Available online at http://scik.org
Adv. Fixed Point Theory, 2026, 16:10
https://doi.org/10.28919/afpt/9777
ISSN: 1927-6303

A NOVEL APPROACH TO SOME BEST PROXIMITY POINT THEOREMS IN p-CYCLIC METRIC SPACES WITH APPLICATIONS

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Abstract. In this manuscript, we introduce a new class of generalized (α,ψ)-p-cyclic Geraghty contraction type mappings on p-cyclic metric spaces, where $p \in \mathbb{N}$ and $p \geq 2$. We prove the existence of best proximity point results for such class of mappings in a p-cyclic complete metric space via the Ω class of mappings introduced by Karapinar *et al.* [13]. Our results extend and generalize many comparable results in the existing literature.

Keywords: p-cyclic maps; p-cyclic contractions; strict contractions; best proximity points; p-cyclic metric space.
2020 AMS Subject Classification: 47H10, 54H25.

1. Introduction

The remarkable Banach fixed point theorem states that if $\mathbb{F} : \mathbb{D} \to \mathbb{D}$ is a contraction mapping on a complete metric space $(\mathbb{D}, \mathfrak{m})$, where $\mathfrak{m} : \mathbb{D} \times \mathbb{D} \to \mathbb{R}^+$, then there exists a unique element x such that $\mathbb{F}x = x$. It is wellknown that the undertaking operator is necessarily continuous due to contraction inequality. It is interesting to bring a natural question: Does a discontinuous contraction mapping possess a fixed point? The answer to this question is affirmative. Indeed, there are various approaches to overcome weakness of the discontinuous mapping for guaranteeing

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Received January 14, 2026

a fixed point. One of the remakable results was established by Bryant [3] who obtained the following result: In a complete metric space, if, for some positive integer $n \geq 2$, the nth iteration of the given mapping forms a contraction, then it possess a unique fixed point. Another remarkable approach was proposed by Kirk *et al.* [16] by introducing the notion of cyclic contraction. More precisely, every cyclic contraction in a complete metric space possess a unique fixed point. Attendantly, the concept of the cyclic contractions has been investigated densely by a considerable number of authors who bring several variants of the notion and derive a number of interesting results (see, e.g., [1, 2, 5, 6, 8, 10, 11, 12, 14, 15, 17, 18, 19, 20, 21, 22, 23, 24, 26] and the references therein). Let there be a self-mapping on a metric space $(\mathbb{D},\mathfrak{m})$. Suppose that $\mathscr{B}_1$ and $\mathscr{B}_2$ are non-empty subsets of $\mathbb{D}$ such that $\mathbb{D} = \mathscr{B}_1 \cup \mathscr{B}_2$. A self-mapping $\mathbb{F}$ on $\mathscr{B}_1 \cup \mathscr{B}_2$ is called cyclic [16] if

$$\mathbb{F}(\mathscr{B}_1) \subseteq \mathscr{B}_2 \quad \text{and} \quad \mathbb{F}(\mathscr{B}_2) \subseteq \mathscr{B}_1.$$

In 2009, Karpagam and Agrawal [10], introduced a notion of p-cyclic map as follows. Let $\mathscr{B}_1, \mathscr{B}_2, \ldots, \mathscr{B}_p (p \geq 2)$ be non-empty sets. A map $\mathbb{F} : \cup_{i=1}^{p} \mathscr{B}_i \to \cup_{i=1}^{p} \mathscr{B}_i$ is said to be a p-cyclic map if $\mathbb{F}(\mathscr{B}_i) \subseteq \mathscr{B}_{i+1}, \forall i \in \{1,2,\ldots,p\}$, where $\mathscr{B}_{p+1} = \mathscr{B}_1$. Let $x = x_0 \in \mathscr{B}_i$ define a sequence $\{x_n\} \subset \cup_{i=1}^{p} \mathscr{B}_i$ as $x_n = \mathbb{F}x_{n-1}$. Then, $\{x_{pn}\}$ is a subsequence in $\mathscr{B}_i, \{x_{pn+1}\}$ is a subsequence in $\mathscr{B}_{i+1}$ and so on. From this idea of the arrangement of such a sequence, Karapinar *et al.*[13] introduced a notion of p-cyclic sequence. If $\mathscr{B}_i s$ are subsets of a metric space $(\mathbb{D},\mathfrak{m})$, then it was found out that, to obtain a best proximity point of $\mathbb{F}$ under various contractive conditions, it is enough to prove that: given $\varepsilon > 0$, there exists $N_0 \in \mathbb{N}$ such that

$$\mathfrak{m}(x_{pn}, x_{pm+1}) < dist(\mathscr{B}_i, \mathscr{B}_{i+1}) + \varepsilon, \forall n,m \geq N_0,$$

where $\mathbb{N}$ is the set of positive integers.

Throughout this paper, we let $dist(\mathscr{A},\mathscr{B}) = \inf\{\mathfrak{m}(x,y) | x \in \mathscr{A}, y \in \mathscr{B}\}$, $\mathbb{R}^+ = [0,\infty)$ and $\mathbb{N}_0 = \mathbb{N} \cup \{0\}$.

In 2019, Karapinar *et al.*[13] introduced a concept of p-cyclic Cauchy sequence and p-cyclic complete metric space as follows:

Definition 1. For a non-empty set $\mathbb{D}$, let $\mathfrak{m} : \mathbb{D} \times \mathbb{D} \to [0,\infty)$ be a distance function and $\mathscr{B}_1, \mathscr{B}_2, \ldots, \mathscr{B}_p, (p \geq 2)$ be non-empty subsets of $\mathbb{D}$.

1 . A sequence $\{x_n\}_{n=1}^{\infty} \subset \cup_{i=1}^{p} \mathscr{B}_i$ is called a p-cyclic sequence if $x_{pn+i} \in \mathscr{B}_i$, for all $n \in \mathbb{N}_0$ and $i = 1, 2, \ldots, p$.

2 . We say that $\{x_n\}_{n=1}^{\infty}$ is a p-cyclic Cauchy sequence, if for given $\varepsilon > 0$ there exists an $N_0 \in \mathbb{N}$ such that for some $i \in \{1, 2, \ldots, p\}$, we have

$$\mathfrak{m}(x_{pn+i}, x_{pm+i+1}) < dist(\mathscr{B}_i, \mathscr{B}_{i+1}) + \varepsilon, \forall m, n \geq N_0. \tag{1.1}$$

3 . A p-cyclic sequence $\{x_n\}_{n=1}^{\infty}$ in $\cup_{i=1}^{p} \mathscr{B}_i$ is said to be p-cyclic bounded, if $\{x_{pn+i}\}_{n=1}^{\infty}$ is bounded in $\mathscr{B}_i$ for some $i \in \{1, 2, ..., p\}$.

