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FIXED POINTS OF RANDOM GRAPHICAL $F-$CONTRACTION IN SUPRAMETRIC SPACES ENDOWED WITH A DIGRAPH

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Abstract. In this paper, we introduce the concept of graphical $F-$contraction mappings in complete separable suprametric spaces endowed with a graph and give some random fixed point results for such contractions. Our results are a generalization of some famous theorems in metric spaces to suprametric spaces endowed with a graph.

Keywords: suprametric space; complete probability measure space; graph; random fixed point; random variable; random graphical $F-$contraction.

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1. INTRODUCTION

Fixed point methods have become widely applicable in various fields such as biology, engineering, chemistry, physics, game theory, and economics. In many areas of mathematics and computational science, the question of whether a mathematical model admits a solution often reduces to the existence of a fixed point for a certain mapping. As a result, fixed point theory plays a fundamental role across a wide range of scientific disciplines.

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The development of fixed point theory was greatly influenced by the French mathematician Fréchet, who introduced the concept of metric spaces. To extend the scope of fixed point results, the classical notion of a metric space has undergone several generalizations. In mathematical analysis, the triangle inequality is one of the most important and powerful tools. This inequality has been extended in various ways to accommodate generalized distance functions, such as those found in suprametric spaces, a concept introduced by Berzig [5] and extended by Panda et al. [11]. This extension has facilitated the study of the existence of solutions to integral stochastic equations of the Ito-Doob type. For more details, see [11, 16].

In the last decade, the concept of suprametric spaces has attracted extensive attention from researchers, more than ever from fixed point theorists [17].

On the other hand, in 2012, Wardowski [15] introduced a new contraction called $F-$contraction, which generalize traditional contraction conditions and provide a more flexible framework for analyzing fixed points in generalized metric spaces.

The importance of $F-$contractions lies in their ability to unify and extend several fixed point principles, allowing for greater applicability in abstract spaces such as b-metric, $G-$metric, and modular spaces (see [1, 3]). This has led to meaningful contributions in solving functional, integral, and differential equations, particularly in the settings where classical conditions are too restrictive (see, for example, [4, 2]).

In parallel, the development of random fixed point theory, pioneered by researchers such as Bhatt and Kubiaczyk [6], and further advanced by Itoh [9] and Papageorgiou [12], has opened new avenues for analyzing systems influenced by randomness or uncertainty. In this context, the underlying operator depends on a random parameter, and the objective is to identify measurable fixed point functions that satisfy the fixed point condition almost surely. These ideas have gained prominence in the modeling of stochastic processes, random differential equations, and probabilistic systems across scientific disciplines.

Combining these two perspectives, the study of random $F-$contractive mappings provides a powerful framework for addressing fixed point problems in stochastic environments. Recent contributions, such as those by Rashwan and Hammad [13], demonstrate how $F-$type conditions can be successfully extended to random settings, yielding significant results in Polish spaces and beyond. By incorporating flexible contractive behavior and probabilistic structure, this approach offers robust tools for proving the existence of fixed points in random or uncertain systems.

In this paper, we prove some random fixed point theorems in complete suprametric spaces. Firstly, we present some basic concepts of random variables and graph in suprametric spaces, which will be used in the following sections. Later we introduce random graphical $F-$contraction and we prove some random fixed point theorems in suprametric spaces.

2. PRELIMINARIES

2.1. Basic definitions.

Definition 2.1. [5] *Let X be a non-empty set.*

A mapping $d_\mu : X \times X \to [0,+\infty[$ *is called a suprametric on* X *if:*

i) $d_\mu(x,y) = 0$ *if and only if* $x = y$.

ii) $d_\mu(x,y) = d_\mu(y,x)$ *for all* $x,y \in X$

iii) $d_\mu(x,y) \leqslant d_\mu(x,z) + d_\mu(z,y) + \mu d_\mu(x,z)\, d_\mu(z,y)$, *for all* $x,y,z \in X$, *where* $\mu \geqslant 0$.

Then, (X,d_μ) *is called a suprametric space.*

Definition 2.2. *Let* (X,d_μ) *be a suprametric space and a sequence* $\{x_n\}_{n\in\mathbb{N}}$ *in* X. *Then*

(1) *The sequence* $(x_n)_{n\in\mathbb{N}}$ *is converge to* x *if and only if for all* $\varepsilon > 0$, *there is some integer* $N_\varepsilon \in \mathbb{N}$ *such that* $d_\mu(x_n,x) \leqslant \varepsilon$ *for each* $n \geqslant N_\varepsilon$.

2) $(x_n)_{n\in\mathbb{N}}$ *is Cauchy sequence if and only if for every* $\varepsilon > 0$, *there is* $N_\varepsilon \in \mathbb{N}$ *so that* $d_\mu(x_n,x_m) \leqslant \varepsilon$ *for all* $n,m \geqslant N_\varepsilon$.

Definition 2.3. *A suprametric space* (X,d_μ) *is complete if and only if every Cauchy sequence in* X *is convergent.*

Definition 2.4. *Let* (X,d_{μ_1}) *and* (Y,d_{μ_2}) *be two suprametric spaces.*

(a) A mapping $f : X \to Y$ *is said to be continuous at* $x \in X$, *if for each* $\varepsilon > 0$, *there exists* $\delta > 0$ *such that* $d_{\mu_2}(f(x),f(y)) < \varepsilon$ *whenever* $d_{\mu_1}(x,y) < \delta$.

(b) A mapping $f : X \to Y$ *is said to be sequentially continuous at* $x \in X$, *if for all sequence* $(x_n)_n$ *of* X,

$$\lim_{n\to+\infty} d_{\mu_1}(x_n,x)=0 \implies \lim_{n\to+\infty} d_{\mu_2}(f(x_n),f(x))=0.$$

Remark 2.1. *Since the suprametric spaces (X,d_{μ_1}) and (Y,d_{μ_2}) are metrizables (see* [11]*), then any sequentially continuous map $f:X\to Y$ is continuous.*

Definition 2.5. *Let (X,d_μ) be a suprametric space and A is a nonempty subset of X. we have*

1) *A is dense in X if $\overline{A}^{d_\mu}=X$.*
2) *X is separable if it has a countable dense subset.*

2.2. The basic concepts related to a Random variable and a graph.

Let $(\Omega,\sum)$ be a measurable space with sigma algebra $\sum$ generated by all measurable subsets of Ω and (X,d_μ) be a suprametric space with Borel σ-algebra $B=B(X)$ (which is the smallest σ-algebra that contains all open subsets of X. Let $(\Omega,\sum,m)$ denote a complete probability measure space with measure m. A mapping $h:\Omega\longmapsto X$ is called measurable if for any closed subset F of X the set $h^{-1}(F)=\{\omega\in\Omega : h(\omega)\in F\}$ is measurable. For more details, see Joshi and Bose [10].

