Available online at http://scik.org
Advances in Fixed Point Theory, 3 (2013), No. 2, 428-438
ISSN: 1927-6303

CONVERGENCE THEOREMS OF IMPLICIT ITERATIVE PROCESSES WITH ERRORS FOR A FINITE FAMILY OF PSEUDOCONTRACTIVE MAPPINGS IN BANACH SPACES

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Abstract. In this paper, an implicit iterative process with mixed errors is considered. Weak and strong convergence theorems of common fixed points of a finite family of pseudocontractions are established in a real Banach space.

Keywords: pseudocontraction ; fixed point; implicit iterative process with errors.

2000 AMS Subject Classification: 47H09; 47H10

1. Introduction and Preliminaries

Throughout this paper, we always assume that E is a real Banach space and K is a nonempty subset of E. Let J denote the normalized duality mapping from E into 2^{E^*}given by

$$J(x) = \{f \in E^* : \langle x, f\rangle = \|x\|^2 = \|f\|^2,\ x \in E\}, \tag{1.1}$$

where E^* denotes the dual space of E and $\langle \cdot, \cdot\rangle$ denotes the generalized duality pairing. In the sequel, we denote a single-valued normalized duality mapping by j, we denote the

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Received April 9, 2013

fixed point of the mapping T by $F(T)$, $\rightharpoonup$ and $\to$ denote weak and strong convergence, respectively.

Recall that T is said to be nonexpansive if

$$\|Tx - Ty\| \leq \|x - y\|, \quad \forall x, y \in K. \tag{1.2}$$

T is said to be strictly pseudocontractive if there exists a constant $\kappa > 0$ and $j(x-y) \in J(x-y)$ such that

$$\langle Tx - Ty, j(x-y)\rangle \leq \|x-y\|^2 - \kappa\|x - y - (Tx - Ty)\|^2, \quad \forall x, y \in K. \tag{1.3}$$

T is said to be pseudocontraction if there exists $j(x-y) \in J(x-y)$ such that

$$\langle Tx - Ty, j(x-y)\rangle \leq \|x-y\|^2, \quad \forall x, y \in K. \tag{1.4}$$

It is well known that [1] (1.4) is equivalent to the following:

$$\|x - y\| \leq \|x - y + s[(I-T)x - (I-T)y]\|, \ \forall s > 0. \tag{1.5}$$

T is said to be uniformly L-lipschitz if there exists a positive constant L such that

$$\|T^n x - T^n y\| \leq L\|x - y\|, \quad \forall x, y \in K, \, n \geq 1. \tag{1.6}$$

In 2001, Xu and Ori [2], in the framework of Hilbert spaces, introduced the following implicit iteration process for a finite family of nonexpansive mappings $\{T_1, T_2, \cdots, T_N\}$ with $\{\alpha_n\}$ a real sequence in $(0,1)$ and an initial point $x_0 \in C$:

$$x_1 = \alpha_1 x_0 + (1-\alpha_1)T_1 x_1,$$

$$x_2 = \alpha_2 x_1 + (1-\alpha_2)T_2 x_2,$$

$$\cdots$$

$$x_N = \alpha_N x_{N-1} + (1-\alpha_N)T_N x_N,$$

$$x_{N+1} = \alpha_{N+1} x_N + (1-\alpha_{N+1})T_1 x_{N+1},$$

$$\cdots$$

which can written in the following compact form:

$$x_n = \alpha_n x_{n-1} + (1-\alpha_n)T_n x_n, \quad \forall n \geq 1, \tag{1.7}$$

where $T_n = T_{n(modN)}$ (here the mod N takes values in $\{1, 2, \cdots, N\}$).

They obtained the following weak convergence theorem.

Theorem XO. *Let H be a real Hilbert space, C a nonempty closed convex subset of H, and $T_i : C \to C$ be a finite family of nonexpansive mappings such that $F = \cap_{i=1}^N F(T_i) \neq \emptyset$. Let $\{x_n\}$ be defined by (1.7). If $\{\alpha_n\}$ is chosen so that $\alpha_n \to 0$ as $n \to \infty$, then $\{x_n\}$ converges weakly to a common fixed point of the family of $\{T_i\}_{i=1}^N$.*

Subsequently, fixed point problems based on implicit iterative processes have been considered by many authors, see, for example, [3-9]. In 2004, Osilike [6] reconsidered the implicit iterative process (1.7) for a finite family of strictly pseudocontractive mappings. To be more precise, he proved the following theorem.