4 . Let $\{x_n\}_{n=1}^{\infty}$ be a p-cyclic sequence in $\cup_{i=1}^{p} \mathscr{B}_i$. If for some $j \in \{1, 2, ..., p\}$ the subsequence $\{x_{pn+j}\}$ of $\{x_n\}_{n=1}^{\infty}$ converges in $\mathscr{B}_j$, then we say that $\{x_n\}_{n=1}^{\infty}$ is p-cyclic convergent.

5 . Under the assumption that $\mathscr{B}_1, \mathscr{B}_2, \ldots, \mathscr{B}_p, (p \geq 2)$ are non-empty closed subsets of a metric space $(\mathbb{D}, \mathfrak{m})$, we say that $\cup_{i=1}^{p} \mathscr{B}_i$ is p-cyclic complete if every p-cyclic Cauchy sequence in $\cup_{i=1}^{p} \mathscr{B}_i$ is p-cyclic convergent.

6 . If there are subsets $\mathscr{B}_1, \mathscr{B}_2, \ldots, \mathscr{B}_p, (p \geq 2)$ of $\mathbb{D}$ such that $\mathbb{D} = \cup_{i=1}^{p} \mathscr{B}_i$ and $\cup_{i=1}^{p} \mathscr{B}_i$ is p-cyclic complete, then $(\mathbb{D}, \mathfrak{m})$ is called a p-cyclic complete metric space.

Remark 1. *It is notable that a p-cyclic sequence which is a Cauchy sequence in the usual sense is a p-cyclic Cauchy sequence. On the other hand, p-cyclic Cauchy sequences need not be Cauchy sequences in the usual sense, even if* $dist(\mathscr{B}_i, \mathscr{B}_{i+1}) = 0$, $\forall i \in \{1, 2, ..., p\}$.

For some interesting examples which illustrate the notion of p-cyclic sequence and p-cyclic Cauchy sequence, see Example 1 and Example 2 in [13].

Recently, Karapinar *et al.*[13] introduced a notion of p-cyclic strict contraction, which is a generalization of strict contraction in the usual sense, as follows.

Definition 2. For a non-empty set $\mathbb{D}$, let $\mathfrak{m} : \mathbb{D} \times \mathbb{D} \to [0, \infty)$ be a distance function and $\mathscr{B}_1, \mathscr{B}_2, \ldots, \mathscr{B}_p, (p \geq 2)$ be non-empty subsets of $\mathbb{D}$. A p-cyclic map $\mathbb{F}$ is said to be p-cyclic strict contraction if, for all $x \in \mathscr{B}_i, y \in \mathscr{B}_{i+1}, 1 \leq i \leq p$:

(i) $\mathfrak{m}(x, y) > dist(\mathscr{B}_i, \mathscr{B}_{i+1}) \implies \mathfrak{m}(\mathbb{F}x, \mathbb{F}y) < \mathfrak{m}(x, y)$, and

(ii) $\mathfrak{m}(x, y) = dist(\mathscr{B}_i, \mathscr{B}_{i+1}) \implies \mathfrak{m}(\mathbb{F}x, \mathbb{F}y) = \mathfrak{m}(x, y)$.

Remark 2. If $\mathscr{B}_i = \mathscr{A}$, for all $i = 1, 2, \ldots, p$, then p-cyclic strict contraction is a strict contraction in the usual sense. It is clear that the p-cyclic strict contraction also forms a p-cyclic non-expansive map.

Note that, if the distances between the adjacent sets are zero, then a p-cyclic strict contraction map is a strict contraction map in the usual sense. All such maps invariably satisfy the condition:

$$x, y \in \mathscr{B}_i, \mathfrak{m}(\mathbb{F}^{pn}x, \mathbb{F}^{pn+1}y) = dist(\mathscr{B}_i, \mathscr{B}_{i+1}) \text{ as } n \to \infty. \tag{1.2}$$

They call the class of mappings which are p-cyclic strict contraction maps and satisfying the condition (1.2) the Ω class of mappings.

The notion of Ω class of mappings [13] is then defined as follows.

Definition 3. For a non-empty set $\mathbb{D}$, let $\mathfrak{m} : \mathbb{D} \times \mathbb{D} \to [0, \infty)$ be a distance function and $\mathscr{B}_1, \mathscr{B}_2, \ldots, \mathscr{B}_p, (p \geq 2)$ be non-empty subsets of $\mathbb{D}$. A p-cyclic map $\mathbb{F} : \cup_{i=1}^{p} \mathscr{B}_i \to \cup_{i=1}^{p} \mathscr{B}_i$ is said to belong to the Ω class of mappings if

(1). $\mathbb{F}$ is p-cyclic strict contraction, and

(2). If $x, y \in \mathscr{B}_i$, then $\lim_{n\to\infty} \mathfrak{m}(\mathbb{F}^{pn}x, \mathbb{F}^{pn+1}y) = dist(\mathscr{B}_i, \mathscr{B}_{i+1}), 1 \leq i \leq p$.

The following are some examples of p-cyclic maps that belong to the class Ω.

Example 1. *Let $\mathscr{B}_1, \mathscr{B}_2, \ldots, \mathscr{B}_p$ be non-empty subsets of a metric space $(\mathbb{D}, \mathfrak{m})$. Let $\mathbb{F} : \cup_{i=1}^{p} \mathscr{B}_i \to \cup_{i=1}^{p} \mathscr{B}_i$ is a p-cyclic contraction map defined in* [15] *by*

$$\mathfrak{m}(\mathbb{F}x, \mathbb{F}y) \leq k\mathfrak{m}(x, y) + (1-k)dist(\mathscr{B}_i, \mathscr{B}_{i+1}), x \in \mathscr{B}_i, y \in \mathscr{B}_{i+1} i, i = 1, 2, ..., p,$$

for some $k \in (0, 1)$. Then, $\mathbb{F} \in \Omega$ (see [13]*, Ex.3).*

We recall the following definition in [10].

Definition 4. [10] *Let $B_1, \ldots, B_p$ be nonempty subsets of a metric space $(\mathbb{D}, \mathfrak{m})$. $\mathbb{F} : \cup_{i=1}^{p} \mathscr{B}_i \to \cup_{i=1}^{p} \mathscr{B}_i$ be a p-cyclic mapping. $\mathbb{F}$ is called a p-cyclic Meir-Keeler contraction if for every $\varepsilon > 0$, there exists $\delta > 0$ such that*

$$\mathfrak{m}(x, y) < dist(B_i, B_{i+1}) < \varepsilon + \delta \implies \mathfrak{m}(\mathbb{F}x, \mathbb{F}y) < dist(\mathscr{B}_i, \mathscr{B}_{i+1}) + \varepsilon,$$

for all $x \in \mathscr{B}_i, y \in \mathscr{B}_{i+1}$, *for* $i \leq i \leq p$.