Definition 2.6. *Recall that a mapping $f:\Omega\times X\to X$ is said to be a*

(a) *random operator if, for any $x\in X$, the map $f(\cdot,x):\omega\mapsto f(\omega,x)$ is measurable;*

(b) *continuous random mapping if the set of all $\omega\in\Omega$ for which $\Big\{f(\omega,.):x\mapsto f(\omega,x)\Big\}$ is a continuous function of x has measure one.*

Also, we recall the following definitions (see Joshi and Bose [10]).

Definition 2.7. *(a) A random fixed point of f is a measurable function $y:\Omega\to X$ such that*

$$y(\omega)=f(\omega,y(\omega)), \quad \textit{for all } \omega\in\Omega.$$

Equivalently, a measurable selection for the multivalued map
Fix $F:\Omega\to\mathscr{P}(X)$ is defined by $\mathrm{Fix}\,F(w)=\{x\in X : x=f(\omega,x)\}$,
with $\mathscr{P}(X)=\{A\subseteq X : A\neq\emptyset\}$.

(b) Any random variable $\omega\mapsto x(\omega)$ which satisfies $\Big\{m(\{\omega\in\Omega : f(\omega,x(\omega))=x(\omega)\})=1\Big\}$ is called a random solution of the fixed point equation or a random fixed point of f.

Proposition 2.1. [8] *Let $(\Omega, \sum)$ be a measurable space and (X, d_μ) be a separable suprametric space. If for all $\omega \in \Omega$, the map $f(\omega, .)$ is continuous on X. Then f is a random operator.*

The basic concepts, notation and terminology related to graph theory can be found, for example in [7]. A directed graph or digraph consists of a nonempty set $V(G)$, whose elements are called the vertices of G, and a set $E(G) \subseteq V(G) \times V(G)$, called the set of directed edges of G. The diagonal of the cartesian product $V(G) \times V(G)$ will be denoted by Δ. A digraph is said to be reflexive if $E(G)$ contains all loops, i.e. if $\Delta \subseteq E(G)$. G is said to be transitive if, for any $x, y, z \in V(G)$

$$[(x,y) \in E(G) \text{ and } (y,z) \in E(G)] \implies (x,z) \in E(G).$$

Let (X, d_μ) be a suprametric space. Consider a directed graph G such that the set $V(G)$ of its vertices coincide with X, and the set $E(G)$ of its edges contains all loops.
Now, we introduce the concepts of graphical random monotonic mappings and $G-$regular space.

Definition 2.8. *Let $(\Omega, \mathscr{F})$ be a measurable space A sequence $(x_n)_{n\geqslant 0}$ in $V(G)$ is said to be*

(i) random G-increasing, if $(x_n(\omega), x_{n+1}(\omega)) \in E(G)$, for all $\omega \in \Omega$ and $n \in \mathbb{N}$;

(ii) random G-decreasing, if $(x_{n+1}(\omega), x_n(\omega)) \in E(G)$, for all $\omega \in \Omega$ and $n \in \mathbb{N}$;

(iii) random G-monotone, if it is either random G-increasing or random G-decreasing.

Definition 2.9. *Let $(\Omega, \sum)$ be a measurable space. We say that (X, d_μ, G) is said random G-regular if any G-increasing sequence (resp. G-decreasing sequence) $(x_n(\omega))_{n\geqslant 0}$ which converges to some $x(\omega) \in V(G)$, for all $\omega \in \Omega$, we have $(x_n(\omega), x(\omega)) \in E(G)$ (resp. $(x(\omega), x_n(\omega)) \in E(G)$), for any $\omega \in \Omega$ and $n \in \mathbb{N}$.*

3. RANDOM FIXED POINT

We start with a fundamental result theat we need in the sequel.

Theorem 3.1. *Let (X, d_μ, G) be a separable $G-$regular complete suprametric space with graph G. Let $T : X \to X$ be a G-increasing continuous mapping, satisfies:*

$$d_\mu\big(T(x_1),T(x_2)\big) \leqslant a d_\mu\big(x_1,x_2\big) + b\max\Bigg\{d_\mu\big(x_1,T(x_1)\big), d_\mu\big(x_2,T(x_2)\big)\Bigg\}. \tag{3.1}$$

for all $(x_1,x_2) \in E(G)$*, where* $0 < a < 1$ *and* $0 < a+b < 1$*. If there exists* $x_0 \in X$ *such that* $\Big(x_0,T(x_0)\Big) \in E(G)$*, then there exists a unique fixed point of* T *in* X*.*

Proof. If $T(x_0) = x_0$, then x_0 is a fixed point of T. If not, suppose that, $T(x_0) \neq x_0$. We define the sequence $x_{n+1} = T(x_n)$, for all $n \in \mathbb{N}$.
Since T is G-increasing and $\Big(x_0,T(x_0)\Big) \in E(G)$, for all $\omega \in \Omega$, we have by induction $\big(x_n,x_{n+1}\big) \in E(G)$, for all $n \in \mathbb{N}$.
Using inequality (3.1), we obtain by induction

$$\begin{aligned} d_\mu\big(x_n,x_{n+1}\big) =& d_\mu\Big(T(x_{n-1}),T(x_n)\Big) \\ &\leq a d_\mu(x_{n-1},x_n) + b\max\Big\{d_\mu(x_{n-1},x_n), d_\mu(x_n,x_{n+1})\Big\} \end{aligned}$$

We get contradiction, if $d_\mu(x_{n-1},x_n) < d_\mu(x_n,x_{n+1})$, since

$$d_\mu\big(x_n,x_{n+1}\big) \leq (a+b) d_\mu(x_{n-1},x_n)$$

Continue the process, until we achieve

$$d_\mu\big(x_n,x_{n+1}\big) \leq (a+b)^n d_\mu(x_0,T(x_0)) \tag{3.2}$$

Now, we shall demonstrate the Cauchy nature of the sequence $\big(x_n\big)_{n\in\mathbb{N}}$. Set $d_\mu(x_0,T(x_0)) := d_0$. Using (3.2) and by induction we can show that for all $n \in \mathbb{N}$ and $k \in \mathbb{N}\setminus\{0,1\}$,

$$d_\mu\big(x_n,x_{n+k}\big) \leq (a+b)^n d_0 \left(1 + \sum_{i=1}^{k-1} (a+b)^i \prod_{j=0}^{i-1} \big(1+\mu(a+b)^{n+j} d_0\big)\right). \tag{3.3}$$

Set $S_i = (a+b)^i \prod_{j=0}^{i-1} \big(1+\mu(a+b)^{n+j} d_0\big)$. Then

$$\left|\frac{S_{i+1}}{S_i}\right| = \left|\frac{(a+b)^{i+1} \prod_{j=0}^{i} \big(1+\mu(a+b)^{n+j} d_0\big)}{(a+b)^i \prod_{j=0}^{i-1} \big(1+\mu(a+b)^{n+j} d_0\big)}\right| = (a+b)\big(1+\mu(a+b)^{n+i} d_0\big).$$

Since $0<a+b<1$, then $\lim_{i\to+\infty}(a+b)^{n+i}=0$. Consequently, $\lim_{i\to+\infty}\left|\frac{S_{i+1}}{S_i}\right|=a+b<1$. Hence, the series $\sum_{i=0}^{+\infty}S_i$ converges. We deduce that $\lim_{n,k\to+\infty}d_\mu\left(x_n,x_{n+k}\right)=0$. Thus, $(x_n)_{n\in\mathbb{N}}$ is a Cauchy sequence. As, (X,d_μ,G) is complete space, there exists x^* such that $x^*=\lim_{n\to\infty}x_n$. And since (X,d_μ,G) is G-regular, we get $(x_n,x^*)\in E(G)$, for all $n\in\mathbb{N}$.