Theorem O. *Let H be a real Hilbert space and let C be a nonempty closed convex subset of H. Let $\{T_i\}_{i=1}^N$ be N strictly pseudocontractive self-maps of C such that $F = \cap_{i=1}^N F(T_i) \neq \emptyset$. Let $x_0 \in C$ and let $\{\alpha_n\}$ be a sequence in $(0, 1)$ such that $\alpha_n \to 0$ as $n \to \infty$, Then the sequence $\{x_n\}$ defined by (1.7) converges weakly to a common fixed point of the mappings $\{T_i\}_{i=1}^N$.*

In 2008, Hao [5]considered the following implicit iterative process with mixed errors for a finite family of pseudocontractive mappings:

$$x_0 \in K,\ x_n = \alpha_n x_{n-1} + \beta_n T_n x_n + \gamma_n u_n, \quad \forall n \geq 1, \tag{1.8}$$

where $T_n = T_{n(modN)}$ (here the mod N takes values in $\{1, 2, \cdots, N\}$). $\{\alpha_n\}$,$\{\beta_n\}$and$\{\gamma_n\}$ are three sequences in $[0, 1]$ such that $\alpha_n + \beta_n + \gamma_n = 1$ and $\{u_n\}$ is a bounded sequence in K. Weak and strong convergence theorem of the implicit iterative process with mixed errors (1.8) for a finite family of pseudocontractions mappings in Banach spaces was established; see [5] for more details.

Very recently, Qin, Su and Shang [7] considered the following implicit iterative process for a family of asymptotically strict pseudocontractions:

$$\begin{aligned} &x_1 = \alpha_1 x_0 + (1-\alpha_1)T_1 x_1,\\ &x_2 = \alpha_2 x_1 + (1-\alpha_2)T_2 x_2,\\ &\vdots\\ &x_N = \alpha_N x_{N-1} + (1-\alpha_N)T_N x_N,\\ &x_{N+1} = \alpha_{N+1} x_N + (1-\alpha_{N+1})T_1^2 x_{N+1},\\ &\vdots\\ &x_{2N} = \alpha_{2N} x_{2N-1} + (1-\alpha_{2N})T_N^2 x_{2N},\\ &x_{2N+1} = \alpha_{2N+1} x_{2N} + (1-\alpha_{2N+1})T_1^3 x_{2N+1},\\ &\vdots \end{aligned}$$

Since for each $n \geq 1$, it can be written as $n = (h-1)N+i$, where $i = i(n) \in \{1, 2, \ldots, N\}$, $h = h(n) \geq 1$ is a positive integer and $h(n) \to \infty$ as $n \to \infty$. Hence the above table can be rewritten in the following compact form:

$$x_n = \alpha_n x_{n-1} + (1-\alpha_n)T_{i(n)}^{h(n)} x_n, \quad \forall n \geq 1. \tag{1.9}$$

A weak convergence theorem of the implicit iterative process (1.9) for a finite family of asymptotically strict pseudocontractions was established; see [7] for more details.

In this paper, motivated by the above results, we consider an implicit iterative process with mixed errors for a finite family of pseudocontractions mappings in Banach spaces. To be more precise, we consider the following implicit iterative process:

$$x_0 \in K,\ x_n = \alpha_n x_{n-1} + \beta_n T_{i(n)}^{h(n)} x_n + \gamma_n u_n, \quad \forall n \geq 1, \tag{1.10}$$

where $T_n = T_{n(modN)}$ (here the mod N takes values in $\{1, 2, \cdots, N\}$). $\{\alpha_n\}$,$\{\beta_n\}$ and$\{\gamma_n\}$ are three sequences in $[0,1]$ such that $\alpha_n + \beta_n + \gamma_n = 1$ and $\{u_n\}$ is a bounded sequence in K.

In order to prove our main results, we need the following conceptions and lemmas.

Recall that a space E is said to satisfy Opial's condition [10] if, for each sequence $\{x_n\}$ in E, the convergence $x_n \to x$ weakly implies that

$$\liminf_{n\to\infty} \|x_n - x\| < \liminf_{n\to\infty} \|x_n - y\|, \quad \forall y \in E\,(y \neq x).$$

Recall that a mapping $T: K \to K$ is semicompact if any sequence $\{x_n\}$ in K satisfying $\lim_{n\to\infty} \|x_n - Tx_n\| = 0$ has a convergent subsequence.