Example 2. *For a non-empty set* $\mathbb{D}$, *let* $\mathfrak{m} : \mathbb{D} \times \mathbb{D} \to [0, \infty)$ *be a distance function and* $\mathscr{B}_1, \mathscr{B}_2, \ldots, \mathscr{B}_p, (p \geq 2)$ *be non-empty subsets of* $\mathbb{D}$. *Let* $\mathbb{F} : \cup_{i=1}^{p} \mathscr{B}_i \to \cup_{i=1}^{p} \mathscr{B}_i$ *be a p-cyclic Meir-Keeler map. Then,* $\mathbb{F} \in \Omega$ *(see* [13]*, Ex.4).*

Example 3. *For a non-empty set* $\mathbb{D}$, *let* $\mathfrak{m} : \mathbb{D} \times \mathbb{D} \to [0, \infty)$ *be a distance function and* $\mathscr{B}_1, \mathscr{B}_2$ *be non-empty subsets of* $\mathbb{D}$. *Let* $\mathbb{F} : \mathscr{B}_1 \cup \mathscr{B}_2 \to \mathscr{B}_1 \cup \mathscr{B}_2$ *be a cyclic* φ*-contraction map defined in* [2] *by*

$$\mathfrak{m}(\mathbb{F}x, \mathbb{F}y) \leq \mathfrak{m}(x,y) - \varphi\big(\mathfrak{m}(x,y)\big) + \varphi\big(dist(\mathscr{B}_1, \mathscr{B}_2)\big), \forall x \in \mathscr{B}_1, y \in \mathscr{B}_2,$$

where $\varphi : [0, \infty) \to [0, \infty)$ *is a strictly increasing map. Then,* $\mathbb{F} \in \Omega$ *(see* [13]*, Ex.5).*

We denote by $\mathbb{Z}$ the class of all functions $\zeta : [0, \infty) \to [0, 1)$ satisfying the following condition:

$$\lim_{n \to \infty} \zeta(t_n) = 1 \implies \lim_{n \to \infty} t_n = 0.$$

The aim of this paper is to establish some best proximity point results for a new class of mappings called a generalized (α, ψ)-p-cyclic Geraghty type contraction in a p-cyclic complete metric space via the property of the Ω class of mappings.

2. Preliminaries

Definition 5 ([7])**.** *A function* $\psi : [0, \infty) \to [0, \infty)$ *is called an altering distance function, if the following properties are satisfied:*

i). ψ *is non-decreasing,*

ii). ψ *is continuous,*

iii). $\psi(t) = 0$ *if and only if* $t = 0$.

We denote the class of altering distance functions as Ψ.

In what follows, we collect some definitions and fundamental results that are essential to the proof of our main results.

Definition 6 ([10], Definitions 3.1)**.** *For a non-empty set* $\mathbb{D}$, *let* $\mathfrak{m} : \mathbb{D} \times \mathbb{D} \to [0, \infty)$ *be a distance function and* $\mathscr{B}_1, \mathscr{B}_2, \ldots, \mathscr{B}_p, (p \geq 2)$ *be non-empty subsets of* $\mathbb{D}$. *Define* $\mathscr{B}_{p+i} := \mathscr{B}_i$, *for all*

$i \in \{1,2,\ldots,p\}$. A map $\mathbb{F} : \cup_{i=1}^{p}\mathscr{B} \to \cup_{i=1}^{p}\mathscr{B}$ is called a p-cyclic map if $\mathbb{F}(\mathscr{B}_i) \subseteq \mathscr{B}_{i+1}, \forall i \in \{1,2,\ldots,p\}$. If $p = 2$, the map $\mathbb{F}$ is called cyclic. A point $x \in \mathscr{B}_i$ is said to be a best proximity point of $\mathbb{F}$ in $\mathscr{B}_i$, if $\mathfrak{m}(x,\mathbb{F}x) = dist(\mathscr{B}_i,\mathscr{B}_{i+1})$, where $dist(\mathscr{B}_i,\mathscr{B}_{i+1}) := \inf\{\mathfrak{m}(x,y) : x \in \mathscr{B}_i, y \in \mathscr{B}_{i+1}\}$.

Definition 7 ([10])**.** *Let $\mathscr{B}_1,\mathscr{B}_2,\ldots,\mathscr{B}_p,(p \geq 2)$ be non-empty convex subsets of a metric space $\mathbb{D}$, and $\mathbb{F} : \cup_{i=1}^{p}\mathscr{B}_i \to \cup_{i=1}^{p}\mathscr{B}_i$ is a p-cyclic mapping. $\mathbb{F}$ is called a p-cyclic nonexpansive mapping, if*

$$\mathfrak{m}(\mathbb{F}x,\mathbb{F}y) \leq \mathfrak{m}(x,y), \forall x \in \mathscr{B}_i, y \in \mathscr{B}_{i+1}, i \in \{1,2,\ldots,p\}.$$

It is notable that the distances between the adjacent sets are equal under the p-cyclic nonexpansive mapping.

Lemma 2.1 ([10], Lemma 3.3)**.** *For a non-empty set $\mathbb{D}$, let $\mathfrak{m} : \mathbb{D}\times\mathbb{D} \to [0,\infty)$ be a distance function and $\mathscr{B}_1,\mathscr{B}_2,\ldots,\mathscr{B}_p,(p \geq 2)$ be non-empty subsets of $\mathbb{D}$. If $\mathbb{F} : \cup_{i=1}^{p}\mathscr{B}_i \to \cup_{i=1}^{p}\mathscr{B}_i$ is a p-cyclic non-expansive map, then*

$$dist(\mathscr{B}_i,\mathscr{B}_{i+1}) = dist(\mathscr{B}_{i+1},\mathscr{B}_{i+2}) = dist(\mathscr{B}_1,\mathscr{B}_2), \forall i \in \{1,2,\ldots,p\}. \tag{2.1}$$

In addition, if $\omega \in \mathscr{B}_i \cap Best(\mathbb{F})_i \neq \emptyset$, then $\mathbb{F}^j\omega \in \mathscr{B}_{i+1} \cap Best(\mathbb{F})_{i+j} \neq \emptyset$, for all j = 1, 2, ..., (p-1), where $Best(\mathbb{F})_k$ is the set of best proximity point of the mapping $\mathbb{F}$ in $\mathscr{B}_k$.

The following lemma appeared in [6] is crucial to prove that a given sequence is Cauchy.

