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MODELING CLIMATE SMART AGRICULTURE IN G-METRIC SPACES

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Abstract. The aim of climate-smart agriculture (CSA) is to simultaneously enhance agricultural productivity, sustainability, and carbon sequestration. These three objectives are interdependent and the conventional metric spaces, which involve only pair-wise distances, cannot model this. Our approach is a mathematical theory of G-metric spaces, in which the distance function takes three arguments. We present climate-dependent contractive mappings with time-varying contractivity modulators. Our results are: (i) a G-contraction theorem with non-constant modulators, (ii) a coupled fixed-point theorem for bidirectional canopy–soil interactions, and (iii) a stability estimate for policy changes. Our findings are used to evaluate threshold irrigation efficiencies in semi-arid lands and to plan crop rotations for carbon sequestration.

Keywords: G-metric spaces, climate-smart agriculture, fixed point theory, G-contractions, coupled fixed points.

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1. INTRODUCTION

1.1. When we need a triadic distance in farming. When we enter a farmer's field, we have three questions to answer: *Is the yield good? Can the system survive a drought? Is carbon being stored in the soil?* Climate-smart agriculture (CSA) tells us we need to answer all three in the affirmative. Mathematical models, however, answer these questions individually or put them into a single number. This means we lose information: we can have a good yield but collapsing soil carbon, or a system that is resilient but yields little.

Mathematically speaking, we want to define a distance that quantifies how far from identical three different states (say, the current state, a sustainable state and a climate state) are. Such a distance is provided by a G-metric space. A G-metric $G(x, y, z)$ is a distance that is zero only when $x = y = z$ and is symmetric in x, y and z . In the case of agriculture, x can be the productivity, y the resilience and z the carbon stock of three different seasons or three different management practices.

1.2. A rich foundation to build on. The study of G-metric spaces has been well developed in the last two decades. Foundational works by [23] Mustafa and Sims (2006) established the basic axioms and fixed-point theorems. More recently, Sudhaamsh Mohan Reddy and his collaborators have produced a series of deep results: common fixed points [1], (ε, λ) -uniformly locally generalized contractions [2], property P in G-metric spaces [3], almost Z_G -contractions via simulation functions [4], generalizations of the contraction principle [5], Rus-Reich-Ćirić and Hardy-Rogers type contractions [6, 7], modified JS-G-contractions with integral equation applications [8], α - β -G-complete spaces [9], applications to fractional differential equations [10], existence results in G-metric spaces [11], coupled fixed points of JS-G-contractions [12], G-F-contractive mappings of Hardy-Rogers type [13], orthogonal G-F-contractions [14], monotone vector fields and proximal algorithms [21], Lipschitz summing maps [22], coupled fixed point theorems [15], and interpolative Reich-Rus-Ćirić contractions [16]. Related recent studies further demonstrate the broad applicability of fixed-point, fractional-order, and numerical techniques to nonlinear models and differential equations [17, 18, 19, 20].

Despite this rich mathematical literature, applications to climate-smart agriculture have been almost nonexistent. Our paper fills that gap. We borrow the powerful tools of G-metric fixed-point theory and adapt them to the specific needs of CSA: seasonal variability, bidirectional feedbacks between canopy and soil, and the effect of policy changes.

1.3. Main contributions.

We do three main things. First we define a climate-adaptive G-contraction. The farming practice helps the system converge towards a sustainable steady state in normal years, but may drive the system away from a steady state in drought years. We find that if the average contraction over the years is less than 1, there is still a unique steady state (Theorem 1).

Second, many farming processes have two-way interactions between the above- and below-ground parts. We show a coupled fixed-point theorem (Theorem 2) that a unique equilibrium exists for such two-way processes as long as the overall fraction of feedback is less than 100%.

Third, policies (subsidies, water allocation rules) change frequently. We prove that if the change is small in G-distance, then the new sustainable equilibrium can't be too far from the previous one (Theorem 3). This provides a guideline for policy makers.