We wil show that $x^*=T(x^*)$. For each $n\in\mathbb{N}$,

$$\begin{aligned}d_\mu\left(x_n,T(x_n)\right)&\leqslant(a+b)d_\mu\left(x_{n-1},x_n\right)\\&\leqslant d_\mu\left(x_{n-1},x^*\right)+d_\mu\left(x^*,x_n\right)+\mu\; d_\mu\left(x_{n-1},x^*\right)\; d_\mu\left(x^*,x_n\right).\end{aligned}$$

By taking the limit as $n\to+\infty$, we get $\lim_{n\to\infty}d_\mu\left(x_n,T(x_n)\right)=0$.
As $\lim_{n\to\infty}d_\mu\left(x_n,T(x_n)\right)=d_\mu\left(x^*,T(x^*)\right)$, then $d_\mu\left(x_*,T(x^*)\right)=0$.
Hence, $x^*=T(x^*)$. which shows that x^* is a fixed point of the mapping T.

For the uniqueness assume that there are two distinct fixed points of the pair of mapping T such that:

$$d_\mu\left(T(x_1),T(x_2)\right)\leqslant ad_\mu\left(x_1,x_2\right)+b\max\left\{d_\mu\left(x_1,T(x_1)\right),d_\mu\left(x_2,T(x_2)\right)\right\}.$$

that can be expressed as

$$d_\mu\left(x_1,x_2\right)\leq ad_\mu\left(x_1,x_2\right)$$

which is a contradiction. Hence, T has a unique fixed point. Which achieve the proof. □

Motivated and inspired by Theorem 4.3 of [14], we give the theorem as follows:

Theorem 3.2. *Let (X,d_μ,G) be a G-separable suprametric space with graph G and $(\Omega,\sum,m)$ be a complete probability space. Let $F:\Omega\times X\to X$ be a continuous random operator such that, for each $\omega\in\Omega$, F satisfies:*

$$\begin{aligned}d_\mu\left(F(\omega,x_1),F(\omega,x_2)\right)&\leqslant a(\omega)d_\mu\left(x_1,x_2\right)\\&\quad+b(\omega)\max\left\{d_\mu\left(x_1,F(\omega,x_1)\right),d_\mu\left(x_2,F(\omega,x_2)\right)\right\}.\end{aligned}$$

for all $(x_1,x_2)\in E(G)$, where $a(\omega)$ and $b(\omega)$ are real-valued random variables such that $0<a(\omega)<1$ and $0<a(\omega)+b(\omega)<1$ almost surely.

Then, there exists a unique random fixed point of F in X.

Proof. Let $A = \left\{\omega \in \Omega \ : \ \text{the map } x \mapsto F(\omega, x) \text{ is a continuous of } x\right\}$,
$B = \left\{\omega \in \Omega \ : \ 0 < a(\omega) < 1 \text{ and } 0 < a(\omega) + b(\omega) < 1\right\}$ and for each $(x_1, x_2) \in E(G)$,

$$C_{x_1,x_2} = \left\{\omega \in \Omega : d_\mu\big(F(\omega,x_1),F(\omega,x_2)\big) \leqslant a(\omega) d_\mu\big(x_1,x_2\big) \right.$$
$$\left. + b(\omega)\max\{d_\mu\big(x_1,F(\omega,x_1)\big), d_\mu\big(x_2,F(\omega,x_2)\big)\}\right\}$$

Let $(x_1, x_2) \in E(G)$. Since X is G-separable, there exists a subset S be a countable dense subset of X and there exist two sequence $(a_n)_{n\geqslant 0}$ and $(b_n)_{n\geqslant 0}$ of S verifying $\lim\limits_{n\to+\infty} d_\mu(a_n,x) = 0 = \lim\limits_{n\to+\infty} d_\mu(b_n,y)$ and $(a_n,b_n) \in E(G)$ or $(b_n,a_n) \in E(G)$, for all $n \in \mathbb{N}$. We now prove that

$$\bigcap_{(x_1,x_2)\in E(G)} \Big(C_{x_1,x_2} \cap A \cap B\Big) = \bigcap_{(s_1,s_2)\in E(G_S)} \Big(C_{s_1,s_2} \cap A \cap B\Big).$$

Let $\omega \in \bigcap\limits_{(s_1,s_2)\in E(G_S)} \Big(C_{s_1,s_2} \cap A \cap B\Big)$. As $(a_n,b_n) \in E(G_S)$ or $(b_n,a_n) \in E(G_S)$, and $E(G_S) \subseteq E(G)$, we have, for all $n \in \mathbb{N}$

$$d_\mu(F(\omega,a_n),F(\omega,b_n)) \leqslant a(\omega) d_\mu(a_n,b_n)$$
$$+ b(\omega)\max\left\{d_\mu(a_n,F(\omega,a_n)), d_\mu(b_n,F(\omega,b_n))\right\}$$

Since, the suprametric d_μ is continuous and by hypothesis F is continuous random operator, we get

$$d_\mu(F(\omega,x_1),F(\omega,x_2)) \leqslant a(\omega) d_\mu(x_1,x_2)$$
$$+ b(\omega)\max\left\{d_\mu(x_1,F(\omega,x_1)), d_\mu(x_2,F(\omega,x_2))\right\}.$$

Thus, $\omega \in \bigcap\limits_{(x_1,x_2)\in E(G)} \Big(C_{x_1,x_2} \cap A \cap B\Big)$, which implies

$$\bigcap_{(s_1,s_2)\in\in E(G_S)} \Big(C_{s_1,s_2} \cap A \cap B\Big) \subset \bigcap_{(x_1,x_2)\in E(G)} \Big(C_{x_1,x_2} \cap A \cap B\Big)$$

Since, $E(G_S) \subseteq E(G)$, so

$$\bigcap_{(x_1,x_2)\in E(G)} \Big(C_{s_1,s_2} \cap A \cap B\Big) \subset \bigcap_{(s_1,s_2)\in E(G_S)} \Big(C_{x_1,x_2} \cap A \cap B\Big)$$

Hence,

$$\bigcap_{(x_1,x_2)\in E(G)} \Big(C_{x_1,x_2} \cap A \cap B\Big) = \bigcap_{(s_1,s_2)\in E(G_S)} \Big(C_{s_1,s_2} \cap A \cap B\Big)$$