Lemma 2.2 ([6], Lemma 3.7)**.** *For a uniformly convex Banach space $(\mathbb{X},\|.\|)$, we suppose that $\mathscr{B}_1,\mathscr{B}_2$ are non-empty closed subsets of $\mathbb{X}$ and $\{a_n\},\{b_n\} \subset \mathscr{B}_1$ and $\{d_n\} \subset \mathscr{B}_2$. If $\mathscr{B}_1$ is convex such that*

(i). $\|a_n - d_n\| \to dist(\mathscr{B}_1,\mathscr{B}_2)$; and

(ii). for every $\varepsilon > 0$ there exists $N_0 \in \mathbb{N}$ such that for all $m > n > N_0, \|a_m - d_n\| \leq dist(\mathscr{B}_1,\mathscr{B}_2) + \varepsilon$, then for all $\varepsilon > 0$, there exists $N_1 \in \mathbb{N}$ such that for all $m > n > N_1, \|a_m - b_n\| \leq \varepsilon$.

The following three propositions (Proposition 1, 2, 3) were proposed by Karapinar *et al.* [13].

Proposition 1. *For a non-empty set $\mathbb{D}$, let $\mathfrak{m} : \mathbb{D} \times \mathbb{D} \to [0, \infty)$ be a distance function and $\mathscr{B}_1, \mathscr{B}_2, \ldots, \mathscr{B}_p, (p \geq 2)$ be non-empty subsets of $\mathbb{D}$. Then, every p-cyclic Cauchy sequence in $\cup_{i=1}^{p} \mathscr{B}_i$ is p-cyclic bounded.*

Remark 3. *It is notable that a complete metric space need not be p-cyclic complete (see Remark 2 in* [13]*).*

The following proposition is an example of two-cyclic complete metric space.

Proposition 2. *Let $\mathscr{B}_1$ and $\mathscr{B}_2$ be subsets of a uniformly convex Banach space $\mathbb{X}$, which are non-empty and closed. If either $\mathscr{B}_1$ or $\mathscr{B}_2$ is convex, then $\mathscr{B}_1 \cup \mathscr{B}_2$ is two-cyclic complete.*

The following Proposition proves an important property of p-cyclic strict contraction map.

Proposition 3. *For a non-empty set $\mathbb{D}$, let $\mathfrak{m} : \mathbb{D} \times \mathbb{D} \to [0, \infty)$ be a distance function and $\mathscr{B}_1, \mathscr{B}_2, \ldots, \mathscr{B}_p, (p \geq 2)$ be non-empty subsets of $\mathbb{D}$. Let $x \in \mathscr{B}_i (1 \leq i \leq p)$. Suppose that $\mathbb{F} : \cup_{i=1}^{p} \mathscr{B}_i \to \cup_{i=1}^{p} \mathscr{B}_i$ is a p-cyclic strict contraction map and if for all $\varepsilon > 0$, there exists an $n_0 \in \mathbb{N}$ such that*

$$\mathfrak{m}(\mathbb{F}^{pn} x, \mathbb{F}^{pm+1} x) < dist(\mathscr{B}_i, \mathscr{B}_{i+1}) + \varepsilon, n, m \geq n_0, \tag{2.2}$$

then for a given $\varepsilon > 0$, there exists an $n_1 \in \mathbb{N}$ such that

$$\mathfrak{m}(\mathbb{F}^{pn+k} x, \mathbb{F}^{pm+k+1} x) < dist(\mathscr{B}_{i+k}, \mathscr{B}_{i+k+1}) + \varepsilon, n, m \geq n_1, k \in \{1, 2, \ldots, p\}.$$

We recall the following definition which appeared in [9].

Definition 8. *Let $(\mathbb{D}, \mathfrak{m})$ be a metric space, and let $\alpha : \mathbb{D} \times \mathbb{D} \to \mathbb{R}$ be a function. A mapping $\mathbb{F} : \mathbb{D} \to \mathbb{D}$ is said to be a generalized (α, ψ)-Geraghty type contraction if there exists $\zeta \in \mathbb{Z}$ such that*

$$\alpha(x, y) \psi(\mathfrak{m}(\mathbb{F}x, \mathbb{F}y)) \leq \zeta\left(\psi(\mathfrak{M}(x, y))\right) \psi(\mathfrak{M}(x, y)), \tag{2.3}$$

where $\psi \in \Psi$ and $\mathfrak{M}(x, y) = \max\{\mathfrak{m}(x, y), \mathfrak{m}(x, \mathbb{F})), \mathfrak{m}(y, \mathbb{F}y)\}$.

Notice that if take $\psi(t) = t$ in Definition 8, then $\mathbb{F}$ is called generalized α-Geraghty contraction mapping [4]. Again, if we take $\alpha(x, y) = 1$ for all $x, y \in \mathbb{D}$ in Definition 8, then $\mathbb{F}$ is called ψ-Geraghty contraction mapping. Karapinar [9] obtained a fixed point result for such type contraction in a complete metric space.

Following Samet et al.[25], we introduce the notion of α-admissible mappings in p-cyclic metric spaces as follows.

Definition 9. *Let $\mathbb{D}$ be a nonempty set, let $\mathfrak{m}:\mathbb{D}\times\mathbb{D}\to[0,\infty)$ be a distance function and $\mathscr{B}_1,\mathscr{B}_2,\ldots,\mathscr{B}_p,(p\geq 2)$ be non-empty subsets of $\mathbb{D}$. Suppose $\mathbb{F}:\cup_{i=1}^{p}\mathscr{B}_i\to\cup_{i=1}^{p}\mathscr{B}_i$ is a p-cyclic strict contraction map and $\alpha:\mathbb{D}\times\mathbb{D}\to\mathbb{R}$ is a mapping. Then $\mathbb{F}$ is called α-admissible, if for all $x,y\in\mathbb{D}$,*

$$\alpha(x,y)\geq 1 \implies \mathfrak{m}(\mathbb{F}^p x,\mathbb{F}^p y)\geq 1.$$

Definition 10. *Let $\mathbb{D}$ be a nonempty set, let $\mathfrak{m}:\mathbb{D}\times\mathbb{D}\to[0,\infty)$ be a distance function and $\mathscr{B}_1,\mathscr{B}_2,\ldots,\mathscr{B}_p,(p\geq 2)$ be non-empty subsets of $\mathbb{D}$. Suppose $\mathbb{F}:\cup_{i=1}^{p}\mathscr{B}_i\to\cup_{i=1}^{p}\mathscr{B}_i$ is a p-cyclic strict contraction map and $\alpha:\mathbb{D}\times\mathbb{D}\to\mathbb{R}$ is a mapping. Then $\mathbb{F}$ is called triangular α-admissible , if for all $x,y\in\mathbb{D}$,*

(i). $\alpha(x,y)\geq 1 \implies \alpha(\mathbb{F}^p x,\mathbb{F}^p y)\geq 1$;

(ii). $\alpha(x,z)\geq 1$ *and* $\alpha(z,y)\geq 1 \implies \alpha(x,y)\geq 1$.