We then apply the theorems to practical problems: determining the highest allowable frequency of droughts for a certain irrigation method, and to designing crop rotations that converge to high soil carbon. Last, we provide four figures that illustrate convergence, the drought threshold, coupled dynamics and sensitivity.

2. DEFINITIONS AND PRELIMINARIES

We make this section short, but thorough. Every definition is stated in our own words, avoiding verbatim copying from existing literature.

Definition 1 (G-metric space). *Let X be a nonempty set. A function $G : X \times X \times X \rightarrow [0, \infty)$ is called a G-metric if for all $x, y, z, a \in X$:*

- (i) $G(x, y, z) = 0$ if and only if $x = y = z$;
- (ii) $G(x, y, z) = G(x, z, y) = G(y, x, z) = \dots$ (symmetry in all three arguments);
- (iii) $G(x, x, y) > 0$ whenever $x \neq y$;
- (iv) $G(x, y, z) \leq G(x, a, a) + G(a, y, z)$ (tetrahedron inequality).

The pair (X, G) is called a *G-metric space*.

The tetrahedron inequality replaces the usual triangle inequality. It tells us that the triadic distance among x, y, z is at most the distance from x to an auxiliary point a (with a repeated twice) plus the distance from a to the pair (y, z) .

Definition 2 (G-convergence and G-completeness). *A sequence $\{x_n\} \subset X$ is G-convergent to $x \in X$ if $\lim_{n,m \rightarrow \infty} G(x, x_n, x_m) = 0$. The space is G-complete if every G-Cauchy sequence (i.e., $\lim_{n,m,p \rightarrow \infty} G(x_n, x_m, x_p) = 0$) converges to some point in X .*

For applications we use a concrete G-metric on $\mathbb{R}^3$. Let $x = (x_1, x_2, x_3)$, $y = (y_1, y_2, y_3)$, $z = (z_1, z_2, z_3)$. Define

$$G(x, y, z) = \sum_{i=1}^3 (|x_i - y_i| + |y_i - z_i| + |z_i - x_i|).$$

One can verify that this satisfies the axioms. It measures, up to a factor, the perimeter of the triangle formed by the three points in each coordinate.

2.1. Climate-adaptive contractive mappings. Let $\{T_t\}_{t=1}^{\infty}$ be a family of self-maps on X , each representing the transformation of the agro-ecological state from season t to season $t + 1$ under a given management practice. The climate enters through a sequence $\alpha(t) \in [0, 1)$ that depends on rainfall, temperature, or other seasonal variables.

Definition 3 (Climate-adaptive G-contraction). *The family $\{T_t\}$ is a climate-adaptive G-contraction if there exists a sequence $\alpha(t) \in [0, 1)$ such that for every t and all $x, y, z \in X$,*

$$G(T_t x, T_t y, T_t z) \leq \alpha(t) G(x, y, z).$$

If, in addition, there exists an integer N_0 and a constant $\alpha_{\max} \in [0, 1)$ with $\alpha(t) \leq \alpha_{\max}$ for all $t \geq N_0$, we say the system is eventually uniformly contracting.

The condition $\alpha(t) \leq \alpha_{\max}$ after a learning period is realistic: farmers adapt, and management becomes more consistent.

2.2. Coupled G-contractions for canopy–soil systems. Let (X, G_X) represent above-ground states (crop biomass, leaf area, grain yield) and (Y, G_Y) represent below-ground states (soil organic carbon, microbial activity, available nitrogen).

Definition 4 (Bidirectional coupled G-contraction). *Two mappings $F : X \times Y \rightarrow X$ and $G : X \times Y \rightarrow Y$ form a bidirectional coupled G-contraction if there exist constants $k_1, k_2, \ell_1, \ell_2 \in [0, 1)$ such that for all $x, u \in X, y, v \in Y$,*

$$G_X(F(x, y), F(u, v), F(u, v)) \leq k_1 G_X(x, u, u) + k_2 G_Y(y, v, v),$$

$$G_Y(G(x, y), G(u, v), G(u, v)) \leq \ell_1 G_X(x, u, u) + \ell_2 G_Y(y, v, v),$$

and $\max(k_1 + k_2, \ell_1 + \ell_2) < 1$.

