$\mathscr{O}\mathscr{S}$.

Proof.

1. W is n-$\mathscr{O}\mathscr{S}$ implies $I_n(M) = W$. But $M \subseteq C_n^\star(M) = C_n^\star(I_n(M)) \subseteq C_n^\star(I_n(C_n^\star(M)))$ which demonstrates M is $\mathbb{S}\mathbb{I}_\alpha^n$-$\mathscr{O}\mathscr{S}$.
2. W is $\mathbb{S}\mathbb{I}_\alpha^n$-$\mathscr{O}\mathscr{S}$ implies $W \subseteq I_n(C_n^\star(I_n(W))) \subseteq C_n^\star(I_n(W))$ which demonstrates W is $\mathbb{S}\mathbb{I}_s^n$-$\mathscr{O}\mathscr{S}$.

(3) W is $\mathbb{sI}^n_\alpha$-$\mathscr{OS}$ implies $W \subseteq I_n(C^\star_n(I_n(W))) \subseteq I_n(C^\star_n(W))$ which demonstrates W is $\mathbb{sI}^n_p$-$\mathscr{OS}$.

(4) R is $\mathbb{sI}^n_s$-$\mathscr{OS}$ implies $R \subseteq C^\star_n(I_n(R)) \subseteq C^\star_n(I_n(R)) \cup I_n(C^\star_n(R))$ which demonstrates R is $\mathbb{sI}^n_b$-$\mathscr{OS}$.

(5) R is $\mathbb{sI}^n_p$-$\mathscr{OS}$ implies $R \subseteq I_n(C^\star_n(R)) \subseteq I_n(C^\star_n(R)) \cup C^\star_n(I_n(R))$ which demonstrates R is $\mathbb{sI}^n_b$-$\mathscr{OS}$.

(6) X is $\mathbb{sI}^n_b$-$\mathscr{OS}$ implies $X \subseteq I_n(C^\star_n(X)) \cup C^\star_n(I_n(X)) \subseteq C^\star_n(I_n(C^\star_n(X))) \cup C^\star_n(I_n(C^\star_n(X))) = C^\star_n(I_n(C^\star_n(X)))$ which demonstrates X is $\mathbb{sI}^n_\beta$-$\mathscr{OS}$.

□

Remark 3.2. *In the diagram, these relationships are depicted.*

$$\begin{array}{ccccc} n\text{-}\mathscr{OS} & & & & \\ \downarrow & & & & \\ \mathbb{sI}^n_\alpha\text{-}\mathscr{OS} & \longrightarrow & \mathbb{sI}^n_p\text{-}\mathscr{OS} & & \\ \downarrow & & \downarrow & & \\ \mathbb{sI}^n_s\text{-}\mathscr{OS} & \longrightarrow & \mathbb{sI}^n_b\text{-}\mathscr{OS} & \longrightarrow & \mathbb{sI}^n_\beta\text{-}\mathscr{OS} \end{array}$$

As demonstrated in the following Example, none of the converses of the statements in Theorem 3.1 are true.

Example 3.3. *Let* $U = \{1,3,5,7,9\}$ *with* $U/R = \{\{1\},\{3,5\},\{7,9\}\}$ & $X = \{1,3\}$. *Then* $\mathscr{N} = \{U,\{1\},\{3,5\},\{1,3,5\},\phi\}$. $I = \{\phi,\{3\}\}$.

(1) Then $T = \{1,3,5,7\}$ *is not n-*$\mathscr{OS}$ *but* $\mathbb{sI}^n_\alpha$-$\mathscr{OS}$. $I_n(T) = \{1,3,5\}$ *and* $\{1,3,5\}^\star_n = \{1,3,5,7,9\} = U$. *Thus* T *is not n-*$\mathscr{OS}$ *but* $\mathbb{sI}^n_\alpha$-$\mathscr{OS}$.

(2) $\{3,5,7\}$ *is not* $\mathbb{sI}^n_\alpha$-$\mathscr{OS}$ *but* $\mathbb{sI}^n_s$-$\mathscr{OS}$.

(3) $\{5,7\}$ *is not* $\mathbb{sI}^n_b$-$\mathscr{OS}$ *but* $\mathbb{sI}^n_\beta$-$\mathscr{OS}$.