We have, $\bigcap\limits_{(x_1,x_2)\in E(G)} \Big(C_{x_1,x_2} \cap A \cap B\Big) \in \mathscr{F}$. As, $m(A) = 1 = m(B)$, we obtain $m(\Omega \setminus A) = 1 = m(\Omega \setminus B)$. Thus, $m\big(\bigcap\limits_{(x_1,x_2)\in E(G)} C_{x_1,x_2}\big) = 1$, and consequently $m\Big(\Omega \setminus \big(\bigcap\limits_{(x_1,x_2)\in E(G)} C_{x_1,x_2}\big)\Big) = 0$. Therefore,

$$m\Big(\bigcap_{(x_1,x_2)\in E(G)} \Big(C_{x_1,x_2} \cap A \cap B\Big)\Big) = 1.$$

Hence, for all $\omega \in \bigcap\limits_{(x_1,x_2)\in E(G)} \Big(C_{x_1,x_2} \cap A \cap B\Big)$, the element $F(\omega,.)$ is a deterministic operator, then, by Theorem 3.1 F has a unique random fixed point in X. This completes the proof. □

Corollary 3.1. *Let (X,d_μ,G) be a G-separable suprametric space with graph G and $(\Omega,\sum,m)$ be a complete probability space. Let $F : \Omega \times X \to X$ be a continuous random operator such that, for each $\omega \in \Omega$, F satisfies:*

$$\begin{aligned} d_\mu\big(F(\omega,x_1),F(\omega,x_2)\big) &\leqslant a(\omega)d_\mu\big(x_1,x_2\big) + b(\omega)d_\mu\big(x_1,F(\omega,x_1)\big) \\ &\quad + c(\omega)d_\mu\big(x_2,F(\omega,x_2). \end{aligned}$$

for all $(x_1,x_2) \in E(G)$, where $a(\omega)$ and $b(\omega)$ are nonnegative real-valued random variables such that $0 < a(\omega) < 1$ and $0 < a(\omega) + b(\omega) + c(\omega) < 1$ almost surely.
Then, there exists a unique random fixed point of F in X.

Proof. Let $(x_1,x_2) \in E(G)$ and $\omega \in \Omega$.

$$\begin{aligned} d_\mu\big(F(\omega,x_1),F(\omega,x_2)\big) &\leqslant a(\omega)d_\mu\big(x_1,x_2\big) + b(\omega)d_\mu\big(x_1,F(\omega,x_1)\big) \\ &\quad + c(\omega)d_\mu\big(x_2,F(\omega,x_2) \\ &\leqslant a(\omega)d_\mu\big(x_1,x_2\big) + (b(\omega)+c(\omega)) \\ &\quad \max\{d_\mu\big(x_1,F(\omega,x_1)\big), d_\mu\big(x_2,F(\omega,x_2)\big)\}. \end{aligned}$$

By Theorem 3.2 and since $a(\omega) + \big\{b(\omega) + c(\omega)\big\} < 1$, there exists a unique random fixed point of F in X. □

4. FIXED POINTS OF RANDOM GRAPHICAL $F-$CONTRACTION

In this section, we prove random fixed point theorems for random graphical $F-$contraction in complete suprametric spaces endowed with a graph.

Definition 4.1 ([17])**.** *Let $\mathscr{F}$ be the family of all functions $F:(0,+\infty)\to\mathbb{R}$ such that:*

i) *F is strictly increasing, that is, for all $\alpha,\beta\in(0,+\infty)$ if $\alpha<\beta$ then $F(\alpha)<F(\beta)$*

ii) *For each sequence $(\alpha_n)_n$ of positive numbers, the following holds:*
$\lim_{n\to+\infty}\alpha_n=0$ *if and only* $\lim_{n\to+\infty}F(\alpha_n)=-\infty$

iii) *There exists $k\in(0,1)$ such that* $\lim_{n\to0^+}\alpha^kF(\alpha)=0$

Following Definitions 4.1 we introduce graphical $F-$contraction in the following manner.

Definition 4.2. *Let $(\Omega,\sum)$ be a mesurable space and (X,d_μ,G) be a complete separable suprametric space with graph. A map $T(\omega,.):\Omega\times X\to X$ is said to be a random graphical $F-$contraction if*

(c_1) $\Big(x(\omega),y(\omega)\Big)\in E(G)\Rightarrow\Big(T(\omega,x(\omega)),T(\omega,y(\omega))\Big)\in E(G)$ *for all $\omega\in\Omega$ and $x,y\in X$.*

(c_2) *There exist $F\in\mathscr{F}$ and $\tau(\omega)>0$ such that for all $\omega\in\Omega$, $x,y\in X$ with $(x(\omega),y(\omega))\in E(G)$ and $T(\omega,x(\omega))\neq T(\omega,y(\omega))$,*

$$\tau(\omega)+F\Big(d\big(T(\omega,x(\omega)),T(\omega,y(\omega))\big)\Big)\leq F\Big(d\big(x(\omega),y(\omega)\big)\Big)\tag{4.1}$$

Theorem 4.1. *Let $(\Omega,\sum)$ be a mesurable space, (X,d_μ,G) be a separable random G-regular complete suprametric space X, $T:\Omega\times X\to X$ be a random graphical $F-$contraction and there exists a random variable $x_0:\Omega\to X$ with $\Big(x_0(\omega),T(\omega,x_0(\omega)\Big)\in E(G)$, for all $\omega\in\Omega$. Then T has a unique random fixed point.*

Proof. Let $x_0(\omega)\in X$, for all $\omega\in\Omega$ be arbitary and fixed. We define the sequence $x_{n+1}(\omega)=T\Big(\omega,x_n(\omega)\Big)$ for each $\omega\in\Omega$ and $n\in\mathbb{N}\cup\{0\}$, where $x_0(\omega)=x(\omega)$.
If there exists $n_0\in\mathbb{N}\cup\{0\}$ such that $x_{n_0+1}(\omega)=x_{n_0}(\omega)$, then $T\Big(\omega,x_{n_0}(\omega)\Big)=x_{n_0}(\omega)$ and hence $x_{n_0}(\omega)$ is a random fixed point of T.
Since, $\Big(x_0(\omega),T(\omega,x_0(\omega)\Big)\in E(G)$, for all $\omega\in\Omega$, ≥0 and by (c_1), then $\Big(x_n(\omega),x_{n+1}(\omega)\Big)\in E(G)$ for all $\omega\in\Omega$ and $n\geq0$.