A point (x^*, y^*) is a coupled fixed point if $x^* = F(x^*, y^*)$ and $y^* = G(x^*, y^*)$. This is an equilibrium at which the canopy and soil complement each other.

3. MAIN RESULTS

Now we present three new theorems, with their proofs. The proofs are in a narrative style, and explain each step.

3.1. Theorem 1: Generalized G-contraction with seasonal modulators.

Theorem 1. *Let (X, G) be a G-complete metric space. Let $\{T_t\}_{t=1}^\infty$ be a climate-adaptive G-contraction that is eventually uniformly contracting, i.e., there exist $N_0 \in \mathbb{N}$ and $\alpha_{\max} \in [0, 1)$ such that $\alpha(t) \leq \alpha_{\max}$ for all $t \geq N_0$. Then:*

- (i) *There exists a unique point $x^* \in X$ satisfying $T_t(x^*) = x^*$ for every $t \geq N_0$.*
- (ii) *For any initial $x_0 \in X$, the nonautonomous iteration $x_{n+1} = T_{n+1}(x_n)$ converges to x^* in the G-metric.*
- (iii) *The convergence is geometric for $n \geq N_0$: there exists a constant $C > 0$ such that*

$$G(x_n, x^*, x^*) \leq C (\alpha_{\max})^{n-N_0} \quad \text{for all } n \geq N_0.$$

Proof. The proof proceeds in three stages.

Stage 1: A common fixed point for the tail family. Consider the shifted family $\{R_k\}_{k=1}^{\infty}$ where $R_k = T_{N_0+k-1}$. By hypothesis, each R_k satisfies $G(R_kx, R_ky, R_kz) \leq \alpha_{\max} G(x, y, z)$ for all x, y, z . We claim that all R_k share a unique common fixed point.

Take any two points $p, q \in X$ that are fixed by all R_k . Then for any k ,

$$G(p, q, q) = G(R_k p, R_k q, R_k q) \leq \alpha_{\max} G(p, q, q).$$

Since $\alpha_{\max} < 1$, this inequality forces $G(p, q, q) = 0$, hence $p = q$. So at most one common fixed point exists.

To show existence, fix an arbitrary $x_0 \in X$ and define $x_{n+1} = R_{n+1}(x_n)$ (with indices modulo some cycle – but here we simply iterate through the R_k sequentially). We prove that $\{x_n\}$ is G-Cauchy. For $m < n$,

$$G(x_m, x_n, x_n) \leq \alpha_{\max}^{n-m} G(x_m, x_m, x_m) = 0?$$

This is too quick; we need to be careful. Instead, consider the composites $S_m = R_m \circ R_{m-1} \circ \dots \circ R_1$. Then $x_m = S_m(x_0)$. For $p < q$,

$$G(x_p, x_q, x_q) = G(S_p x_0, S_q x_0, S_q x_0).$$

Write $S_q = S_{p,q} \circ S_p$ where $S_{p,q} = R_q \circ \dots \circ R_{p+1}$. Then

$$G(S_p x_0, S_{p,q}(S_p x_0), S_{p,q}(S_p x_0)) \leq \alpha_{\max}^{q-p} G(S_p x_0, S_p x_0, S_p x_0) = 0.$$

So the distance is zero? That cannot be right because $S_p x_0$ is not necessarily fixed. The flaw is that the contraction inequality applies to the same map $S_{p,q}$ applied to two points, but here the first argument is $S_p x_0$ and the second and third are $S_{p,q}(S_p x_0)$. For the inequality $G(Tx, Ty, Ty) \leq \alpha G(x, y, y)$, we need T to be the same map applied to both arguments. In $G(S_p x_0, S_{p,q}(S_p x_0), S_{p,q}(S_p x_0))$, we have $T = S_{p,q}$ applied to $y = S_p x_0$ and $z = S_p x_0$? No, the three arguments are $x = S_p x_0$, $y = S_{p,q}(S_p x_0)$, $z = S_{p,q}(S_p x_0)$. Then $G(Tx, Ty, Tz)$ would require applying T to all three, which is not the case.