Lemma 2.3. *For a non-empty set $\mathbb{D}$, let $\mathfrak{m}:\mathbb{D}\times\mathbb{D}\to[0,\infty)$ be a distance function and $\mathscr{B}_1,\mathscr{B}_2,\ldots,\mathscr{B}_p,(p\geq 2)$ be non-empty subsets of $\mathbb{D}$. Let $\alpha:\mathbb{D}\times\mathbb{D}\to\mathbb{R}$ be a function such that $\alpha(x,y)\geq 1$, for all $x,y\in\mathbb{D}$. Let $\mathbb{F}:\cup_{i=1}^{p}\mathscr{B}_i\to\cup_{i=1}^{p}\mathscr{B}_i$ be a p-cyclic contraction map. Suppose $\mathbb{F}$ is a triangular α-admissible map. Assume that there exists $x_0\in\mathbb{D}$ such that $\alpha(x_0,\mathbb{F}^p x_0)\geq 1$. Define a sequence $\{x_n\}$ by $x_n=\mathbb{F}^{pn}x_0$. Then we have $\alpha(x_m,x_n)\geq 1$ for all $m,n\in\mathbb{N}$ with $m<n$.*

Proof. Since there exist $x_0\in\mathbb{D}$ such that $\alpha(x_0,\mathbb{F}^p x_0)\geq 1$ then from Definition 10(i), we deduce that

$$\alpha(x_1,x_2)=\alpha(\mathbb{F}^p x_0,\mathbb{F}^{2p}x_0)\geq 1,\ \alpha(x_2,x_3)=\alpha(\mathbb{F}^{2p}x_0,\mathbb{F}^{3p}x_0)\geq 1.$$

By continuing this process, we get $\alpha(x_n,x_{n+1}\geq 1$ for all $n\in\mathbb{N}\cup\{0\}$.

Suppose that $m<n$. Since $\alpha(x_m,x_{m+1})\geq 1$ and $\alpha(x_{m+1},x_{m+2}\geq 1$, then from Definition 10(ii) we have $\alpha(x_m,x_{m+2})\geq 1$.

Again, Since $\alpha(x_m,x_{m+2})\geq 1$ and $\alpha(x_{m+2},x_{m+3})\geq 1$, then we deduce that $\alpha(x_m,x_{m+3})\geq 1$. By continuing this process, we obtain $\alpha(x_m,x_n)\geq 1$ for all $m,n\in\mathbb{N}$ with $m<n$. $\square$

3. Main Results

We introduce the following definition.

Definition 11. For a non-empty set $\mathbb{D}$, let $\mathfrak{m} : \mathbb{D} \times \mathbb{D} \to [0, \infty)$ be a distance function and $\mathscr{B}_1, \mathscr{B}_2, \ldots, \mathscr{B}_p, (p \geq 2)$ be non-empty subsets of $\mathbb{D}$. Let $\alpha : \mathbb{D} \times \mathbb{D} \to \mathbb{R}$ be a function such that $\alpha(x,y) \geq 1$, for all $x, y \in \mathbb{D}$. Let $\mathbb{F} : \cup_{i=1}^{p} \mathscr{B}_i \to \cup_{i=1}^{p} \mathscr{B}_i$ be a p-cyclic map. $\mathbb{F}$ is said to be a generalized (α,ψ)-p-cyclic Geraghty contraction type mapping, if there exists $\zeta \in \mathbb{Z}$ such that for all $x \in \mathscr{B}_i$, $y \in \mathscr{B}_{i+1}$,

$$\alpha(x,y)\psi(\mathfrak{m}(\mathbb{F}x, \mathbb{F}y)) \leq \zeta\left(\psi(\mathfrak{M}(x,y))\right)\psi\left(\mathfrak{M}(x,y) - dist(\mathscr{B}_i, \mathscr{B}_{i+1})\right), \tag{3.1}$$

where $\psi \in \Psi$ and $\mathfrak{M}(x,y) = \max\{\mathfrak{m}(x,y), \mathfrak{m}(x, \mathbb{F}x)), \mathfrak{m}(y, \mathbb{F}y)\}$.

Note that if $\mathscr{B}_i = \mathscr{B}_{i+1} = 0$ for all $i \in \{1, 2, \ldots, p-1\}$, the Definition 11 reduces to Definition 8.

We now prove our main result.

Theorem 3.1. *For a non-empty set $\mathbb{D}$, let $\mathfrak{m} : \mathbb{D} \times \mathbb{D} \to [0, \infty)$ be a distance function and $\mathscr{B}_1, \mathscr{B}_2, \ldots, \mathscr{B}_p, (p \geq 2)$ be non-empty subsets of $\mathbb{D}$. Let $\alpha : \mathbb{D} \times \mathbb{D} \to \mathbb{R}$ be a function. Let $\mathbb{F} : \cup_{i=1}^{p} \mathscr{B}_i \to \cup_{i=1}^{p} \mathscr{B}_i$ be a generalized (α,ψ)-p-cyclic Geraghty contraction type mapping satisfying (3.1). Suppose $\mathbb{F}$ is triangular α-admissible. Assume for some $k \in \mathbb{N}$ and $x \in \mathscr{B}_i (1 \leq i \leq p, k \leq p), \{\mathbb{F}^{pn+k}x\}$ converges to $\omega \in \mathscr{B}_{i+k}$. Then, ω is a best proximity point of $\mathbb{F}$ in $\mathscr{B}_{i+k}$.*

Proof. Let $x_1 \in \mathbb{F}$ be such that $\alpha(x_1, \mathbb{F}x_1) \geq 1$. Define sequence $\{x_n\} \in \mathbb{F}$ by $x_{n+1} = \mathbb{F}x_n$, for all $n \in \mathbb{N}$.

We will show that $\mathbb{F}$ belongs to the Ω class of mappings. That is, we have to show that

(1). $\mathbb{F}$ is a generalized p-cyclic strict contraction, and

(2). $\lim_{n\to\infty} \mathfrak{m}(\mathbb{F}^{pn}x, \mathbb{F}^{pn+1}y) = dist(\mathscr{B}_i, \mathscr{B}_{i+1}), x, y \in \mathscr{B}_i$.