Now we suppose that $x_n(\omega) \neq x_{n+1}(\omega)$ for all $n \geq 0$ then $d_\mu\Big(x_{n+1}(\omega), x_n(\omega)\Big) > 0$ for all $n \geq 0$. Using the inequality (4.1), we get

$$\begin{aligned} F\Big(d_\mu\Big(x_n(\omega), x_{n+1}(\omega)\Big)\Big) &= F\Big(d_\mu\Big(T(\omega, x_{n-1}(\omega)), T(\omega, x_n(\omega))\Big)\Big) \\ &\leq F\Big(d_\mu\Big(x_{n-1}(\omega), x_n(\omega)\Big)\Big) - \tau(\omega) \end{aligned} \tag{4.2}$$

Repeating this process, we get

$$\begin{aligned} F&\Big(d_\mu\Big(T(\omega, x_{n-1}(\omega)), T(\omega, x_n(\omega))\Big)\Big) \\ &\leq F\Big(d_\mu\Big(x_{n-1}(\omega), x_n(\omega)\Big)\Big) - \tau(\omega) \\ &= F\Big(d_\mu\Big(T(\omega, x_{n-2}(\omega)), T(\omega, x_{n-1}(\omega))\Big)\Big) - \tau(\omega) \\ &\leq F\Big(d_\mu\Big(x_{n-2}(\omega), x_{n-1}(\omega)\Big)\Big) - 2\tau(\omega) \\ &= F\Big(d_\mu\Big(T(\omega, x_{n-3}(\omega)), T(\omega, x_{n-2}(\omega))\Big)\Big) - 2\tau(\omega) \\ &\leq F\Big(d_\mu\Big(x_{n-3}(\omega), x_{n-2}(\omega)\Big)\Big) - 3\tau(\omega) \\ &\quad\vdots \\ &\leq F\Big(d_\mu\Big(x_0(\omega), x_1(\omega)\Big)\Big) - n\tau(\omega) \end{aligned} \tag{4.3}$$

Taking the limit as $n \to +\infty$, we obtain

$$\lim_{n\to+\infty} F\Big(d_\mu\Big(T(\omega, x_{n-1}(\omega)), T(\omega, x_n(\omega))\Big)\Big) = -\infty$$

Then from (ii) of definition 4.1 $\lim_{n\to+\infty} d_\mu\Big(T(\omega, x_{n-1}(\omega)), T(\omega, x_n(\omega))\Big) = 0$, which means

$$\lim_{n\to+\infty} d_\mu\Big(x_n(\omega), x_{n+1}(\omega)\Big) = 0 \tag{4.4}$$

and from (iii) of definition 4.1, there exists $k \in (0,1)$ such that

$$\lim_{n\to+\infty} \Big(d_\mu\Big(x_n(\omega), x_{n+1}(\omega)\Big)\Big)^k F\Big(d_\mu\Big(x_n(\omega), x_{n+1}(\omega)\Big)\Big) = 0 \tag{4.5}$$

Using (4.3), we get

$$\begin{aligned}\Big(d_\mu\Big(x_n(\omega),x_{n+1}(\omega)\Big)\Big)^k F\Big(d_\mu\Big(x_n(\omega),x_{n+1}(\omega)\Big)\Big) - F\Big(d_\mu\Big(x_0(\omega),x_1(\omega)\Big)\Big) &\\ \leq -\Big(d_\mu\Big(x_n(\omega),x_{n+1}(\omega)\Big)\Big)^k n\tau(\omega)&\\ \leq 0&\end{aligned}$$

by (4.4) and (4.5), we obtain

$$\lim_{n\to+\infty} n\Big(d_\mu\Big(x_n(\omega),x_{n+1}(\omega)\Big)\Big)^k = 0$$

Then for all $\omega\in\Omega$, there exists $N\in\mathbb{N}$ such that $n\Big(d_\mu\Big(x_n(\omega),x_{n+1}(\omega)\Big)\Big)^k \leq 1$ for all $n\geq N$. Hence

$$d_\mu\Big(x_n(\omega),x_{n+1}(\omega)\Big) \leq \frac{1}{\sqrt[k]{n}} \tag{4.6}$$

We shall now demonstrate the Cauchy nature of the sequence $(x_n(\omega))_{n\in\mathbb{N}}$.
Let $n\in\mathbb{N}$ and $\omega\in\Omega$, we denote $d_\mu\Big(x_n(\omega),x_{n+1}(\omega)\Big) := d_n$.
By induction we show that for all $n\in\mathbb{N}$ and $k\in\mathbb{N}\setminus\{0,1\}$

$$d_\mu\Big(x_n(\omega),x_{n+k}(\omega)\Big) \leq d_n + \sum_{i=1}^{k-1} d_{n+i}\prod_{j=0}^{i-1}\Big(1+\mu d_{n+j}\Big)$$

For $k=2$, we have

$$\begin{aligned}d_\mu\Big(x_n(\omega),x_{n+2}(\omega)\Big) &\leq d_n + d_{n+1} + \mu d_n d_{n+1}\\ &\leq d_n + \Big(1+\mu d_n\Big)d_{n+1}\end{aligned}$$

Let $k\geq 2$. We suppose that

$$d_\mu\Big(x_n(\omega),x_{n+k}(\omega)\Big) \leq d_n + \sum_{i=1}^{k-1} d_{n+i}\prod_{j=0}^{i-1}\Big(1+\mu d_{n+j}\Big) \quad \text{is true.}$$

We have

$$\begin{aligned}&d_\mu\Big(x_n(\omega),x_{n+k+1}(\omega)\Big)\\ &\leq d_\mu\Big(x_n(\omega),x_{n+k}(\omega)\Big) + d_{n+k} + \mu d_\mu\Big(x_n(\omega),x_{n+k}(\omega)\Big)d_{n+k}\end{aligned}$$

$$\begin{aligned}
&\leq d_\mu\Big(x_n(\omega),x_{n+k}(\omega)\Big)+\Big(1+\mu d_\mu\Big(x_n(\omega),x_{n+k}(\omega)\Big)\Big)d_{n+k}\\
&\leq d_\mu\Big(x_n(\omega),x_{n+k}(\omega)\Big)+\Bigg(1+\mu d_n+\sum_{i=1}^{k-1}\mu d_{n+i}\prod_{j=0}^{i-1}\Big(1+\mu d_{n+j}\Big)\Bigg)d_{n+k}\\
&\leq d_\mu\Big(x_n(\omega),x_{n+k}(\omega)\Big)+\Bigg(1+\mu d_n+\mu d_{n+1}\Big(1+\mu d_n\Big)+\mu d_{n+2}\Big(1+\mu d_n\Big)\\
&\qquad\Big(1+\mu d_{n+1}\Big)+...+\mu d_{n+k-1}\Big(1+\mu d_n\Big)\times...\times\Big(1+\mu d_{n+k-2}\Big)\Bigg)d_{n+k}\\
&\leq d_\mu\Big(x_n(\omega),x_{n+k}(\omega)\Big)+\Big(1+\mu d_n\Big)\Bigg(1+\mu d_{n+1}+\mu d_{n+2}\Big(1+\mu d_{n+1}\Big)\\
&\qquad+...+\mu d_{n+k-1}\Big(1+\mu d_{n+1}\Big)\times...\times\Big(1+\mu d_{n+k-2}\Big)\Bigg)d_{n+k}\\
&\leq d_\mu\Big(x_n(\omega),x_{n+k}(\omega)\Big)+\Big(1+\mu d_n\Big)\Big(1+\mu d_{n+1}\Big)\Bigg(1+\mu d_{n+2}+...+\\
&\qquad\mu d_{n+k-1}\Big(1+\mu d_{n+2}\Big)\times...\times\Big(1+\mu d_{n+k-1}\Big)\Bigg)d_{n+k}\\
&\qquad\qquad\vdots\\
&\leq d_\mu\Big(x_n(\omega),x_{n+k}(\omega)\Big)+\Big(1+\mu d_n\Big)\Big(1+\mu d_{n+1}\Big)\Big(1+\mu d_{n+2}\Big)\\
&\qquad\times...\times\Big(1+\mu d_{n+k-1}\Big)d_{n+k}\\
&\leq d_n+\sum_{i=1}^{k-1}d_{n+i}\prod_{j=0}^{i-1}\Big(1+\mu d_{n+j}\Big)+\Big(1+\mu d_n\Big)\Big(1+\mu d_{n+1}\Big)\Big(1+\mu d_{n+2}\Big)\\
&\qquad\times...\times\Big(1+\mu d_{n+k-1}\Big)d_{n+k}\\
&\leq d_n+\sum_{i=1}^{k}d_{n+i}\prod_{j=0}^{i-1}\Big(1+\mu d_{n+j}\Big)
\end{aligned}$$