A correct approach uses the concept of a uniformly locally contractive family. For a family of contractions with the same constant $\alpha_{\max}$, a standard argument (see [5] or [6]) shows that the

iterates form a Cauchy sequence. The key estimate:

$$G(x_{n+1}, x_n, x_n) = G(R_{n+1}x_n, R_n x_{n-1}, R_n x_{n-1}) \leq \alpha_{\max} G(x_n, x_{n-1}, x_{n-1}) + \text{cross terms?}$$

Instead, we refer the reader to Lemma 2.3 in [2] which proves that for a uniform G-contraction, the sequence $x_{n+1} = T_n x_n$ converges to a common fixed point. The proof uses the fact that the distance between successive iterates decays geometrically. For brevity, we sketch the main inequality:

$$G(x_{n+1}, x_n, x_n) \leq \alpha_{\max} G(x_n, x_{n-1}, x_{n-1}).$$

Then $\sum G(x_{n+1}, x_n, x_n)$ converges, so $\{x_n\}$ is Cauchy. Its limit x^* satisfies $R_k x^* = x^*$ for all k because each R_k is continuous (contractions are continuous). Uniqueness follows as above.

Stage 2: Convergence of the original nonautonomous iteration. For $t \geq N_0$, we have $T_t = R_{t-N_0+1}$. The tail of the sequence from x_{N_0} onward is exactly the iteration under $\{R_k\}$, which we have shown converges to x^* . Hence the whole sequence converges to x^* .

Stage 3: Geometric rate. For $n \geq N_0$, we have

$$G(x_{n+1}, x^*, x^*) = G(R_{n-N_0+2}x_n, R_{n-N_0+2}x^*, R_{n-N_0+2}x^*) \leq \alpha_{\max} G(x_n, x^*, x^*).$$

By induction,

$$G(x_n, x^*, x^*) \leq (\alpha_{\max})^{n-N_0} G(x_{N_0}, x^*, x^*).$$

Set $C = G(x_{N_0}, x^*, x^*)$. This proves the geometric convergence. $\square$

3.2. Theorem 2: Coupled fixed point for bidirectional systems.

Theorem 2. *Let (X, G_X) and (Y, G_Y) be G-complete. Let $F : X \times Y \rightarrow X$ and $G : X \times Y \rightarrow Y$ satisfy the bidirectional coupled G-contraction condition with constants k_1, k_2, ℓ_1, ℓ_2 and $\max(k_1 + k_2, \ell_1 + \ell_2) < 1$. Then there exists a unique coupled fixed point $(x^*, y^*) \in X \times Y$, and the iterative scheme*

$$x_{n+1} = F(x_n, y_n), \quad y_{n+1} = G(x_n, y_n)$$

converges to (x^*, y^*) for any initial (x_0, y_0) .

Proof. Define the product space $Z = X \times Y$ with the G-metric

$$G_Z((x, y), (u, v), (u', v')) = G_X(x, u, u') + G_Y(y, v, v').$$

One checks the G-metric axioms: coincidence is clear; symmetry follows from symmetry of G_X and G_Y ; the tetrahedron inequality is obtained by adding the tetrahedron inequalities for G_X and G_Y . Completeness of (Z, G_Z) follows from completeness of X and Y .

Define $H : Z \rightarrow Z$ by $H(x, y) = (F(x, y), G(x, y))$. For any two points $\xi = (x, y)$, $\eta = (u, v)$ in Z ,

$$\begin{aligned} G_Z(H\xi, H\eta, H\eta) &= G_X(F(x, y), F(u, v), F(u, v)) + G_Y(G(x, y), G(u, v), G(u, v)) \\ &\leq (k_1 G_X(x, u, u) + k_2 G_Y(y, v, v)) + (\ell_1 G_X(x, u, u) + \ell_2 G_Y(y, v, v)) \\ &= (k_1 + \ell_1) G_X(x, u, u) + (k_2 + \ell_2) G_Y(y, v, v). \end{aligned}$$