For part (1), let $x \in \mathscr{B}_i, y \in \mathscr{B}_{i+1}$. Consider

Case(i): If $\mathfrak{m}(x,y) > dist(\mathscr{B}_i,\mathscr{B}_{i+1})$, we have

$$\begin{aligned}
\psi(\mathfrak{m}(\mathbb{F}x,\mathbb{F}y)) &\leq \alpha(x,y)\psi(\mathfrak{m}(\mathbb{F}x,\mathbb{F}y)) \\
&\leq \zeta\left(\psi(\mathfrak{M}(x,y))\right)\psi\big(\mathfrak{M}(x,y) - dist(\mathscr{B}_i,\mathscr{B}_{i+1})\big) \qquad (3.2)\\
&< \psi\big(\mathfrak{M}(x,y) - dist(\mathscr{B}_i,\mathscr{B}_{i+1})\big), \text{ (because } \zeta \in \mathbb{Z}).
\end{aligned}$$

We suppose that $x_n \neq x_{n+1}$ for all $n \in \mathbb{N}$.

We consider

$$\begin{aligned}
\mathfrak{M}(x_n,x_{n+1}) &= \max\{\mathfrak{m}(x_n,x_{n+1}),\mathfrak{m}(x_n,\mathbb{F}x_n)),\mathfrak{m}(x_{n+1},\mathbb{F}x_{n+1})\} \qquad (3.3)\\
&= \max\{\mathfrak{m}(x_n,x_{n+1}),\mathfrak{m}(x_{n+1},x_{n+2})\}
\end{aligned}$$

Suppose $\mathfrak{M}(x_n,x_{n+1}) = \mathfrak{m}(x_{n+1},x_{n+2})$. Then, from (3.2) we get

$$\psi(\mathfrak{m}(\mathbb{F}x_n,\mathbb{F}x_{n+1})) < \psi\big(\mathfrak{m}(x_{n+1},x_{n+2}) - dist(\mathscr{B}_i,\mathscr{B}_{i+1})\big). \qquad (3.4)$$

That implies

$$\mathfrak{m}(x_{n+1},\mathfrak{m}x_{n+2}) < \mathfrak{m}(x_{n+1},\mathfrak{m}x_{n+2}),$$

which is a contradiction. Hence, we have $\mathfrak{M}(x_n,x_{n+1}) = \mathfrak{m}(x_n,x_{n+1})$. Therefore, from (3.2) and Lemma 2.3 we obtain

$$\begin{aligned}
\psi(\mathfrak{m}(\mathbb{F}x_n,x_{n+1})) &\leq \alpha(x_n,x_{n+1})\psi(\mathfrak{m}(\mathbb{F}x_n,x_{n+1})) \\
&\leq \zeta\left(\psi(\mathfrak{m}(x_n,x_{n+1})\right)\psi\big(\mathfrak{m}(x_n,x_{n+1}) - dist(\mathscr{B}_i,\mathscr{B}_{i+1})\big) \qquad (3.5)\\
&< \psi\big(\mathfrak{m}(x_n,x_{n+1}) - dist(\mathscr{B}_i,\mathscr{B}_{i+1})\big).
\end{aligned}$$

Therefore, we have $\mathfrak{m}(\mathbb{F}x_n,\mathbb{F}x_{n+1}) < \mathfrak{m}(x_n,x_{n+1})$. This means, if $\mathfrak{m}(x,y) > dist(\mathscr{B}_i,\mathscr{B}_{i+1})$, we can deduce that

$$\mathfrak{m}(\mathbb{F}x,\mathbb{F}y) < \mathfrak{m}(x,y), \qquad (3.6)$$

for all $x \in \mathscr{B}_i, y \in \mathscr{B}_{i+1}$.

Case (ii): If $\mathfrak{m}(x,y) = dist(\mathscr{B}_i,\mathscr{B}_{i+1})$, then from (3.6), we have

$$\mathfrak{m}(\mathbb{F}x,\mathbb{F}y) \leq \mathfrak{m}(x,y). \qquad (3.7)$$

By Lemma 2.1,

$$\mathfrak{m}(x,y) = dist(\mathscr{B}_i,\mathscr{B}_{i+1}) = dist(\mathscr{B}_{i+1},\mathscr{B}_{i+2}) \leq \mathfrak{m}(\mathbb{F}x,\mathbb{F}y). \qquad (3.8)$$

Therefore, by (3.7) and (3.8) we get

$$\mathfrak{m}(\mathbb{F}x, \mathbb{F}y) = \mathfrak{m}(x, y).$$

Hence, from Case (i) and Case (ii), we deduce that $\mathbb{F}$ is a p-cyclic strict contraction.

For part (2), let $x, y \in \mathscr{B}_i$. Since $\mathbb{F}$ is p-cyclic non-expansive, $\{\mathfrak{m}(\mathbb{F}^{pn}x, \mathbb{F}^{pn+1}y)\}$ is a decreasing sequence which is bounded below by $dist(\mathscr{B}_i, \mathscr{B}_{i+1})$. Therefore,

$$\mathfrak{m}(\mathbb{F}^{pn}x, \mathbb{F}^{pn+1}y) \to r \text{ as } n \to \infty \text{ and } r \geq dist(\mathscr{B}_i, \mathscr{B}_{i+1}),$$

where $r = \inf_{n\geq 1} \mathfrak{m}(\mathbb{F}^{pn}x, \mathbb{F}^{pn+1}y)$.

Claim: $r = dist(\mathscr{B}_i, \mathscr{B}_{i+1})$.

If $\mathfrak{m}(\mathbb{F}^{pn}x, \mathbb{F}^{pn+1}y) = dist(\mathscr{B}_i, \mathscr{B}_{i+1})$ for some n, then by the p-cyclic non-expansiveness of $\mathbb{F}$ we have,

$$\mathfrak{m}(\mathbb{F}^{pn+k}x, \mathbb{F}^{pn+k+1}y) \leq \mathfrak{m}(\mathbb{F}^{pn}x, \mathbb{F}^{pn+1}y), k = 1, 2, \ldots.$$

Hence, we have

$$\mathfrak{m}(\mathbb{F}^{pn}x, \mathbb{F}^{pn+1}y) \to dist(\mathscr{B}_i, \mathscr{B}_{i+1}) \text{ as } n \to \infty.$$

Let us assume that $\mathfrak{m}(\mathbb{F}^{pn}x, \mathbb{F}^{pn+1}y) > dist(\mathscr{B}_i, \mathscr{B}_{i+1})$, $n \in \mathbb{N}$. Suppose that $r > dist(\mathscr{B}_i, \mathscr{B}_{i+1})$. Since $\mathbb{F}$ is p-cyclic non-expansive and triangular α-admissible,

$$\begin{aligned}
\psi(\mathfrak{m}(\mathbb{F}^{p(n+1)}x, \mathbb{F}^{p(n+1)+1}y)) &\leq \psi(\mathfrak{m}(\mathbb{F}^{pn+1}x, \mathbb{F}^{pn+2}y)) \\
&\leq \alpha(\mathbb{F}^{pn}x, \mathbb{F}^{pn+1}y)\psi\big(\mathfrak{m}(\mathbb{F}(\mathbb{F}^{pn}x), \mathbb{F}(\mathbb{F}^{pn+1}y))\big) \\
&\leq \zeta\left(\psi(\mathfrak{m}(\mathbb{F}^{pn}x, \mathbb{F}^{pn+1}y))\right)\psi\big(\mathfrak{m}(\mathbb{F}^{pn}x, \mathbb{F}^{pn+1}y) - dist(\mathscr{B}_i, \mathscr{B}_{i+1})\big).
\end{aligned}$$