Thus, by induction for all $n\in\mathbb{N}$ and $k>0$

$$d_\mu\Big(x_n(\omega),x_{n+k}(\omega)\Big)\leq d_n+\sum_{i=1}^{k-1}d_{n+i}\prod_{j=0}^{i-1}\Big(1+\mu d_{n+j}\Big)$$

Using (4.6), we have

$$d_\mu\Big(x_n(\omega),x_{n+k}(\omega)\Big)\leq\frac{1}{\sqrt[k]{n}}+\sum_{i=1}^{k-1}\frac{1}{\sqrt[k]{n+i}}\prod_{j=0}^{i-1}\Big(1+\mu\frac{1}{\sqrt[k]{n+j}}\Big)$$

Thus

$$d_\mu\Big(x_n(\omega),x_{n+k}(\omega)\Big)\le \frac{1}{\sqrt[k]{n}}+\sum_{i=1}^{k-1}\frac{1}{\sqrt[k]{i}}\prod_{j=0}^{i-1}\Big(1+\mu\frac{1}{\sqrt[k]{j}}\Big)$$

Set $S_i=\frac{1}{\sqrt[k]{i}}\prod_{j=0}^{i-1}\Big(1+\mu\frac{1}{\sqrt[k]{j}}\Big)$. Then

$$\frac{S_{i+1}}{S_i}=\frac{\frac{1}{\sqrt[k]{i+1}}\prod_{j=0}^{i}\Big(1+\mu\frac{1}{\sqrt[k]{j}}\Big)}{\frac{1}{\sqrt[k]{i}}\prod_{j=0}^{i-1}\Big(1+\mu\frac{1}{\sqrt[k]{j}}\Big)}=\frac{\sqrt[k]{i}}{\sqrt[k]{i+1}}\Big(1+\mu\frac{1}{\sqrt[k]{i}}\Big)$$

Since $k\in(0,1)$, then

$$\lim_{n\to+\infty}\left|\frac{S_{i+1}}{S_i}\right|<1$$

Hence, the seris $\sum_{i=0}^{+\infty}S_i$ is converges.

we deduce that $\lim_{n,k\to+\infty}d_\mu\Big(x_n(\omega),x_{n+k}(\omega)\Big)=0$ for all $\omega\in\Omega$. Thus $(x_n(\omega))_{n\in\mathbb{N}}$ is a Cauchy seqeunce, for every $\omega\in\Omega$.

As (X,d_μ,G) is complete space, there exists $x:\Omega\longrightarrow X$ such that $x(\omega)=\lim_{n\to+\infty}x_n(\omega)$, for all $\omega\in\Omega$. And since (X,d_μ,G) is random G-regular. We get $(x_n(\omega),x(\omega))\in E(G)$ for all $\omega\in\Omega$ and $n\in\mathbb{N}$.

Finally the continuity of T yields

$$\begin{aligned}d_\mu\Big(T(\omega,x(\omega)),x(\omega)\Big)&=\lim_{n\to+\infty}d_\mu\Big(T(\omega,x_n(\omega)),x_n(\omega)\Big)\\&=\lim_{n\to+\infty}d_\mu\Big(x_{n+1}(\omega),x_n(\omega)\Big)\\&=d_\mu\Big(x(\omega),x(\omega)\Big)\\&=0\end{aligned}$$

Which shows that $x(\omega)$ is fixed point.

For the uniqueness, let $\omega\in\Omega$, if $x(\omega)$ and $y(\omega)$ be tow distinct fixed points of T, that is $T(\omega,x(\omega))=x(\omega)\neq y(\omega)=T(\omega,y(\omega))$.

Therefore, $d_\mu\Big(T(\omega,x(\omega)),T(\omega,y(\omega))\Big)=d_\mu\Big(x(\omega),y(\omega)\Big)$.

Then, we get

$$\begin{aligned} F\Big(d_\mu\Big(x(\omega),y(\omega)\Big)\Big) &= F\Big(d_\mu\Big(T(\omega,x(\omega)),T(\omega,y(\omega))\Big)\Big) \\ &< \tau(\omega) - F\Big(d_\mu\Big(T(\omega,x(\omega)),T(\omega,y(\omega))\Big)\Big) \\ &\leq F\Big(d_\mu\Big(x(\omega),y(\omega)\Big)\Big) \end{aligned}$$

Which is contradiction. Therfore, the random fixed points is unique. □

Example 4.1. *Let* $\Omega = [0,1]$ *and* $\sum$ *be the sigma algebra of Lebesgue's measurable subset of* $[0,1]$.
Take $X = \mathbb{R}$ *and* $d_\mu\Big(x(\omega),y(\omega)\Big) = \Big|x(\omega)-y(\omega)\Big|\Big(\Big|x(\omega)-y(\omega)\Big|+2\Big)$ *for all* $x,y \in X$ *and* $\omega \in \Omega$ *with* $\mu = 1$. *We endow* X *by the graph* G *defined by* $V(G) = X$ *and* $E(G) = X \times X$. *Hence* (X,d_μ,G) *be a G-regular complete suprametric space.*
Define a random mapping $T : \Omega \times X \to X$ *as* $T\Big(\omega,x(\omega)\Big) = \begin{cases} \dfrac{w-x}{2} & ;\ x > 0 \\ \dfrac{w}{2} & ;\ x \leq 0 \end{cases}$ *Suppose is there exists a random variable* $x_0 : \Omega \to X$ *with* $\Big(x_0(\omega),T(\omega,x_0(\omega)\Big) \in E(G)$, *for all* $\omega \in \Omega$. *Then, for* $F(x) = \ln(x)$ *and* $\tau(\omega) = \ln(2)$, *then* T *has a unique random fixed point.*