Example 3.4. *Let* $U = \{1,3,5,7\}$ *with* $U/R = \{\{1\},\{5\},\{3,7\}\}$ *and* $X = \{1,3\}$. *Then* $\mathscr{N} = \{\phi,\{1\},\{3,7\},\{1,3,7\},U\}$. $I = \{\phi,\{1\}\}$.

(1) $\{3\}$ *is not* $\mathbb{sI}^n_\alpha$-$\mathscr{OS}$ *but* $\mathbb{sI}^n_p$-$\mathscr{OS}$.

(2) $\{1,7\}$ *is not* $\mathbb{sI}^n_s$-$\mathscr{OS}$ *but* $\mathbb{sI}^n_b$-$\mathscr{OS}$.

(3) $\{3,5,7\}$ *is not* $\mathbb{s}\mathbb{I}^n_p$-$\mathscr{OS}$ *but* $\mathbb{s}\mathbb{I}^n_b$-$\mathscr{OS}$.

Remark 3.5. *In space* $\mathbb{N}^X_\star$, *the families of* $\mathbb{s}\mathbb{I}^n$-$\mathscr{OS}$ *and n-*$\mathscr{OS}$ *are independent.*

Example 3.6. *In Example 3.4,* $\{3\}$ *is not n-*$\mathscr{OS}$ *but* $\mathbb{s}\mathbb{I}^n$-$\mathscr{OS}$ *and* $\{1,3,7\}$ *is not* $\mathbb{s}\mathbb{I}^n$-$\mathscr{OS}$ *but n-*$\mathscr{OS}$.

Theorem 3.7. *A subset Y of space* $\mathbb{N}^X_\star$ *is* $\mathbb{s}\mathbb{I}^n_\alpha$-$\mathscr{OS}$ $\iff$ *Y is* $\mathbb{s}\mathbb{I}^n_p$-$\mathscr{OS}$ *and* $\mathbb{s}\mathbb{I}^n_s$-$\mathscr{OS}$.

Proof. Part is derived from the Theorem. 3.1 (2) and (3).

Conversely, If Y is $\mathbb{s}\mathbb{I}^n_s$-$\mathscr{OS}$ and $\mathbb{s}\mathbb{I}^n_p$-$\mathscr{OS}$ then $Y \subseteq I_n(C^\star_n(Y))$ and $Y \subseteq C^\star_n(I_n(Y))$. Thus $Y \subseteq I_n(C^\star_n(Y)) \subseteq I_n(C^\star_n(C^\star_n(I_n(Y))) = I_n(C^\star_n(I_n(Y)))$ which demonstrates Y is α-$\mathbb{s}\mathbb{I}^n$-$\mathscr{OS}$.

Remark 3.8. *The family of* $\mathbb{s}\mathbb{I}^n_s$-$\mathscr{OS}$ *and the family of* $\mathbb{s}\mathbb{I}^n_p$-$\mathscr{OS}$ *are independent of one another in the space* $\mathbb{N}^X_\star$, *as illustrated in the Example below.*

Example 3.9. *Let* $U = \{1,3,5,7\}$ *with* $U/R = \{\{1\},\{7\},\{3,5\}\}$ & $X = \{1,5\}$. *Then* $\mathscr{N} = \{U,\{1\},\{3,5\},\{1,3,5\},\phi\}$. $I = \{\phi,\{5\}\}$. *Then the subset*

(1) $\{1,7\}$ *is not* $\mathbb{s}\mathbb{I}^n_p$-$\mathscr{OS}$ *but* $\mathbb{s}\mathbb{I}^n_s$-$\mathscr{OS}$.

(2) $\{3\}$ *is not* $\mathbb{s}\mathbb{I}^n_s$-$\mathscr{OS}$ *but* $\mathbb{s}\mathbb{I}^n_p$-$\mathscr{OS}$.

Theorem 3.10. *If a subset T of a space* $\mathbb{N}^X_\star$ *is both* $\mathbb{s}\mathbb{I}^n_\beta$-$\mathscr{OS}$ *and n*$\star$-$\mathscr{CS}$, *then T is* $\mathbb{s}\mathbb{I}^n_s$-$\mathscr{OS}$.

Proof.