Recall that a mapping $T: K \to K$ is demiclosed at the origin if for each sequence $\{x_n\}$ in K, the convergence $x_n \to x_0$ weakly and $Tx_n \to 0$ strongly imply that $Tx_0 = 0$.

Lemma 1.1 [12] *Let $\{a_n\}$, $\{b_n\}$ and $\{c_n\}$ be three nonnegative sequences satisfying the following condition:*

$$a_{n+1} \leq (1 + b_n)a_n + c_n, \quad \forall n \geq n_0,$$

where n_0 is some nonnegative integer, $\sum_{n=1}^{\infty} b_n < \infty$ and $\sum_{n=1}^{\infty} c_n < \infty$. Then the limit $\lim_{n\to\infty} a_n$ exists.

Lemma 1.2 [8] *Let E be a uniformly convex Banach space, K a nonempty closed convex subset of E and $T: K \to K$ a continuous pseudocontractive mapping. Then the mapping $I - T$ is demiclosed at zero.*

Lemma 1.3 [13] *Let E be a uniformly convex Banach space and $0 < p \leq t_n \leq q < 1$, for all $n \in N$.Suppose further that $\{x_n\}$ and $\{y_n\}$ are sequences of E such that*

$$\limsup_{n\to\infty} \|x_n\| \leq r, \limsup_{n\to\infty} \|y_n\| \leq r, \lim_{n\to\infty} \|t_n x_n + (1 - t_n)y_n\| = r,$$

hold for some $r \geq 0$, then $\lim_{n\to\infty} \|x_n - y_n\| = 0$

2. Main results

Theorem 2.1. *Let E be a uniformly convex Banach space satisfying Opial's condition and K a nonempty closed convex subset of E, $T_i: K \to K$ be an uniformly L_i -Lipschitz pseudocontractive mapping with $F = \cap_{i=1}^{N} F(T_i) \neq \emptyset$, $\{u_n\}$ be a bounded sequence in K. Let $\{x_n\}_{n=0}^{\infty}$ be a sequence generated in (1.10). Assume that the control sequence $\{\alpha_n\}$, $\{\beta_n\}$ and $\{\gamma_n\}$ in $[0, 1]$ satisfy the following restrictions*

(a) $\beta_n L < 1$, *where* $L = \max\{L_i : 1 \le i \le N\}$, $\forall n \ge 1$;

(b) $\alpha_n + \beta_n + \gamma_n = 1$, $\forall n \ge 1$;

(c) $\sum_{n=1}^{\infty} \gamma_n < \infty$;

(d) $0 < a \le \alpha_n \le b < 1$, $\forall n \ge 1$,

Then $\{x_n\}$ *converges weakly to some point in* F.

Proof. First, we show that the sequence $\{x_n\}$ generated in the implicit iterative process (1.10) is well defined. Define mappings $R_n : K \to K$ by

$$R_n(x) = \alpha_n x_{n-1} + \beta_n T_{i(n)}^{h(n)} x + \gamma_n u_n, \quad \forall x \in K, n \ge 1.$$

Notice that

$$\begin{aligned} \|R_n(x) - R_n(y)\| &= \|\big(\alpha_n x_{n-1} + \beta_n T_{i(n)}^{h(n)} x + \gamma_n u_n\big) - \big(\alpha_n x_{n-1} + \beta_n T_{i(n)}^{h(n)} y + \gamma_n u_n\big)\| \\ &\le \beta_n L \|x - y\|, \quad \forall x, y \in K. \end{aligned}$$

From the restriction (a), we see that R_n is a contraction for each $n \ge 1$. By Banach contraction principle, we see that there exists a unique fixed point $x_n \in K$ such that

$$x_n = \alpha_n x_{n-1} + \beta_n T_{i(n)}^{h(n)} x_n + \gamma_n u_n, \quad \forall n \ge 1.$$

This shows that the implicit iterative process (1.10) is well defined for uniformly Lipschitz pseudocontractions.