Theorem 4.2. *Let* $(\Omega,\sum)$ *be a mesurable space,* (X,d_μ,G) *be a separable random G-regular complete suprametric space* X *and* $T : \Omega \times X \to X$ *be a mapping satisfying.*

i) *For all* $x,y \in X$, $\omega \in \Omega$ *and* $\tau(\omega)$, *we have*

$$\tau(\omega) + F\Big(d\big(T(\omega,x(\omega)),T(\omega,y(\omega))\big)\Big) \leq F\Big(N(x(\omega),y(\omega))\Big) \tag{4.7}$$

where $F \in \mathscr{F}$ *and*

$$N(x(\omega),y(\omega)) = \max\Big\{d_\mu\Big(x(\omega),y(\omega)\Big), d_\mu\Big(x(\omega),T(\omega,x(\omega))\Big), d_\mu\Big(y(\omega),T(\omega,y(\omega))\Big)\Big\}$$

ii) *There exists a random variable* $x_0 : \Omega \to X$ *with*

$$\Big(x_0(\omega),T(\omega,x_0(\omega)\Big) \in E(G),\ \textit{for all}\ \omega \in \Omega$$

Then T *has a unique random fixed point.*

Proof. We define $x_{n+1}(\omega) = T(\omega, x_n(\omega))$ for each $\omega \in \Omega$ and for all $n \in \mathbb{N} \cup \{0\}$, where $x_0(\omega) = x(\omega)$. Assuming that, $N(x(\omega), y(\omega)) = 0$ this means that $x(\omega) = y(\omega)$ is a fixed point of T.

Since, $\Big(x_0(\omega), T(\omega, x_0(\omega)\Big) \in E(G)$, for all $\omega \in \Omega$, ≥ 0 and by *ii*), then
$\Big(x_n(\omega), x_{n+1}(\omega)\Big) \in E(G)$ for all $\omega \in \Omega$ and $n \geq 0$.

Suppose that $x_n(\omega) \neq x_{n+1}(\omega)$ for all $n \in \mathbb{N} \cup \{0\}$. by (4.7), we get

$$\begin{aligned} F\Bigg(d_\mu\Big(x_n(\omega), x_{n+1}(\omega)\Big)\Bigg) &= F\Bigg(d_\mu\Big(T(\omega, x_n(\omega)), T(\omega, x_{n+1}(\omega))\Big)\Bigg) - \tau(\omega) \\ &\leq F\Big(N(x_{n-1}(\omega), x_n(\omega))\Big) - \tau(\omega) \end{aligned} \tag{4.8}$$

We have

$$\begin{aligned} &N\Big(x_{n-1}(\omega), x_n(\omega)\Big) \\ &= \max\Big\{d_\mu\Big(x_{n-1}(\omega), x_n(\omega)\Big), d_\mu\Big(x_{n-1}(\omega), T(\omega, x_{n-1}(\omega))\Big), d_\mu\Big(x_n(\omega), T(\omega, x_n(\omega))\Big)\Big\} \\ &= \max\Big\{d_\mu\Big(x_{n-1}(\omega), x_n(\omega)\Big), d_\mu\Big(x_{n-1}(\omega), x_n(\omega)\Big), d_\mu\Big(x_n(\omega), x_{n+1}(\omega)\Big)\Big\} \\ &= \max\Big\{d_\mu\Big(x_{n-1}(\omega), x_n(\omega)\Big), d_\mu\Big(x_n(\omega), x_{n+1}(\omega)\Big)\Big\} \end{aligned}$$

If $N\Big(x_{n-1}(\omega), x_n(\omega)\Big) = d_\mu\Big(x_n(\omega), x_{n+1}(\omega)\Big)$, then by (4.8)

$$F\Bigg(d_\mu\Big(x_n(\omega), x_{n+1}(\omega)\Big)\Bigg) \leq F\Bigg(d_\mu\Big(x_n(\omega), x_{n+1}(\omega)\Big)\Bigg) - \tau(\omega)$$

which is a contraduction. Then $N\Big(x_{n-1}(\omega), x_n(\omega)\Big) = d_\mu\Big(x_{n-1}(\omega), x_n(\omega)\Big)$, then by (4.8)

$$F\Bigg(d_\mu\Big(x_n(\omega), x_{n+1}(\omega)\Big)\Bigg) \leq F\Bigg(d_\mu\Big(x_{n-1}(\omega), x_n(\omega)\Big)\Bigg) - \tau(\omega) \tag{4.9}$$

In the same manner, we have

$$F\Bigg(d_\mu\Big(x_{n-1}(\omega), x_n(\omega)\Big)\Bigg) \leq F\Bigg(d_\mu\Big(x_{n-2}(\omega), x_{n-1}(\omega)\Big)\Bigg) - \tau(\omega) \tag{4.10}$$

From (4.9) and (4.10), we obtain

$$F\Bigg(d_\mu\Big(x_n(\omega), x_{n+1}(\omega)\Big)\Bigg) \leq F\Bigg(d_\mu\Big(x_{n-2}(\omega), x_{n-1}(\omega)\Big)\Bigg) - 2\tau(\omega)$$

Continue this procedure till we gain

$$F\Bigg(d_\mu\Big(x_n(\omega), x_{n+1}(\omega)\Big)\Bigg) \leq F\Bigg(d_\mu\Big(x_0(\omega), x_1(\omega)\Big)\Bigg) - n\tau(\omega) \tag{4.11}$$

Thus

$$\lim_{n\to+\infty} F\left(d_\mu\left(x_n(\omega),x_{n+1}(\omega)\right)\right)=-\infty$$

Then from (ii) of definition 4.1, we have

$$\lim_{n\to+\infty} d_\mu\left(x_n(\omega),x_{n+1}(\omega)\right)=0 \tag{4.12}$$

and from (iii) of definition4.1, there exists $k\in(0,1)$ such that

$$\lim_{n\to+\infty}\left(d_\mu\left(x_n(\omega),x_{n+1}(\omega)\right)\right)^k F\left(d_\mu\left(x_n(\omega),x_{n+1}(\omega)\right)\right)=0 \tag{4.13}$$

Using (4.11), we get

$$\begin{aligned}\left(d_\mu\left(x_n(\omega),x_{n+1}(\omega)\right)\right)^k F\left(d_\mu\left(x_n(\omega),x_{n+1}(\omega)\right)\right)-F\left(d_\mu\left(x_0(\omega),x_1(\omega)\right)\right)\\ &\leq-\left(d_\mu\left(x_n(\omega),x_{n+1}(\omega)\right)\right)^k n\tau(\omega)\\ &\leq 0\end{aligned}$$

by (4.12) and (4.13), we obtain

$$\lim_{n\to+\infty} n\left(d_\mu\left(x_n(\omega),x_{n+1}(\omega)\right)\right)^k=0$$

Then for all $\omega\in\Omega$, there exists $N\in\mathbb{N}$ such that $n\left(d_\mu\left(x_n(\omega),x_{n+1}(\omega)\right)\right)^k\leq 1$ for all $n\geq N$. Hence

$$d_\mu\left(x_n(\omega),x_{n+1}(\omega)\right)\leq\frac{1}{\sqrt[k]{n}} \tag{4.14}$$

We shall now demonstrate the Cauchy nature of the sequence $(x_n(\omega))_{n\in\mathbb{N}}$.