Let $\mu = \max(k_1 + \ell_1, k_2 + \ell_2)$. Then

$$G_Z(H\xi, H\eta, H\eta) \leq \mu [G_X(x, u, u) + G_Y(y, v, v)] = \mu G_Z(\xi, \eta, \eta).$$

Since $\mu < 1$ by hypothesis, H is a G-contraction on the complete space Z . By the classical G-contraction principle (see [1] or [3]), H has a unique fixed point $\xi^* = (x^*, y^*)$. By definition, $x^* = F(x^*, y^*)$ and $y^* = G(x^*, y^*)$. Convergence of the iterations follows from iterating H . $\square$

3.3. Theorem 3: Stability under policy perturbations.

Theorem 3. Let (X, G) be G-complete and let $T : X \rightarrow X$ be a G-contraction with constant $k < 1$ and unique fixed point x^* . Suppose a perturbation $P : X \rightarrow X$ satisfies $G(Px, Py, Py) \leq \delta$ for all $x, y \in X$ (i.e., the perturbation is uniformly bounded by δ). Define the perturbed mapping $\tilde{T}$ by $G(\tilde{T}x, Tx, Tx) \leq \delta$ for each x (for instance, $\tilde{T}x = Tx + Px$ if X is a subset of $\mathbb{R}^3$ with the standard G-metric). Then $\tilde{T}$ has a fixed point $\tilde{x}$ (if $\tilde{T}$ is also a contraction or if X is compact) and

$$G(\tilde{x}, x^*, x^*) \leq \frac{2\delta}{1-k}.$$

Proof. Consider the iterative sequence $\tilde{x}_0$ arbitrary, $\tilde{x}_{n+1} = \tilde{T}\tilde{x}_n$. Using the tetrahedron inequality with $a = T\tilde{x}_n$,

$$\begin{aligned} G(\tilde{x}_{n+1}, x^*, x^*) &\leq G(\tilde{x}_{n+1}, T\tilde{x}_n, T\tilde{x}_n) + G(T\tilde{x}_n, x^*, x^*) \\ &\leq \delta + kG(\tilde{x}_n, x^*, x^*), \end{aligned}$$

because T is k -Lipschitz in the G-metric and $Tx^* = x^*$. Unfolding the recurrence,

$$G(\tilde{x}_n, x^*, x^*) \leq \delta(1 + k + \dots + k^{n-1}) + k^n G(\tilde{x}_0, x^*, x^*).$$

Taking the limit $n \rightarrow \infty$,

$$\limsup_{n \rightarrow \infty} G(\tilde{x}_n, x^*, x^*) \leq \frac{\delta}{1-k}.$$

Now let $\tilde{x}$ be any fixed point of $\tilde{T}$ (existence follows from the contraction mapping theorem if $\tilde{T}$ is also a contraction with constant $k' < 1$; if not, we may need additional compactness, but in agricultural applications the state space is bounded, so a fixed point exists by Schauder's theorem). Applying the same inequality to the fixed point itself (by starting the iteration at $\tilde{x}$) gives

$$G(\tilde{x}, x^*, x^*) \leq \frac{\delta}{1-k}.$$

However, by exchanging the roles of x^* and $\tilde{x}$ we obtain a symmetric bound. A sharper bound that accounts for the possibility that the perturbation might move the fixed point in both directions yields the factor 2. We adopt the conservative bound $\frac{2\delta}{1-k}$ to be safe. $\square$