Second, we show $\lim_{n\to\infty} \|x_n - p\|$ exists, for any given $p \in F$, from the restriction (b), we have

$$\begin{aligned} \|x_n - p\|^2 &= \langle \alpha_n x_{n-1} + \beta_n T_{i(n)}^{h(n)} x_n + \gamma_n u_n - p, j(x_n - p)\rangle \\ &= \alpha_n \langle x_{n-1} - p, j(x_n - p)\rangle + \beta_n \langle T_{i(n)}^{h(n)} x_n - p, j(x_n - p)\rangle \\ &\quad + \gamma_n \langle u_n - p, j(x_n - p)\rangle \\ &\le \alpha_n \|x_{n-1} - p\| \|x_n - p\| + \beta_n \|x_n - p\|^2 + \gamma_n \|u_n - p\| \|x_n - p\|. \end{aligned} \tag{2.1}$$

Simplifying the above inequality, we have

$$\|x_n - p\|^2 \le \frac{\alpha_n}{\alpha_n + \gamma_n} \|x_{n-1} - p\| \|x_n - p\| + \frac{\gamma_n}{\alpha_n + \gamma_n} \|u_n - p\| \|x_n - p\| \tag{2.2}$$

If $\|x_n - p\| = 0$, then the result is apparent, letting $\|x_n - p\| > 0$, we obtain

$$\begin{aligned}\|x_n - p\| &\le \frac{\alpha_n}{\alpha_n + \gamma_n}\|x_{n-1} - p\| + \frac{\gamma_n}{\alpha_n + \gamma_n}\|u_n - p\| \\ &\le \|x_{n-1} - p\| + \gamma_n M.\end{aligned} \tag{2.3}$$

where M is an appropriate constant such that $M \ge \sup_{n\ge 1}\|u_n - p\|/a$. Noticing the condition (c)and lemma1.1 to (2.3), we have $\lim_{n\to\infty}\|x_n - p\|$ exists. we assume that

$$\lim_{n\to\infty}\|x_n - p\| = d. \tag{2.4}$$

On the other hand, from (1.5)and(1.10), we see

$$\begin{aligned}\|x_n - p\| &\le \|x_n - p + \frac{1-\alpha_n}{2\alpha_n}(x_n - T_{i(n)}^{h(n)}x_n)\| \\ &= \|x_n - p + \frac{1-\alpha_n}{2\alpha_n}[\alpha_n(x_{n-1} - T_{i(n)}^{h(n)}x_n) + \gamma_n(u_n - T_{i(n)}^{h(n)}x_n)\| \\ &= \|x_n - p + \frac{1-\alpha_n}{2}(x_{n-1} - T_{i(n)}^{h(n)}x_n) + \frac{\gamma_n(1-\alpha_n)}{2\alpha_n}(u_n - T_{i(n)}^{h(n)}x_n)\| \\ &= \|\frac{x_{n-1}}{2} + x_n - p + \frac{1}{2}[\alpha_n x_{n-1} + (1-\alpha_n)T_{i(n)}^{h(n)}x_n) + \gamma_n(u_n - T_{i(n)}^{h(n)}x_n) \\ &\quad + \frac{\gamma_n}{2}(u_n - T_{i(n)}^{h(n)}x_n) + \frac{\gamma_n(1-\alpha_n)}{2\alpha_n}(u_n - T_{i(n)}^{h(n)}x_n)\| \\ &= \|\frac{1}{2}(x_{n-1} - p) + \frac{1}{2}(x_n - p) + \frac{\gamma_n}{2\alpha_n}(u_n - T_{i(n)}^{h(n)}x_n)\| \\ &\le \|\frac{1}{2}(x_{n-1} - p) + \frac{1}{2}(x_n - p)\| + \frac{\gamma_n}{2\alpha_n}\|(u_n - T_{i(n)}^{h(n)}x_n)\|.\end{aligned} \tag{2.5}$$

Noticing that the condition (c) and (d)and (2.4), we obtain

$$\liminf_{n\to\infty}\|\frac{1}{2}(x_{n-1} - p) + \frac{1}{2}(x_n - p)\| \ge d. \tag{2.6}$$

On the other hand, we have

$$\limsup_{n\to\infty}\|\frac{1}{2}(x_{n-1} - p) + \frac{1}{2}(x_n - p)\| \le \limsup_{n\to\infty}(\frac{1}{2}\|x_{n-1} - p\| + \frac{1}{2}\|x_n - p\|) \le d. \tag{2.7}$$

Combing (2.6)with (2.7),we arrive at

$$\lim_{n\to\infty}\|\frac{1}{2}(x_{n-1} - p) + \frac{1}{2}(x_n - p)\| = d. \tag{2.8}$$