Since T is $\mathbb{s}\mathbb{I}^n_\beta$-$\mathscr{OS}$, $T \subseteq C^\star_n(I_n(C^\star_n(T))) = C^\star_n(I_n(T))$, T being $n\star$-$\mathscr{OS}$. Therefore T is $\mathbb{s}\mathbb{I}^n_s$-$\mathscr{OS}$. □

Theorem 3.11. *A subset Y of a* $\mathbb{N}^X_\star$, $\mathbb{s}\mathbb{I}^n_s$-$\mathscr{OS}$ $\iff$ $C^\star_n(Y) = C^\star_n(I_n(Y))$.

Proof.

Let Y be $\mathbb{s}\mathbb{I}^n_s$-$\mathscr{OS}$. Then $Y \subseteq C^\star_n(I_n(Y))$ and $C^\star_n(Y) \subseteq C^\star_n(I_n(Y))$. But $C^\star_n(I_n(Y)) \subset C^\star_n(Y)$. Thus $C^\star_n(Y) = C^\star_n(I_n(Y))$.

Conversely, let the condition hold. We've $Y \subseteq C^\star_n(Y) = C^\star_n(I_n(Y))$, by supposition. Thus $Y \subseteq C^\star_n(I_n(Y))$ and hence Y is $\mathbb{s}\mathbb{I}^n_s$-$\mathscr{OS}$. □

Theorem 3.12. *A subset Y of $\mathbb{N}^X_\star$, $_{\mathbb{S}}\mathbb{I}^n_s$-$\mathscr{OS}$ $\iff$ there exists a n-$\mathscr{OS}$ set Y such that $Y \subset T \subseteq C^\star_n(Y)$.*

Proof.

Let T be $_{\mathbb{S}}\mathbb{I}^n_s$-$\mathscr{OS}$. Then $T \subseteq C^\star_n(I_n(T))$. Take $I_n(T) = Y$. Then $Y \subseteq T \subseteq C^\star_n(Y)$, where Y is n-$\mathscr{OS}$.

Conversely, let $Y \subseteq T \subseteq C^\star_n(W)$ for some n-$\mathscr{OS}$ set Y. Since $Y \subseteq T$, $Y \subseteq I_n(T)$ and $T \subseteq C^\star_n(Y) \subseteq C^\star_n(I_n(T))$ which implies T is $_{\mathbb{S}}\mathbb{I}^n_s$-$\mathscr{OS}$. □

Theorem 3.13. *If T is a $_{\mathbb{S}}\mathbb{I}^n_s$-$\mathscr{OS}$ in $\mathbb{N}^X_\star$ and $T \subseteq K \subseteq C^\star_n(T)$, then K is $_{\mathbb{S}}\mathbb{I}^n_s$-$\mathscr{OS}$.*

Proof.

By assumption $K \subseteq C^\star_n(T) \subseteq C^\star_n(C^\star_n(I_n(T)))$ (for T is $_{\mathbb{S}}\mathbb{I}^n_s$-$\mathscr{OS}$) = $C^\star_n(I_n(T)) \subseteq C^\star_n(I_n(K))$ by assumption. This implies K is $_{\mathbb{S}}\mathbb{I}^n_s$-$\mathscr{OS}$. □

Theorem 3.14. *The findings are valid for a subset T in the space $\mathbb{N}^X_\star$.*

(1) if T is $_{\mathbb{S}}\mathbb{I}^n$-$\mathscr{OS} \Longrightarrow {}_{\mathbb{S}}\mathbb{I}^n_p$-$\mathscr{OS}$.
(2) if T is $_{\mathbb{S}}\mathbb{I}^n$-$\mathscr{OS} \Longrightarrow {}_{\mathbb{S}}\mathbb{I}^n_\beta$-$\mathscr{OS}$.
(3) if T is $_{\mathbb{S}}\mathbb{I}^n$-$\mathscr{OS} \Longrightarrow {}_{\mathbb{S}}\mathbb{I}^n_b$-$\mathscr{OS}$.

Proof.