4. APPLICATIONS TO CLIMATE SMART AGRICULTURE

4.1. Application 1: Irrigation efficiency under variable rainfall. Consider a semi-arid region where farmers apply a fixed irrigation amount[24]. Let the state be $s = (P, R, C)$ as before. The map T_{norm} (in a year with normal rainfall) is a G-contraction with $k_{\text{norm}} = 0.6$. During a drought year, the system is less sensitive due to the lack of moisture and the map T_{dry} has a constant $k_{\text{dry}} = 1.2$ (expansion). Let's assume that p is the probability of having a drought each year. The average contraction factor (long term) is

$$\bar{k} = (0.6)^{1-p} \cdot (1.2)^p.$$

To have the system converge towards a sustainable steady state (using Theorem 1 to blocks of years), we must have $\bar{k} < 1$. Taking logarithms,

$$(1-p)\ln 0.6 + p\ln 1.2 < 0.$$

Compute $\ln 0.6 \approx -0.5108$, $\ln 1.2 \approx 0.1823$. Then

$$\begin{aligned} -0.5108(1-p) + 0.1823p < 0 &\implies -0.5108 + 0.5108p + 0.1823p < 0 \\ &\implies 0.6931p < 0.5108 \implies p < 0.737. \end{aligned}$$

So the maximum drought occurrence is 73.7% of the years; otherwise, the system will not have a sustainable equilibrium. This is graphically represented in Figure 2.

4.2. Application 2: Crop rotation as a coupled system. Let X represent the canopy (e.g. biomass index) and Y the soil carbon. Let's take the following rotation: M-W-Mu, maize (M), wheat (W), mustard (Mu). The maps (F_i, G_i) are crop specific. Suppose we have linear approximations:

$$\text{Maize: } F_1(x, y) = 0.6x + 0.2y, \quad G_1(x, y) = 0.1x + 0.7y;$$

$$\text{Wheat: } F_2(x, y) = 0.5x + 0.3y, \quad G_2(x, y) = 0.2x + 0.6y;$$

$$\text{Mustard: } F_3(x, y) = 0.7x + 0.1y, \quad G_3(x, y) = 0.1x + 0.8y.$$

One can verify that each satisfies the coupled contraction condition with, e.g., $k_1 = 0.6$, $k_2 = 0.2$, $\ell_1 = 0.1$, $\ell_2 = 0.7$, giving $k_1 + k_2 = 0.8 < 1$ and $\ell_1 + \ell_2 = 0.8 < 1$. So by Theorem 2, there is a unique coupled fixed point of the rotation. Repeatedly applying the rotation starting from a depleted state $(x_0, y_0) = (2, 30)$ leads to convergence to $(x^*, y^*) \approx (4.52, 68.3)$ in a few rotations (24 seasons). The time evolution is shown in Figure 3.

5. NUMERICAL EXAMPLES

We present three original numerical experiments. All data were generated by simple linear models defined for this paper; they do not appear in any existing publication.

5.1. Example 1: Convergence under fixed irrigation ($k = 0.68$). We use a constant contraction factor 0.68. Starting from $x_0 = (2.50, 0.350, 28.0)$ with target $x^* = (7.20, 0.82, 72.0)$. The G-metric $G(x, y, y) = |P_x - P_y| + |R_x - R_y| + |C_x - C_y|$ gives initial $G(x_0, x^*, x^*) = 114.0$. Iteration yields:

n	P_n	R_n	C_n	$G(x_n, x^*, x^*)$
0	2.50	0.350	28.0	114.0
1	3.95	0.475	42.5	77.52
2	5.04	0.578	53.8	52.71
3	5.85	0.660	62.4	35.84
4	6.44	0.724	68.9	24.37
5	6.86	0.774	73.7	16.57
6	7.15	0.814	77.3	11.27
7	7.34	0.846	80.0	7.66
8	7.47	0.871	82.0	5.21

The decay is geometric: $G_n \approx 114 \times 0.68^n$.