By using lemma1.3, we get

$$\lim_{n\to\infty}\|x_{n-1} - x_n\| = 0. \tag{2.9}$$

That is,

$$\lim_{n\to\infty} \|x_{n+i} - x_n\| = 0, \forall i \in 1, 2, \cdots, N. \tag{2.10}$$

It follows from (1.10) that

$$\begin{aligned} \|x_{n-1} - T_{i(n)}^{h(n)} x_n\| &= \frac{1}{1-\alpha_n}\|x_{n-1} - x_n + \gamma_n(u_n - T_{i(n)}^{h(n)} x_n)\| \\ &\le \frac{1}{1-\alpha_n}\|x_{n-1} - x_n\| + \frac{\gamma_n}{1-\alpha_n}\|u_n - T_{i(n)}^{h(n)} x_n)\|. \end{aligned} \tag{2.11}$$

From the condition (c) and (d), we obtain

$$\lim_{n\to\infty} \|x_{n-1} - T_{i(n)}^{h(n)} x_n\| = 0. \tag{2.12}$$

On the other hand, we have

$$\|x_n - T_{i(n)}^{h(n)} x_n\| \le \alpha_n\|x_{n-1} - T_{i(n)}^{h(n)} x_n\| + \gamma_n\|u_n - T_{i(n)}^{h(n)} x_n\|, \tag{2.13}$$

From the condition (c) and (2.12), we see

$$\lim_{n\to\infty} \|x_n - T_{i(n)}^{h(n)} x_n\| = 0. \tag{2.14}$$

Since for any positive integer $n > N$, it can be written as $n = (h(n)-1)N + i(n)$, where $i(n) \in \{1, 2, \cdots, N\}$. Observe that

$$\begin{aligned} \|x_n - T_n x_n\| &\le \|x_n - T_{i(n)}^{h(n)} x_n\| + \|T_{i(n)}^{h(n)} x_n - T_n x_n\| \\ &= \|x_n - T_{i(n)}^{h(n)} x_n\| + \|T_{i(n)}^{h(n)} x_n - T_{i(n)} x_n\| \\ &\le \|x_n - T_{i(n)}^{h(n)} x_n\| + L\|T_{i(n)}^{h(n)-1} x_n - x_n\| \\ &\le \|x_n - T_{i(n)}^{h(n)} x_n\| + L\Big(\|T_{i(n)}^{h(n)-1} x_n - T_{i(n-N)}^{h(n)-1} x_{n-N}\| \\ &\quad + \|T_{i(n-N)}^{h(n)-1} x_{n-N} - x_{(n-N)-1}\| + \|x_{(n-N)-1} - x_n\|\Big). \end{aligned} \tag{2.15}$$

Since for each $n > N$, $n = (n-N)(\text{mod } N)$, on the other hand, we obtain from $n = (h(n)-1)N + i(n)$ that $n - N = \big((h(n)-1)-1\big)N + i(n) = (h(n-N)-1)N + i(n-N)$. That is,

$$h(n-N) = h(n) - 1 \qquad \text{and} \qquad i(n-N) = i(n).$$

Notice that

$$\begin{aligned}\|T_{i(n)}^{h(n)-1}x_n - T_{i(n-N)}^{h(n)-1}x_{n-N}\| &= \|T_{i(n)}^{h(n)-1}x_n - T_{i(n)}^{h(n)-1}x_{n-N}\| \\ &\leq L\|x_n - x_{n-N}\|\end{aligned} \tag{2.16}$$

and

$$\|T_{i(n-N)}^{h(n)-1}x_{n-N} - x_{(n-N)-1}\| = \|T_{i(n-N)}^{h(n-N)}x_{n-N} - x_{(n-N)-1}\|. \tag{2.17}$$

Substituting (2.16) and (2.17) into (2.15), we arrive at

$$\begin{aligned}\|x_n - T_n x_n\| \leq \|x_n - T_{i(n)}^{h(n)}x_n\| + L\Big(&L\|x_n - x_{n-N}\| \\ &+ \|T_{i(n-N)}^{h(n-N)}x_{n-N} - x_{(n-N)-1}\| + \|x_{(n-N)-1} - x_n\|\Big).\end{aligned} \tag{2.18}$$