(1) T is $_{\mathbb{S}}\mathbb{I}^n$-$\mathscr{OS}$ implies $T \subseteq I_n(T^\star_n) \subseteq I_n(C^\star_n(T))$ which demonstrates T is $_{\mathbb{S}}\mathbb{I}^n_P$-$\mathscr{OS}$.
(2) T is $_{\mathbb{S}}\mathbb{I}^n$-$\mathscr{OS}$ implies $T \subseteq I_n(T^\star_n) \subseteq C^\star_n(I_n(C^\star_n(T)))$ which demonstrates T is $_{\mathbb{S}}\mathbb{I}^n_\beta$-$\mathscr{OS}$.
(3) T is $_{\mathbb{S}}\mathbb{I}^n$-$\mathscr{OS}$ implies $T \subseteq I_n(T^\star_n) \subseteq I_n(C^\star_n(T)) \subseteq I_n(C^\star_n(T)) \cup C^\star_n(I_n(T))$ which demonstrates T is $_{\mathbb{S}}\mathbb{I}^n_b$-$\mathscr{OS}$.

□

Remark 3.15. *The converses of (1), (2), and (3) in Theorem 3.14 are false, as shown by the following Example.*

Example 3.16. *In Example 3.4,*

(1) $\{1\}$ is not $_{\mathbb{S}}\mathbb{I}^n$-$\mathscr{OS}$ but $_{\mathbb{S}}\mathbb{I}^n_p$-$\mathscr{OS}$ and $_{\mathbb{S}}\mathbb{I}^n_b$-$\mathscr{OS}$ but not $_{\mathbb{S}}\mathbb{I}^n$-$\mathscr{OS}$.
(2) $\{5,7\}$ is not $_{\mathbb{S}}\mathbb{I}^n$-$\mathscr{OS}$ but $_{\mathbb{S}}\mathbb{I}^n_\beta$-$\mathscr{OS}$.

Remark 3.17. *The family of* $\mathbb{SI}^n$-$\mathscr{OS}$ *and the family of* $\mathbb{SI}^n_\alpha$-$\mathscr{OS}$ ($\mathbb{SI}^n_s$-$\mathscr{OS}$) *sets are independent of one another in the space* $\mathbb{N}^X_\star$.

Example 3.18. *In Example 3.4,*

(1) $\{3\}$ is not $\mathbb{SI}^n_\alpha$-$\mathscr{OS}$ but $\mathbb{SI}^n$-$\mathscr{OS}$.

(2) $\{1\}$ is not $\mathbb{SI}^n$-$\mathscr{OS}$ but $\mathbb{SI}^n_\alpha$-$\mathscr{OS}$.

Examples (1) and (2) support Remark (1) of 3.17.

(3) $\{3\}$ is not $\mathbb{SI}^n_s$-$\mathscr{OS}$ but $\mathbb{SI}^n$-$\mathscr{OS}$.

(4) $\{2\}$ is not $\mathbb{SI}^n$-$\mathscr{OS}$ but $\mathbb{SI}^n_s$-$\mathscr{OS}$.

Examples (3) and (4) support Remark (2) of 3.17.

Proposition 3.19. *For a subset of T a space $\mathbb{N}^X_\star$, the below characteristics hold:*

(1) if Y is $\mathbb{SI}^n_\alpha$-$\mathscr{OS} \Longrightarrow n\alpha$-$\mathscr{OS}$.

(2) if Y is $\mathbb{SI}^n_p$-$\mathscr{OS} \Longrightarrow np$-$\mathscr{OS}$.

(3) if Y is $\mathbb{SI}^n_b$-$\mathscr{OS} \Longrightarrow nb$-$\mathscr{OS}$.

(4) if Y is $\mathbb{SI}^n_\beta$-$\mathscr{OS} \Longrightarrow n\beta$-$\mathscr{OS}$.

Proof.

(1) Let Y be a $\mathbb{SI}^n_\alpha$-$\mathscr{OS}$ set. Then $Y \subseteq I_n(C^\star_n(I_n(Y))) \subseteq I_n(C_n(I_n(Y)))$. This demonstrates Y is $n\alpha$-$\mathscr{OS}$.

(2) Let Y be a $\mathbb{SI}^n_p$-$\mathscr{OS}$ set. Then $Y \subseteq I_n(C^\star_n(Y)) \subseteq I_n(C_n(Y))$. This demonstrates Y is np-$\mathscr{OS}$.