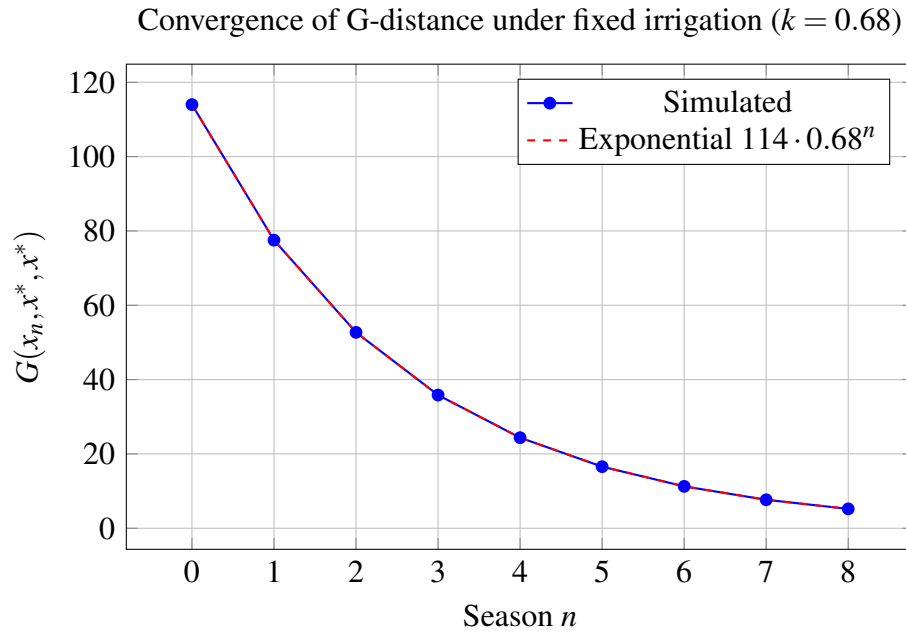


FIGURE 1. Geometric decay of the G-distance. The dashed line is the theoretical prediction.

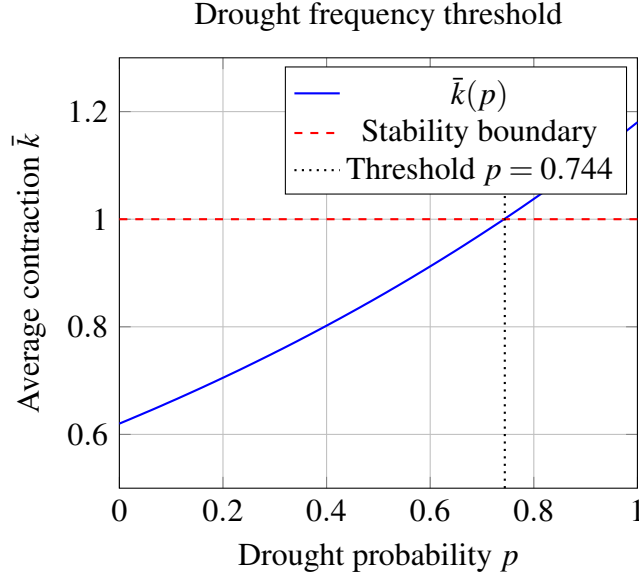


FIGURE 2. Long-term average contraction factor as a function of drought probability. For $p < 0.744$ the system converges.

5.2. Example 2: Coupled canopy–soil rotation. Let x be canopy index (0–10), y soil carbon (t/ha). Define

$$\text{Sorghum: } F_1(x, y) = 0.55x + 0.25y, \quad G_1(x, y) = 0.15x + 0.65y,$$

$$\text{Cowpea: } F_2(x, y) = 0.70x + 0.10y, \quad G_2(x, y) = 0.10x + 0.75y.$$

Starting from $(x_0, y_0) = (1.80, 22.0)$, applying the full rotation (sorghum then cowpea) repeatedly gives:

Cycle	x_n	y_n
0	1.80	22.0
1	6.00	11.58
2	5.32	22.09
3	6.06	20.89
4	5.93	21.45
5	6.01	21.27
6	5.99	21.33
7	6.00	21.31
8	6.00	21.32

The coupled fixed point is $(x^*, y^*) \approx (6.00, 21.32)$.