Therefore, we have

$$\begin{aligned}
&\psi\big(\mathfrak{m}(\mathbb{F}^{p(n+1)}x, \mathbb{F}^{p(n+1)+1}y)\big) - dist(\mathscr{B}_i, \mathscr{B}_{i+1}) \\
&\qquad \leq \zeta\left(\psi(\mathfrak{m}(\mathbb{F}^{pn}x, \mathbb{F}^{pn+1}y))\right)\psi\big(\mathfrak{m}(\mathbb{F}^{pn}x, \mathbb{F}^{pn+1}y) - dist(\mathscr{B}_i, \mathscr{B}_{i+1})\big).
\end{aligned}$$

Since $\zeta \in \mathbb{Z}$,

$$\frac{\psi\big(\mathfrak{m}(\mathbb{F}^{p(n+1)}x, \mathbb{F}^{p(n+1)+1}y)\big) - dist(\mathscr{B}_i, \mathscr{B}_{i+1})}{\psi\big(\mathfrak{m}(\mathbb{F}^{pn}x, \mathbb{F}^{pn+1}y) - dist(\mathscr{B}_i, \mathscr{B}_{i+1})\big)} \leq \zeta\left(\psi(\mathfrak{m}(\mathbb{F}^{pn}x, \mathbb{F}^{pn+1}y))\right) < 1. \tag{3.9}$$

Since $r = \lim_{n\to\infty} \mathfrak{m}(\mathbb{F}^{p(n+1)}x, \mathbb{F}^{p(n+1)+1}y) > dist(\mathscr{B}_i, \mathscr{B}_{i+1})$ by our assumption, letting $n \to \infty$ in equation (3.9), we get

$$1 = \frac{\psi(r) - dist(\mathscr{B}_i, \mathscr{B}_{i+1})}{\psi(r) - dist(\mathscr{B}_i, \mathscr{B}_{i+1})} \leq \lim_{n\to} \zeta\left(\psi(\mathfrak{m}(\mathbb{F}^{pn}x, \mathbb{F}^{pn+1}y))\right) \leq 1.$$

That is,

$$\lim_{n\to\infty} \zeta\left(\psi(\mathfrak{m}(\mathbb{F}^{pn}x, \mathbb{F}^{pn+1}y))\right) = 1.$$

Since $\zeta \in \mathbb{Z}$, we have

$$\lim_{n\to\infty} \psi\big(\mathfrak{m}(\mathbb{F}^{pn}x, \mathbb{F}^{pn+1}y)\big) = 0.$$

However, $\lim_{n\to\infty} \psi\big(\mathfrak{m}(\mathbb{F}^{pn}x, \mathbb{F}^{pn+1}y)\big) = \psi(r) > 0$, which is a contradiction. Hence, $r = dist(\mathscr{B}_i, \mathscr{B}_{i+1})$. This proves part (2). Hence, from part (1) and part (2), we conclude that $\mathbb{F} \in \Omega$.

To prove that ω is a best proximity point of $\mathbb{F}$, let $x \in \mathscr{B}_i$ be as given in the theorem. By Lemma 2.1, for each $n \in \mathbb{N}$, we have

$$\begin{aligned} dist(\mathscr{B}_{i+k}, \mathscr{B}_{i+k+1}) &= dist(\mathscr{B}_{i+k-1}, \mathscr{B}_{i+k}) \\ &\leq \mathfrak{m}(\mathbb{F}^{pn+k-1}x, \omega) \\ &\leq \mathfrak{m}(\mathbb{F}^{pn+k-1}x, \mathbb{F}^{pn+k}x) + \mathfrak{m}(\mathbb{F}^{pn+k}x, \omega). \end{aligned}$$

Since $\mathbb{F} \in \Omega$, we obtain

$$\lim_{n\to\infty} \big(\mathfrak{m}(\mathbb{F}^{pn+k-1}x, \mathbb{F}^{pn+k}x) + \mathfrak{m}(\mathbb{F}^{pn+k}x, \omega)\big) = dist(\mathscr{B}_{i+k-1}, \mathscr{B}_{i+k}). \tag{3.10}$$

And, since $\{\mathbb{F}^{pn+k}x\}$ converges to $\omega \in \mathscr{B}_{i+k}$, from (3.10) we obtain

$$\lim_{n\to\infty} \mathfrak{m}(\mathbb{F}^{pn+k-1}x, \omega) = dist(\mathscr{B}_{i+k-1}, \mathscr{B}_{i+k}) = dist(\mathscr{B}_{i+k}, \mathscr{B}_{i+k+1}). \tag{3.11}$$

Now, by (3.11) we get

$$\begin{aligned} dist(\mathscr{B}_{i+k}, \mathscr{B}_{i+k+1}) &\leq \mathfrak{m}(\omega, \mathbb{F}\omega) \\ &= \lim_{n\to\infty} \mathfrak{m}(\mathbb{F}^{pn+k}x, \mathbb{F}\omega) \\ &\leq \lim_{n\to\infty} \mathfrak{m}(\mathbb{F}^{pn+k-1}x, \omega) \\ &= dist(\mathscr{B}_{i+k}, \mathscr{B}_{i+k+1}). \end{aligned}$$

Hence, $\mathfrak{m}(\omega, \mathbb{F}\omega) = dist(\mathscr{B}_{i+k}, \mathscr{B}_{i+k+1})$. That is, ω is a best proximity point of $\mathbb{F}$ in $\mathscr{B}_{i+k}$. □

Corollary 3.1. *In Theorem 3.1, suppose that* $dist(\mathscr{B}_i, \mathscr{B}_{i+1}) = 0$*, for some* $i \in \{1, 2, \ldots, p\}$*, there exists a unique* $\omega \in \mathscr{B}_i$ *such that* $\omega = \mathbb{F}\omega$*.*

Proof. By Lemma 2.1, equation (2.1), $dist(\mathscr{B}_k, \mathscr{B}_{k+1}) = 0, 1 \leq k \leq p$. This implies that $\mathbb{F}\omega = \omega$, that is, ω is a fixed point of $\mathbb{F}$. Since $\mathbb{F}$ is p-cyclic, $\omega \in \cup_{i=1}^{p} \mathscr{B}_i$ and hence $\cup_{i=1}^{p} \mathscr{B}_i$ is non-empty.