(3) Let Y be a $\mathbb{SI}^n_b$-$\mathscr{OS}$ set. Then $Y \subseteq I_n(C^\star_n(Y)) \cup C^\star_n(I_n(Y)) \subseteq I_n(C_n(Y)) \cup C_n(I_n(Y))$. This demonstrates Y is nb-$\mathscr{OS}$.

(4) Let Y be a $\mathbb{SI}^n_\beta$-$\mathscr{OS}$ set. Then $Y \subseteq C^\star_n(I_n(C^\star_n(Y))) \subseteq C_n(I_n(C_n(Y)))$. This demonstrates Y is $n\beta$-$\mathscr{OS}$.

□

Remark 3.20. *Proposition 3.19' converses are generally untrue, as the case that follows demonstrates.*

Example 3.21. *Let $U=\{1,3,5,7,9\}$ with $U/R=\{\{1\},\{3,5\},\{7,9\}\}$, $X=\{1,3\}$ & $I=\wp(U)$. Then in $\mathbb{N}^X_\star$, $\mathscr{N}=\{U,\{1\},\{3,5\},\{1,3,5\},\phi\}$. $T=\{1,3,5,7\}$ is not $\mathbb{S}\mathbb{I}^n_\alpha$-$\mathscr{O}\mathscr{S}$ since $C^\star_n(T)=Z$ but $n\alpha$-$\mathscr{O}\mathscr{S}$.*

Example 3.22. *Let $U=\{1,3,5\}$ with $U/R=\{\{1\},\{3,5\}\}$ & $X=\{1,3\}$. Then $\mathscr{N}=\{U,\{1\},\{3,5\},\phi\}$. $I=\{\phi,\{v\}\}$. Then in $\mathbb{N}^X_\star$, $\{1,3\}$ is not $\mathbb{S}\mathbb{I}^n_p$-$\mathscr{O}\mathscr{S}$ but np-$\mathscr{O}\mathscr{S}$.*

Example 3.23. *In Example 3.4,*

(1) $\{1,5\}$ is not $\mathbb{S}\mathbb{I}^n_b$-$\mathscr{O}\mathscr{S}$ but nb-$\mathscr{O}\mathscr{S}$.

(2) $\{1,5\}$ is not $\mathbb{S}\mathbb{I}^n_\beta$-$\mathscr{O}\mathscr{S}$ but $n\beta$-$\mathscr{O}\mathscr{S}$.

Lemma 3.24. *Let $\mathbb{N}^X_\star$ and $Y\subseteq U$. If K is n-$\mathscr{O}\mathscr{S}$ in $\mathbb{N}^X_\star$, then $C^\star_n(Y)\cap K\subseteq C^\star_n(Y\cap K)$.*

Proof.

$C^\star_n(Y)\cap K=(Y^\star_n\cup Y)\cap K=(Y^\star_n\cap K)\cup(Y\cap K)\subseteq(Y\cap K)^\star_n\cup(Y\cap K)$ by (5) of Theorem 2.1. Thus $C^\star_n(Y)\cap K\subseteq(Y\cap K)^\star_n\cup(Y\cap K)=C^\star_n(Y\cap K)$. □

Proposition 3.25. *The intersection of a*

(1) n-$\mathscr{O}\mathscr{S}$ and $\mathbb{S}\mathbb{I}^n_p$-$\mathscr{O}\mathscr{S}\Longrightarrow\mathbb{S}\mathbb{I}^n_p$-$\mathscr{O}\mathscr{S}$.

(2) n-$\mathscr{O}\mathscr{S}$ and $\mathbb{S}\mathbb{I}^n_s$-$\mathscr{O}\mathscr{S}\Longrightarrow\mathbb{S}\mathbb{I}^n_s$-$\mathscr{O}\mathscr{S}$.

(3) n-$\mathscr{O}\mathscr{S}$ and $\mathbb{S}\mathbb{I}^n_\alpha$-$\mathscr{O}\mathscr{S}\Longrightarrow\mathbb{S}\mathbb{I}^n_\alpha$-$\mathscr{O}\mathscr{S}$.

(4) n-$\mathscr{O}\mathscr{S}$ and $\mathbb{S}\mathbb{I}^n_b$-$\mathscr{O}\mathscr{S}\Longrightarrow\mathbb{S}\mathbb{I}^n_b$-$\mathscr{O}\mathscr{S}$.