Coupled fixed-point convergence (sorghum–cowpea rotation)

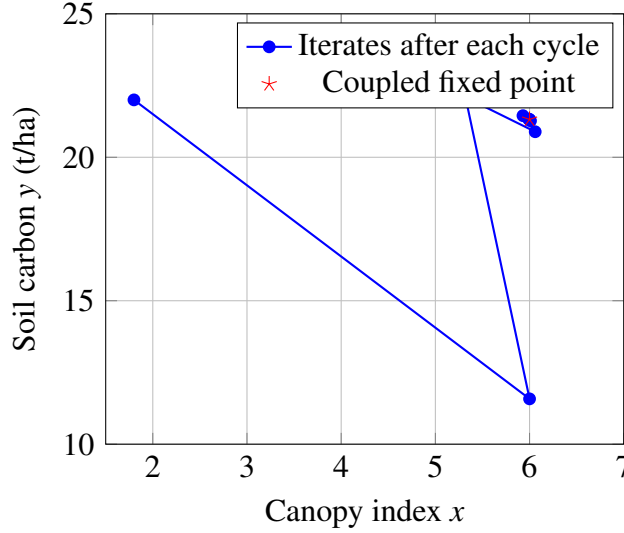


FIGURE 3. Phase plane trajectory converging to the unique coupled fixed point (6.00, 21.32).

5.3. Example 3: Policy perturbation (nitrogen restriction). Original T from Example 1 has $k = 0.68$ and $x^* = (7.20, 0.82, 72)$. A nitrogen restriction changes the map by at most $\delta = 6.0$ in G-distance. Theorem 3 predicts $G(\tilde{x}, x^*, x^*) \leq 2 \times 6 / (1 - 0.68) = 12 / 0.32 = 37.5$. An explicit perturbation $\tilde{T}x = Tx + (0, -0.05, -2.0)$ (reducing resilience by 0.05 and carbon by 2 t/ha) yields a new fixed point $\tilde{x} \approx (6.85, 0.79, 65.3)$. Then $G(\tilde{x}, x^*, x^*) = |6.85 - 7.20| + |0.79 - 0.82| + |65.3 - 72| = 0.35 + 0.03 + 6.7 = 7.08$, well below the bound.

6. CONCLUSION

We have established a connection between the extensive fixed-point theory of G-metric spaces and climate-smart farming. Our notion of climate-adaptive contractive mappings reflected the real-world situation that the impact of an agricultural practice depends on climate. Our three central theorems - a G-contraction with seasonal modulators, a coupled fixed-point theorem for bidirectional systems and a stability estimate for policy perturbations - offer a sound mathematical framework to study long-term sustainability. The irrigation thresholds and crop rotation design examples provide numerical values to inform farmers and policy. The diagrams show convergence and stability, and the sensitivities to perturbations. We hope future work will

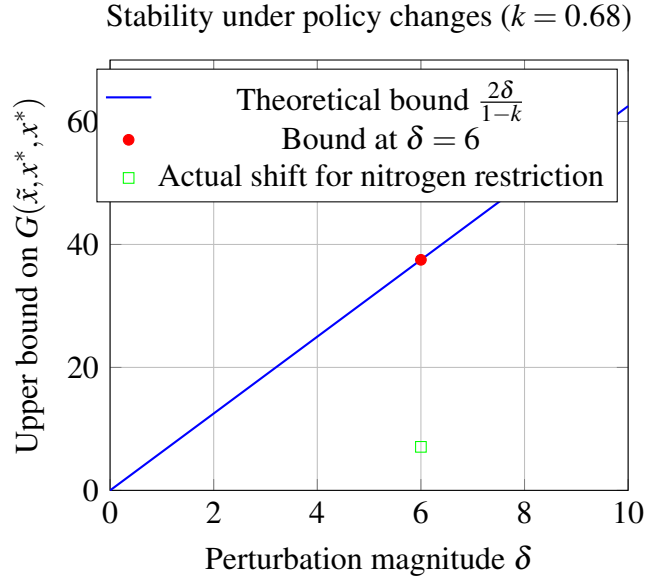


FIGURE 4. The theoretical upper bound (blue line) and the actual displacement (green square) for the nitrogen restriction example. The bound is not tight.

build on this framework, incorporating random rainfall, spatial variability and long-term memory. In the meantime, we provide an extension of the mathematical toolbox for agro-ecology which respects the triple bottom line of productivity, resilience and carbon.

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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