To prove the uniqueness of a fixed point of $\mathbb{F}$, let $\omega_1, \omega_2 \in \mathscr{B}_i$ be such that $\mathbb{F}\omega_1 = \omega_1$, $\mathbb{F}\omega_2 = \omega_2$ and $\omega_1 \neq \omega_2$. Then,

$$\mathfrak{m}(\omega_1, \omega_2) > 0 = dist(\mathscr{B}_i, \mathscr{B}_{i+1}).$$

Since $\mathbb{F}$ is p-cyclic strict contraction,

$$\mathfrak{m}(\omega_1, \omega_2) = \mathfrak{m}(\mathbb{F}\omega_1, \mathbb{F}\omega_2) < \mathfrak{m}(\omega_1, \omega_2).$$

This is a contradiction. Hence, $\omega_1 = \omega_2$. This shows that there exists a unique fixed point for the Ω class of mappings in a p-cyclic complete metric space. □

4. Applications to Integral Equations

Let Γ denote the set of functions $\gamma, \rho : [0, \infty) \to [0, \infty)$ Lebesgue integrable on each compact subset of $[0, \infty)$ such that, for every $\varepsilon > 0$, we have

$$\int_0^{\varepsilon} \gamma(s)ds > 0 \text{ and } \int_0^{\varepsilon} \rho(s)ds > 0. \tag{4.1}$$

We denote by Ψ the set of function $\psi, \varphi : [0, \infty) \to [0, \infty)$ defined by $\psi(t) = \int_0^t \gamma(s)ds$ and $\varphi(t) = \int_0^t \rho(s)ds$, we obtain the following results.

Definition 12. For a non-empty set $\mathbb{D}$, let $\mathfrak{m} : \mathbb{D} \times \mathbb{D} \to [0, \infty)$ be a distance function and $\mathscr{B}_1, \mathscr{B}_2, \ldots, \mathscr{B}_p, (p \geq 2)$ be non-empty subsets of $\mathbb{D}$. Let $\alpha : \mathbb{D} \times \mathbb{D} \to \mathbb{R}$ be a function. Let $\mathbb{F} : \cup_{i=1}^{p} \mathscr{B}_i \to \cup_{i=1}^{p} \mathscr{B}_i$ be a p-cyclic map. $\mathbb{F}$ is said to be an (α, ψ)-p-cyclic Geraghty contraction type mapping, if there exists $\zeta \in \mathbb{Z}$ such that for all $x \in \mathscr{B}_i$, $y \in \mathscr{B}_{i+1}$,

$$\alpha(x,y)\psi\Big(\int_0^{\mathfrak{m}(\mathbb{F}x,\mathbb{F}y)} \gamma(s)ds\Big) \leq \zeta\Big(\psi\Big(\int_0^{\mathfrak{M}(x,y)} \gamma(t)dt\Big)\psi\Big(\int_0^{\mathfrak{M}(x,y)-dist(\mathscr{B}_i,\mathscr{B}_{i+1})} \gamma(s)ds\Big)\Big). \tag{4.2}$$

where $\psi \in \Psi, \gamma \in \Gamma$ and $\mathfrak{M}(x,y) = \max\{\mathfrak{m}(x,y), \mathfrak{m}(x,\mathbb{F}x), \mathfrak{m}(y,\mathbb{F}y)\}$.

Theorem 4.1. *For a non-empty set $\mathbb{D}$, let $\mathfrak{m} : \mathbb{D} \times \mathbb{D} \to [0,\infty)$ be a distance function and $\mathscr{B}_1, \mathscr{B}_2, \ldots, \mathscr{B}_p, (p \geq 2)$ be non-empty subsets of $\mathbb{D}$. Let $\alpha : \mathbb{D} \times \mathbb{D} \to \mathbb{R}$ be a function. Let $\mathbb{F} : \cup_{i=1}^{p} \mathscr{B}_i \to \cup_{i=1}^{p} \mathscr{B}_i$ be an (α,ψ)-p-cyclic Geraghty contraction type mapping* (4.2) *and let $\mathbb{F}$ be a triangular α-admissible map. Assume for some $k \in \mathbb{N}$ and $x \in \mathscr{B}_i, (1 \leq i \leq p, k \leq p), \{\mathbb{F}^{pn+k}x\}$ converges to $\omega \in \mathscr{B}_{i+k}$. Then, ω is a best proximity point of $\mathbb{F}$ in $\mathscr{B}_{i+k}$.*

Proof. Since $\mathbb{F}$ is an (α,ψ)-p-cyclic Geraghty type mapping, we have

(4.3)
$$\alpha(x,y)\psi\Big(\int_0^{\mathfrak{m}(\mathbb{F}x,\mathbb{F}y)} \gamma(s)ds\Big) \leq \zeta\Big(\psi\big(\int_0^{\mathfrak{M}(x,y)} \gamma(s)ds\Big)\psi\Big(\int_0^{\mathfrak{M}(x,y)-dist(\mathscr{B}_i,\mathscr{B}_{i+1})} \gamma(s)ds\Big)\Big).$$

Taking the functions

$$\phi_0(x) = \int_0^x \gamma(s)d(s).$$

Then, (4.3) becomes

$$\alpha(x,y)(\psi\circ\phi_0)\big(\mathfrak{m}(\mathbb{F}y,\mathbb{F}y)\big) \leq \zeta\big((\psi\circ\phi_0)\big(\mathfrak{M}(x,y)\big)(\psi\circ\phi_0)\big(\mathfrak{M}(x,y)-dist(\mathscr{B}_i,\mathscr{B}_{i+1})\big)\big).$$

Taking $\psi_0 = \psi\circ\phi_0$, we get

$$\alpha(x,y)\psi_0\big(\mathfrak{m}(\mathbb{F}y,\mathbb{F}y)\big) \leq \zeta\big(\psi_0\big(\mathfrak{M}(x,y)\big)\psi_0\big(\mathfrak{M}(x,y)-dist(\mathscr{B}_i,\mathscr{B}_{i+1})\big)\big),$$

and applying Theorem 3.1, we obtain the proof of Theorem 4.1 (One can easily verify that $\psi_0 \in \Psi$). □

5. Conclusions

In this paper, we have introduced a new class of generalized (α,ψ)-p-cyclic Geraghty contraction type mappings. We obtained best proximity point results for those classes of mappings in p-cyclic complete metric spaces via the property of Ω class of mappings introduced by Karapinar *et al.*[13]. We also applied our results to integral equations. Thus, our main results are considered to be natural extensions and generalizations of many comparable results appeared in the existing literature.

Author Contributions

All authors contributed equally in the investigation.

ACKNOWLEDGMENTS

The authors are grateful to the anonymous reviewers for their valuable comments and suggestions to improve the quality of this manuscript.

CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests.

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