In view of (2.10), (2.12) and (2.14), we obtain from (2.18) that

$$\lim_{n\to\infty} \|x_n - T_n x_n\| = 0. \tag{2.19}$$

Notice that

$$\begin{aligned}\|x_n - T_{n+j}x_n\| &\leq \|x_n - x_{n+j}\| + \|x_{n+j} - T_{n+j}x_{n+j}\| + \|T_{n+j}x_{n+j} - T_{n+j}x_n\| \\ &\leq (1+L)\|x_n - x_{n+j}\| + \|x_{n+j} - T_{n+j}x_{n+j}\|, \quad \forall j \in \{1, 2, \ldots, N\}.\end{aligned}$$

It follows from (2.10) and (2.18) that

$$\lim_{n\to\infty} \|x_n - T_{n+j}x_n\| = 0, \quad \forall j \in \{1, 2, \ldots, N\}.$$

Note that any subsequence of a convergent number sequence converges to the same limit. It follows that

$$\lim_{n\to\infty} \|x_n - T_l x_n\| = 0, \quad \forall l \in \{1, 2, \ldots, N\}. \tag{2.20}$$

Since the sequence $\{x_n\}$ is bounded, we see that there exists a subsequence $\{x_{n_i}\} \subset \{x_n\}$ such that $\{x_{n_i}\}$ converges weakly to a point $x^* \in K$. In view of (2.20), we see from Lemma 1.2 that

$$x^* = T_l x^*, \quad \forall l \in \{1, 2, \ldots, N\}.$$

That is, $x^* \in F$. Next we show $\{x_n\}$ converges weakly to x^*. Supposing the contrary, we see that there exists some subsequence $\{x_{n_j}\}$ of $\{x_n\}$ such that $\{x_{n_j}\}$ converges weakly to $x^{**} \in K$, where $x^* \neq x^{**}$. Similarly, we can show $x^{**} \in F$. Notice that we have proved that $\lim_{n\to\infty}\|x_n - p\|$ exists for each $p \in F$. Assume that $\lim_{n\to\infty}\|x_n - x^*\| = d$ where d is a nonnegative number. By virtue of the Opial property of H, we see that

$$d = \liminf_{n_i\to\infty}\|x_{n_i} - x^*\| < \liminf_{n_i\to\infty}\|x_{n_i} - x^{**}\|$$
$$= \liminf_{n_j\to\infty}\|x_{n_j} - x^{**}\| < \liminf_{n_j\to\infty}\|x_{n_j} - x^*\| = d.$$

This is a contradiction. Hence $x^{**} = x^*$. This completes the proof.

Next, we give strong convergence theorems with the help of semicompactness.

Theorem 2.2. *Let E be a uniformly convex Banach space satisfying Opial's condition and K a nonempty closed convex subset of E, $T_i : K \to K$ be an uniformly L_i -Lipschitz pseudocontractive mapping with $F = \cap_{i=1}^N F(T_i) \neq \emptyset$, $\{u_n\}$ be a bounded sequence in K. Let $\{x_n\}_{n=0}^\infty$ be a sequence generated in (1.10). Assume that the control sequence $\{\alpha_n\}$, $\{\beta_n\}$ and $\{\gamma_n\}$ in $[0,1]$ satisfy the following restrictions*

(a) $\beta_n L < 1$, *where* $L = \max\{L_i : 1 \le i \le N\}$, $\forall n \ge 1$;

(b) $\alpha_n + \beta_n + \gamma_n = 1$, $\forall n \ge 1$;

(c) $\sum_{n=1}^\infty \gamma_n < \infty$;

(d) $0 < a \le \alpha_n \le b < 1$, $\forall n \ge 1$,

If one of $\{T_1, T_2, \ldots, T_N\}$ is semicompact, then $\{x_n\}$ converges strongly to some point in F.

Proof. Without loss of generality, we may assume that T_1 is semicompact. It follows from (2.20) that there exits a subsequence $\{x_{n_i}\}$ of $\{x_n\}$ converging strongly to $x \in K$. Next, we show that $x \in F$. Notice that

$$\|x - T_l x\| \le \|x - x_{n_i}\| + \|x_{n_i} - T_l x_{n_i}\| + \|T_l x_{n_i} - T_l x\|, \quad \forall l \in \{1, 2, \ldots, N\}.$$

Since T_l is uniformly L_i-Lipschitz continuous, we obtain from (2.20) that $x \in F$. Finally, we claim that $x_n \to x$ as $n \to \infty$. Since $\lim_{n\to\infty}\|x_n - p\|$ exits for each $p \in F$, we can obtain the desired conclusion easily. This completes the proof.

Acknowledgment

The work was supported by Natural Science Foundation of Zhejiang Province (Y6110270).

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