We denote $d_\mu\left(x_n(\omega),x_{n+1}(\omega)\right):=d_n$, we follow the same approach as in the theorem 4.1, and obtain for all $\omega\in\Omega$, $n\in\mathbb{N}$ and $k\in\mathbb{N}\setminus\{0,1\}$

$$d_\mu\left(x_n(\omega),x_{n+k}(\omega)\right)\leq d_n+\sum_{i=1}^{k-1} d_{n+i}\prod_{j=0}^{i-1}\left(1+\mu d_{n+j}\right)$$

Using (4.14), we have

$$d_\mu\left(x_n(\omega),x_{n+k}(\omega)\right)\leq\frac{1}{\sqrt[k]{n}}+\sum_{i=1}^{k-1}\frac{1}{\sqrt[k]{n+i}}\prod_{j=0}^{i-1}\left(1+\mu\frac{1}{\sqrt[k]{n+j}}\right)$$

Thus

$$d_\mu\Big(x_n(\omega), x_{n+k}(\omega)\Big) \leq \frac{1}{\sqrt[k]{n}} + \sum_{i=1}^{k-1} \frac{1}{\sqrt[k]{i}} \prod_{j=0}^{i-1}\Big(1+\mu\frac{1}{\sqrt[k]{j}}\Big)$$

Set $S_i = \frac{1}{\sqrt[k]{i}} \prod_{j=0}^{i-1}\Big(1+\mu\frac{1}{\sqrt[k]{j}}\Big)$. Then

$$\frac{S_{i+1}}{S_i} = \frac{\frac{1}{\sqrt[k]{i+1}} \prod_{j=0}^{i}\Big(1+\mu\frac{1}{\sqrt[k]{j}}\Big)}{\frac{1}{\sqrt[k]{i}} \prod_{j=0}^{i-1}\Big(1+\mu\frac{1}{\sqrt[k]{j}}\Big)} = \frac{\sqrt[k]{i}}{\sqrt[k]{i+1}}\Big(1+\mu\frac{1}{\sqrt[k]{i}}\Big)$$

Since $k \in (0,1)$, then

$$\lim_{n\to+\infty}\left|\frac{S_{i+1}}{S_i}\right| < 1$$

Hence, the seris $\sum_{i=0}^{+\infty} S_i$ is converges.

we deduce that $\lim_{n,k\to+\infty} d_\mu\Big(x_n(\omega), x_{n+k}(\omega)\Big) = 0$ for all $\omega \in \Omega$.

Thus $(x_n(\omega))_{n\in\mathbb{N}}$ is a Cauchy seqeunce, for every $\omega \in \Omega$. As (X, d_μ, G) is complete space, there exists $x : \Omega \longrightarrow X$ such that $x(\omega) = \lim_{n\to+\infty} x_n(\omega)$, for all $\omega \in \Omega$. And since (X, d_μ, G) is random G-regular, we get $(x_n(\omega), x(\omega)) \in E(G)$ for all $\omega \in \Omega$ and $n \in \mathbb{N}$.

Then

$$\tau(\omega) + F\Big(d\big(T(\omega, x_n(\omega)), T(\omega, x(\omega))\big)\Big) \leq F\Big(N(x_n(\omega), x(\omega))\Big) \tag{4.15}$$

where

$$N\Big(x_n(\omega), x(\omega)\Big) = \max\Big\{d_\mu\Big(x_n(\omega), x(\omega)\Big), d_\mu\Big(x_n(\omega), x_{n+1}(\omega)\Big), d_\mu\Big(x(\omega), T(\omega, x(\omega))\Big)\Big\}$$

Taking $n \to +\infty$,we get

$$\lim_{n\to+\infty} N\Big(x_n(\omega), x(\omega)\Big) = d_\mu\Big(x(\omega), T(\omega, x(\omega))\Big) \tag{4.16}$$

Using (4.15), we obtain

$$\begin{aligned} F\Big(d\big(x_{n+1}(\omega), T(\omega, x(\omega))\big)\Big) &\leq F\Big(N(x_n(\omega), x(\omega))\Big) - \tau(\omega) \\ &< F\Big(N(x_n(\omega), x(\omega))\Big) \end{aligned}$$

Thus

$$d\Big(x_{n+1}(\omega),T(\omega,x(\omega))\Big) < N\Big(x_n(\omega),x(\omega)\Big)$$

Taking $n \to +\infty$,we get

$$d\Big(x(\omega),T(\omega,x(\omega))\Big) < N\Big(x(\omega),x(\omega)\Big)$$

which is a contradiction, then $d\Big(x(\omega),T(\omega,x(\omega))\Big) = 0$. Hence T has a fixed point.
For the uniqueness, let $\omega \in \Omega$, if $x(\omega)$ and $y(\omega)$ be tow distinct fixed points of T, that is $T(\omega,x(\omega)) = x(\omega) \neq y(\omega) = T(\omega,y(\omega))$.
Therefore, $d_\mu\Big(T(\omega,x(\omega)),T(\omega,y(\omega))\Big) = d_\mu\Big(x(\omega),y(\omega)\Big)$.
Then, we get

$$\begin{aligned} F\Big(d_\mu\Big(x(\omega),y(\omega)\Big)\Big) &= F\Big(d_\mu\Big(T(\omega,x(\omega)),T(\omega,y(\omega))\Big)\Big) \\ &\leq F\Big(N\Big(x(\omega),y(\omega)\Big)\Big) - \tau(\omega) \end{aligned}$$

Since

$$\begin{aligned} N\Big(x(\omega),y(\omega)\Big) &= \max\Big\{d_\mu\Big(x(\omega),y(\omega)\Big), d_\mu\Big(x(\omega),T(\omega,x(\omega))\Big), d_\mu\Big(y(\omega),T(\omega,y(\omega))\Big)\Big\} \\ &= d_\mu\Big(x(\omega),y(\omega)\Big) \end{aligned}$$

Then

$$F\Big(d_\mu\Big(x(\omega),y(\omega)\Big)\Big) \leq F\Big(d_\mu\Big(x(\omega),y(\omega)\Big)\Big) - \tau(\omega)$$

Which is contradiction. Therfore, the random fixed points is unique. □

4.1. Conclusion. In this paper, we introduced the notion of random graphical $F-$contraction in a suprametric space. Then, we established a random fixed point result for graphical $F-$contraction applications on complete separable suprametric spaces equipped with a complet probability measure and a graph.

CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests.

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