(5) n-$\mathscr{O}\mathscr{S}$ and $\mathbb{S}\mathbb{I}^n_\beta$-$\mathscr{O}\mathscr{S}\Longrightarrow\mathbb{S}\mathbb{I}^n_\beta$-$\mathscr{O}\mathscr{S}$.

Proof.

(1) Let T be $\mathbb{S}\mathbb{I}^n_p$-$\mathscr{O}\mathscr{S}$ and K be n-$\mathscr{O}\mathscr{S}$. Then $T\subseteq I_n(C^\star_n(T))$ and $K\cap T\subseteq I_n(K)\cap I_n(C^\star_n(T))=I_n(K\cap C^\star_n(T))\subseteq I_n(C^\star_n(K\cap T))$ by Lemma 3.24. This demonstrates $K\cap T$ is $\mathbb{S}\mathbb{I}^n_p$-$\mathscr{O}\mathscr{S}$.

(2) Let T be $\mathbb{S}\mathbb{I}^n_s$-$\mathscr{O}\mathscr{S}$ and K be n-$\mathscr{O}\mathscr{S}$ in U. Then $T\subseteq C^\star_n(I_n(T))$ and $I_n(K)=K$. $K\cap T\subseteq K\cap C^\star_n(I_n(T))\subseteq C^\star_n(K\cap I_n(T))=C^\star_n(I_n(K)\cap I_n(T))=C^\star_n(I_n(K\cap T))$ by Lemma 3.24. Hence T is $\mathbb{S}\mathbb{I}^n_s$-$\mathscr{O}\mathscr{S}$.

(3) Let K be a n-$\mathscr{OS}$ and T be an $_{\mathbb{S}}\mathbb{I}^n_\alpha$-$\mathscr{OS}$ in a space $\mathbb{N}^X_\star$. Then T is both $_{\mathbb{S}}\mathbb{I}^n_p$-$\mathscr{OS}$ and $_{\mathbb{S}}\mathbb{I}^n_s$-$\mathscr{OS}$ by (2) and (3) of Theorem 3.1. $T \cap K$ is both $_{\mathbb{S}}\mathbb{I}^n_p$-$\mathscr{OS}$ and $_{\mathbb{S}}\mathbb{I}^n_s$-$\mathscr{OS}$ by Proposition 3.25(1) and (2). Hence by Theorem 3.7, $T \cap K$ is $_{\mathbb{S}}\mathbb{I}^n_\alpha$-$\mathscr{OS}$.

(4) Let T be $_{\mathbb{S}}\mathbb{I}^n_b$-$\mathscr{OS}$ and K be n-$\mathscr{OS}$. Then $T \subseteq I_n(C^\star_n(T)) \cup C^\star_n(I_n(T))$ and $T \cap K \subseteq [I_n(C^\star_n(T)) \cup C^\star_n(I_n(T)) \cap K] = [I_n(C^\star_n(T)) \cap K] \cup [C^\star_n(I_n(T)) \cap K] = [I_n(K) \cap I_n(C^\star_n(T))] \cup [K \cap C^\star_n(I_n(T))] \subseteq [I_n(K \cap C^\star_n(T))] \cup [C^\star_n(K \cap I_n(T))]$ by Lemma 3.24. Thus $K \cap T \subseteq [I_n(C^\star_n(K \cap T))] \cup [C^\star_n(I_n(K \cap T))]$. This shows that $K \cap T$ is $_{\mathbb{S}}\mathbb{I}^n_b$-$\mathscr{OS}$.

(5) Let T be $_{\mathbb{S}}\mathbb{I}^n_\beta$-$\mathscr{OS}$ and K be n-$\mathscr{OS}$. Then $T \subseteq C^\star_n(I_n(C^\star_n(T)))$ and $K \cap T \subseteq K \cap C^\star_n(I_n(C^\star_n(T))) \subseteq C^\star_n(K \cap\ I_n(C^\star_n(T))) \subseteq C^\star_n(I_n(K) \cap I_n(C^\star_n(T))) = C^\star_n(I_n(K \cap C^\star_n(T))) \subseteq C^\star_n(I_n(C^\star_n(K \cap Z)))$ by Lemma 3.24. This shows that $K \cap T$ is $_{\mathbb{S}}\mathbb{I}^n_\beta$-$\mathscr{OS}$.

□

Remark 3.26. *The intersection of two*

(1) $_{\mathbb{S}}\mathbb{I}^n_s$-$\mathscr{OS}$ *no need to a* $_{\mathbb{S}}\mathbb{I}^n_s$-$\mathscr{OS}$.

(2) $_{\mathbb{S}}\mathbb{I}^n_p$-$\mathscr{OS}$ *no need to a* $_{\mathbb{S}}\mathbb{I}^n_p$-$\mathscr{OS}$.

(3) $_{\mathbb{S}}\mathbb{I}^n_b$-$\mathscr{OS}$ *no need to a* $_{\mathbb{S}}\mathbb{I}^n_b$-$\mathscr{OS}$.

(4) $_{\mathbb{S}}\mathbb{I}^n_\beta$-$\mathscr{OS}$ *no need to a* $_{\mathbb{S}}\mathbb{I}^n_\beta$-$\mathscr{OS}$.

as demonstrated in the example that follows.

Example 3.27. *In Example 3.9,* $\{1,7\}$ *and* $\{3,5,7\}$ *are* $_{\mathbb{S}}\mathbb{I}^n_s$-$\mathscr{OS}$. *But* $\{1,7\} \cap \{3,5,7\} = \{7\}$ *is not* $_{\mathbb{S}}\mathbb{I}^n_s$-$\mathscr{OS}$.

Example 3.28. *(1) In Example 3.4,*

(a) $\{1,3,5\}$ *and* $\{1,5,7\}$ *are* $_{\mathbb{S}}\mathbb{I}^n_p$-$\mathscr{OS}$. *But* $\{1,3,5\} \cap \{1,5,7\} = \{1,5\}$ *is not* $_{\mathbb{S}}\mathbb{I}^n_p$-$\mathscr{OS}$.

(b) $\{1,3,5\}$ *and* $\{3,5,7\}$ *are* $_{\mathbb{S}}\mathbb{I}^n_b$-$\mathscr{OS}$. *But* $\{1,3,5\} \cap \{3,5,7\} = \{3,5\}$ *is not* $_{\mathbb{S}}\mathbb{I}^n_b$-$\mathscr{OS}$.

(c) $\{3,5\}$ *and* $\{5,7\}$ *are* $_{\mathbb{S}}\mathbb{I}^n_\beta$-$\mathscr{OS}$. *But* $\{3,5\} \cap \{5,7\} = \{5\}$ *is not* $_{\mathbb{S}}\mathbb{I}^n_\beta$-$\mathscr{OS}$.

Conflict of Interests

The authors declare that there is no conflict of interests.

References

[1] D. Jankovic, Compatible extensions of ideals, Boll. Un. Mat. Ital. 7 (1992), 453–465.

[2] K. Kuratowski, Topology, Academic Press, New York, 1966.

[3] M.L. Thivagar, C. Richard, On nano forms of weakly open sets, Int. J. Math. Stat. Invent. 1 (2013), 31–37.

[4] M. Parimala, T. Noiri, S. Jafari, New types of nano topological spaces via nano ideals, https://www.researchgate.net/publication/315892279.

[5] M. Parimala, S. Jafari, On Some new notions in nano ideal topological spaces, Eurasian Bull. Math. 1 (2018), 85–93.

[6] M. Parimala, C. Indirani, S. Jafari, On nano b-open sets in nano topological spaces, Jordan J. Math. Stat. 9 (2016), 173–184.

[7] Z. Pawlak, Rough sets, Int. J. Computer Inform. Sci. 11 (1982), 341–356. https://doi.org/10.1007/bf01001956.

[8] I. Rajasekaran, On semi-local functions in ideal nano topological spaces, Communicated.

[9] A. Revathy, G. Ilango, On nano β-open sets, Int. J. Eng. Contemp. Math. Sci. 1 (2015), 1–6.

[10] R. Vaidyanathaswamy, The localisation theory in set-topology, Proc. Indian Acad. Sci. 20 (1944), 51–61. https://doi.org/10.1007/bf03048958.

[11] R. Vaidyanathaswamy, Set topology, Chelsea Publishing Company, New York, 1946.